Midterm 1

MATH 211 (A01), Spring 2015 (Siefken)

Date: ____________

Name: ________________________________

ID Number: ________________________________

This is a 50 minute test. It has 6 pages including this cover page.

Q1		/10
Q2		/10
Q3		/10
Q4		/10
Q5		/10
Total		/50

1 (10pts) Complete each of the following sentences with a mathematically precise definition.

(a) (2pts) A non-empty subset $V \subseteq \mathbb{R}^n$ is a **subspace** if

(b) (2pts) The set of vectors $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is **linearly independent** if

(c) (2pts) The set of vectors $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is a **basis for the subspace** V if

(d) (2pts) The vector $\vec{a}$ is a **unit vector** if

(e) (2pts) The vector $\vec{a}$ is **orthogonal** to the vector $\vec{b}$ if

2 (10pts) Let

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{c} = \begin{bmatrix} -3 \\ 0 \\ -1 \end{bmatrix}.$$

(a) (3pts) Compute $\vec{a} \cdot (\vec{b} - \vec{c})$.

(b) (2pts) Compute $-\vec{a} + 2\vec{b}$.

(c) (5pts) Is the set of vectors $X = \{\vec{a}, \vec{b}, \vec{c}\}$ linearly independent or linearly dependent? Explain your answer.

3 (10pts)

(a) (7pts) Use an augmented matrix to solve the following system of equations

$$\begin{array}{rcrcrcr} & & y & + & z & = & 1 \\ x & + & 2y & + & z & = & 1 \\ 2x & + & 4y & + & 3z & = & 4 \end{array}.$$

(b) (3pts) Let $\vec{a} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$. Express $\vec{v}$ as a linear combination of $\vec{a}$, $\vec{b}$, and $\vec{c}$.
Hint, these vectors closely relate to the system of equations in part (a).

4 (10pts) Below you are given the matrix A and its reduced row echelon form.

$$A = \begin{bmatrix} 1 & 2 & -1 & -1 & 4 \\ 2 & 4 & 1 & 0 & 4 \\ 1 & 2 & 1 & -1 & 4 \end{bmatrix} \qquad \operatorname{rref}(A) = \begin{bmatrix} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

(a) (2pts) What is the rank of A?

(b) (2pts) Write down the *homogeneous system* of linear equations corresponding to A.

(c) (6pts) Write down the *general solution to* the homogeneous system of linear equations corresponding to A.

5 (10pts) For each of the following subsets of $\mathbb{R}^2$, either prove that the subset is a subspace or provide an example that shows it is not a subspace.

(a) (5pts) $U = \{\vec{x} \in \mathbb{R}^2 : \vec{x} \cdot \vec{e}_1 + \vec{x} \cdot \vec{e}_2 = 1\}$.

(b) (5pts) $V = \{\vec{x} \in \mathbb{R}^2 : \vec{x} \cdot \vec{e}_1 + \vec{x} \cdot \vec{e}_2 = 0\}$.