We are nearly experts at typing by now. Though you are not required to type your homework, it's strongly encouraged. You can even download the `tex` *file for this homework and type your answers below each problem. Using the* `\begin{quote}` *and* `\end{quote}` *environment will indent anything you type inbetween. Perfect for typing answers!*

1. Let A and B be $n \times n$ invertible matrices and let $X = AB$. Does $X^{-1} = A^{-1}B^{-1}$ or does $X^{-1} = B^{-1}A^{-1}$ or neither? Explain.

2. For each of the following sets, determine whether or not it is a subspace. Explain your answer.

   (a) $A = \left\{ \vec{x} : \vec{x} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 0 \right\} \subseteq \mathbb{R}^2$

   (b) $B \subseteq \mathbb{R}^3$ is the plane with normal vector $\vec{n} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ passing through the point $\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$.

   (c) $C \subseteq \mathbb{R}^3$ is the x-axis.

   (d) $D \subseteq \mathbb{R}^3$ is the plane given in vector form as $\vec{x} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$.

   (e) $E = \{(x, y) : y = 3x + 4\} \subseteq \mathbb{R}^2$.

   (f) $F = \text{span}\{\vec{u}_1, \vec{u}_2\} \subseteq \mathbb{R}^2$ where $\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

   (g) $G \subseteq \mathbb{R}^4$ is the set of all solutions to the matrix equation $\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

   (h) $H = \{(x, y) : xy = 0\} \subseteq \mathbb{R}^2$.

3. For every set in problem 1 that is a vector space, find a basis.

4. For every set in problem 1 that is a vector space, find its dimension.

5. Let $\mathcal{P}$ be the plane in $\mathbb{R}^3$ given in vector form by $\vec{x} = t\vec{d}_1 + s\vec{d}_2 + \vec{p}$ for unknown vectors $\vec{d}_1, \vec{d}_2, \vec{p}$.

   (a) Show that $\mathcal{P}$ is a subspace if and only if $\vec{p} = \vec{0}$. That is, show that if $\vec{p} = \vec{0}$, then $\mathcal{P}$ is a subspace, and if $\vec{p} \neq \vec{0}$, then $\mathcal{P}$ is not a subspace.

   (b) Suppose that $\vec{d}_1$ and $\vec{d}_2$ were accidentally chosen to be linearly dependent. Is it still the case that $\mathcal{P}$ is a subspace exactly when $\vec{p} = \vec{0}$, or is the outcome different now?

6. Let $\mathcal{V}$ be the subspace spanned by $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \vec{v}_5 = \begin{bmatrix} -5 \\ 4 \\ 9 \end{bmatrix}$.

   (a) Find a basis for $\mathcal{V}$ and call your basis vectors $\vec{b}_1$, $\vec{b}_2$, etc.

   (b) Describe $\mathcal{V}$ geometrically.

   (c) Let $V = [\vec{v}_1|\vec{v}_2|\vec{v}_3|\vec{v}_4|\vec{v}_5]$ be the matrix whose columns are the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5$, let $B = [\vec{b}_1|\vec{b}_2|\cdots]$ be the matrix whose columns are your basis vectors from part (a), and let $\vec{v} \in \mathcal{V}$.
   Without computing, how many solutions does the equation $V\vec{x} = \vec{v}$ have? How about $B\vec{x} = \vec{v}$?

7. Suppose A is an invertible matrix and $\vec{c}_1, \vec{c}_2, \ldots, \vec{c}_n$ are its columns. Is $\{\vec{c}_1, \vec{c}_2, \ldots, \vec{c}_n\}$ a basis? If so, what is it a basis of? Explain.