

*Note: Your PAR (both initial and final drafts) must be typed in  $\text{\LaTeX}$ .*

### Problem Statement

Recall the setup from PAR 1 and 5. The newly-appointed queen of a newly-discovered land has commissioned a new expedition to explore her territory. This time, she sends two explorers: Emily and Aaron. The explorers have their own equipment and their own quirks.

**Emily** Emily is a careful explorer with an accurate compass. When Emily measures distance in a cardinal direction, it too is accurate.

**Aaron** Aaron has miscalibrated *all* of his equipment; he measures neither distances nor directions accurately, but he can convert a location in his coordinates into Emily's coordinates using the matrix  $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ .

The queen also decrees that she will expand her territory by means of a linear transformation  $T$ ; people living at position vector  $\vec{v}$  will have to move to  $T(\vec{v})$ . The realm's primary roads are Main Street (along the line  $y = 2x$ ) and First Avenue (along the line  $y = -2x$ ), intersecting at the royal palace. The queen wishes to set up  $T$  so that everyone living on Main Street will stay on Main Street but move twice as far from the palace, and everyone living on First Avenue will stay on First Avenue but move three times as far from the palace.

1. Draw a map showing the locations of Main Street, First Avenue, and the locations of at least 3 villagers before the expansion. Draw a second map showing Main Street, First Avenue, and the same 3 villagers after the expansion. For each villager in each picture, label their coordinates both in **true** coordinates (i.e. Emily's), and with what Aaron thinks their coordinates are.
2. The queen now relocates all of her citizens. Emily wants to use a matrix to help find everyone's new address. Find a matrix  $T_E$  such that if a citizen lives at position vector  $\begin{bmatrix} x \\ y \end{bmatrix}$  (given in Emily's coordinates), then their new location is  $T_E \begin{bmatrix} x \\ y \end{bmatrix}$ .
3. Aaron wants to use a similar matrix to find people's new addresses, but in his coordinates. Find a matrix  $T_A$  such that if a citizen's position vector is  $\begin{bmatrix} a \\ b \end{bmatrix}$  (in Aaron's coordinates), then their new location in Aaron's coordinates is  $T_A \begin{bmatrix} a \\ b \end{bmatrix}$ .
4. Explain, geometrically, why  $AT_AA^{-1} = T_E$ .  
(Geometrically, two matrices are called *similar* if they represent the same transformation in different bases (*bases* is the plural of basis). Algebraically, the matrix  $P$  is similar to the matrix  $Q$  if there is an invertible matrix  $X$  so  $XPX^{-1} = Q$ .)

### Reflection

Turn the page and check off the icons for things you think you did well; circle the icons for things you would like feedback on.

**Suggestions**

**Communication**

**Strengths**



Show All Steps



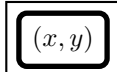
Explain Why,  
Not Just What



Avoid Pronouns



Use Correct  
Definitions



Define Variables,  
Units, etc.

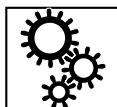


Create Diagrams

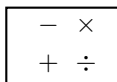
**Suggestions**

**Accuracy**

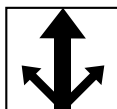
**Strengths**



Correct Setup



Accurate Calculations



Solve Multiple Ways



Answer Reasonable



Other  
(Write Below)