

We are nearly experts at typing by now. Though you are not required to type your homework, it's strongly encouraged. You can even download the `tex` file for this homework and type your answers below each problem. Using the `\begin{quote}` and `\end{quote}` environment will indent anything you type inbetween. Perfect for typing answers!

- Let A and B be $n \times n$ invertible matrices and let $X = AB$. Does $X^{-1} = A^{-1}B^{-1}$ or does $X^{-1} = B^{-1}A^{-1}$ or neither? Explain.
- For each of the following sets, determine whether or not it is a subspace. Explain your answer.
 - $A = \left\{ \vec{x} : \vec{x} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 0 \right\} \subseteq \mathbb{R}^2$
 - $B \subseteq \mathbb{R}^3$ is the x -axis.
 - $C \subseteq \mathbb{R}^3$ is the plane given in vector form as $\vec{x} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$.
 - $D \subseteq \mathbb{R}^3$ is the plane with normal vector $\vec{n} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ passing through the point $\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$.
 - $E = \{(x, y) : y = 3x + 4\} \subseteq \mathbb{R}^2$.
 - $F = \text{span}\{\vec{u}_1, \vec{u}_2\} \subseteq \mathbb{R}^2$ where $\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.
 - $G \subseteq \mathbb{R}^4$ is the set of all solutions to the matrix equation $\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.
 - $H = \{(x, y) : xy = 0\} \subseteq \mathbb{R}^2$.
- For every set in problem 1 that is a vector space, find a basis.
- For every set in problem 1 that is a vector space, find its dimension.
- Let \mathcal{P} be the plane in \mathbb{R}^3 given in vector form by $\vec{x} = t\vec{d}_1 + s\vec{d}_2 + \vec{p}$ for unknown vectors $\vec{d}_1, \vec{d}_2, \vec{p}$.
 - Show that if $\vec{p} = \vec{0}$, then \mathcal{P} is a subspace.
 - What if $\vec{p} \neq \vec{0}$? Could \mathcal{P} still be a subspace? Give conditions on \vec{p} that determine whether or not \mathcal{P} is a subspace. That is, give a condition so that if it is true, \mathcal{P} is a subspace and if it is false, \mathcal{P} is not a subspace.
- Let \mathcal{V} be the subspace spanned by $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \vec{v}_5 = \begin{bmatrix} -5 \\ 4 \\ 9 \end{bmatrix}$.
 - Find a basis for \mathcal{V} and call your basis vectors \vec{b}_1, \vec{b}_2 , etc.
 - Describe \mathcal{V} geometrically.
 - Let $V = [\vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4 | \vec{v}_5]$ be the matrix whose columns are the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5$, let $B = [\vec{b}_1 | \vec{b}_2 | \dots]$ be the matrix whose columns are your basis vectors from part (a), and let $\vec{v} \in \mathcal{V}$.
 Without computing, how many solutions does the equation $V\vec{x} = \vec{v}$ have? How about $B\vec{x} = \vec{v}$?
- Suppose A is an invertible matrix and $\vec{c}_1, \vec{c}_2, \dots, \vec{c}_n$ are its columns. Is $\{\vec{c}_1, \vec{c}_2, \dots, \vec{c}_n\}$ a basis? Describe $\text{span}\{\vec{c}_1, \vec{c}_2, \dots, \vec{c}_n\}$. Explain your reasoning.