1. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.
   (a) Show that the null space of T is a subspace of $\mathbb{R}^n$.
   (b) Show that the range of T is a subspace of $\mathbb{R}^m$.
2. (a) For a 4×3 matrix M, must the column space of M be identical to the column space of $\text{rref}(M)$?
   (b) For a 3×3 matrix N with $\text{rank}(N) = 3$, must the column space of N be identical to the column space of $\text{rref}(N)$? Can the assumption that $\text{rank}(N) = 3$ be dropped?
3. For a linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^3$, we have the following information:

$$L \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \qquad L \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix} \qquad L \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

   (a) Write down a matrix for L.
   (b) Describe the range of L as a point, line, plane, or hyperplane and give a basis for the range of L.
   (c) Describe the null space of L as a point, line, plane, or hyperplane and give a basis for the null space of L.