Midterm 2

MATH 211 (A01), Spring 2015 (Siefken)

Date: ____________

Name: ________________________________

ID Number: ________________________________

This is a 50 minute test. It has 6 pages including this cover page.

Q1		/10
Q2		/10
Q3		/10
Q4		/10
Q5		/10
Total		/50

1 (10pts) Complete each of the following sentences with a mathematically precise definition.

(a) (2pts) A non-empty subset $V \subseteq \mathbb{R}^n$ is a **subspace** if

(b) (2pts) A function $T : \mathbb{R}^n \to \mathbb{R}^m$ is a **linear transformation** if

(c) (2pts) The **null space** of a matrix M is

(d) (2pts) The **inverse** of a matrix A is
(Hint: you will get no points if all you write is A^{-1})

(e) (2pts) The **range** (or **image**) of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is

2 (10pts) Given

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{bmatrix} \qquad C^{-1} = \begin{bmatrix} 9 & -3/2 & -5 \\ -5 & 1 & 3 \\ -2 & 1/2 & 1 \end{bmatrix}$$

(a) (2pts) Compute AB.

(b) (2pts) Compute A^{-1}.

(c) (2pts) Compute C^T.

(d) (4pts) Solve the equation $C\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

3 (10pts) For each of the following, indicate **true** or **false**. You *do not* need to explain how you arrived at your answer.

Let M be an $n \times n$ matrix. If M is invertible, then we *must* have

(i) $\text{row}(M) = \{\vec{0}\}$

(ii) $\text{col}(M) = \mathbb{R}^n$

(iii) $\text{null}(M) = \{\vec{0}\}$

(iv) $\text{rank}(M) = n$

(v) $M = M^T$

4 (10pts) Let $\mathcal{F} : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects vectors across the line with direction vector $\vec{e}_2$. Let $\mathcal{P} : \mathbb{R}^2 \to \mathbb{R}^2$ be projection onto the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(a) (4pt) Compute $\mathcal{F} \begin{bmatrix} 7 \\ -3 \end{bmatrix}$ and $\mathcal{P} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(b) (6pts) Find a matrix for $\mathcal{F} \circ \mathcal{P}$.

5 (10pts) For each of the following transformations, either prove that the transformation is linear or provide an example that shows it is not linear.

(a) (5pts) $U : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $U \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ x + y \end{bmatrix}$.

(b) (5pts) $V : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $V \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ x - 1 \end{bmatrix}$.