1. Is the set $X = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \right\}$ a basis for $\mathbb{R}^3$? Justify your answer using technical linear algebra vocabulary (including definitions).

2. Recall the set $\mathcal{G}$ of all 4×4 grayscale images (like in Lab 1). Is $\mathcal{G}$ a subspace? If so, find a basis for it and its dimension.

3. (a) Give an example of a 3×3 non-homogeneous system of equations that (i) has one solution, (ii) has infinitely many solutions, (iii) has no solutions. Make sure to explain why you know each system has the desired property.
   (b) Can you give an example of a 3×3 homogeneous system for properties (i), (ii), and (iii)? Explain why or why not.

4. (a) Use an augmented matrix to solve
$$\begin{aligned} x + y &= 7 \\ 2x - 3y &= 13. \end{aligned}$$
   Are there any values you could replace the right hand side of the equations with such that there would be no solution? Explain using technical linear algebra terms like basis, span, linear independence/dependence, subspace, etc. (Yes, this probablem should look just like the one from Homework 1!)
   (b) Consider the system given by the augmented matrix
$$C = \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 2 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right].$$
   and call the variables in this system x_1, x_2, x_3, x_4, x_5. Write all solutions to this system in vector form.
   (c) There are 10 ways to pick two things from the set $\{x_1, x_2, x_3, x_4, x_5\}$. For each of the ten ways, determine whether that pair is a valid choice of free variables for C.
   (d) Write down all solutions to the homogeneous system corresponding to C. How does this set of solutions correspond to the solutions to C?
   (e) Let H be the homogeneous system corresponding to C. For every choice of free variables you make when solving H, you get solutions of the form
$$\vec{x} = t\vec{d}_1 + s\vec{d}_2.$$
   Let $\Gamma = \{\vec{d}_1, \vec{d}_2\}$ be the set of vectors $\vec{d}_1$ and $\vec{d}_2$ corresponding to one choice of free variables and let $\Delta = \{\vec{d}_1, \vec{d}_2\}$ be the set of vectors $\vec{d}_1$, $\vec{d}_2$ arising from a *different* choice of free variables.
   Will the set $\Gamma \cup \Delta$ be linearly independent or dependent? Does this depend on your choice? Give a basis for span$(\Gamma \cup \Delta)$. (Hint: if this is hard to think about, work out some examples with some actual choices.)

5. Let X be an unknown 3×3 matrix. List every possible reduced row echelon form for X. You may use the ? or $*$ symbols as place holders for "any number," but if a certain position in the matrix must be filled with a particular number, put that number. For each reduced row echelon form, identity how many free variables there are and the rank of X. (Hint: there are a fair number of possible reduced row echelon forms for a 3×3).

6. Let X be an unknown 3×3 system and let $S = \{\text{solutions to } X\}$. Show that S is a subspace if and only if X is a homogeneous system.