1. A rocket follows a straight path. Its position along the path is t^2 meters from the origin at time t. The radius of the exhaust pipe at time t is $r(t) = 8 - t^{1/3}$.

   (a) The EPA wants an estimate of the total volume of exhaust from $t = 0$ to $t = 8$ seconds. They request you estimate this volume by sampling the radius of the exhaust pipe and the length of the exhaust column at 8 regularly spaced **times**.

   Write and evaluate a summation for the EPA's requested estimate.

   (b) Use an integral to produce a more accurate estimate of the total amount of exhaust (you may assume the volume of exhaust in an exhaust column of height h and radius r is $\pi r^2 h$).

2. For this problem, you will be using MATLAB/OCTAVE. Please include a printout of your code along with your assignment.

   (a) Let $f(x) = x^2$. Set up a right-endpoint Riemann sum to estimate the value of $\int_0^3 f(x)\,\mathrm{d}x$.

   (b) Use MATLAB/OCTAVE to evaluate the Riemann sum from part (a) using 10, 100, and 1000 intervals. Report the deviation from the exact integral for each case.

   (c) Consider the *parameterization* $p : \mathbb{R} \to \mathbb{R}$ of the real numbers given by $p(t) = t^{1/3}$. Find an interval $[a, b]$ such that $p([a, b]) = [0, 3]$. (If you're unfamiliar with this notation, for a set X, $p(X)$ is the *image* of X under p. That is, $p(X) = \{y : y = p(x) \text{ for some } x \in X\}$).

   (d) The command `stairs(xs, ys)` can be used to graph a step function in MATLAB/OCTAVE. Perfect for visualizing Riemann sums! We're going to compare two ways of approximating f. Use `stairs` to plot $(t, f(t))$ for 50 equally spaced points in $[0, 3]$. Again, use `stairs` to plot $(p(t), f \circ p(t))$ for 50 equally spaced points in your interval $[a, b]$ from above.

   How do the domain and range of your t-values compare? How do the x and y values for each of your plots compare?

   (e) Set up a sum that estimates the area under the curve $(p(t), f \circ p(t))$ when t ranges over your previously-established interval $[a, b]$. Use MATLAB/OCTAVE to compute the value of this sum for 10, 100, and 1000 regularly spaced t values. Should you expect these estimates to be close to $\int_0^3 f(x)\,\mathrm{d}x$? How do your estimates compare to part (b)?

   (f) Set up and evaluate an integral to find the exact area under the curve $(p(t), f \circ p(t))$ where t ranges over your previously-established interval $[a, b]$. The bounds of your integral should be a and b. (*Hint: set up a general sum to estimate the area and then take a limit as* $\Delta t \to 0$).

3. You travel, starting from the origin and heading in the positive direction, along the x-axis with a speed given by $s(x) = \sqrt{x}$ units per second, where x is your position along the x-axis. You sample the height of a function as you travel and discover $h(t) = (2 - t)^2$ where t is time in seconds.

   (a) How long does it take you to get to $x = 10$?

   (b) Give a relationship between x and t.

   (c) Reparameterize h in terms of x.

   (d) Write an integral formula for the area under h from $x = 0$ to $x = 10$.

Recall that Work $= \vec{F} \cdot \vec{d}$ where $\vec{F}$ is a force vector and $\vec{d}$ is a displacement vector.

4. A turbulent river pushes a particle at the point (x, y) with a force

$$\vec{F}(x, y) = (-yx, x).$$

   You are pushing a box through the river from $(0, 0)$ to $(1, 1)$ along a straight path.

(a) Does $\vec{F}(0,0) \cdot (1,1)$ give you the total work done? How about $\vec{F}(1,1) \cdot (1,1)$? Why or why not?

(b) Suppose you take tiny steps of size $\sqrt{2}/n$ on your way from $(0,0)$ to $(1,1)$. Write a summation that represents the total amount of work done traveling from $(0,0)$ to $(1,1)$. If it's not the case already, rewrite your summation so it involves a dot product.

(c) Write down the integral that results from your summation in (b) when you let $n \to \infty$.

(d) What is the total work done?

5. A vector moves along the path $\vec{r}(t) = \begin{bmatrix} \sin t \\ 2\cos t \\ t \end{bmatrix}$ through a force field given by $\vec{F}(x,y,z) = \begin{bmatrix} yz \\ xz \\ xy \end{bmatrix}$. Find the total work done between $t = 0$ and $t = \pi$.