Final exam Fundamentals of Photonics

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Part I
Closed book questions

1. (5 points) For the first three questions select the correct answer from multiple choices and for the last two questions fill the correct answer in the blank area.

   (a) (1 point) The phase difference (φ) and path difference (ΔL) are related by $\varphi =$

   Solution
   $\frac{2\pi}{\lambda}\Delta L$. Phase difference is a quantity with degrees or radians as units, so we can drop option b and d. Since a full wavelength is 2π, we need answer a.

   (b) (1 point) A phase difference π between two interfering beams is equivalent to a path difference

   Solution
   $\lambda/2$. Plug π in the equation of subquestion a and solve for the path difference.

   (c) (1 point) When a light wave incident perpendicular to a surface is reflected at the surface of an optically denser medium, then the change in the phase difference is

   Solution
   π.

   (d) (1 point) If D is the distance to the screen in Young's double slit experiment, then fringe width β decreases with ... in D.

   Solution
   decrease

   (e) (1 point) If two waves maintain constant phase difference or equal phase at any two points on a wavefront is known as

*This exam was probably meant as the resit for 2019, but wasn't used. It was used as the 2020 final exam instead.
†Altough these solution were written carefully, I could've made mistakes. If you find one, please let me know.

> **Solution**
> spatial coherence.

2. (5 points) What is an optical mode in a waveguide/fiber? Sketch $m = 0$, $m = 1$, and $m = 2$ modes in a symmetric slab waveguide.

> **Solution**
> An optical mode in a waveguide is the field pattern of propagating waves. In the next picture, the first three modes are illustrated. Optical modes can have it's electric field or magnetic field entirely transverse. These modes are then called TE or TM modes respectively.

3. (5 points) One of the methods to couple input light into an optical waveguide is prism coupling where you place a glass prism on top the optical waveguide. Explain why a prism is needed in this configuration?

> **Solution**
> In an optical waveguide, we need light to undergo total internal reflection to make it propagate to the other side of the guide. If the light comes in at γ_m (fixed) and has to go out at γ_m, then we need the prisms to assure the light is coming in the waveguide at such an angle that the internal reflections occur at the critical angle to prevent loss of the light. At the end, a prism is needed to get the light out of the waveguide and turn the evanescent wave of the total internal reflection into a propagating wave at angle γ_m.
>
> A prism also makes alignment of the beam and film easier without having to deal with very small precision. With a prism, the numerical aperture also doesn't have to match between the beam and the film. Using a prism coupler, the coupled beam can be much wider than the thickness of the waveguide.

4. (5 points) What is index of refraction? How does it change with the wavelength of light? Assume normal dispersion regime.

> **Solution**
> Refractive index n describes how fast light travels through a medium as it's relation is $n = \frac{c}{v}$, where v is the measured speed. It effectively changes when the wavelength changes: $n_{effective} = \frac{2\pi}{\lambda} n$. If λ gets bigger, the effective refractive index gets smaller.

5. (5 points) In an optical microscope, if we change the objective lens to an objective lens with a higher numerical aperture, how does the beam waist and depth of focus change?

> **Solution**
> An optical microscope makes use of guassian beam. Here's an illustration of such a beam. b is the depth of focus and is related to the numerical aperture NA by $b = 2n\omega_0/\text{NA} = 2Z_R$. So with a higher NA, the depth of focus gets smaller. Also, ω_0 is considered the beam waist and is roughly related to NA by $\omega_0 \propto \frac{1}{\text{NA}}$, which results in a smaller beam waist if the numerical aperature gets higher.

6. (5 points) A beam of light with intensity I0 passes through two linear polarizers as shown above. The intensity of the final beam is $1/8I_0$. Write down one possible polarization state of the input light that can result in this power output.

> **Solution**
>
> For this questoin we need Malus' law:
>
> $$I_1 = I_0 \cos^2 \theta_i.$$
>
> Solve for θ:
>
> $$\begin{aligned} \frac{1}{2} I_0 &= I_0 \cos^2 \theta \\ \theta &= \arccos \frac{1}{\sqrt{2}} \\ &= 45°. \end{aligned}$$
>
> So we need linearly polarized light at angle 45°.

Part II

Open book questions

1. (10 points) Above you see a liquid level sensor, which is comprised of two fibers and a prism. Explain the working principle of this system.

> **Solution**
>
> In the left picture, we see that the light reflects and transmits at a glass-air interface, for example. Some of the light reflects to the right and then up and some of the light transmits into the air. The detector measures the intensity.
>
> Now, in the right image, the prism is submerged in the liquid, meaning other portions of the light will reflect and transmit according to the Fresnel equations. The detector will measure a lower intensity as the light will leak out more at the prism interfaces when put in liquid than in air. If the differenc is significant, the user will be notified.

2. (15 points) An unpolarized light with an intensity of I_0 is incident on two linear polarizers (45° and 60°) and a quarter-wave plate (QWP).

   (a) What is I_1 in terms of I_0? Explain your answer.

   > **Solution**
   >
   > Unpolarized light contains polarized light in all possible directions. When using Malus'law to calculate $I_1 = I_0 \cos^2 \theta$, we need to know the average of $\cos^2 \theta$, because we want to know what happens to 'each polarized part in the unpolarized light'. The average of $\cos^2 \theta = \frac{1}{2}$, so $I_1 = \frac{1}{2} I_0$.

   (b) What is the polarization state of the light after the QWP?

Solution
Quarter wave plates make circular polarized light of polarized light. So circularly polarized.

(c) What is the intensity of light I_2 after the QWP?

Solution
Waveplates don't alter the intensity of incoming light. So $I_2 = I_1 = \frac{1}{2} I_0$

(d) Assume that $E_0 = E_0 \sin(kz - \omega t)$ Write down E_x and E_y in terms of E_1 after the QWP.

Solution
The x part of the wave gets retarded and thus a little extra phase of 90°. We then have

$$\begin{aligned} E_y &= E_0 \cos\left(kz - \omega t\right) \\ E_x &= E_0 \cos\left(kz - \omega t + \frac{\pi}{2}\right), \end{aligned}$$

assuming we need to write E in terms of E_0 instead of E_1. If not, then

$$\begin{aligned} E_y &= E_1 \\ E_x &= E1 \frac{\sin\left(kz - \omega t + \frac{\pi}{2}\right)}{\sin\left(kz - \omega t\right)} \\ &= E_1 \cot\left(kz - \omega t\right). \end{aligned}$$

(e) What is the polarization state of the light after the 60° polarizer?

Solution
All the light other than at 60° will be blocked, resulting in linearly polarized light at 60°.

(f) If we replace the 60° polarizer with another QWP with the fast axis oriented along the x direction and the slow axis along the y direction, what will be the polarization of the light after the second QWP?

Solution
The E_y part will be retarded by $+\frac{\pi}{2}$ phase making the phase difference between the y and x part zero, polarizing it again to 45°. The circular polarization gets undone.

(g) What will be the intensity of the light after the second QWP in terms of I_1?

Solution
QWPs don't alter the intensity, thus the intensity will be $I_3 = I_2 = I_1 = \frac{1}{2} I_0$.

3. (10 points) Below two electromagnetic waves with same frequency and different field amplitudes are given.

$$\begin{aligned} E_1 &= E_{10} \sin(\omega t) \\ E_2 &= E_{20} \sin(\omega t + \varphi) \end{aligned}$$

If these two waves superimpose ate a point P, show that the intensity at P is $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\varphi)$.

Solution

First, we change the notation to it's complex form.

$$
\begin{aligned}
E_1 &= E_{10}\sin(\omega t) = E_{10} \cdot \Im\left(e^{i\omega t}\right) \\
E_2 &= E_{20}\sin(\omega t + \varphi) = E_{20} \cdot \Im\left(e^{i\omega t + \varphi}\right)
\end{aligned}
$$

When we calculate the intensity, we need to add the fields up before squaring.

$$
\begin{aligned}
I &= (E_1 + E_2)^2 \\
&= \left[E_{10}e^{i\omega t} + E_{20}e^{(\omega t + \varphi)}\right]^2 \\
&= \left[e^{i\omega t}\right]^2 \left[E_{10} + E_{20}e^{i\varphi}\right]^2 \\
&= E_{10}^2 + E_{20}^2 + 2E_{10}E_{20}e^{i\varphi} \\
&= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi
\end{aligned}
$$

We take the real part of the complex term in the last step and take $I_i = E_{i0}^2$. Note that squaring a complex number means multiplying by it's complex conjugate, e.g. $\left|e^{i\varphi}\right|^2 = e^{i\varphi}e^{-i\varphi} = 1$.

4. (10 points) Waveguide coupler

(a) (3 points) Explain the general operating principle of the waveguide coupler shown above.

Solution

A waveguide coupler brings two waveguides close to eachother. Because of this, the evanescent field from one light out of the other waveguide gets into the other waveguide and can propagate further until it hops to the other waveguide again. This hopping of the light occurs until the end of the coupler. Because the two waveguides are close to each other, we say they are coupled. If the two waveguides were single mode, then the collection is a two-mode structure and there's an antisymmetric and symmetric mode.

(b) (2 points) Explain how the waveguide coupler can be used to split the incoming signal in two equal output signals A and B.

Solution

Because of the hopping nature of the structure, the light can be partly in one of the waveguides. If this part of the light is equal to the other part in the other waveguide, then we can stop the coupling there and the output signals at A and B are equal.

(c) (2 points) In order to make the coupling ratio variable, what kind of a mechanism can be used?

Solution

It is possible to make the coupling ration variable if we make use of electro-optic material in the place of the coupler. When an electric field is applied to this material, the optical path changes, which results in a different distribution of light at the output.

(d) (3 points) If we increase the width of waveguide B, how would this change the device performance?

Solution

This makes the waveguide asymmetric. The light will never fully escape waveguide B and if the increase in width of B is big enough, the power in A will never be higher than B. In that case, it's also not possible to have the two outputs be exactly the same. Also, if B is wide enough, then it won't be single mode, so there will be more scattering in the core which is not desirable.

5. (10 points) White light in air shines on an oil film that floats on water with an incident angle of 90° as shown below. When looking straight down at the film, the reflected light is red, with a wavelength of 636 nm. What is the minimum possible thickness of the film?

Solution

White light contains a continious spectrum of wavelengths, but if we only want to have constructive interference after reflection, we have to meet the constructive interference condition for $\lambda = 636\,\text{nm}$. The air and the water both have lower refractive indices than oil, so at the first reflection there will be a π phase shift. At the second there won't be one. We got

$$2n_{oil}t_m = \left(m + \frac{1}{2}\right)\lambda$$

$$t_m = \frac{1}{2n_{oil}}\left(m + \frac{1}{2}\right)\lambda.$$

The minimum possible thickness is then

$$\begin{aligned} t_0 &= \frac{1}{2}\frac{1}{2}\frac{636}{1.5}\text{nm} \\ &= 106\,\text{nm}. \end{aligned}$$

6. (15 points) Sketch a time-domain optical coherence tomography (OCT) system based on the next interferometers. On your schematic write down each components' name explicitly and mention their function.

(a) Mach Zehnder interferometer (MZI)

Solution

If there's no sample then the two beams will constructively interfere at detector 1 and destructively interfere at detector 2. If we put a sample inside the left beam, this sample beam will be slightly phaseshifted and thus there will be different interference patterns at the detector. From this, it's possible to deduce the phase shift caused by the sample.

(b) Michelson interferometer (MI)

Solution

Here we have a low coherent light source going through a collimating lens so we can bounce the light useful off mirrors. The beam gets split to the sample and a reference arm. The beam to the reference arm can be rotated so different points of the sample get imaged. The reference mirror will move to see how the interference at the detector is changed.

(c) What is the advantage of using an MZI compared to MI? Indicate the additional components that are needed to configure an MZI based OCT.

Solution

In the MZI each path is only taken once, which doesnt allow particles going in opposite directions interact strangely with each other as in the MI. For MZI we need one detector more and one more beam splitter. When using the MI, part of the light goes back to the light source which can't be used.

(d) If one needs to build an ultra-high resolution OCT system (axial resolution $\sim 1\,\mu\mathrm{m}$), what kind of a light source will be needed? What is the biggest challenge of an ultra-high OCT system?

Solution

The axial resolution is

$$l_c \approx 0.44 \frac{\lambda_0^2}{\Delta\lambda},$$

so the higher the bandwidth, the shorter the coherence length resulting in a higher resolution. Maximizing this bandwidth is the biggest challenge nex to lowering λ_0^2 as much as possible. When λ_0 gets very small, very much energy is required.