MATH 240 **Due: Thursday, November 17**
HOMEWORK 6 (*November 7, 2016*)

We are nearly experts at typing by now. Though you are not required to type your homework, it's strongly encouraged. You can even download the `tex` *file for this homework and type your answers below each problem. Using the* `\begin{quote}` *and* `\end{quote}` *environment will indent anything you type inbetween. Perfect for typing answers!*

1. Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 & 3 \\ 1 & 2 & 3 & -1 & -1 \\ 3 & 1 & 4 & 2 & 7 \end{bmatrix}$. Let $\mathcal{C}$, $\mathcal{R}$, and $\mathcal{N}$ be the column, row, and null spaces of A, respectively.

   (a) Find a basis for $\mathcal{C}$

   (b) Find a basis for $\mathcal{R}$.

   (c) Find a basis for $\mathcal{N}$.

   (d) Find the dot product of each of your basis vectors for $\mathcal{R}$ with each of your basis vectors for $\mathcal{N}$. What can you say geometrically about $\mathcal{R}$ and $\mathcal{N}$?

2. Let L be the line $x = y = z$ in $\mathbb{R}^3$.

   (a) Find a 3×3 matrix B_1 whose column space is the xy-plane and whose null space is L.

   (b) Find a 3×3 matrix B_2 whose column space is the xz-plane and whose null space is L.

   (c) Find the row space of B_1 and the row space of B_2. How do they compare? Explain.

3. A linear transformation T has the following effects:

   - Along the line $y = x$, it "shrinks" everything by a factor of 2—points along this line move halfway to the origin.
   - Along the line $y = -3x$, it reflects everything over the origin—if $\vec{v}$ is the position vector of a point on this line, then it moves to $-\vec{v}$.

   (a) Find a matrix A such that $T(\vec{v}) = A\vec{v}$ for all $\vec{v} \in \mathbb{R}^2$.

   (b) Find A^{-1}.

   (c) What are the "stretch factors" and "stretch directions" of the transformation given by A^{-1}?

4. Recall the "italicizing N" matrix that you found in class: $A = \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & \frac{4}{3} \end{bmatrix}$. Find all eigenvalues ("stretch factors") and eigenvectors ("stretch directions") of A.