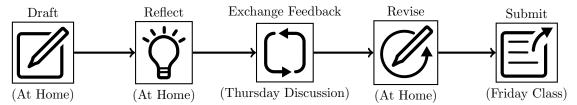
## The PAR Process



## **Problem Statement**

Let M be a  $3 \times 4$  matrix with unknown entries. You do know, however, that  $\operatorname{rref}(M) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ .

- 1. Is there a vector  $\vec{b}$  such that the matrix equation  $M\vec{x} = \vec{b}$  is consistent? Explain why or why not, or whether you'd need more information to tell.
- 2. Is there a vector  $\vec{b}$  such that the matrix equation  $M\vec{x} = \vec{b}$  is *inconsistent*? Explain why or why not, or whether you'd need more information to tell.
- 3. Let  $\vec{c_1}$ ,  $\vec{c_2}$ ,  $\vec{c_3}$ , and  $\vec{c_4}$  be the columns of the matrix M. For which i and j with  $i \neq j$  is the set  $\{\vec{c_i}, \vec{c_j}\}$  linearly dependent? For which is it linearly independent? Explain how you know.
- 4. Can a set of four vectors in  $\mathbb{R}^3$  ever be linearly independent? Explain.
- 5. Suppose  $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4 \in \mathbb{R}^3$ . Let  $\mathcal{V} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$  and let  $X = [\vec{v}_1 | \vec{v}_2 | \vec{v}_3 | \vec{v}_4]$  be the matrix whose columns are  $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ . Suppose further that every subset of  $\mathcal{Y} \subset \mathcal{V}$  of size two is linearly independent. Explain what  $\operatorname{rref}(X)$  must look like in this case. (Hint: you won't be able to pin down exact numbers for every entry of  $\operatorname{rref}(X)$ , but you might know things like whether the entry can be zero or not, etc.)

Feedback Provided By:\_



Show All Steps



Explain Why, Not Just What



Avoid Pronouns



Use Correct Definitions



Define Variables, Units, etc.



Create Diagrams

Suggestions Accuracy Strengths



Correct Setup



Accurate Calculations



Solve Multiple Ways



Answer Reasonable



Other (Write Below)