

1. Find unit vectors in the directions $\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\vec{v} = \hat{x} + 3\hat{y}$, $\vec{w} = -\hat{x} - \hat{y} - \hat{z}$, and $\vec{r} = \frac{\vec{v}}{\|\vec{v}\|} + \frac{\vec{w}}{\|\vec{w}\|}$.
2. Use the algebraic definition of the dot product to show for any $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$
 - (i) $\vec{a} \cdot (\alpha \vec{b}) = \alpha(\vec{a} \cdot \vec{b})$
 - (ii) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

(See page 7 of the Evans text for the definition of $\alpha \vec{v}$ and $\vec{u} + \vec{v}$.)
3. For vectors $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$, is $\vec{a} \cdot \vec{b} \cdot \vec{c}$ defined? Explain your answer in terms of the definition of the dot product.
4. Consider the plane \mathcal{P} defined as the set of solutions to the equation

$$3x - 2y + z = 4.$$

Let $\vec{p} \in \mathcal{P}$ be a point in \mathcal{P} interpreted as a vector.

- (a) What is $\vec{p} \cdot \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$? Explain.
 - (b) The plane \mathcal{Q} is the plane \mathcal{P} translated in the \hat{y} direction by one unit. Let $\vec{q} \in \mathcal{Q}$. What is $\vec{q} \cdot \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$?
5. Let $\vec{r} = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$ and consider the sets

$$A = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = 2t\vec{r} \text{ for some } t \in \mathbb{Z}\} \quad B = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = 3t\vec{r} \text{ for some } t \in \mathbb{Z}\}$$

$$C = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = \vec{a} + \vec{b} \text{ for some } \vec{a} \in A \text{ and } \vec{b} \in B\}.$$

- (a) On separate axes, draw A , B , and C .
 - (b) Prove or disprove the following statements:
 - (i) $A = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = t\vec{r} \text{ for some } t \in \mathbb{Z}\}$
 - (ii) $C = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = t\vec{r} \text{ for some } t \in \mathbb{Z}\}$
 - (iii) $C = \{\vec{x} \in \mathbb{R}^2 : \vec{x} = t\vec{r} \text{ for some } t \in \mathbb{R}\}$

Make sure to include any relevant definitions. (*Hint:* you may take it as a fact that if $a, b \in \mathbb{Z}$ then $a \pm b \in \mathbb{Z}$ and $ab \in \mathbb{Z}$.)
6. Let's get started learning how to type mathematics! Read about L^AT_EX on the course webpage. Then, do one of the following:
 - (a) type this homework assignment, or
 - (b) create a L^AT_EX document that contains the sentence, "Hi, my name is *your name*, and my favorite equation is $y = x^2 + \frac{\alpha}{2}$," and turn in a printout with your homework. You are free to substitute your actual favorite equation in for $y = x^2 + \frac{\alpha}{2}$.