1. For each of the following statements, produce a counterexample to show that the statement is **false**.

   (a) If A and B are square matrices, $AB = BA$.

   (b) If $AB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then A and B are 2×2 matrices.

   (c) If $AB = I$ then $BA = I$.

   (d) If $A^2 = 0$, then $A = 0$.

2. Let $R = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

   (a) Find all solutions to the matrix equation $R \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$.

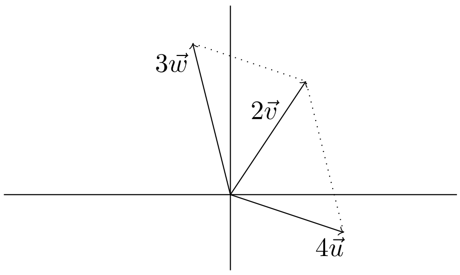
   (b) Prove that the set $X = \{\vec{x} \in \mathbb{R}^3 : R\vec{x} = \vec{0}\}$ is a subspace.

3. Suppose E is a 4×3 matrix with columns $\vec{c}_1, \vec{c}_2, \vec{c}_3$ and rows $\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4$. Let $\vec{v} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$.

   (a) Express $E\vec{v}$ as a linear combination of $\vec{c}_1, \vec{c}_2, \vec{c}_3$.

   (b) Supposing $\vec{r}_1 \cdot \vec{v} = 1$, $\vec{r}_2 \cdot \vec{v} = 6$, $(\vec{r}_3 + \vec{r}_4) \cdot \vec{v} = 2$, and $(\vec{r}_3 - \vec{r}_4) \cdot \vec{v} = -2$, compute $E\vec{v}$.

4. Suppose that $\vec{u}$, $\vec{v}$, and $\vec{w}$ are vectors in $\mathbb{R}^2$ that are related by the following diagram.



   Let $A = [\vec{u}|\vec{v}|\vec{w}]$ be the matrix with columns $\vec{u}$, $\vec{v}$, and $\vec{w}$.

   (a) What is the rank of A?

   (b) Find all solutions to the equation $A\vec{x} = \vec{0}$.

   (c) Find a basis for the subspace $V = \{\vec{x} \in \mathbb{R}^3 : A\vec{x} = \vec{0}\}$.