Before next class:

- **Watch videos 9.4, 9.5, 9.6**

Computation practice: integration by substitution

Use substitutions to compute:

(A) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

(B) $\displaystyle\int e^x \cos\left(e^x\right)dx$

(C) $\displaystyle\int \cot x\,dx$

(D) $\displaystyle\int x^2\sqrt{x+1}\,dx$

Computation practice: integration by substitution

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(C) $\displaystyle\int \cot x \, dx$

(D) $\displaystyle\int x^2\sqrt{x+1}\, dx$

(E) $\displaystyle\int \frac{e^{2x}}{\sqrt{e^x+1}}\, dx$

(F) $\displaystyle\int \frac{\left(\ln\ln x\right)^2}{x\ln x}\, dx$

(G) $\displaystyle\int xe^{-x^2}\, dx$

(H) $\displaystyle\int e^{-x^2}\, dx$

Definite integral via substitution

This final answer is right, but the write-up is WRONG. Why?

Calculate $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

Wrong answer

Substitution: $u = x^3 + 1,\ du = 3x^2 dx.$

$$
\begin{aligned}
I &= \frac{1}{3}\int_0^2 \sqrt{x^3+1}\,(3x^2 dx) & &= \frac{1}{3}\int_0^2 u^{1/2}\, du \\
&= \frac{1}{3}\,\frac{2}{3}\ u^{3/2}\Big|_0^2 & &= \frac{1}{9}\left(x^3+1\right)^{2/3}\Big|_0^2 \\
&= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}\left(0+1\right)^{3/2} & &= \frac{52}{9}
\end{aligned}
$$

Integral of products of sin and cos

We want to compute

$$I = \int \sin^3 x \cos^2 x \, dx$$

(A) Attempt the substitution $u = \sin x$

(B) Attempt the substitution $u = \cos x$

(C) One worked better than the other. Which one? Why? Finish the problem.

Integral of products of sin and cos

We want to compute

$$I = \int \sin^3 x \cos^2 x \, dx$$

(A) Attempt the substitution $u = \sin x$

(B) Attempt the substitution $u = \cos x$

(C) One worked better than the other. Which one? Why? Finish the problem.

(D) Assume we want to compute

$$\int \sin^n x \cos^m x \, dx$$

When will the substitution $u = \sin x$ be helpful?
When will the substitution $u = \cos x$ be helpful?

Before next class:

- **Watch videos 9.7, 9.8, 9.9**

Computation practice: Integration by parts

Use integration by parts (possibly in combination with other methods) to compute:

(A) $\int xe^{-2x}dx$

(B) $\int x^2 \sin x \, dx$

(C) $\int \ln x \, dx$

(D) $\int \sin \sqrt{x} \, dx$

Computation practice: Integration by parts

Use integration by parts (possibly in combination with other methods) to compute:

(A) $\int xe^{-2x} dx$

(B) $\int x^2 \sin x \, dx$

(C) $\int \ln x \, dx$

(D) $\int \sin \sqrt{x} \, dx$

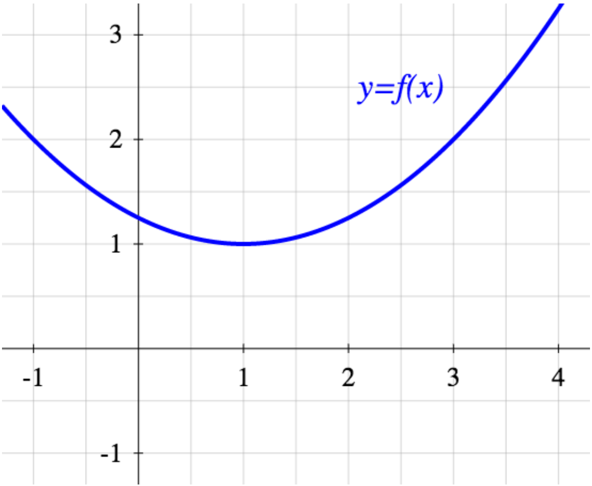
(E) $\int x \arctan x \, dx$

(F) $\int x^2 \arcsin x \, dx$

(G) $\int e^{\cos x} \sin^3 x \, dx$

(H) $\int e^{ax} \sin(bx) dx$

Integrals from a graph



Estimate:

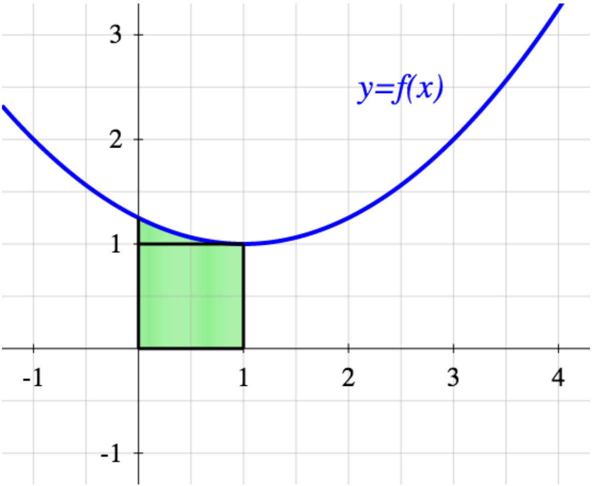
(A) $\int_0^1 f(x)dx$

(B) $\int_0^1 f'(x)dx$

(C) $\int_0^3 x\, f'(x)dx$

(D) $\int_0^1 f(3x)dx$

Integrals from a graph



Estimate:

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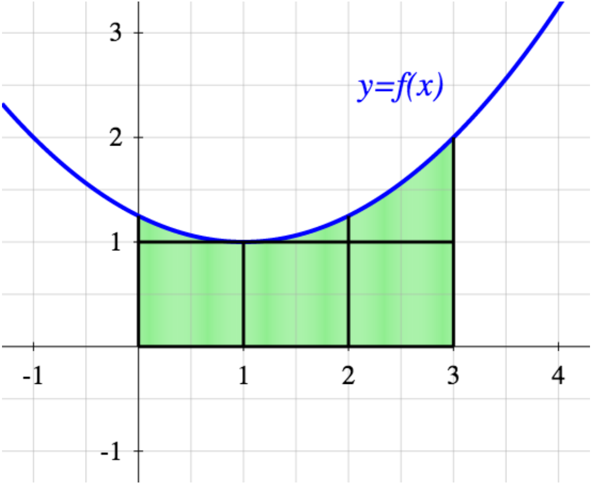
Integrals from a graph



Estimate:

(A) $\int_0^1 f(x)dx$

(B) $\int_0^1 f'(x)dx$

(C) $\int_0^3 x\,f'(x)dx$

(D) $\int_0^1 f(3x)dx$

The error function

The following function is tabulated.

$$E(x) = \int_0^x e^{-t^2} dt.$$

Write the following quantities in terms of E:

(A) $\int_1^2 e^{-t^2} dt$

(B) $\int_0^x t^2 e^{-t^2} dt$

(C) $\int_0^x e^{-2t^2} dt$

The error function

The following function is tabulated.

$$E(x) = \int_0^x e^{-t^2} dt.$$

Write the following quantities in terms of E:

(A) $\int_1^2 e^{-t^2} dt$

(B) $\int_0^x t^2 e^{-t^2} dt$

(C) $\int_0^x e^{-2t^2} dt$

(D) $\int_0^1 e^{-t^2+6t} dt$

(E) $\int_{x_1}^{x_2} e^{-\frac{(t-\mu)^2}{\sigma^2}} dt$

(F) $\int_1^2 \frac{e^{-t}}{\sqrt{t}} dt$

MAT137 Lecture 47 — Integration of Products of Trig. Functions

Before next class:

- **Watch videos 9.10, 9.11, 9.12**

Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (Once you get them to a form from where you see a path to finish them, even if long, you may stop.)

(A) $\displaystyle\int \sin^{10} x \cos x \; dx$

(B) $\displaystyle\int \sin^{10} x \cos^7 x \, dx$

(C) $\displaystyle\int e^{\cos x} \cos x \sin^3 x \, dx$

(D) $\displaystyle\int \cos^2 x \, dx$

(E) $\displaystyle\int \cos^4 x \, dx$

(F) $\displaystyle\int \csc x \, dx$

Useful trig identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\boxed{}$, then use the substitution $u = \tan x$.
- If $\boxed{}$, then use the substitution $u = \sec x$.

Hint: You will need

- $\dfrac{d}{dx}[\tan x] = \ldots$
- $\dfrac{d}{dx}[\sec x] = \ldots$
- The trig identity involving sec and tan

The error function

The following function is tabulated.

$$E(x) = \int_0^x e^{-t^2} dt.$$

Write the following quantities in terms of E:

(A) $\displaystyle\int_1^2 e^{-t^2} dt$

(B) $\displaystyle\int_0^x t^2 e^{-t^2} dt$

(C) $\displaystyle\int_0^x e^{-2t^2} dt$

The error function

The following function is tabulated.

$$E(x) = \int_0^x e^{-t^2} dt.$$

Write the following quantities in terms of E:

(A) $\displaystyle\int_1^2 e^{-t^2} dt$

(B) $\displaystyle\int_0^x t^2 e^{-t^2} dt$

(C) $\displaystyle\int_0^x e^{-2t^2} dt$

(D) $\displaystyle\int_0^1 e^{-t^2+6t} dt$

(E) $\displaystyle\int_{x_1}^{x_2} e^{-\frac{(t-\mu)^2}{\sigma^2}} dt$

(F) $\displaystyle\int_1^2 \frac{e^{-t}}{\sqrt{t}} dt$

Before next class:

- **Watch videos 10.1**

Rational integrals

(A) Calculate $\displaystyle\int \frac{1}{x+a}\,dx$

(B) Reduce to common denominator $\displaystyle\frac{2}{x} - \frac{3}{x+3}$

(C) Calculate $\displaystyle\int \frac{-x+6}{x^2+3x}\,dx$

(D) Calculate $\displaystyle\int \frac{1}{x^2+3x}\,dx$

(E) Calculate $\displaystyle\int \frac{1}{x^3-x}\,dx$

Repeated factors

(A) Calculate $\displaystyle\int \frac{1}{(x+1)^n}\,dx \quad \text{for } n > 1$

(B) Calculate $\displaystyle\int \frac{(x+1)-1}{(x+1)^2}\,dx$

(C) Calculate $\displaystyle\int \frac{2x+6}{(x+1)^2}\,dx$

(D) Calculate $\displaystyle\int \frac{x^2}{(x+1)^3}\,dx$

The integral of secant

Compute
$$\int \sec x \, dx$$
using the substitution $u = \sin x$.