MAT137 Lecture 42 — Antiderivatives and Indefinite Integrals

Before next class:

- **Watch videos 8.3, 8.4**

The most misunderstood antiderivative

(A) Find the *domain* and the derivative of $F_1(x) = \ln x$

(B) Find the *domain* and the derivative of $F_2(x) = \ln(-x)$

(C) Find the *domain* and the derivative of $F_3(x) = \ln|x|$
Suggestion: Break the domain into two pieces.

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(D) Based on your answers, what is $\displaystyle\int \frac{1}{x}\,dx$?

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(D) Based on your answers, what is $\displaystyle\int \frac{1}{x}\,dx$?

(E) Find the *domain* and the derivative of $F_4(x) = \ln|2x|$
Why doesn't this contradict your answer to 4 ?

Functions defined by integrals

Which ones of these are valid ways to define functions?

(A) $F(x) = \int_0^x \frac{t}{1+t^8}\,dt$

(B) $F(x) = \int_0^x \frac{x}{1+x^8}\,dx$

(C) $F(x) = \int_0^x \frac{x}{1+t^8}\,dt$

(D) $F(x) = \int_0^{x^2} \frac{t}{1+t^8}\,dt$

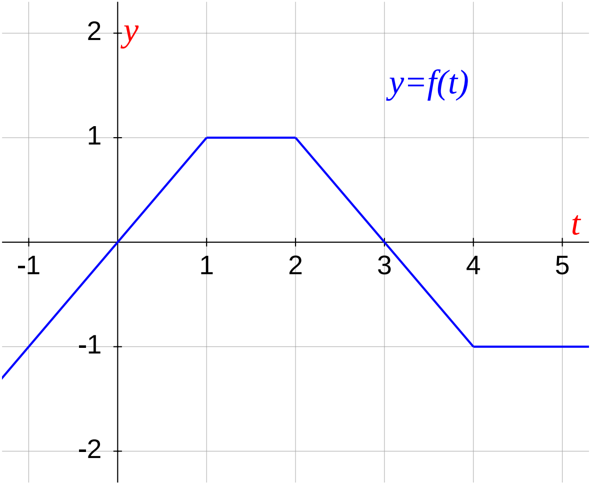
(E) $F(x) = \int_{\sin x}^{e^x} \frac{t}{1+t^8}\,dt$

(F) $F(x) = \int_0^3 \frac{t}{1+x^2+t^8}\,dt$

(G) $F(x) = x\int_{\sin x}^{e^x} \frac{t}{1+x^2+t^8}\,dt$

(H) $F(x) = t\int_{\sin x}^{e^x} \frac{t}{1+x^2+t^8}\,dt$

Towards FTC



Compute:

(A) $\int_0^1 f(t)dt$

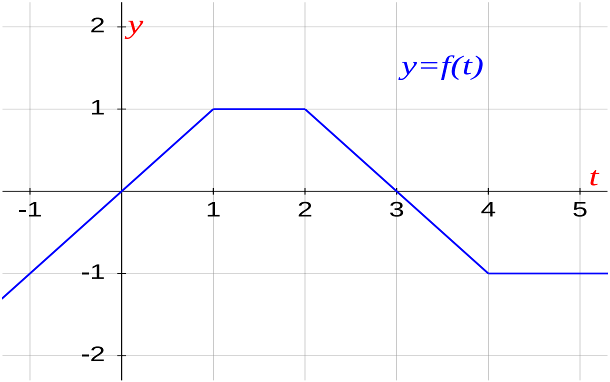
(B) $\int_0^2 f(t)dt$

(C) $\int_0^3 f(t)dt$

(D) $\int_0^4 f(t)dt$

(E) $\int_0^5 f(t)dt$

Towards FTC (continued)



Call $F(x) = \int_0^x f(t)dt$. This is a new function.

- Sketch the graph of $y = F(x)$.
- Using the graph you just sketched, sketch the graph of $y = F'(x)$.

Compute these antiderivatives by guess 'n check

(A) $\displaystyle\int x^5 dx$

(B) $\displaystyle\int \left(3x^8 - 18x^5 + 1\right) dx$

(C) $\displaystyle\int \sqrt[3]{x}\, dx$

(D) $\displaystyle\int \frac{1}{x^9}\, dx$

(E) $\displaystyle\int \sqrt{x}\left(x^2 + 5\right) dx$

(F) $\displaystyle\int \frac{1}{e^{2x}}\, dx$

(G) $\displaystyle\int \sin(3x)\, dx$

(H) $\displaystyle\int \cos(3x + 2)\, dx$

(I) $\displaystyle\int \sec^2 x\, dx$

(J) $\displaystyle\int \sec x \tan x\, dx$

(K) $\displaystyle\int \frac{1}{x}\, dx$

(L) $\displaystyle\int \frac{1}{x+3}\, dx$

Before next class:

- **Watch videos 8.5, .86, 8.7**

True or False?

(A) If f is continuous on the interval $[a, b]$, then

$$\frac{d}{dx}\left(\int_a^b f(t)dt\right) = f(x).$$

(B) If f is differentiable, then

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) \;=\; \int_a^x f'(t)\,dt.$$

Examples of FTC-1

Compute the derivative of the following functions

(A) $F_1(x) = \int_0^1 e^{-t^2} dt.$

(B) $F_2(x) = \int_0^x e^{-\sin t} dt.$

(C) $F_3(x) = \int_1^{x^2} \frac{\sin t}{t^2} dt.$

(D) $F_4(x) = \int_x^7 \sin^3\left(\sqrt{t}\right) dt.$

(E) $F_5(x) = \int_{2x}^{x^2} \frac{1}{1+t^3} dt.$

Creative Guess and Check 1

(A) $\dfrac{d}{dx}[x \sin x] =$

(B) $\dfrac{d}{dx}[\cos x] =$

Use the previous answers to compute

(C) $\displaystyle\int x \cos x \, dx =$

Creative Guess and Check 2

(A) $\dfrac{d}{dx}[xe^x] =$

(B) ???

Use the previous answers to compute

(C) $\displaystyle\int xe^x\,dx =$

Creative Guess and Check 3

(A) $\frac{d}{dx}[x^2 e^{-x}] =$

(B) ???

(C) ???

Use the previous answers to compute

(D) $\int x^2 e^{-x}\, dx =$

Creative Guess and Check 4

(A) $\dfrac{d}{dx}[x \ln x] =$

(B) ???

Use the previous answers to compute

(C) $\displaystyle\int \ln x \, dx =$

A challenge for guess-and-check ninjas

$$\int x\, e^x \cos x\, dx \;=\; ???$$

Before next class:

- **Watch videos 9.1, 9.2, 9.3**

Compute these definite integrals

(A) $\displaystyle\int_1^2 x^3 dx$

(B) $\displaystyle\int_0^1 \left[e^x + e^{-x} - \cos(2x)\right] dx$

(C) $\displaystyle\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}} dx$

(D) $\displaystyle\int_{\pi/4}^{\pi/3} \sec^2 x \, dx$

(E) $\displaystyle\int_1^2 \left[\frac{d}{dx}\left(\frac{\sin^2 x}{1+\arctan^2 x + e^{-x^2}}\right)\right] dx$

Find the error

$$\int_{-1}^{1} \frac{1}{x^4} dx \;=\; \left.\frac{-1}{3x^3}\right|_{-1}^{1} = \frac{-2}{3}$$

However, x^4 is always positive, so the integral should be positive.

Areas

Calculate the area of the bounded region...

(A) ... between the x-axis and $y = 4x - x^2$.

(B) ... between $y = \cos x$, the x-axis, from $x = 0$ to $x = \pi$.

(C) ... between $y = x^2 + 3$ and $y = 3x + 1$.

(D) ... between $y = 1$, the y-axis, and $y = \ln(x + 1)$.

More True or False

Let f and g be differentiable functions with domain $\mathbb{R}$.
Assume that $f'(x) = g(x)$ for all x.
Which of the following statements must be true?

(A) $f(x) = \int_0^x g(t)dt$.

(B) If $f(0) = 0$, then $f(x) = \int_0^x g(t)dt$.

(C) If $g(0) = 0$, then $f(x) = \int_0^x g(t)dt$.

(D) There exists $C \in \mathbb{R}$ such that $f(x) = C + \int_0^x g(t)dt$.

(E) There exists $C \in \mathbb{R}$ such that $f(x) = C + \int_1^x g(t)dt$.