1. We know a system of linear equations can have $0, 1$, or infinitely many solutions.
   (a) Explain why a system of linear equations cannot have exactly 2 solutions.
   (b) For a homogeneous system of linear equations, what are the possibilities for the number of solutions? Explain, and make sure to define *homogeneous system*.
2. Suppose $\mathcal{M}$ is a homogeneous system of 4 linear equations and 3 variables. Let M be the non-augmented matrix of coefficients.
   (a) If $\text{rank}(M) = 3$, how many solutions does the system $\mathcal{M}$ have? (You do not need to define *reduced row echelon form*, but include all other relevant definitions.)
   (b) If $\text{rank}(M) = 2$, how many solutions does the system $\mathcal{M}$ have?
   (c) Could $\text{rank}(M) = 4$? Explain.
3. Consider the equation $\mathcal{E}$ given by
$$x_1 - x_2 + x_3 - x_4 = 0$$
and let $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 : \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \text{ is a solution to } \mathcal{E} \right\}$. Find vectors $\vec{v}_1, \ldots, \vec{v}_n$ such that $V = \text{span}\{\vec{v}_1, \ldots, \vec{v}_n\}$. Explain your process.