Chapter 1

Set Notation
- Given a set in set-builder notation, explicitly specify it by listing its elements or drawing it.
- Identify when one set is a subset of another.
- Compute and specify the result of an expression involving unions and intersections.

Vectors
- Add and scale vectors specified geometrically or component-wise.
- Write the definition of what it means for two vectors to be equal.

Linear Combinations
- Write the definition of a linear combination of two vectors.
- Compute linear combinations.
- Write a vector in $\mathbb{R}^2$ as a linear combination of other vectors in $\mathbb{R}^2$.
- Write the definition of span.
- Identify in simple cases (e.g. $\mathbb{R}^2$ or $\mathbb{R}^3$) if a vector is in the span of others.

Linear Independence
- Write the definition of linear independence and linear dependence.
- Identify whether simple sets are linearly independent or dependent.

Subspace/Basis
- Write the definition of a subspace and basis.
- Produce a proof of whether or not a set (described in words or set notation) is a subspace.
- Identify an invalid proof of subspaceness.
- Define dimension.

Dot product/Length
- Write the definition of the dot product of two vectors written in components as well as the geometric definition of the dot product involving lengths and angles.
- Write the definition of the length of a vector.
- Compute dot products and lengths.
- Use linearity of the dot product to compute other dot products.
- Find the angle between two vectors.
- Find the distance between two vectors.
- Define and produce unit vectors.
- Define orthogonality.

Projections
- Define projection.
- Project vectors onto lines and planes.

Lines & Planes
- Specify lines and planes as spans or in vector form.
- Specify a given line or plane in scalar form.

Chapter 2

SLEs
- Solve SLEs with unique solutions.
- Convert back and forth between a SLE and an augmented matrix.
- Given two solutions to a SLE, produce another.
- Recite a SLE has $0, 1$, or ∞ solutions.
- Identify whether a system is inconsistent.

Row Reduction
- Reduce a matrix to rref.
- Identify whether a matrix is in rref.
- Produce the solution(s) to a SLE from rref form of the augmented matrix.
- List the elementary row operations.

Rank
- Define rank.
- Compute rank of a given matrix.
- Use rank to determine whether a system has a unique solution.

Spanning Sets
- Use row reduction to determine if a set of vectors is linearly independent.
- Use row reduction to find a basis for the span of a set of vectors.
- Use row reduction to determine if a vector is in the span of others.
- Use row reduction to write a vector as a linear combination of things in a basis.

Chapter 3

Matrix Ops

- Index a matrix.
- Add and multiply matrices.
- Produce matrices where the order of multiplication does/doesn't matter.
- Rewrite a SLE as a matrix equation.
- Compute the transpose of a matrix.
- Write out the special matrices I and 0.

Matrix Inverses

- Define the inverse of a matrix.
- Compute the inverse of a matrix.
- Solve a matrix equation using an inverse.
- Relate rank and invertibility.
- Produce examples of invertible/non-invertible matrices.
- Prove the inversion formula for matrix products ($(AB)^{-1} = B^{-1}A^{-1}$).

Elementary Matrices

- Define an elementary matrix.
- Find the inverse of an elementary matrix.
- Decompose an invertible matrix as a product of elementary matrices.

Linear Transformations

- Write the definition of a linear transformation from $\mathbb{R}^n \to \mathbb{R}^m$.
- Compute the image of a vector under a linear transformation described in words or a matrix.
- Produce the standard matrix for a linear transformation described in some other way.
- Determine whether a standard matrix for a linear transformation will be invertible based on the description of the linear transformation (and without actually trying to compute the inverse of a matrix).
- Define and compute the image/range and null space/kernel of a linear transformation.
- Define and compute the row/column space of a matrix.
- Determine whether or not a given transformation is linear.
- State the rank-nullity theorem.
- Use the rank-nullity theorem to determine the dimension of the image and kernel of a given linear transformation.

Chapter 4

Change of Basis

- Write a vector given in the standard basis in another basis.
- Write a linear transformation in a different basis.

Chapter 5

Determinants

- Define the determinant as an oriented volume.
- Relate the determinant to invertibility.
- Compute the determinant of a 2×2, 3×3, or sparse $n \times n$ matrix.
- Use the multiplicativity, inverse property, and transpose property of the determinant to compute the determinant of a composition of matrices.
- Use determinants to compute volumes.
- Compute the determinant of particular linear transformations (those with nice geometric descriptions) without using a matrix.

Chapter 6

Eigen Vectors/Values

- Define eigenvectors/values.
- Compute eigenvectors/values.
- Relate the set of eigenvalues of a particular matrix to its determinant.
- Define and compute the characteristic polynomial of a matrix.
- Create a matrix with given eigenvalues/eigenvectors.
- Diagonalize a matrix.
- Define eigen space.
- Use diagonalization to compute large powers of a matrix.

Chapter 7

Orthogonality

- Use Gram-Schmidt to produce an orthonormal basis for a subspace.