Problems 1–3 should be review. *Review* is not the same as *easy*; it just means that the problems don't require knowledge beyond Math 230. If you feel rusty, please refresh yourself on vectors, dot products, and planes before starting. Note that $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors pointing along the x and y axes respectively. They might have been called $\mathbf{i}$ and $\mathbf{j}$ in your Math 230 class.

1. Find unit vectors in the directions $\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\vec{v} = \mathbf{e}_1 + 3\mathbf{e}_2$, $\vec{w} = -\mathbf{e}_1 - \mathbf{e}_2 - \mathbf{e}_3$, and $\vec{r} = \frac{\vec{v}}{\|\vec{v}\|} + \frac{\vec{w}}{\|\vec{w}\|}$.

2. The dot product of vectors (in $\mathbb{R}^n$) can be defined algebraically by:
$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$
   (a) Rewrite this definition in summation notation.

   Use the algebraic definition of the dot product to show that for any $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$:

   (b) $\vec{a} \cdot (\alpha \vec{b}) = \alpha(\vec{a} \cdot \vec{b})$

   (c) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

3. Consider the plane $\mathcal{P}$ defined as the set of solutions to the equation
$$3x - 2y + z = 4.$$
   Let $\vec{p} \in \mathcal{P}$ be a point in $\mathcal{P}$ interpreted as a vector.

   (a) What is $\vec{p} \cdot \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$? Explain.

   (b) The plane $\mathcal{Q}$ is the plane $\mathcal{P}$ translated in the y-direction (or the $\mathbf{e}_2$-direction) by one unit. Let $\vec{q} \in \mathcal{Q}$. What is $\vec{q} \cdot \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$?

4. Sketch each of the following sets on a number line or in the coordinate plane:

   (a) $\{x \in \mathbb{R} \,:\, 2x \in \mathbb{Z}\}$

   (b) $\{(x, y) \in \mathbb{R}^2 \,:\, x \leq 2y\}$

   Write each of the following sets in set-builder notation:

   (c) The graph of $y = x^2$

   (d) The filled-in triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$

5. A **linear combination** of two vectors $\vec{u}$ and $\vec{v}$ is any vector of the form
$$a\vec{u} + b\vec{v}$$
   where a and b are scalars.

   (a) Write $\begin{bmatrix} 107 \\ 64 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

   (b) Write $\begin{bmatrix} -99 \\ -100 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

   (c) Linear combinations of three vectors are defined similarly. Which vectors in $\mathbb{R}^3$ can be written as a linear combination of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$? Explain how you know.

Don't forget to also complete PAR1, this week's Peer-Assisted Reflection problems.