An equation for volumes by the carrot method

Let $a < b$.
Let f be a continuous, positive function defined on $[a, b]$.
Let R be the region in the first quadrant bounded between the graph of f and the x-axis.
Find a formula for the volume of the solid of revolution obtained by rotation the region R around the x-axis.

Sphere

You know the formula for the volume of a sphere with radius R. Now you are able to prove it!

1. Write an equation for the circle with radius R centered at $(0, 0)$.
2. If you rotate this circle around the x-axis, it will produce a sphere. Compute its volume as an integral by slicing it like a carrot.

Pyramid

Compute the volume of a pyramid with height H and square base with side length L.

Hint: Slice the pyramid like a carrot with cuts parallel to the base.

Many axis of rotation

Let R be the region in the first quadrant bounded between the curves with equations $y = x^3$ and $y = \sqrt{32x}$.
Compute the volume of the solid of revolution obtained by rotating R around...

1. ... the x-axis
2. ... the y-axis
3. ... the line $y = -1$

An equation for volumes by "cylindrical shells"

Let $a < b$.
Let f be a continuous, positive function defined on $[a, b]$.
Let R be the region in the first quadrant bounded between the graph of f and the x-axis.
Find a formula for the volume of the solid of revolution obtained by rotation the region R around the y-axis.

A hat

Let R be the region in the first quadrant bounded between the graphs of $y = x^5 + x - 2$, $x = 2$, and the x-axis.
Compute the volume of the solid of revolution obtained by rotating R around the y-axis.

Doughnut

Let R be the region inside the curve with equation

$$(x-1)^2 + y^2 = 1.$$

Rotate R around the line with equation $x = 4$. The resulting solid is called a *torus*.

1. Draw a picture and convince yourself that a torus looks like a doughnut.
2. Set up the volume of the torus as an integral using x as the variable ("cylindrical shell method"). You do not need to compute the integral.
3. Set up the volume of the torus as an integral using y as the variable ("carrot method"). You do not need to compute the integral.

Challenge

Two cylinders have the same radius R and their axes are perpendicular. Find the volume of their intersection.

Hint: You can slice the resulting solid by parallel cuts in three different directions. One of the three makes the problem much, much simpler than the other two.