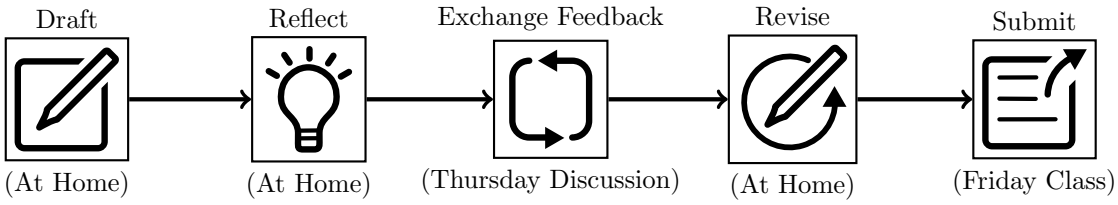
The PAR Process



Note: Your PAR (both initial and final drafts) must be typed in LaTeX.

Problem Statement

Recall the setup from PAR 1. The newly-appointed queen of a newly-discovered land hires three explorers to map her territory: Emily, Jack, and Lila. The explorers have their own equipment and their own quirks.

Jack Jack has a miscalibrated compass—it is off by 45°. When Jack thinks he's walking east, he is actually walking due northeast. When Jack walks in what he thinks is a cardinal direction, he measures distance accurately.

Emily Emily is a careful explorer with an accurate compass. When Emily measures distance in a cardinal direction, it too is accurate.

Lila Lila is an excitable explorer. Her compass is accurate, but when she walks north, she skips and twirls. As a consequence, when Lila walks north, the distance she thinks she travels is *half* the distance she actually traveled.

Further, each explorer only walks in what they think are cardinal directions (i.e., north, east, south, and west). The queen declares that her palace is the center of the nation and that all measurements be made relative to her palace. She then sends the explorers on their way.

With the framework of linear transformations and bases, we have a more powerful framework to analyze this problem.

1. Find linear transformations $\mathcal{S}_J$, $\mathcal{S}_E$ and $\mathcal{S}_L$ that convert Jack, Emily, and Lila's coordinates into true coordinates. (Remember, a linear transformation is a function from vectors to vectors. It is closely related to a matrix, but it is not the same thing as a matrix. Make sure you explain what your *linear transformation* is in each case.)

2. Define the distance function $d : \mathbb{R}^2 \to \mathbb{R}$ by $d(x, y) = \sqrt{x^2 + y^2}$ to be the usual Euclidean distance. Is d a linear transformation? Why or why not?

3. Using the language/notation of function composition, explain how to create distance functions d_J, d_E, and d_L which take each explorer's measured coordinates and returns the *true* distance that they traveled.

4. Jack and Lila are best friends and love to trade coordinates with each other. However, they both know they need to tweak their coordinates to suit the other. Jack invents a function $T_J : \mathbb{R}^2 \to \mathbb{R}^2$ which takes Jack's coordinates as input and outputs the same position in Lila's coordinates. Lila has a similar transformation $T_L : \mathbb{R}^2 \to \mathbb{R}^2$. Are T_J and T_L linear transformations? How do they relate to each other?

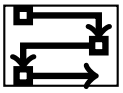
5. Jack, Lila, and Emily got into a fight over whose coordinates are *right*. Each of them claims since they can compute distances accurately and can convert their coordinates to each other's, that their coordinates should be the *Official Royal Coordinates*. Using the language of linear algebra, and in particular the idea of basis, explain what's going on to Jack, Lila, and Emily.

Reflection

Turn the page and check off the icons for things you think you did well; circle the icons for things you would like feedback on.

Feedback Provided By:_____

Suggestions Communication **Strengths**

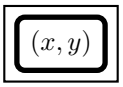


Show All Steps

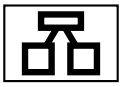
Explain Why,
Not Just What

Avoid Pronouns

Use Correct
Definitions

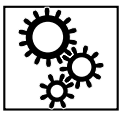


Define Variables,
Units, etc.

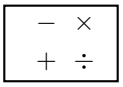


Create Diagrams

Suggestions Accuracy **Strengths**



Correct Setup



Accurate Calculations

Solve Multiple Ways

Answer Reasonable

Other
(Write Below)