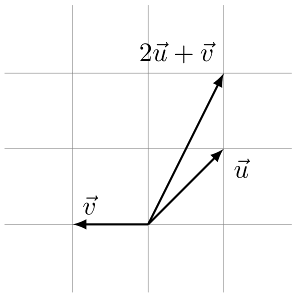
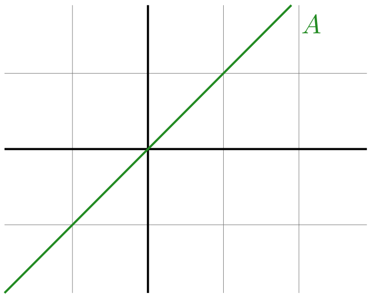
Let $\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$.

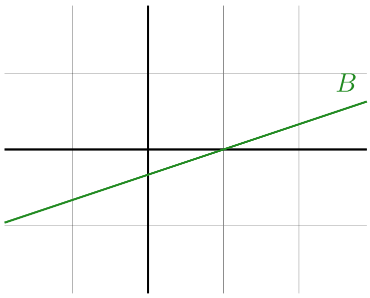
1. Graph the vectors $\vec{u}$, $\vec{v}$, and $2\vec{u} + \vec{v}$.



2. (a) Draw the set $A = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = t\vec{u} \text{ for some } t \in \mathbb{R}\}$.



   (b) Draw the set $B = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = t\vec{u} - (2t+1)\vec{v} \text{ for some } t \in \mathbb{R}\}$.



3. (a) Find values of x, y that satisfy the following relationships:

$$x + y = 7$$
$$2x - 3y = 13.$$

By subtracting twice the first equation from the second, we see that $0x - 5y = -1$ and so $y = \frac{1}{5}$. Plugging this in the first equation, we see $x = 7 - \frac{1}{5} = \frac{34}{5}$.

   (b) Find values of x, y, z that satisfy the following relationships (your answer may involve ugly fractions):

$$x + 2y + 8z = 1$$
$$4x + 5y + 8z = 2.$$

Since we only have two equation, we would expect to be able to solve for at most two of the three variables. By subtracting the first equation from the second, we see that $3x + 3y = 1$ and so $x = \frac{1}{3} - y$. Substituting this in the first equation, we see

$$\tfrac{1}{3} - y + 2y + 8z = 1 \implies z = \tfrac{1}{12} - \tfrac{1}{8}y.$$

We have solved for the variables x and z, and cannot possibly solve for any more. Since we must only come up with one solution, we can pick an arbitrary value for y. Choosing $y = 0$ gives $x = \frac{1}{3}$ and $z = \frac{1}{12}$, and so $(x, y, z) = (\frac{1}{3}, 0, \frac{1}{12})$ is a solution. Verifying, we see indeed

$$\begin{aligned} \tfrac{1}{3} + 2(0) + 8\tfrac{1}{12} &= 1 \\ 4\tfrac{1}{3} + 5(0) + 8\tfrac{1}{12} &= 2. \end{aligned}$$

4. Let $\vec{w} = \begin{bmatrix} 5 \\ -12 \end{bmatrix}$. Find values of a and b so that

$$\vec{w} = a\vec{u} + b\vec{v}.$$

That is, write $\vec{w}$ as a linear combination of $\vec{u}$ and $\vec{v}$.

We need to solve the equation

$$\begin{bmatrix} 5 \\ -12 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} -1 \\ 0 \end{bmatrix},$$

which means we need to find a and b so that the following relationships hold:

$$\begin{aligned} 1a - 1b &= 5 \\ 1a + 0b &= -12. \end{aligned}$$

We deduce that $a = -12$ and $b = -17$, and so

$$\vec{w} = -12\vec{u} - 17\vec{v}.$$

5. Let

$$S = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Is S a point, line, plane, or all of $\mathbb{R}^3$? Explain.

Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and notice that $\vec{v}_3 = \vec{v}_1 - \vec{v}_2$. This means

$$S = \operatorname{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \operatorname{span}\{\vec{v}_1, \vec{v}_2\}.$$

Since $\vec{v}_1$ and $\vec{v}_2$ are linearly independent, their span must be a two dimensional space. Therefore S is a plane.