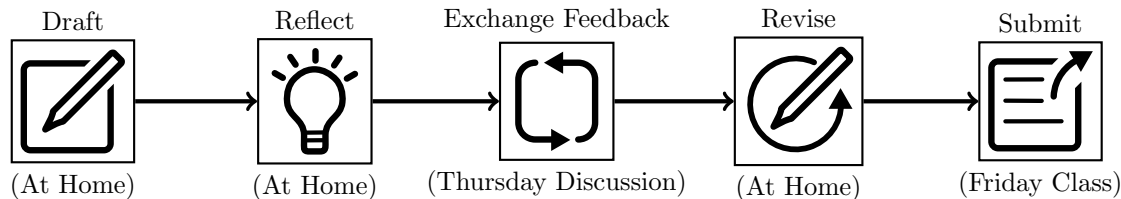


### The PAR Process



### Problem Statement

Let  $M$  be a  $3 \times 4$  matrix with unknown entries. You do know, however, that  $\text{rref}(M) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ .

1. Is there a vector  $\vec{b}$  such that the matrix equation  $M\vec{x} = \vec{b}$  is *consistent*? Explain why or why not, or whether you'd need more information to tell.
2. Is there a vector  $\vec{b}$  such that the matrix equation  $M\vec{x} = \vec{b}$  is *inconsistent*? Explain why or why not, or whether you'd need more information to tell.
3. Let  $\vec{c}_1, \vec{c}_2, \vec{c}_3$ , and  $\vec{c}_4$  be the columns of the matrix  $M$ . For which  $i$  and  $j$  with  $i \neq j$  is the set  $\{\vec{c}_i, \vec{c}_j\}$  linearly dependent? For which is it linearly independent? Explain how you know.
4. Can a set of four vectors in  $\mathbb{R}^3$  ever be linearly independent? Explain.
5. Suppose  $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4 \in \mathbb{R}^3$ . Let  $\mathcal{V} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$  and let  $X = [\vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4]$  be the matrix whose columns are  $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ . Suppose further that every subset of  $\mathcal{V} \subset \mathcal{V}$  of *size two* is linearly independent. Explain what  $\text{rref}(X)$  must look like in this case. (*Hint: you won't be able to pin down exact numbers for every entry of  $\text{rref}(X)$ , but you might know things like whether the entry can be zero or not, etc.*)

### Reflection

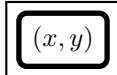
Turn the page and check off the icons for things you think you did well; circle the icons for things you would like feedback on.

**Suggestions****Communication****Strengths**

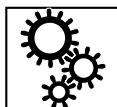
Show All Steps

Explain Why,  
Not Just What

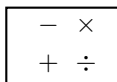
Avoid Pronouns

Use Correct  
DefinitionsDefine Variables,  
Units, etc.

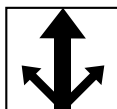
Create Diagrams

**Suggestions****Accuracy****Strengths**

Correct Setup



Accurate Calculations



Solve Multiple Ways



Answer Reasonable

Other  
(Write Below)