

This is the topic list for the final skills check. It includes ALL topics from the first two skills checks, plus some new material. New material is in **bold**.

You should be able to give a mathematically precise definition of the following:

- linear combination
- linearly independent/dependent
- span
- consistent/inconsistent
- free variable
- trivial/nontrivial linear combination
- ... and any other terminology or notation we've discussed in class.
- Linear transformation
- Inverse of a linear transformation
- Inverse of a matrix
- Elementary matrix
- Rank of a matrix
- Subspace
- Basis
- Dimension
- Range
- Null space
- Row space
- Column space
- **Eigenvalue**
- **Eigenvector**
- **Eigenspace**
- **Characteristic polynomial**
- **Diagonalization/diagonalizable**

You should be able to state the following theorems:

- The Rank-Nullity Theorem
- The Invertible Matrix Theorem (or most of it)

You should be able to do the following computations:

- Give mathematically precise definitions for all terms used in class, particularly including the ones used later on this list.
- Do vector arithmetic: addition, subtraction, scalar multiplication, and dot product.
- Multiply matrices.

- Express a vector as a linear combination of other vectors.
- Determine whether or not a set of vectors is linearly independent.
- Determine whether one vector is in the span of others.
- Find the span of a set of vectors.
- Put a matrix into reduced row echelon form, and recognize whether or not it is.
- Convert a system of linear equations into augmented-matrix form and matrix-equation ( $Av=b$ ) form, and vice versa.
- Find all solutions to a system of linear equations.
- Given a linear transformation  $T$ , compute  $T(\vec{v})$ .
- Given a linear transformation  $T$ , find a matrix for  $T$ .
- Find the inverse of a matrix.
- Use several different methods to determine whether or not a matrix is invertible.
- Determine whether or not a transformation is linear.
- Determine whether or not something is a subspace.
- Given a subspace, find a basis for it and compute its dimension.
- Describe the range and null space of a linear transformation.
- Give a basis for the range and null space of a linear transformation.
- Describe the row space, column space, and null space of a matrix.
- Give a basis for the row space, column space, and null space of a matrix.
- **Given a vector in one basis, convert it to another basis.**
- **Given a linear transformation expressed as a matrix in one basis, express it as a matrix in another basis.**
- **Given two bases, find a change-of-basis matrix to convert between them.**
- **Compute the eigenvalues of a linear transformation, and their corresponding eigenvectors.**
- **Given a linear transformation and one of its eigenvectors, find the corresponding eigenvalue.**
- **Given a linear transformation and one of its eigenvalues, find a basis for the corresponding eigenspace.**
- **Given the eigenvalues and eigenvectors of a linear transformation, find a matrix for it.**
- **Given a linear transformation (or a matrix for it), find its characteristic polynomial.**

Also remember that problems from HW7 (or close variations on them) may appear on the skills check!