- 1. Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.
 - (a) Show that the null space of T is a subspace of \mathbb{R}^n .
 - (b) Show that the range of T is a subspace of \mathbb{R}^m .
- 2. (a) For a 4×3 matrix M, must the column space of M be identical to the column space of rref(M)?
 - (b) For a 3×3 matrix N with rank(N) = 3, must the column space of N be identical to the column space of rref(N)? Can the assumption that rank(N) = 3 be dropped?
- 3. For a linear transformation $L: \mathbb{R}^3 \to \mathbb{R}^3$, we have the following information:

$$L \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \qquad L \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix} \qquad L \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

- (a) Write down a matrix for L.
- (b) Describe the range of L as a point, line, plane, or hyperplane and give a basis for the range of L.
- (c) Describe the null space of L as a point, line, plane, or hyperplane and give a basis for the null space of L.