Tersely summarized, my teaching philosophy is: *Students learn when students do the work*. Learning is hard, requires energy, and manifests as physical changes in the brain. Short of situations like *The Matrix*, this is not something a teacher can do *to* students. So, what then is teaching? I endorse the idea that *teaching is arranging conditions to expedite learning*, and exactly what learning I seek to expedite is influenced by my goals for a university education—to produce independent problem solvers (be they technological, intellectual, or social problems) who communicate ideas effectively.

In my opinion, an effective teacher explains things at the right time, provides impetus, motivation, and opportunity for students to work, and ensures students are aware of what they do and do not know.

Explaining Things at the Right Time

An essential component to good timing is to carefully consider what concepts you would like a student to learn and then concoct a lesson. This often means preparing a motivating example that builds a need for mathematical framework, a presentation of the framework, and further examples to explore the consequences of the mathematics.

For example, you might be teaching compactness in an arbitrary topological space with the goal of students differentiating between compact and sequentially compact. Assuming students primarily associate compactness with closed and bounded subsets of $\mathbb{R}^n$, a series of examples showing that the inclusion of all limit points of sequences does not imply closed, closed does not imply convergent subsequences, and boundedness is not a property of an abstract topological space, would set the groundwork for an open-covering definition of compactness. Finally, proving abstract compactness is equivalent to closed and bounded in $\mathbb{R}^n$ would justify its inclusion in mathematics and other consequences could be explored.

Of course, careful planning only structures the primary time an instructor spends with students. Class time is where theory meets practice. Since traditional lectures are the norm, I will focus onwas how I address timing in some non-traditional classroom approaches.

When I used the "Think, Pair-Share" flipped-classroom approach to teach Calculus I, students watched videos and read the textbook before class. During class, students worked on difficult concept-based multiple-choice questions. They would think individually and vote on the correct answer with a clicker. I would then assess the results. If everyone got it, we moved on. If not, I asked students to explain their answer to their neighbor. This was followed by a re-vote, maybe more discussion, and an explanation by me. The students were more engaged than in a typical math course, and, even when I gave an explanation, it was different from an explanation in a lecture. Not that the *explanation* was different, but that the context was different. Students, having already struggled with the problem, were ready to see how the framework of calculus could help them, and because I was aware of which conceptual errors they were making based on their clicker responses, I could carefully target my explanation.

In Linear Algebra classes varying in size from 20–120, I have conducted class by having students work in pairs or small groups on worksheets I created to guide them to the big ideas. While students work, I walk around the room eavesdropping on conversations, injecting questions, and assessing where there are difficulties. I then give explanations targeting students' misunderstandings and tying together the themes of the course with the problems they solved.

Changing the workings of a classroom so drastically is difficult to pull off, but the rewards are enormous. Some students resent not being told ahead of time how to solve their math problems, but most feel transformed. Allowing students to struggle, fail, and succeed on non-routine problems and with help only when they need it is key for this transformation. One student of mine, who had failed calculus twice, had been at it so long that he had memorized all the mechanics but somehow missed the concepts. When he encountered my conceptual questions, he forced himself to go beyond the mechanics and, with extraordinary effort on his part, got an A. In a multi-section Linear Algebra for Engineers course, besides being the top scoring section (out of three) my students were more willing to attempt new problems. The final Matlab project asked them to numerically bound the operator norm of a matrix. Students in the other sections complained that this project was unreasonable, but for my students, attempting something when the algorithmic steps have not been outlined in class was just another normal day.

Fostering Motivation

Motivating students with grades or discipline-specific examples is important but commonplace. For this reason, I will focus on how I foster motivation by talking about university goals, how learning happens, and leveraging cognitive dissonance.

Students come to university for a variety of reasons, but whether it is marketability or self-improvement, the path forward is the same—learn to solve problems and answer questions others cannot. It is easy to lose sight of why homework problem 14 is important, but well-placed reminders help. We are working to change our brains, and that this process is uncomfortable! I have received ovations from a class after explaining how neural networks learn, and how being smart and having a brain as large as a human means that it is a major challenge to avoid memorization in lieu of real learning.

As for cognitive dissonance, if students discover they are wrong about something they thought they understood, they will be motivated to find out what went wrong. For example, many students "understand" linear independence as the vectors not being multiples of each other. However, upon discovering a particular set of three non-parallel vectors is linearly dependent, students will spend energy to resolve their misconception.

Student Awareness of What They Do and Do not Know

Well-designed assessment is essential for helping students gauge their progress, but asking students to explain their techniques is a stepping stone to the self-evaluation required of independent problem solvers. In office hours, I do this in an individualized way. At scale, I rely on group/peer discussion, presentations, and written work. My job is to convince students that if their response to the prompt, "Please define linear independence" is, after some struggle, "Well, I can't write it down, but I'd know it when I see it," then they do not really *know* what linear independence is. I put this into practice by requiring formal write-ups (sometimes typed) of not just the solution to a problem but how it was solved, including relevant definitions and rigorous logic.

Lastly, sometimes my role is to explicitly tell. Solving problem after problem gives students the tools, but the birds eye view is something that comes from a math expert. "You know how to solve any system of equations; now you know how to determine if a set is linearly independent."

Conclusion

As cliché as it is, I enjoy teaching for its reward and challenge. The satisfaction of seeing someone grasp a beautiful idea or helping someone develop a new way of thinking about the world somewhat uniquely comes from teaching. The very best way to accomplish this under the constraints of a university setting is contested and by all accounts unknown, but I approach this challenge head on in hopes of being the best educator I can be.