1. Suppose the matrix equation $A\vec{x} = \begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$ has the general solution

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

   (a) How many rows and how many columns does A have?
   (b) Find null(A).
   (c) Find rank(A).
   (d) Find col(A).
   (e) Find row(A).

2. Let

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{b}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad \vec{b}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \vec{c} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

   and let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ and $\mathcal{S} = \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation and $T(\vec{b}_1) = 2\vec{b}_1$, $T(\vec{b}_2) = 3\vec{b}_2$, and $T(\vec{b}_3) = -\vec{b}_3$.

   (a) Compute $[\vec{c}]_{\mathcal{B}}$.
   (b) Compute $[T\vec{c}]_{\mathcal{B}}$ and $[T\vec{c}]_{\mathcal{S}}$.
   (c) Find a matrix for T in the $\mathcal{B}$ basis (i.e., the matrix $[T]_{\mathcal{B}}$) and a matrix for T in the $\mathcal{S}$ basis (i.e., $[T]_{\mathcal{S}}$).

3. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & x \end{bmatrix}$.

   (a) Compute det(A).
   (b) Compute det(B). For what values of x is B not invertible?

4. Let $A = \begin{bmatrix} 1 & 2 \\ 5 & 9 \end{bmatrix}$.

   (a) Find an equation for the function $p(x) = \det(A - xI)$ (this is called the *characteristic polynomial* of A).
   (b) For what values of x is $A - xI$ non-invertible?
   (c) Compute $p(A)$, the polynomial p with the matrix A plugged into it. When you plug a matrix into a polynomial, replace any constant terms k with the matrix kI. Can you guess why p is called an *annihilating* polynomial for A?