1. Let $L \subset \mathbb{R}^3$ be the line that passes through the points $(0,0,0)$ and $(1,2,3)$ and let $E \subset \mathbb{R}^2$ be ellipse, centered at the origin, with major axis along the x-axis, minor axis along the y-axis, major radius of 2 and a minor radius of 1.

   (a) Find three different parameterizations of L, $\vec{p}_1, \vec{p}_2, \vec{p}_3$ where $\vec{p}_1$ is an arc-length parameterization, $\vec{p}_2$ is a parameterization that moves at speed 2, and $\vec{p}_3$ is a parameterization that passes through the point $(0,0,0)$ at speed 1 and $(1,2,3)$ at speed 2. Make sure to specify the domains of each parameterization.

   (b) Find a parameterization of L by $(0,1)$ (the open unit interval). Can you find a parameterization of L by $[0,1]$ (the closed unit interval)? Explain.

   (c) Find a parameterization of E. Make sure to specify its domain.

   (d) Find a parameterization $\vec{p}$ of E by $[0,1)$ with the added property that

   $$\|\vec{p}'(0)\| = \lim_{t \to 1^-} \|\vec{p}'(t)\| = 1.$$

   In your answer, you may use C to stand for the circumference of the ellipse E. (*Hint: there are many answers; some of them involve piecewise functions.*)

2. An *isometric parameterization* of a 2D surface S is a parameterization $p(t,s)$ that is length preserving *and* area preserving. That is, the speed with respect to the first variable is 1, the speed with respect to the second variable is 1, and the area of the image of the square with corners $(\alpha,\beta), (\alpha+1,\beta), (\alpha,\beta+1), (\alpha+1,\beta+1)$ is 1.

   Consider the surface $S \subset \mathbb{R}^3$ parameterized by

   $$s(x,y) = (x^2, y, |y|^{3/2}).$$

   Produce an isometric parameterization $p : \mathbb{R}^2 \to \mathbb{R}^3$ and verify each property of the parameterization. (Hint, use a computer to visualize the surface and imagine it as a piece of paper. Now choose easy direction vectors to parameterize with. Also, don't forget that $\|\vec{a} \times \vec{b}\|$ gives you the area of the parallelogram with sides $\vec{a}$ and $\vec{b}$.)

3. In homework 2, you developed a theory of line integrals working directly with limits and Riemann sums. Most textbooks present the theory of line integrals as follows:

   > Suppose $C \subset \mathbb{R}^2$ is a one dimensional curve with parameterization $\vec{p} : [a,b] \to \mathbb{R}^2$, and let $f : \mathbb{R}^2 \to \mathbb{R}$ be a scalar-valued function. Then,
   >
   > $$\int_C f = \int_a^b f(\vec{p}(t))\|\vec{p}'(t)\| \mathrm{d}t,$$
   >
   > where $\int_C f$ is the signed area above the curve C and below the surface $(x, y, f(x,y))$.

   Your task is to explain where this formula comes from using the foundational ideas of integral calculus: Riemann sums and limits. You may work with a partner and turn in one writeup between the two of you, or you may work individually. Your explanation should be typed in LaTeX and include the following:

   (a) justification for any Riemann sums or limits you use;

   (b) at least one example of a line integral;

   (c) a discussion (it can be brief, or extended) about the special case of arclength parameterizations in relation to line integrals;

   (d) appropriate citations if you used external resources (the `\footnote{...}` command is your friend!).

   You are strongly encouraged to include pictures. Your target audience is a student who just completed integral calculus. Your writeup will be graded both on correctness and on how well you explain/present the topic.