Comparing Platform Approaches to Common Graph Partitioning Algorithms

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Abstract:

In summary, there were six frameworks chosen for this project and they were chosen to be the best performing, with the same set of kernels using one of three platforms. Hardware was chosen to provide a single example for all tests, although it might not have been the exact same computer between tests. Frog (FR)[1], Graphlt (GI)[2], Gunrock (GR)[3], Hygraph (HY)[4], Polymer (PM)[5], and Totem (TM)[6] are the provided frameworks for testing. Information regarding frameworks will be found under the platform used, while graphs and hardware can be found in their respective sections. Kernels used were breadth first search (BFS), connected components (CC), pagerank (PR), and single source shortest path (SSSP). Platforms used were CPU only, GPU only, and a hybrid approach depending on the graph and task.

I refer the reader to the results section for how everything played out in testing, but it appears that FR and GI performed better with most graphs and kernels. A noticeable runner up would be GR, and TM had a clean sweep with PR but doubt exists. HY would not work finish on any graph and PSC support could not help either. More information can be found reading further, as well as the papers of the respective frameworks.

Introduction:

There are a few dozen or so graph partitioning frameworks to choose from. Most have compared themselves to the previous "best of " group, and displayed their rankings accordingly. Usually with the newest framework performing the best. A reason for my performing these tests was to apply a standardized hardware set, a set of graphs, and different platforms to see if there are any advantages across a small spectrum of the previous results. The small spectrum being the four kernels, or algorithms chosen that the frameworks have in common.

When I am referring to different platforms, I am speaking in regards to how each framework is using either the CPU, GPU/'s, or a hybrid approach. In this project, the frameworks chosen for each platform were the few that performed the best, and had comparable kernels. More information about the platforms can be found in the hardware section and their respective section in the paper.

The choice of graphs was another item to consider for this project, and a variety of available graphs (or sometimes can be referred to as maps) from an online repository were chosen for a variety of traits. In all there were sixteen candidates with chosen characteristics involving the number of connected components, the overall number of vertices and edges, to the largest in pure file size. More information on the graphs chosen can be found in the Graphs section as well as the Appendix.

Hardware used in this project was provided through XSEDE and the Pittsburgh Supercomputing Center (PSC)[7], and will help establish a baseline for current and future hardware capabilities. The chosen hardware was to keep the same set of hardware across all tests, and does not reflect the overall availability of the services at the PSC, or capability of the frameworks presented.

Finally, the last part of the project was the choice of kernels being Breadth First Search (BFS), Connected Components (CC), Page Rank (PR), and Single Source Shortest Path (SSSP). Bellman-Ford

algorithm was used in place of listed SSSP algorithms for each framework. The four selected kernels do not represent all of the options of each framework, but were most common among them all and provide a variety for consideration.

Hardware:

PSC has a list on their respective website detailing more information about hardware availability. The provided components for testing from that website are as follows:

CPUs: 2x Intel Haswell (E5-2695 v3) CPUs; 14 cores/CPU; 2 threads/core; 2.3 - 3.3 GHz

RAM: 128GB, DDR4-2133 Cache: 35MB SmartCache

Node-local Storage: 2 HDDs, 4TB each

Server: HPE Apollo 2000

GPUs: 2x NVIDIA Tesla K80; (1x K80 = 2x 12GB VRAM, 5GHz GDDR5) (equivalent of 4 gpu's)

The NVIDIA Tesla K80 was chosen in part due to its compatibility with more of the frameworks, but mostly for its use on the exact same CPU configuration that is available from PSC Bridges. This would help ensure a standard across all three platforms.

CPU:

GraphIt (GI) - "a new domain specific language for graph computations that generates fast implementations for algorithms with different performance characteristics running on graphs with different sizes and structures". Compiled with Python 2.7 and g++ 5.3.0 with OpenMP as stated from the GitHub readme. The input file format used for GI was the EL and WEL. Results for GI provide each graph a total of ten iterations with each iteration providing an elapsed time. BFS and CC were a strength of GI, with it also doing well on the larger file sizes.

Polymer (PM) - "a NUMA-aware Graph-structured Analytics Framework written in C++". Non Uniform Memory Access(NUMA). Compiled and run with g++ version 5.3.0 with CILK+ as stated on the GitHub readme. Input file format for PM was in the form of an adjacency list. PM did run well on SSSP with the graphs that were about 1Gb in size.

GPU:

Frog (FR) - "asynchronous Graph Processing on GPU with Hybrid Coloring Model". They used a modified Pareto principle about coloring algorithms, also known as the 80-20 rule. Compiled with gcc 5.3.0 and cuda 8.0 as mentioned from their GitHub readme. I used the Process-Large files from my GitHub folder during compiling to allow the use of large file sizes. The framework recommends tuning the buffer size to the hardware that you are running, and for a default value I used 1024*1024 or 1Gb blocks. The input file format required a preprocessing step in the form of converting the matrix market file type into a binary version. FR performed the best when the file size was about 320Mb or less in the category of BFS and SSSP. Where FR performed the worst was with large file sizes, which could be attributed to my choice of buffer size. Another notable feature is that FR ran every test regardless of graph size.

Gunrock (GR) - "is to deliver the performance of customized, complex GPU hardwired graph primitives with a high level model programming model that allows programmers to quickly develop new graph primitives." Their chief problem was how to manage irregularity in work distribution. Compiled with cuda 7.5 and gcc 4.8 as mentioned from my readme. The input format for GR is in the matrix market format which was how the files were downloaded. Results for GR were good overall, and possibly could have been improved if more recent versions of cuda or gcc were used during compilation. Also to be noted that parameters run during tests were set by default or to default.

Hybrid:

HyGraph (HY) - "a novel graph-processing systems for hybrid platforms which delivers performance using CPU and GPU concurrently. It's core feature is a specialized data structure which enables dynamic scheduling of jobs onto both CPU and the GPU's, thus supersedes the need for static workload distribution, provides load balancing, and minimizes inter-process communication overhead by overlapping computation and communication.". Compiled with cuda 8.0 and gcc 5.3.0 as well as the settings mentioned on their GitHub. Unfortunately neither version was successful in finishing any tests. Help was provided from the PSC Bridges help team and a solution was not found. Therefor HY is not found with any results. The input format for HY is the EL type, but can also use a binary version if you want to preprocess the file to further reduce loading time.

Totem (TM)- "a processing engine that provides a convenient environment to implement graph algorithms on hybrid platforms; we show that further significant performance gains can be extracted using partitioning strategies that aim to produce partitions that each matches the strengths of the processing element it is allocated to, and finally, we demonstrate the performance advantages of the hybrid system through comprehensive evaluation that uses real and synthetic scale-free workloads (as large as 16 billion edges), multiple graph algorithms that stress the system in various ways, and a variety of hardware configurations." Compiling TM with gcc 5.3 and cuda 8.0 as mentioned on their GitHub. The input file format needs to be in the *.totem format, which is a sorted EL with the number of nodes, edges, and whether the graph is directed or undirected, commented at the beginning of the file. The performance of TM was evident in the PR results, as no other framework was able to outperform it. More research will need to be done on why it was efficient with that particular kernel.

Graphs:

I collected all the graphs used from the SuiteSparse Matrix Collection formerly the University of Florida Sparse Matrix Collection. All of the graphs were originally downloaded in the matrix market format, and then converted into the edge list format (EL). All of the weighted versions of the graphs used, started in the EL format and were converted to the weighted edge list (WEL) format with the use of Graph Algorithm Processing Benchmark Suite (GAPBS)[8]. From there the graphs were converted into the file type specified for processing with each framework resulting in a use of 576 Gb to store them all. One file (in bold) in particular was so large that it crashed a few of the programs and was not included in the results, but mentioned here for completeness. The following list is the order of graphs used and some information about them. More information can be found in Appendix table 1 for the specific web address of the file, and table 2 for size comparisons of file types.

or Strongly

Name	Directed / Symmetric	# of Rows or Columns	# of Nonzeros	Connected Components	File size in *.el format.
Gh001 - Webbase-2001	D / Non-S	118,142,155	1,019,903,190	41,126,852	3.8Gb
Gh002 - Europe_Osm	UnD / Sym	50,912,018	108,109,320	1	1.89Gb
Gh003 - Sk-2005	D / Non-S	50,636,154	1,949,412,601	8,815,057	24.5Gb
Gh004 - It-2004	D / Non-S	41,291,594	1,150,725,436	6,753,961	19.9Gb
Gh005 - Uk-2005	D / Non-S	39,459,925	936,364,282	5,811,041	11Gb
Gh006 - Road_usa	UnD / Sym	23,947,347	57,708,624	1	.984Gb
Gh007 - Arabic-2005	D / Non-S	22,744,080	639,999,458	4,000,414	10.8Gb
Gh008 - Hugebubbles-00020	UnD / Sym	21,198,119	63,580,358	1	.538Gb
Gh009 - Rgg_n_2_24_s0	UnD / Sym	16,777,216	265,114,400	2	2.2Gb
Gh010 - Delaunay_n24	UnD / Sym	16,777,216	100,663,202	1	.839Gb
Gh011 - Hugetrace-00020	UnD / Sym	16,002,413	47,997,626	1	.398Gb
Gh012 - Hugetrace-00010	UnD / Sym	12,057,441	36,164,358	1	.292Gb
Gh013 - Wb-edu	D / Non-S	9,845,725	57,156,537	4,269,022	.903Gb
Gh014 - Ljournal-2008	D / Non-S	5,363,260	79,023,142	1,119,171	1.2Gb
Gh015 - Cage15	D / Non-S	5,154,859	99,199,551	1	1.4Gb
Gh016 - Soc-LiveJournal1	D / Non-S	4,847,571	68,993,773	971,232	1.0Gb

Results:

Results provided are the times returned from running tests on each graph, and in some cases averaged with other results if performed more than once. In the fifth image with the results, the last column refers to the framework that showed the best times with a map, if one existed. Any blank spaces occur when frameworks fail to run a particular map. In the case of HY, it failed to complete every map.

									Time listed in	milliseconds.			
	BFS	Time listed in	n milliseconds.					CC					
	FROG	CDADUIT	GUNROCK	UVCDADU	DOLVMED	TOTEM		FROG		GUNROCK	HYGRAPH	POLYMER	TOTEM
-6001	179.42	24.21	GUNROCK	HTUKAPH	POLIMER	497.81	gh001	40,105.1					11,809.74
gh001 gh002	10.27	1590.88	1751.53		2468.47	103011.25	gh002	5,436.49		416.91		1,556.32	347,478.36
gh004	4128.62	441.86	1/31.33		2400.47	2840.35	gh004	1,585.71					2,458.30
-	10837.9	9.88	210.69			235.46	gh005	2,292.22					3,432.60
gh005	2.83	664.93	703.99		41	14005.84	gh006	3,718.13				476.95	68,180.00
gh006 gh007	1896.58	4.48	0.54		41	205.3	gh007	1,265.14		2,676.59			1,119.11
	2.91				89.14	52	gh008	5,773.74		253.31		128.00	182.65
gh008		4.43	570.37		77X THE R. P. LEWIS CO.	1 932 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	gh009	18,110.70		349.33		1,078.03	171.02
gh009	9.22	3.39 3.77	454		711.38	7470.56	gh010	3,837.63		250.62		6,460.88	149.32
gh010	4.16		320.36		469417.2	179.69	gh011	3,881.18		177.43		83.87	139.03
gh011	2.26	3.07	524.55		50.58	55.3	gh012	2,618.97	22.78	126.54		59.02	103.26
gh012	1.69	2.4	465.29 0.3		70.84	39.32	gh013	182.89	123.34	749.77			1,554.83
gh013	854.7	63.98			2512.60	60.53	gh014	264.72	120.54	261.54		503.99	224.17
gh014	4.85	28.85	61.08		3512.68	72.92	gh015	143.31	101.40	95.17		728.94	215.47
gh015	319.9	94.64 26.03	68.69 60.07		346.26 4533.49	52.47 36.99	gh016	166.57	159.49	88.07		1,579.25	205.90
gh016	81.29	26.03	60.07		4555.49	30.33							
		Town Hotel discount	-1111										
	PR	Time listed in n	nilliseconas.						Time listed in mil	liseconds.			
		CDADHIT	GUNROCK H	VCDADH	POLYMER	TOTEM		SSSP FROG	GRAPHIT	GUNROCK	UVCDADU	POLYMER	TOTEM
gh001	14,566.51	2,956.08	domicock ii	IGICALLI	CEITIER	612.98	gh001	120.52	28.90	1.52	HTUKAPH	106.49	1,354.92
gh002	1.527.43	1.508.93	3,945.90		4.780.00	911.42	gh002	10.27	27,281.27	8,117.59		3.47	504,486.20
gh004	13,504.58	4,179.59	3,513.50		1,700.00	985.09	gh004	3,391.68	5,230.32	0,117.39		3.47	1,295.42
gh005	11,612.84	1,885.43				713.59	gh005	10,843.76	8.55	5,446.68		44.95	2,677.05
gh006	669.05	714.87	2.148.56		1,890.00	432.36	gh006	2.75	26.712.72	8.101.97		10.21	62,945.60
gh007	7,542.18	1,409.74	3,461.84		208,000.00	537.45	gh007	1.886.19	6.02	0.35		26.85	925.16
gh008	748.52	743.21	2.016.79		2.080.00	360.84	gh008	2.84	4.24	17,864.25		12.34	63.00
gh009	2,090.71	616.02	1,356.39		8,310.00	360.63	gh009	9.19	3.47	106,588.57		27.03	25,055.39
gh010	955.03	796.71	2,665.30		4,750.00	301.97	gh010	4.08	3.87	14,278.50		3.16	149.72
gh011	563.19	522.12	1,419.89		1,510.00	270.32	gh011	2.15	2.77	13,835.40		17.88	83.86
gh012	426.05	387.66	1,049.51		1,170.00	205.01	gh012	1.62	1.74	9,450.34		8.61	45.00
gh013	865.87	355.55	495.39			174.93	gh013	657.05	178.99	0.29		1.36	33.65
gh014	1,088.73	721.23	803.99		5,260.00	133.86	gh014	4.76	384.92	432.81		3.54	219.26
gh015	1,330.57	297.39	364.22		7,150.00	141.65	gh015	242.22	387.57	97.20		7.66	181.84
gh016	999.78	947.37			13,500.00	204.93	gh016	213.62	507.71	486.87		15.98	346.57

The final results are:

1 = Frog					
2=Graphit					
3=Gunrock					
4=Hygraph					
5=Polymer					
6=Totem					
	BFS	CC	PR	SSSP	Framework
gh001	2	2	6	3	2
gh002	1	3	6	5	
gh004	2	3	6	6	6
gh005	2	3	6	2	2
gh006	1	3	6	1	1
gh007	3	2	6	3	3
gh008	1	2	6	1	1
gh009	2	2	6	2	2
gh010	2	2	6	5	2
gh011	1	2	6	1	1
gh012	1	2	6	1	1
gh013	3	2	6	3	3
gh014	1	2	6	5	
gh015	6	3	6	5	6
gh016	2	3	6	5	
Тор	1-2	2	6	5	
Runner Up	3	3		1	

Conclusion:

Overall, I tried to do my best to keep some standard parameters so that testing would yield a more accurate comparison between frameworks and their respective kernels. Some frameworks were easier to work with in that regard, and some were not.

Finding a way to have a more accurate comparison proved challenging, with some frameworks needing a converted file format, while others performed the task at run time. Knowing that the timing has been left up to the program, could mean some results are skewed.

More research would need to be done to figure out exactly which framework was being precise. Further time spent researching the GABPS and Graph Algorithm Iron Law (GAIL)[9], would also be helpful in understanding the benchmarking process.

Also, all hard data and information can be found at my github account: github.com/siegfriedgreg/ Please address any comments or questions there.

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Appendix:

Table 1:

A list of the website links for all of the graphs used in this project. More information can be found there.

Gh002 - https://sparse.tamu.edu/DIMACS10/europe_osm Gh003 - https://sparse.tamu.edu/LAW/sk-2005 Gh004 - https://sparse.tamu.edu/LAW/it-2004 Gh005 - https://sparse.tamu.edu/LAW/uk-2005 Gh006 - https://sparse.tamu.edu/DIMACS10/road_usa Gh007 - https://sparse.tamu.edu/LAW/arabic-2005 Gh008 - https://sparse.tamu.edu/DIMACS10/hugebubbles-00020 Gh009 - https://sparse.tamu.edu/DIMACS10/rgg_n_2_24_s0 Gh010 - https://sparse.tamu.edu/DIMACS10/delaunay_n24

Gh001 - https://sparse.tamu.edu/LAW/webbase-2001

Gh011 - https://sparse.tamu.edu/DIMACS10/hugetrace-00020

Gh012 - https://sparse.tamu.edu/DIMACS10/hugetrace-00010

Gh013 - https://sparse.tamu.edu/Gleich/wb-edu

Gh014 - https://sparse.tamu.edu/LAW/ljournal-2008

Gh015 - https://sparse.tamu.edu/vanHeukelum/cage15

Gh016 - https://sparse.tamu.edu/SNAP/soc-LiveJournal1

Table 2:

All the used maps with their file size listed. The left column, from top to bottom, refers to: Adjacency list, EdgeList, Matrix Market, Binary Matrix Market, Totem file type.

In Bytes.								
	gh001	gh002	gh004	gh005	gh006	gh007	gh008	
Adjacency	20,338,002,852	951,938,529	22,737,538,124	17,583,320,434	494,808,832	11,841,443,753	540,212,173	
Edge List	3,850,992,348	1,897,981,740	19,978,659,994	11,082,967,111	984,970,306	10,893,830,336	538,897,997	
MTX	18,497,838,955	949,001,304	20,241,588,318	16,451,551,195	492,496,838	10,893,833,669	538,907,930	
MTX.BIN	8,631,794,160	636,085,372	9,370,969,884	7,648,753,976	326,623,904	5,210,972,004	339,113,928	
TOTEM	3,848,394,327	1,897,981,785	19,978,660,038	11,082,916,303	984,970,350	10,893,830,077	538,898,020	
MAX	20.338.002.852	1.897.981.785	22 737 538 124	17,583,320,434	984,970,350	11.841.443.753	540,212,173	
AVG	11.033.404.528		18,461,483,272		656,774,046	9,946,781,968	499,206,010	
MIN	3,848,394,327	636,085,372	9,370,969,884	7,648,753,976	326,623,904	5,210,972,004	339,113,928	
IVIIIA								
		,	-,,	.,,	,,	-,,,	,,	
	gh009	gh010	gh011	gh012	gh013	gh014	gh015	gh016
Adjacency						gh014		gh016 3,076,277,487
Adjacency Edge List	gh009	gh010	gh011	gh012	gh013	gh <mark>014</mark> 1,327,137,505	gh015	
	gh009 2,416,696,192	gh010 878,862,062	gh011 404,807,686	gh012 298,943,738	gh013 963,975,564	gh014 1,327,137,505 1,233,480,013	gh015 1,866,329,851	3,076,277,487
Edge List	gh009 2,416,696,192 2,210,494,279	gh010 878,862,062 839,303,889	gh011 404,807,686 398,650,785	gh012 298,943,738 292,150,178	gh013 963,975,564 903,100,945	gh014 1,327,137,505 1,233,480,013	gh015 1,866,329,851 1,475,669,040	3,076,277,487 1,003,771,408
Edge List MTX	gh009 2,416,696,192 2,210,494,279 2,210,504,057	gh010 878,862,062 839,303,889 839,313,191	gh011 404,807,686 398,650,785 398,660,712	gh012 298,943,738 292,150,178 292,160,105	gh013 963,975,564 903,100,945 903,101,884	gh014 1,327,137,505 1,233,480,013 1,233,483,873	gh015 1,866,329,851 1,475,669,040 2,627,728,707 814,215,864	3,076,277,487 1,003,771,408 1,011,609,587
Edge List MTX MTX.BIN TOTEM	gh009 2,416,696,192 2,210,494,279 2,210,504,057 1,127,566,484 2,210,494,243	gh010 878,862,062 839,303,889 839,313,191 469,761,692 839,303,893	gh011 404,807,686 398,650,785 398,660,712 256,000,176 398,650,808	gh012 298,943,738 292,150,178 292,160,105 192,887,216 292,150,201	gh013 963,975,564 903,100,945 903,101,884 496,635,216 902,331,898	gh014 1,327,137,505 1,233,480,013 1,233,483,873 653,638,196 1,233,479,841	gh015 1,866,329,851 1,475,669,040 2,627,728,707 814,215,864 1,475,669,081	3,076,277,487 1,003,771,408 1,011,609,587 571,340,488 1,003,760,104
Edge List MTX MTX.BIN TOTEM	gh009 2,416,696,192 2,210,494,279 2,210,504,057 1,127,566,484 2,210,494,243 2,416,696,192	gh010 878,862,062 839,303,889 839,313,191 469,761,692 839,303,893 878,862,062	gh011 404,807,686 398,650,785 398,660,712 256,000,176 398,650,808 404,807,686	gh012 298,943,738 292,150,178 292,160,105 192,887,216 292,150,201 298,943,738	gh013 963,975,564 903,100,945 903,101,884 496,635,216 902,331,898 963,975,564	gh014 1,327,137,505 1,233,480,013 1,233,483,873 653,638,196 1,233,479,841 1,327,137,505	gh015 1,866,329,851 1,475,669,040 2,627,728,707 814,215,864 1,475,669,081 2,627,728,707	3,076,277,487 1,003,771,408 1,011,609,587 571,340,488 1,003,760,104 3,076,277,487
Edge List MTX MTX.BIN TOTEM	gh009 2,416,696,192 2,210,494,279 2,210,504,057 1,127,566,484 2,210,494,243	gh010 878,862,062 839,303,889 839,313,191 469,761,692 839,303,893	gh011 404,807,686 398,650,785 398,660,712 256,000,176 398,650,808	gh012 298,943,738 292,150,178 292,160,105 192,887,216 292,150,201	gh013 963,975,564 903,100,945 903,101,884 496,635,216 902,331,898	gh014 1,327,137,505 1,233,480,013 1,233,483,873 653,638,196 1,233,479,841	gh015 1,866,329,851 1,475,669,040 2,627,728,707 814,215,864 1,475,669,081 2,627,728,707 1,651,922,509	3,076,277,487 1,003,771,408 1,011,609,587 571,340,488 1,003,760,104

Table 3:

A list of the vertice and edge count.

	gh001	gh002	gh004	gh005	gh006
Vertices	118,142,155	50,912,018	41,291,594	39,459,925	23,947,347
Edges	1,019,903,190	54,054,660	1,150,725,436	936,364,282	28,854,312
	gh007	gh008	gh009	gh010	gh011
Vertices	22,744,080	21,198,119	16,777,216	16,777,216	16,002,413
Edges	639,999,458	31,790,179	132,557,200	50,331,601	23,998,813
	gh012	gh013	gh014	gh015	gh016
Vertices	12,057,441	9,845,725	5,363,260	5,154,859	4,847,571
Edges	18,082,179	57,156,537	79,023,142	99,199,551	68,993,773