

1. **Reading Assignment:** Read and practice the algorithm for finding the inverse of a matrix, found at the end of section 2.2 of the textbook.
2. For each of the following transformations, find a matrix corresponding to the transformation.
 - (a) \mathcal{A} : Rotate 90° counterclockwise around the origin (in \mathbb{R}^2).
 - (b) \mathcal{B} : Send every vector in \mathbb{R}^2 to the zero vector.
 - (c) \mathcal{C} : Project (in \mathbb{R}^3) onto the yz -plane.
 - (d) \mathcal{D} : Project (in \mathbb{R}^3) onto the x -axis.
 - (e) \mathcal{E} : Reflect (in \mathbb{R}^3) across the xy -plane.
 - (f) \mathcal{F} : Stretch by a factor of 2 in the y -direction (in \mathbb{R}^2).
 - (g) \mathcal{G} : Stretch by a factor of 2 in the y -direction (in \mathbb{R}^3).
 - (h) \mathcal{H} : Rotate 90° counterclockwise (as viewed looking “down” from the positive y -axis towards the origin) around the y -axis (in \mathbb{R}^3).
3.
 - (a) For each transformation in problem 2, explain geometrically whether or not the transformation is invertible.
 - (b) Which of the matrices you generated in problem 2 are invertible? (You may wish to use the algorithm you read about.)
 - (c) Suppose f is a transformation that is represented by the matrix M . Is it possible for f to be invertible and M not invertible? Is it possible for M to be invertible and f not be invertible? Explain your reasoning.
4. Each of the following matrices gives a transformation of \mathbb{R}^2 or \mathbb{R}^3 . Describe the effect of that transformation in terms of the transformations from problem 2. (Hint: you may need to combine two, three, or more transformations from problem 2 for some examples.)

$$X = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$Z = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5. Use the definition of a linear transformation to show that if T is a linear transformation, then $T(\vec{0}) = \vec{0}$. Do not appeal to the matrix representation of T .
6. Consider the matrices $A = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.
 - (a) Describe the geometric effect of A . Feel free to use the language of the in-class worksheets.
 - (b) Compute the product BA .
 - (c) Find a matrix C such that $AC = BA$.
 - (d) Describe geometrically why $C \neq B$.

$$7. \text{ Suppose } A \text{ is an unknown matrix, but } A^{-1} = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

- (a) Find a solution to the matrix equation $A\vec{x} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$.
- (b) Are there other solutions besides the one you found in (a)? Explain how you know.