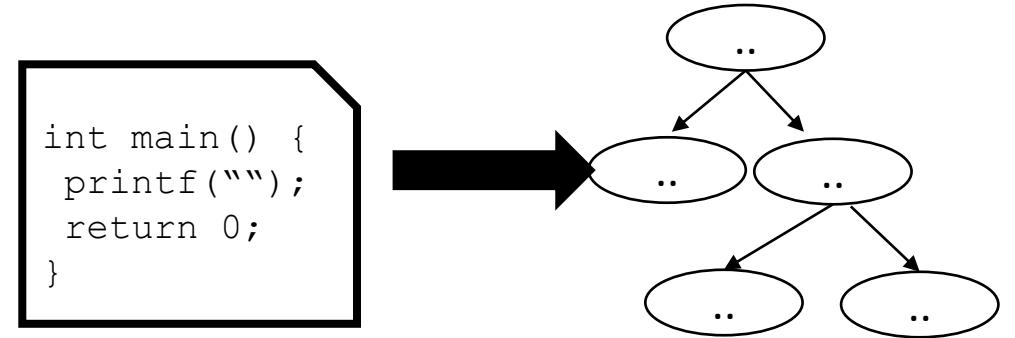


CSE110A: Compilers

Topics:

- *Syntactic Analysis continued*
 - *Derivations*
 - *Parse trees*
 - *Precedence and associativity*



Ambiguous grammars

- What happens when different derivations have different parse trees?

```
1: Statement ::= "if" Expr "then" Statement "else" Statement
2:           |   "if" Expr "then" Statement
3:           |   Assignment
4:           |   ....
```

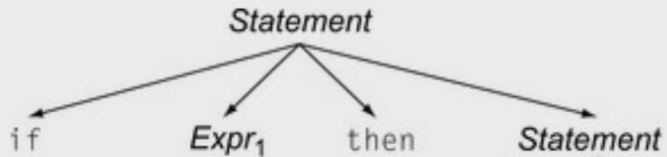
can we derive this string?

`if Expr1 then if Expr2 then Assignment1 else Assignment2`

Ambiguous grammars

```
1: Statement ::= "if" Expr "then" Statement "else" Statement
2:           |   "if" Expr "then" Statement
3:           |   Assignment
4:           |   ....
```

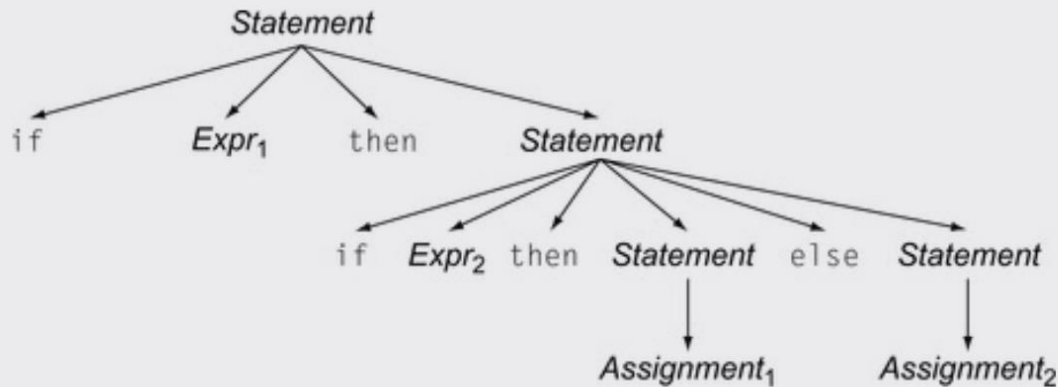
`if Expr1 then if Expr2 then Assignment1 else Assignment2`



Ambiguous grammars

```
1: Statement ::= "if" Expr "then" Statement "else" Statement
2:           |   "if" Expr "then" Statement
3:           |   Assignment
4:           |   ....
```

`if Expr1 then if Expr2 then Assignment1 else Assignment2`

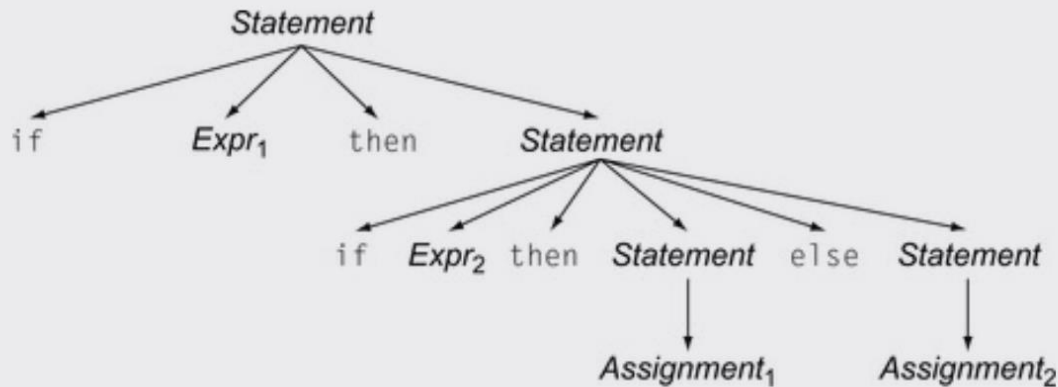


Valid derivation

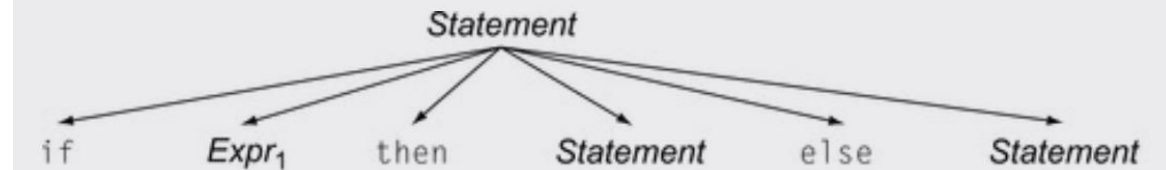
Ambiguous grammars

```
1: Statement ::= "if" Expr "then" Statement "else" Statement
2:           |   "if" Expr "then" Statement
3:           |   Assignment
4:           |   ....
```

`if Expr1 then if Expr2 then Assignment1 else Assignment2`



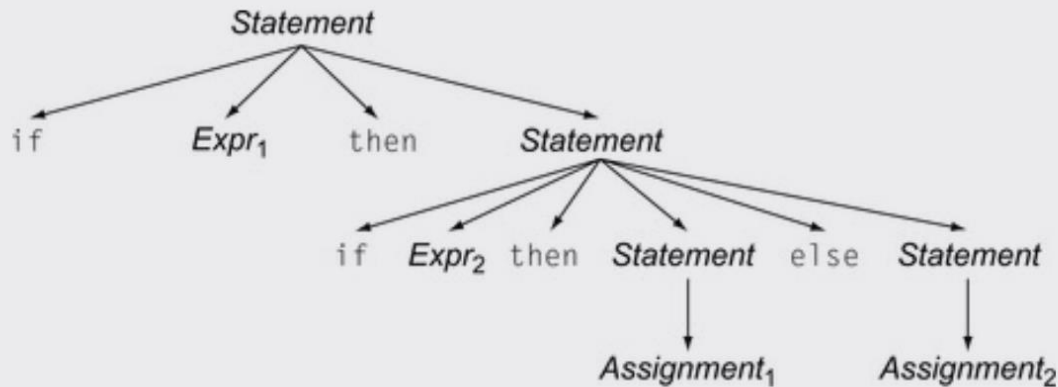
Valid derivation



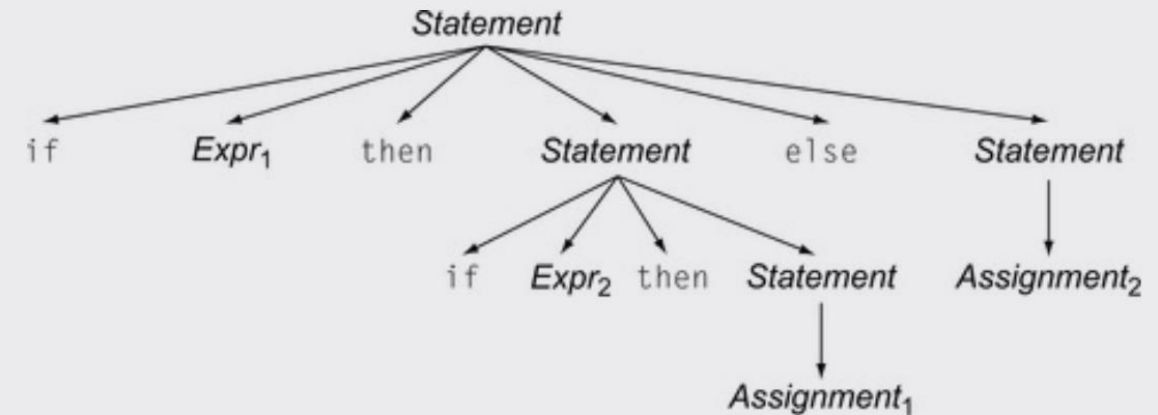
Ambiguous grammars

1: Statement ::= "if" Expr "then" Statement "else" Statement
2: | "if" Expr "then" Statement
3: | Assignment
4: |

`if Expr1 then if Expr2 then Assignment1 else Assignment2`



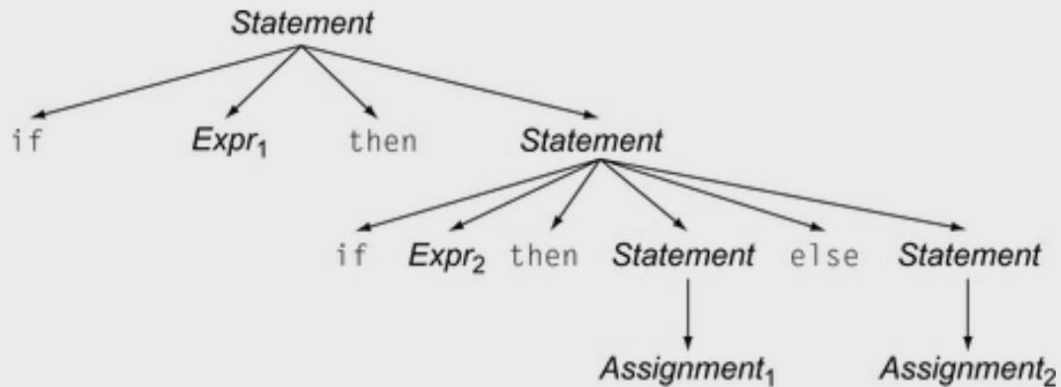
Valid derivation



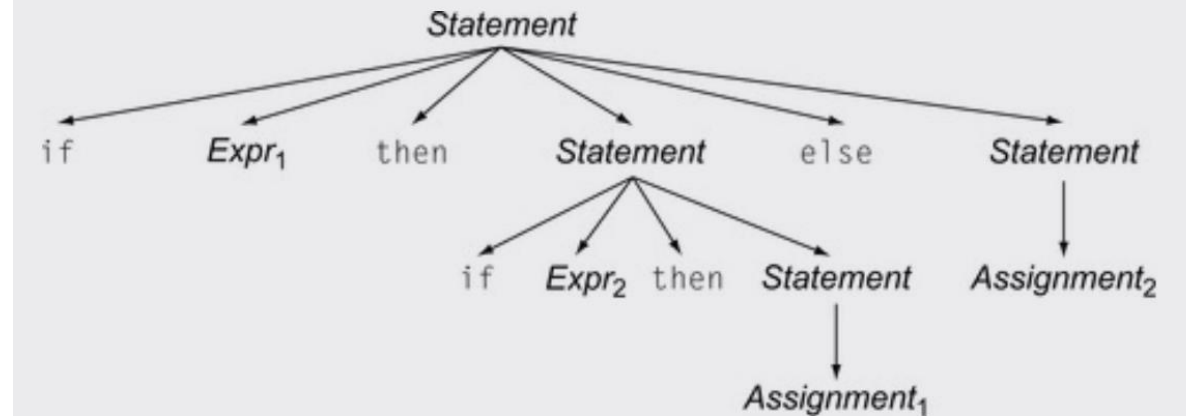
And another valid derivation

Ambiguous grammars

Is this an issue? Don't we only care if a grammar can derive a string?



Valid derivation

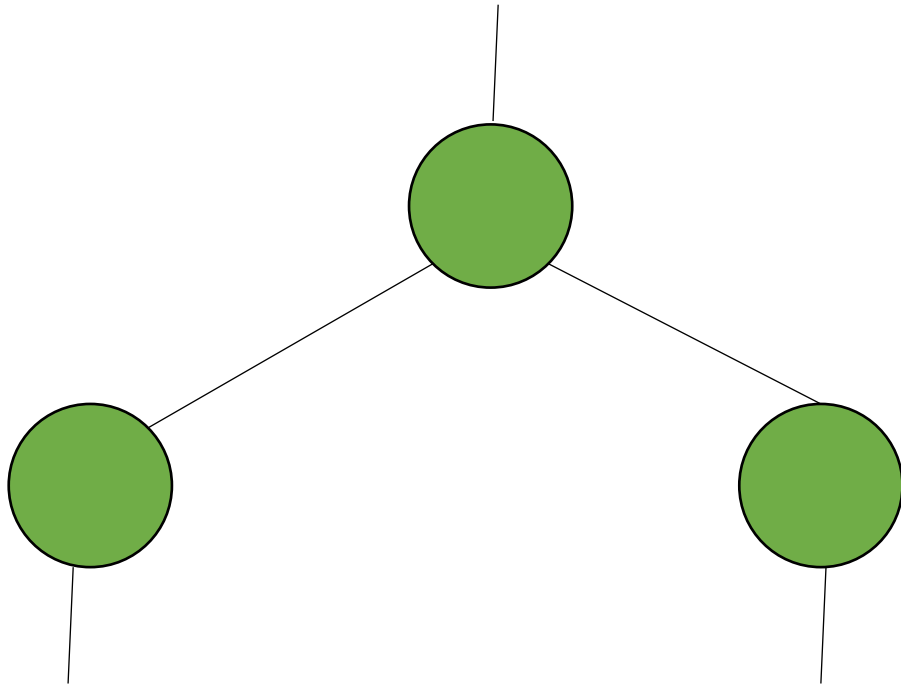


And another valid derivation

Meaning into structure

- We want to start encoding meaning into the parse structure. We will want as much structure as possible as we continue through the compiler
- The structure is that we want evaluation of program to correspond to a post order traversal of the parse tree (also called the natural traversal)

Post order traversal



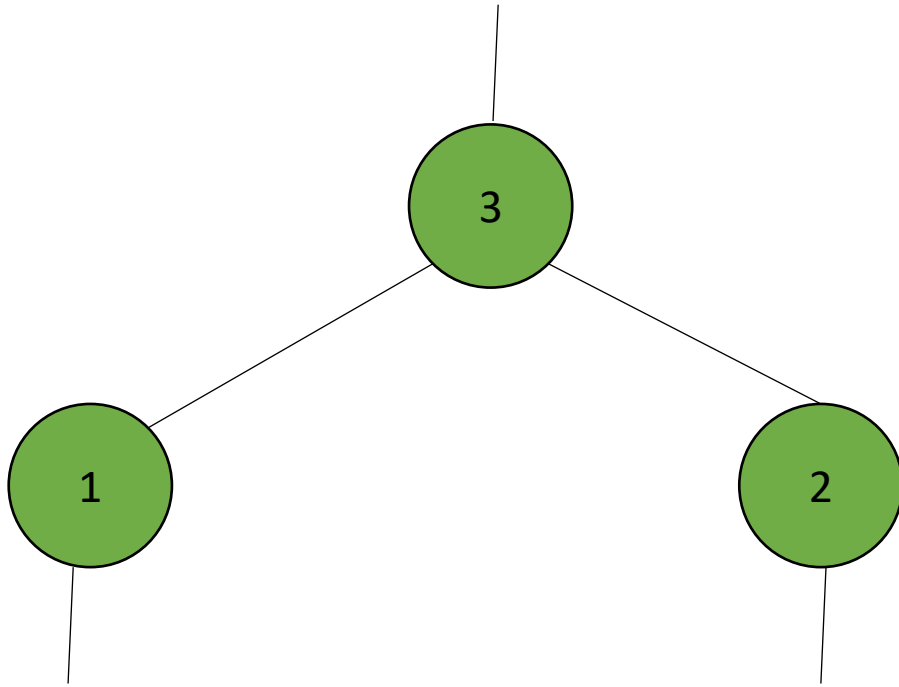
visiting for for different types
of traversals:

pre order?

in order?

post order?

Post order traversal

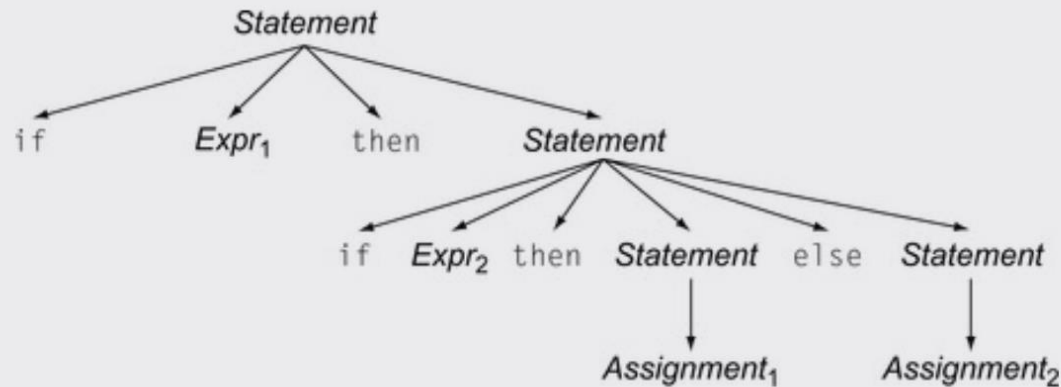


visiting for for different types
of traversals:

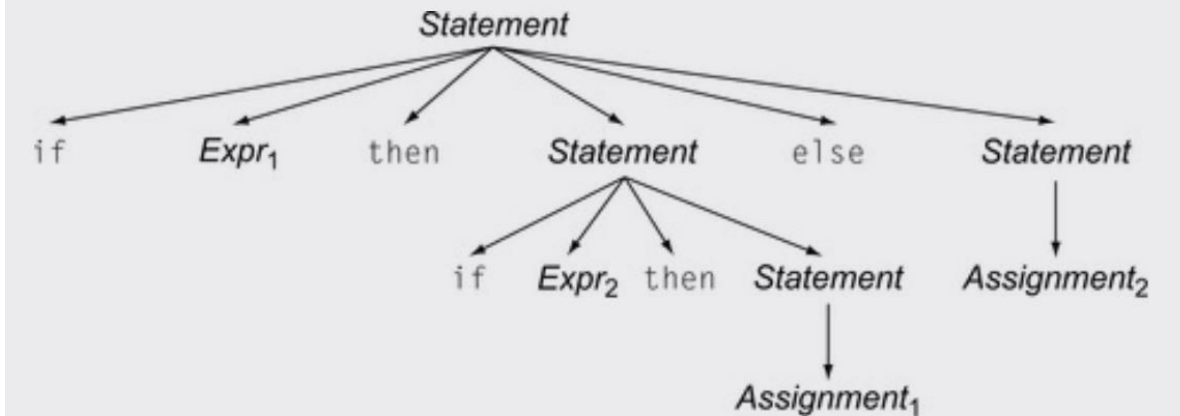
post order

Ambiguous grammars

Encoding meaning into structure can result in very different programs



Valid derivation



Also a valid derivation

Programming language structure

```
int x = 1; //true  
int y = 0; //false  
int check0 = 0;
```

```
if (x)  
if (y)  
pass();  
else  
check0 = 1;
```

pop quiz: what is the value of check0 at the end?

Programming language structure

```
x = 1  
y = 0  
check0 = 0
```

```
if (x):  
    if (y):  
        pass  
    else:  
        check0 = 1
```

```
print(check0)
```

How does Python handle this?

Programming language structure

```
x = 1  
y = 0  
check0 = 0
```

```
if (x):  
if (y):  
pass  
else:  
check0 = 1
```

```
print(check0)
```

How does Python handle this?

```
x = 1  
y = 0  
check0 = 0
```

```
if (x):  
    if (y):  
        pass  
    else:  
        check0 = 1
```

```
print(check0)
```

Invalid syntax, you need to indent, which makes it clear

Ambiguous expressions

- First let us define some tokens:

- NUM = "[0-9]+"
- PLUS = '\+'
- TIMES = '*'
- LP = '\('
- RP = '\)'

- And then define a simple expression language

```
Expr ::= NUM  
      | Expr PLUS Expr  
      | Expr TIMES Expr  
      | LP Expr RP
```

Parse trees examples

input: 5

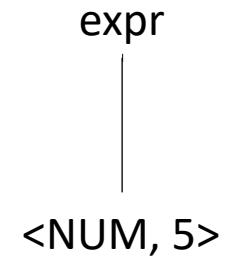
```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

expr
|

Parse trees examples

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

input: 5



Parse trees examples

input: 5*6

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

Parse trees examples

input: 5*6

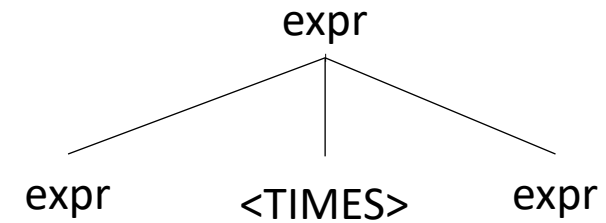
```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

expr

Parse trees examples

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

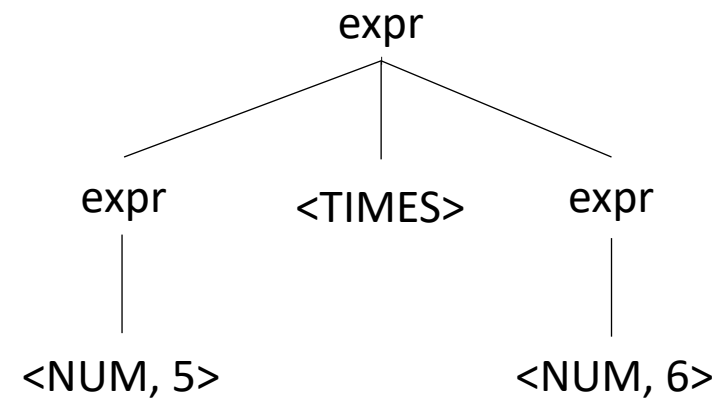
input: 5*6



Parse trees examples

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

input: 5*6



Parse trees examples

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

input: 5**6

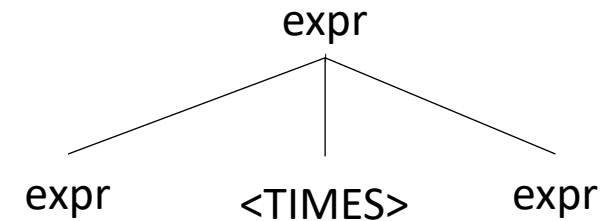
expr
|

What happens
in an error?

Parse trees examples

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

input: 5**6



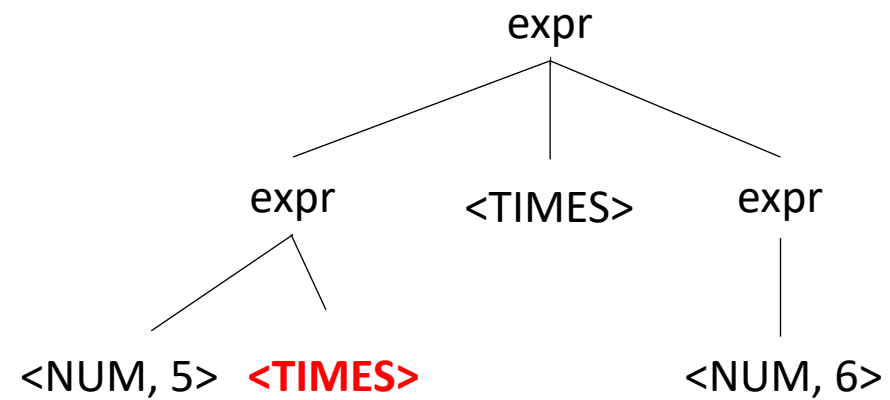
What happens
in an error?

Parse trees examples

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

input: 5**6

What happens
in an error?



Not possible!

Parse trees examples

input: (1+5)*6

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

Parse trees examples

input: (1+5)*6

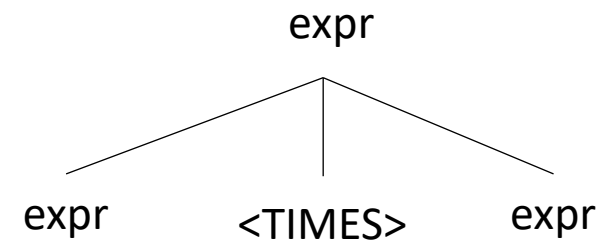
```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

expr

Parse trees examples

input: (1+5)*6

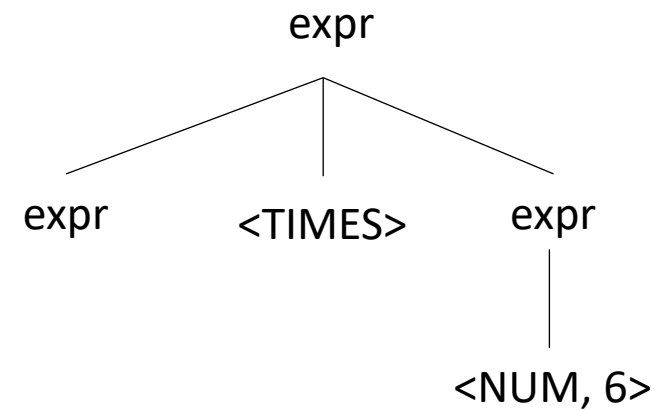
```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```



Parse trees examples

input: (1+5)*6

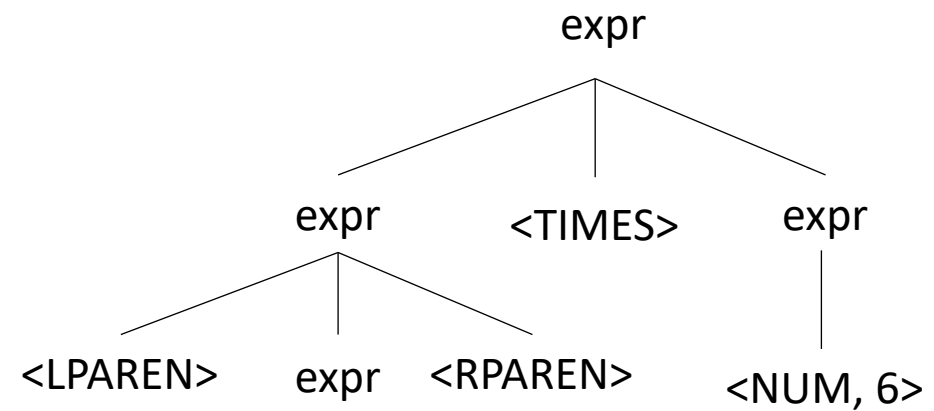
```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```



Parse trees examples

input: (1+5)*6

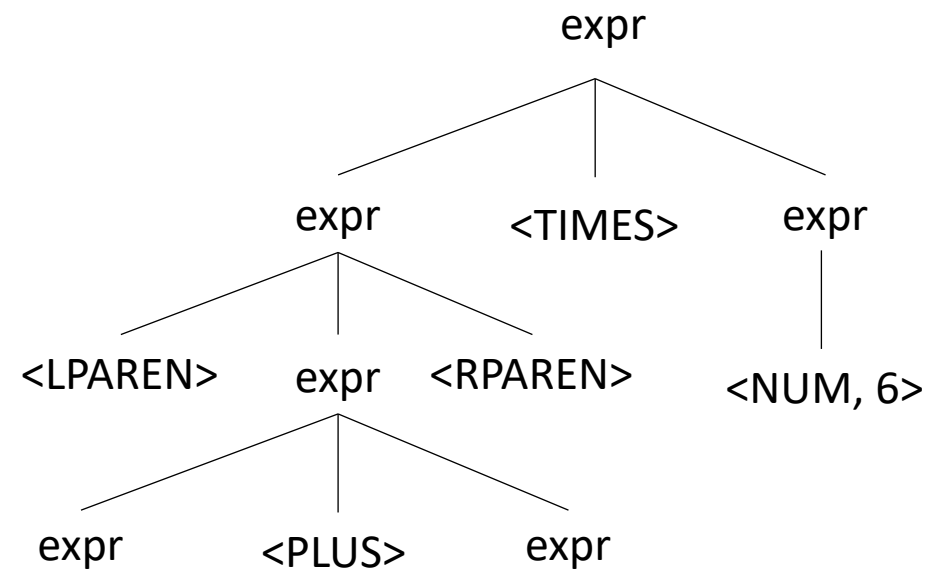
```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```



Parse trees examples

input: (1+5)*6

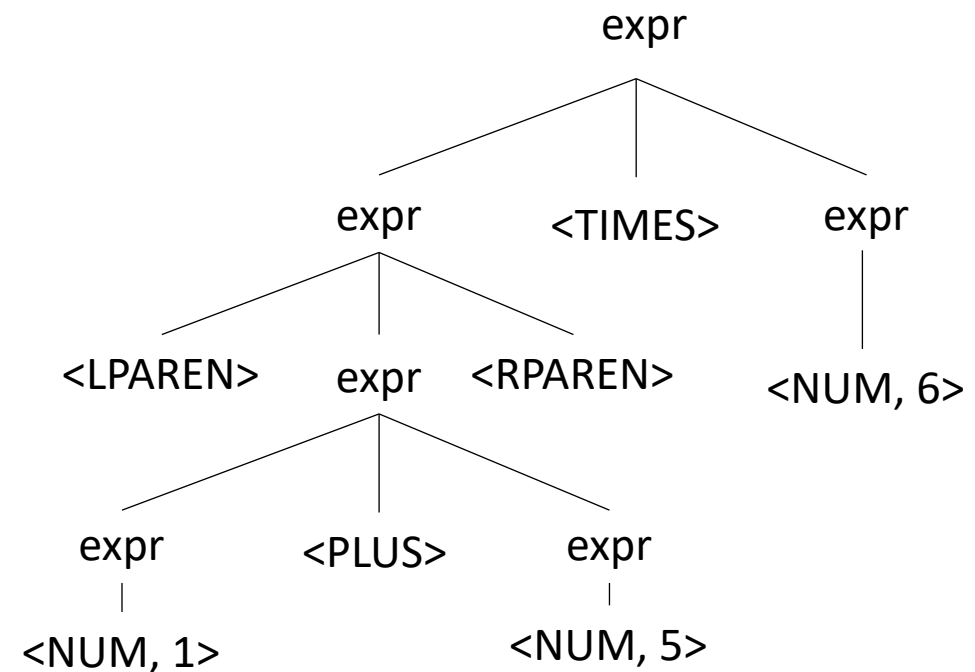
```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```



Parse trees examples

input: (1+5)*6

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

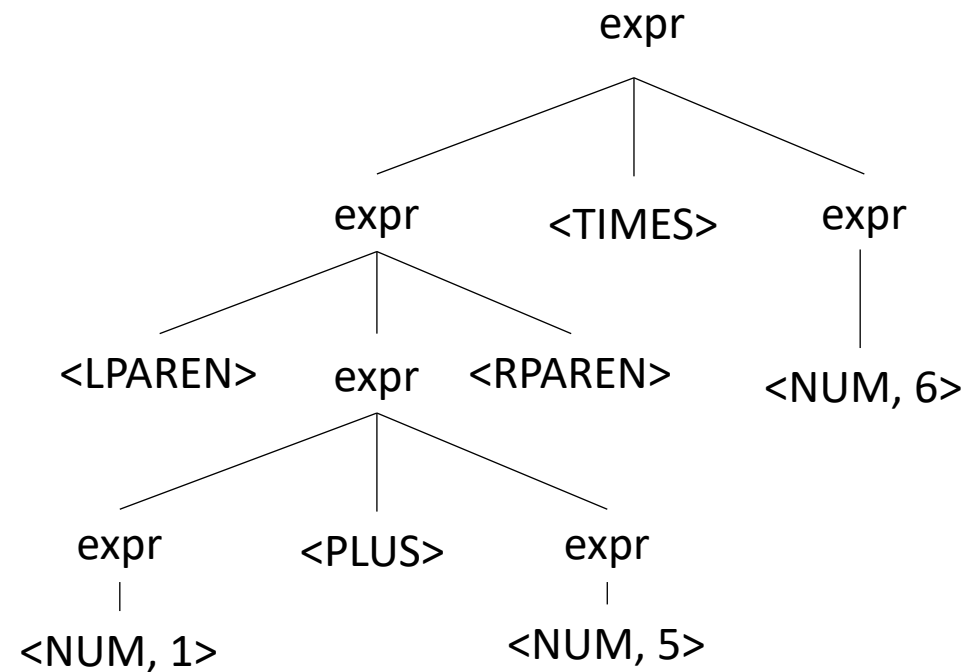


Parse trees examples

Does this parse tree capture the structure we want?

input: (1+5)*6

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```



Parse trees

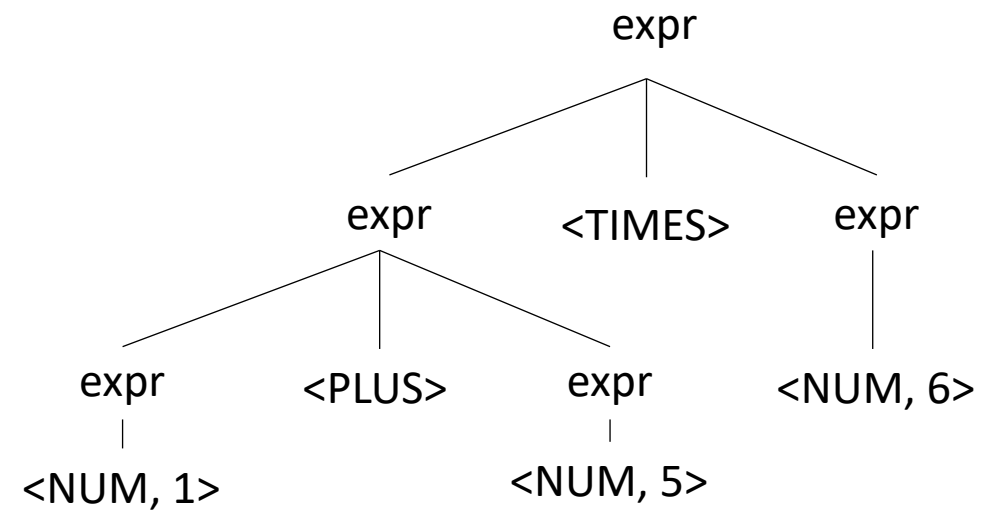
- How about: 1 + 5 * 6

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

Parse trees

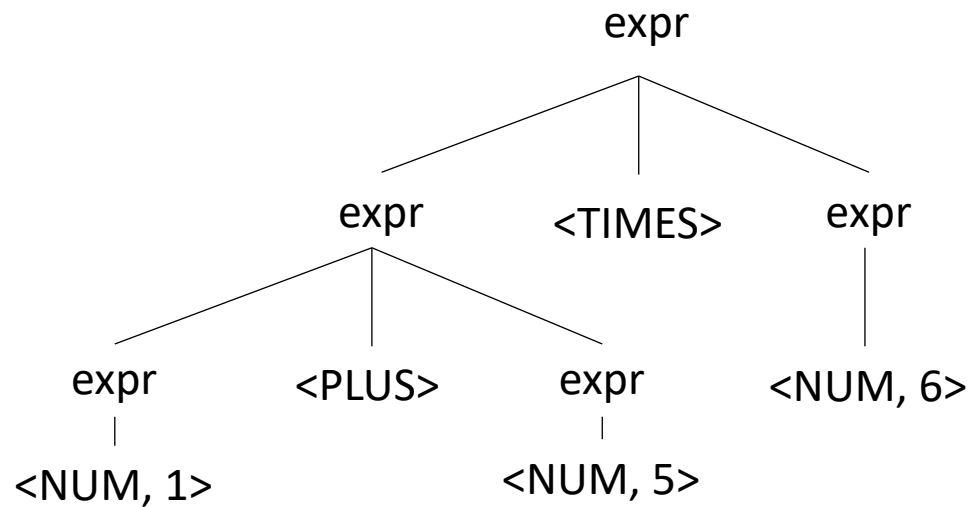
- How about: 1 + 5 * 6

```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```



Ambiguous Grammars

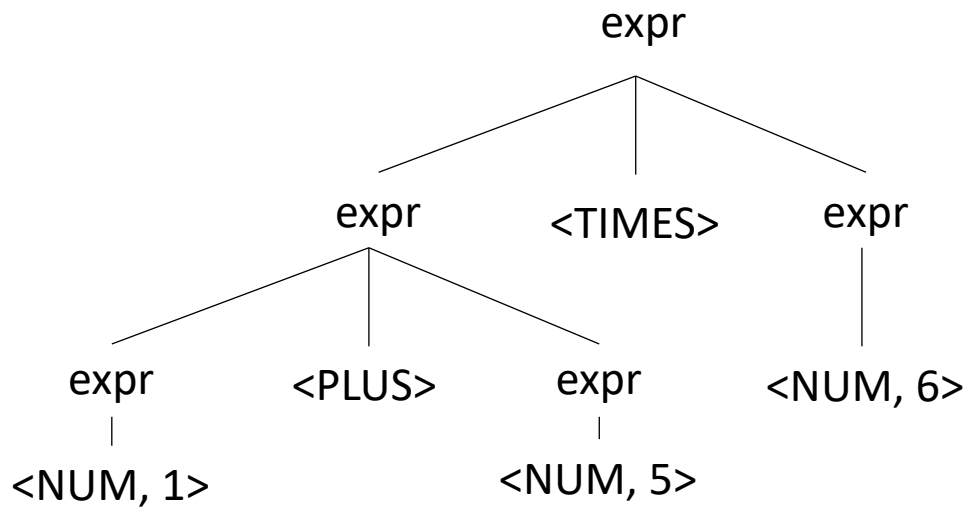
- input: 1 + 5 * 6



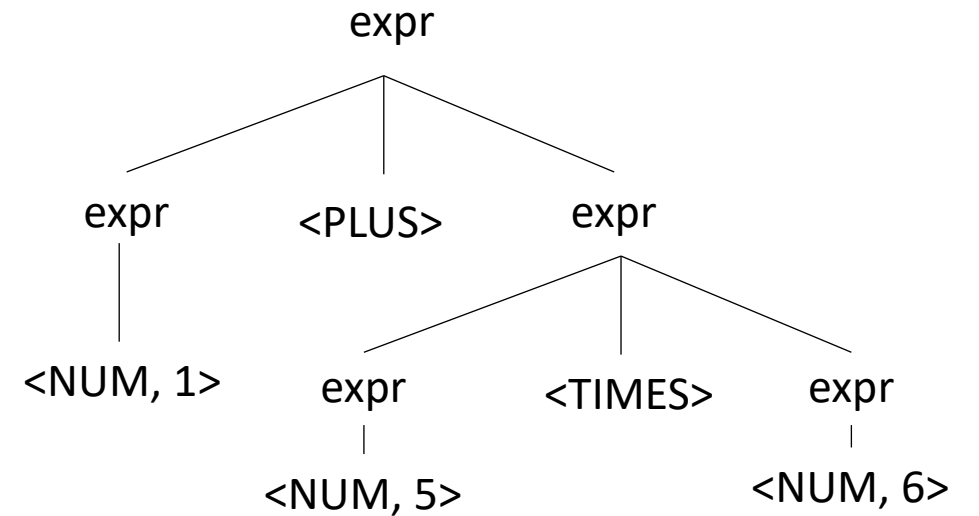
```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```

Ambiguous Grammars

- input: 1 + 5 * 6



```
expr ::= NUM
      | expr PLUS expr
      | expr TIMES expr
      | LPAREN expr RPAREN
```



Avoiding Ambiguity

- How to avoid ambiguity related to precedence?
- Define precedence into the grammar:
 - Ambiguity comes from conflicts. Explicitly define how to deal with conflicts by indicating that:
 - * has higher precedence than +
- Some parser generators support this, e.g. YACC/Bison

Avoiding Ambiguity

- How to avoid ambiguity related to precedence?
- **Second way:** use new production rules
 - One non-terminal for each level of precedence
 - lowest precedence at the top
 - highest precedence at the bottom
- Lets try with expressions and the following:
+ * ()

Avoiding Ambiguity

The second way: new production rules

- One non-terminal for each level of precedence
- lowest precedence at the top
- highest precedence at the bottom

| Operator | Name | Productions |
|----------|--------|---------------------------------|
| + | expr | ::= expr PLUS expr term |
| * | term | ::= term TIMES term factor |
| () | factor | ::= LPAREN expr RPAREN NUM |

Precedence
increases going down



Now let's create a parse tree

input: $1+5*6$

| Operator | Name | Productions |
|----------|--------|---|
| + | expr | $::= \text{expr PLUS expr}$ term |
| * | term | $::= \text{term TIMES term}$ factor |
| () | factor | $::= \text{LPAREN expr RPAREN}$ NUM |

Now let's create a parse tree

input: $1+5*6$

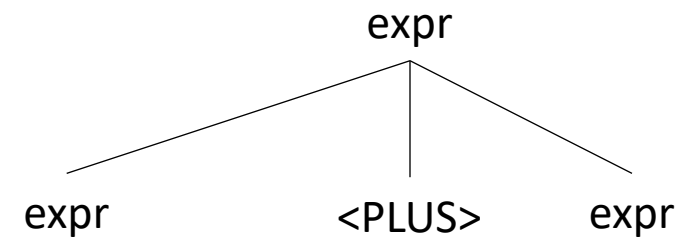
expr

| Operator | Name | Productions |
|----------|--------|---|
| + | expr | $::= \text{expr PLUS expr}$ term |
| * | term | $::= \text{term TIMES term}$ factor |
| () | factor | $::= \text{LPAREN expr RPAREN}$ NUM |

Now let's create a parse tree

input: 1+5*6

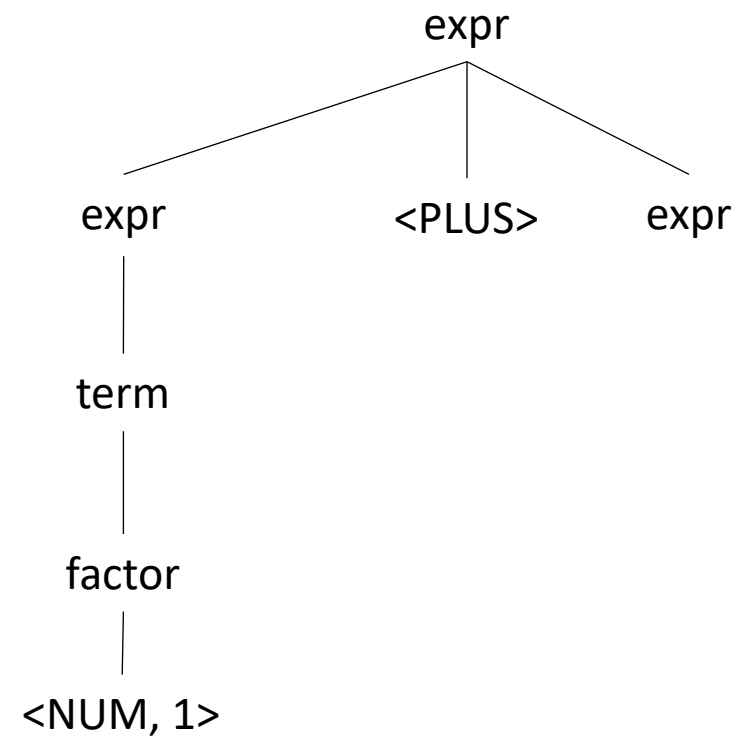
| Operator | Name | Productions |
|----------|--------|---------------------------------|
| + | expr | ::= expr PLUS expr term |
| * | term | ::= term TIMES term factor |
| () | factor | ::= LPAREN expr RPAREN NUM |



Now let's create a parse tree

input: 1+5*6

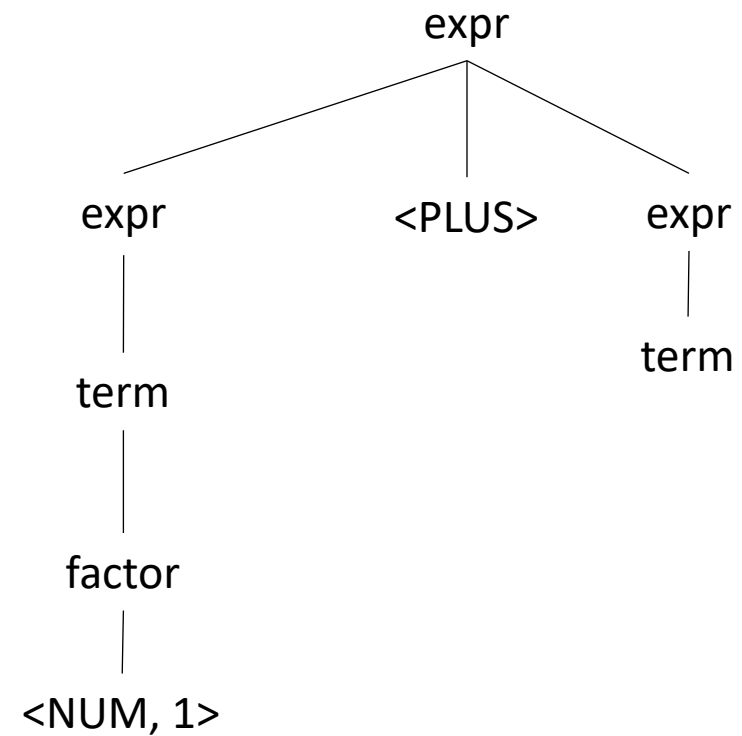
| Operator | Name | Productions |
|----------|--------|---------------------------------|
| + | expr | ::= expr PLUS expr term |
| * | term | ::= term TIMES term factor |
| () | factor | ::= LPAREN expr RPAREN NUM |



Now let's create a parse tree

input: 1+5*6

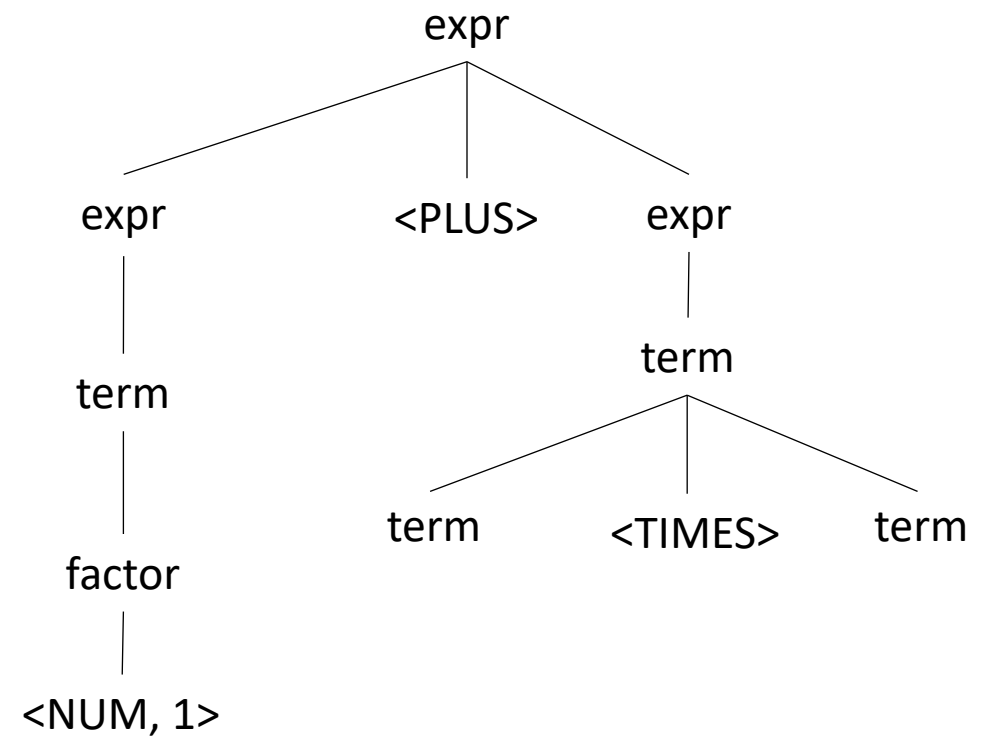
| Operator | Name | Productions |
|----------|--------|---------------------------------|
| + | expr | ::= expr PLUS expr term |
| * | term | ::= term TIMES term factor |
| () | factor | ::= LPAREN expr RPAREN NUM |



Now let's create a parse tree

input: 1+5*6

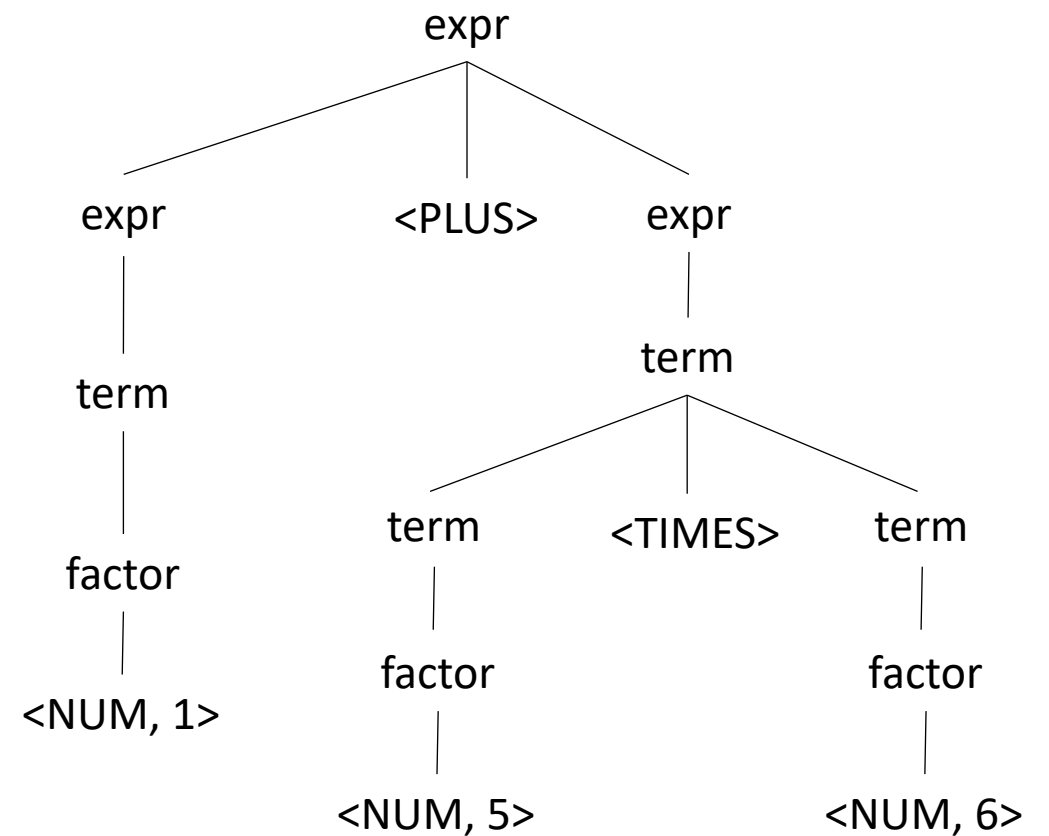
| Operator | Name | Productions |
|----------|--------|---------------------------------|
| + | expr | ::= expr PLUS expr term |
| * | term | ::= term TIMES term factor |
| () | factor | ::= LPAREN expr RPAREN NUM |



Now let's create a parse tree

input: 1+5*6

| Operator | Name | Productions |
|----------|--------|---------------------------------|
| + | expr | ::= expr PLUS expr term |
| * | term | ::= term TIMES term factor |
| () | factor | ::= LPAREN expr RPAREN NUM |



Parsing REs

Let's try it for regular expressions, $\{| \cdot * ()\}$

- *Assume \cdot is concat*

| Operator | Name | Productions |
|----------|--------|------------------------------|
| | choice | choice PIPE choice concat |
| . | concat | concat DOT concat star |
| * | star | star STAR unit |
| () | unit | LPAR choice RPAR CHAR |

Parsing REs

Let's try it for regular expressions, $\{| \cdot * ()\}$

- *Assume \cdot is concat*

| Operator | Name | Productions |
|----------|---------|--|
| | choice | $::= \text{choice PIPE choice}$ concat |
| . | concat | $::= \text{concat DOT concat}$ starred |
| * | starred | $::= \text{starred STAR}$ unit |
| () | unit | $::= \text{LPAREN choice RPAREN}$ CHAR |

Parsing REs

Let's try it for regular expressions, $\{| \cdot * ()\}$

- *Assume \cdot is concat*

input: $a.b \mid c^*$

| Operator | Name | Productions |
|----------|---------|------------------------------------|
| | choice | ::= choice PIPE choice concat |
| . | concat | ::= concat DOT concat starred |
| * | starred | ::= starred STAR unit |
| () | unit | ::= LPAREN choice RPAREN CHAR |

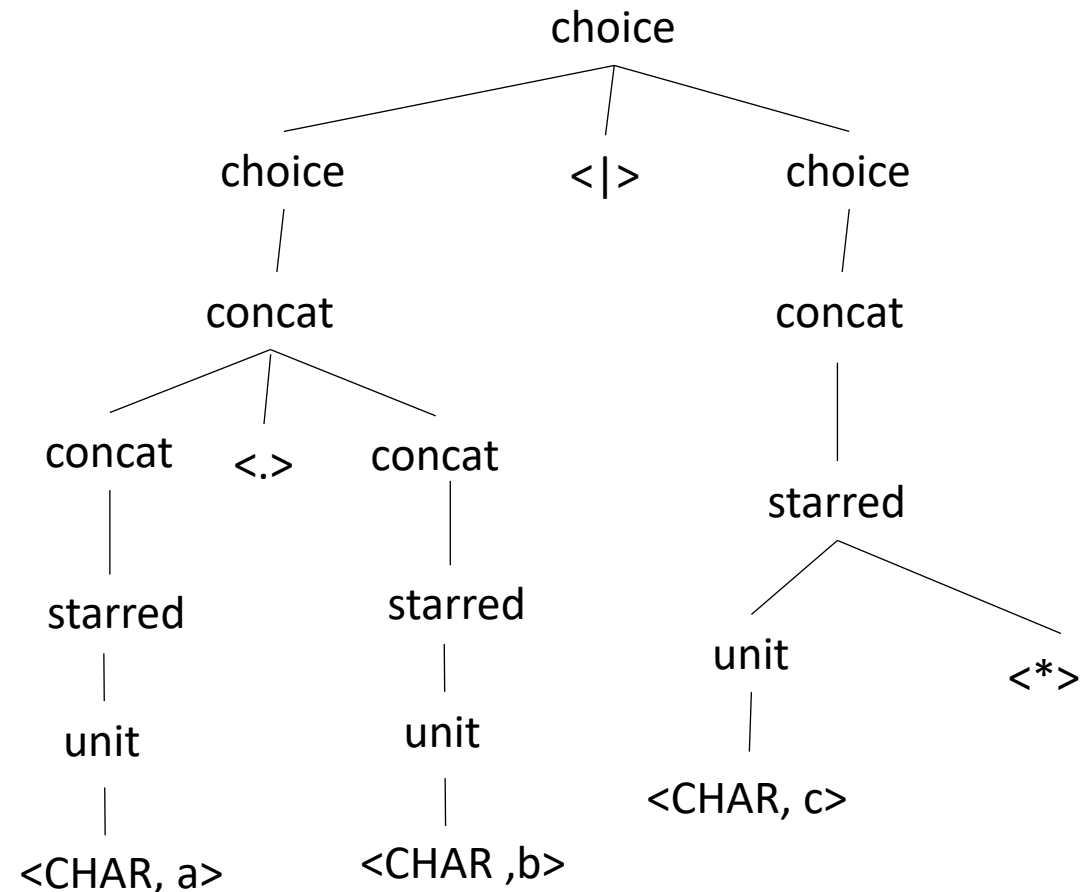
Parsing REs

Let's try it for regular expressions, $\{| \cdot * ()\}$

- Assume \cdot is a concatenate operator

| Operator | Name | Productions |
|----------|---------|------------------------------------|
| | choice | ::= choice PIPE choice concat |
| . | concat | ::= concat DOT concat starred |
| * | starred | ::= starred STAR unit |
| () | unit | ::= LPAREN choice RPAREN CHAR |

input: a.b | c*



How many levels of precedence does C have?

- https://en.cppreference.com/w/c/language/operator_precedence

Have we removed all ambiguity?

Let's make some more parse trees

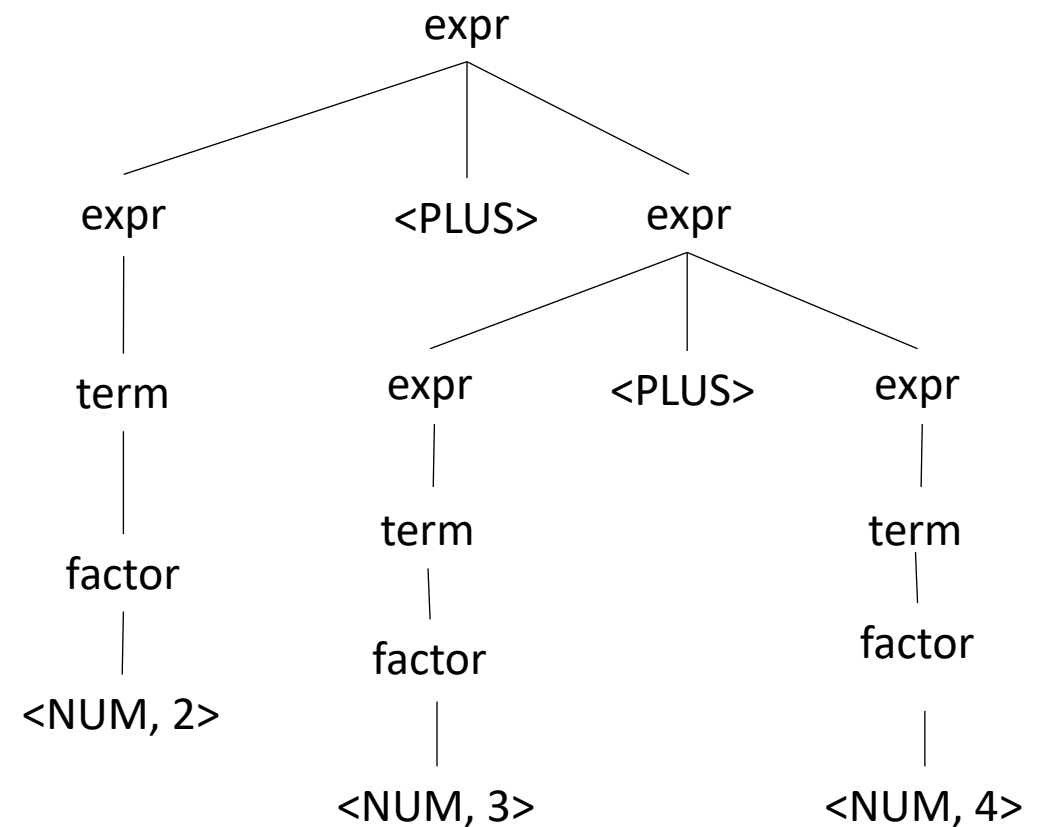
input: 2+3+4

| Operator | Name | Productions |
|----------|--------|-------------------------------|
| + | expr | : expr PLUS expr term |
| * | term | : term TIMES term factor |
| () | factor | : LP expr RP NUM |

Let's make some more parse trees

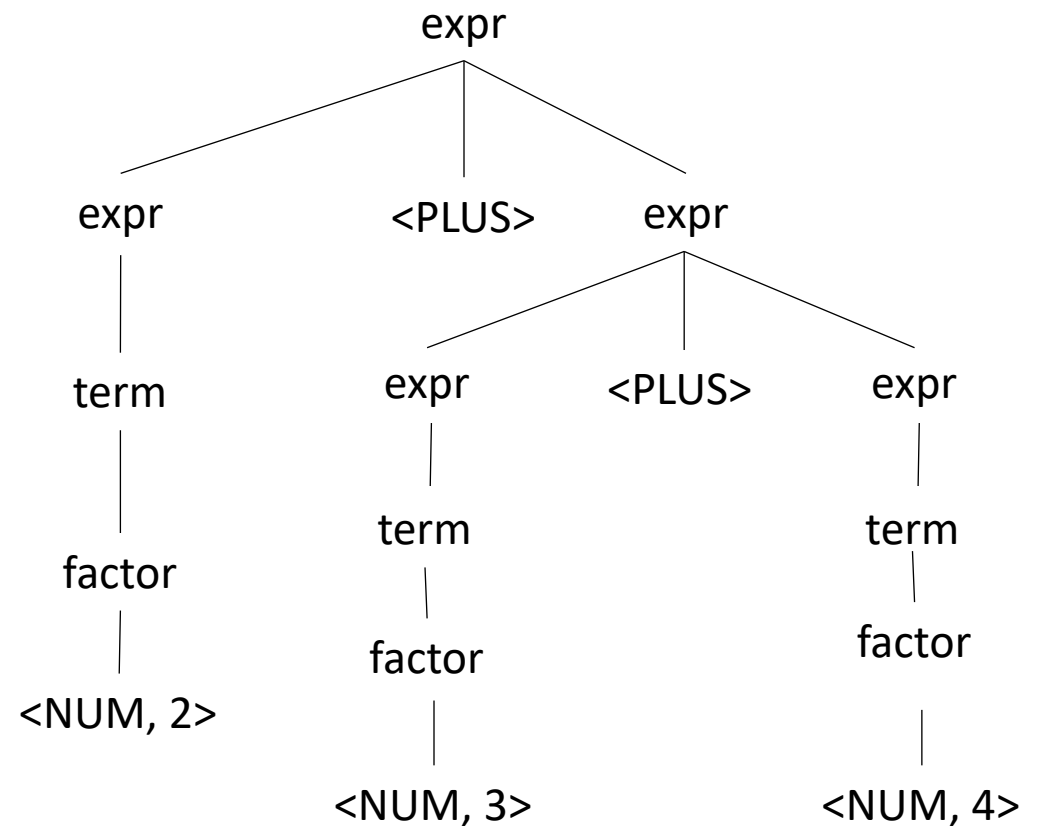
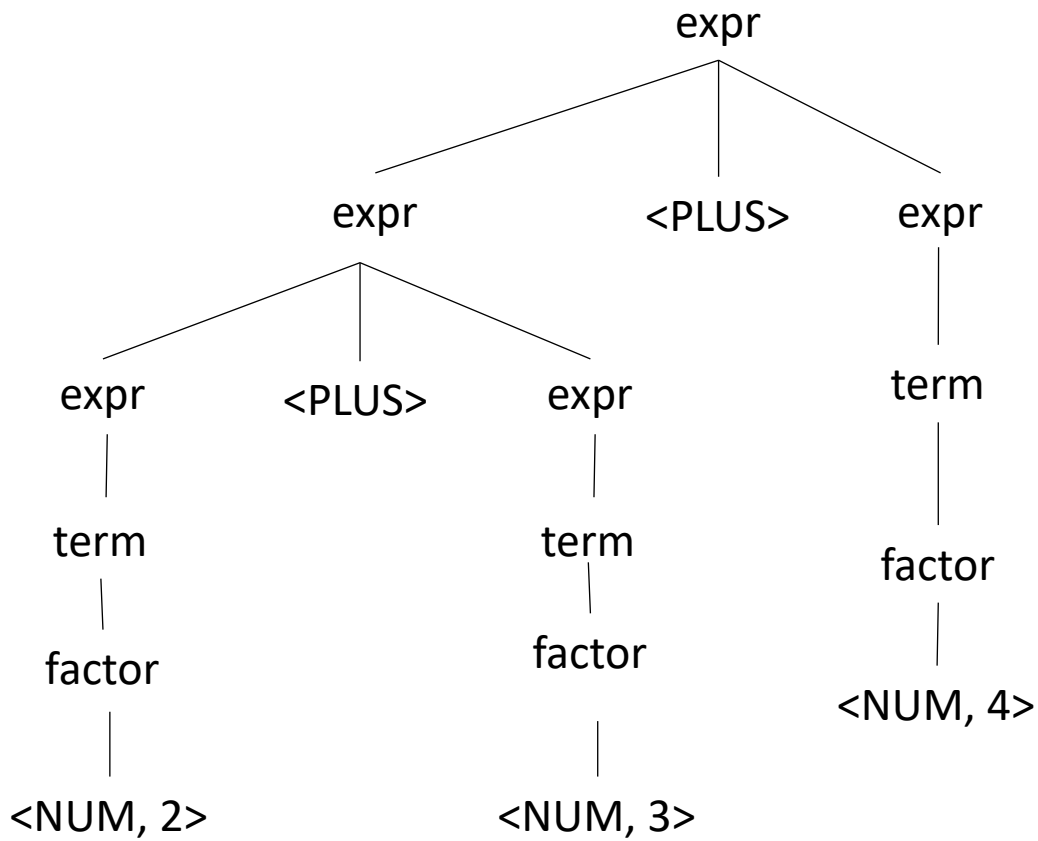
input: 2+3+4

| Operator | Name | Productions |
|----------|--------|-------------------------------|
| + | expr | : expr PLUS expr term |
| * | term | : term TIMES term factor |
| () | factor | : LP expr RP NUM |



This is ambiguous, is it an issue?

input: 2+3+4

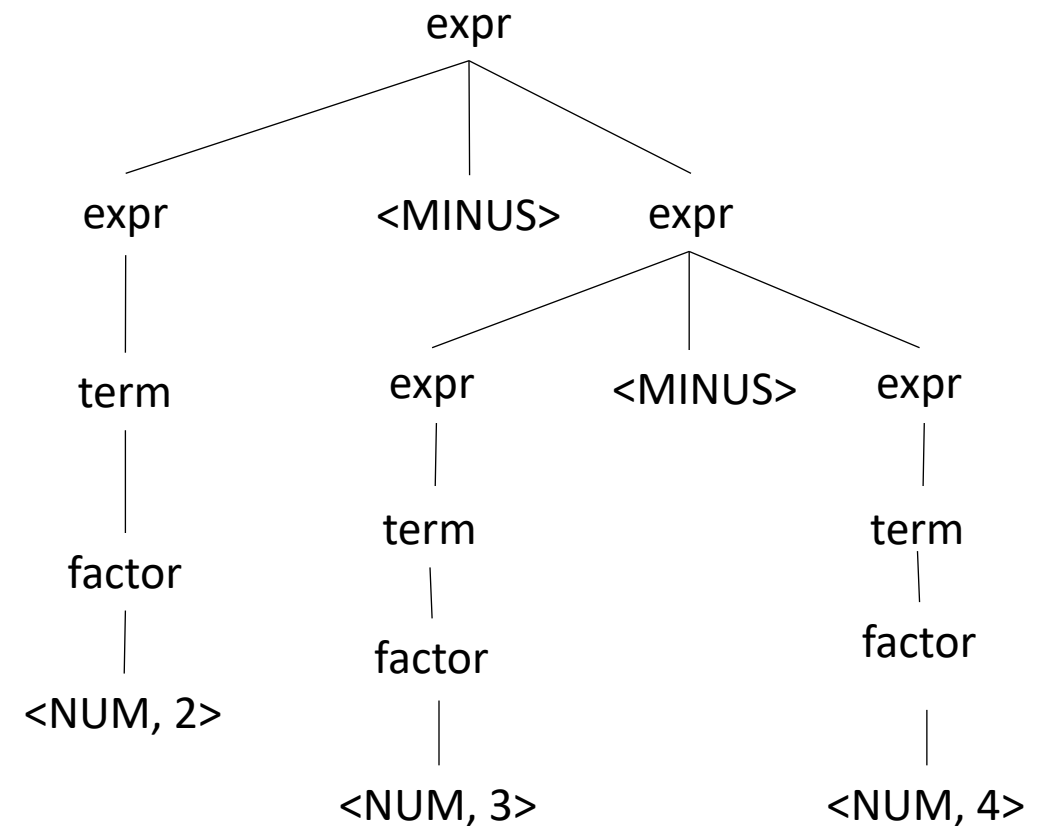
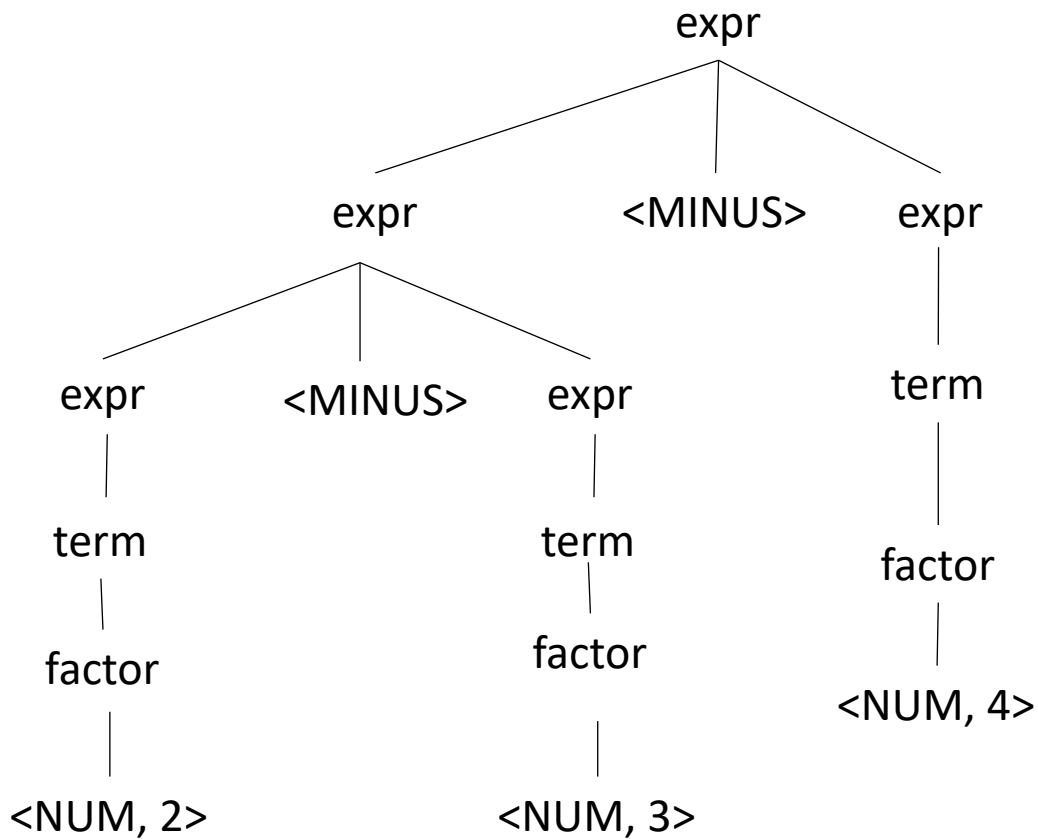


What about for a different operator?

input: 2-3-4

What about for a different operator?

input: 2-3-4



Which one is right?

Associativity

Describes the order in which apply the same operator

Sometimes it doesn't matter:

- When?

Associativity

Describes the order in which apply the same operator

Sometimes it doesn't matter:

- Integer addition
- Integer multiplication

*These operators
are said to be associative*

Good test:

- $((a \text{ OP } b) \text{ OP } c) == (a \text{ OP } (b \text{ OP } c))$

What about floating-point arithmetic?

Associativity

If an operator is not associative then we define

- left to right (left-associative)
 - $2-3-4$ is evaluated as $((2-3) - 4)$
 - What other operators are left-associative
- right-to-left (right-associative)
 - Any operators you can think of?

Associativity

If an operator is not associative then we define

- left to right (left-associative)
 - $2-3-4$ is evaluated as $((2-3) - 4)$
 - What other operators are left-associative
- right-to-left (right-associative)
 - Assignment, power operator

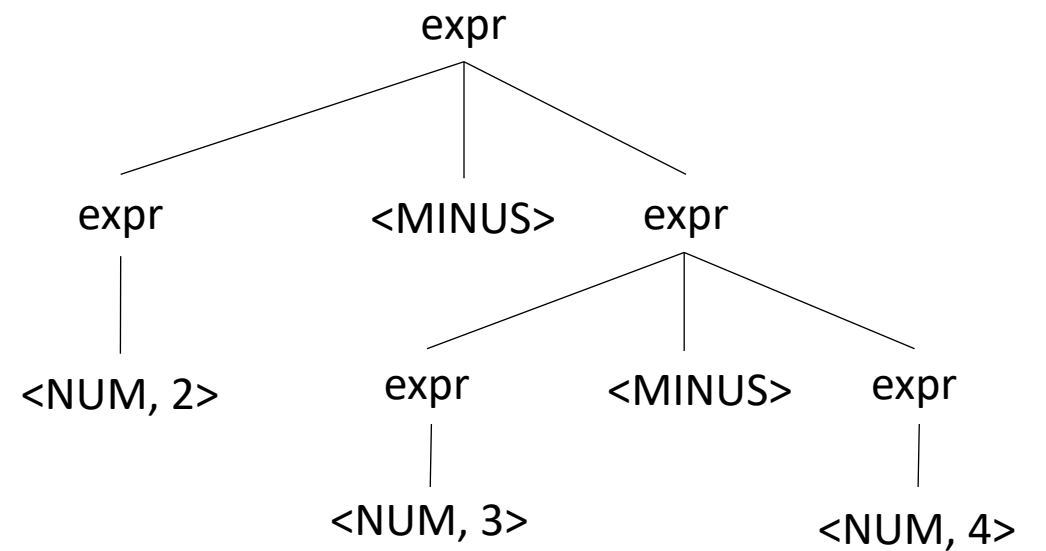
How to encode associativity?

- Like precedence, some tools (e.g. YACC/Bison) allow associativity specification through keywords:
 - “+”: left, “^”: right
- Like precedence, we can also encode it into the production rules

Associativity for a single operator

input: 2-3-4

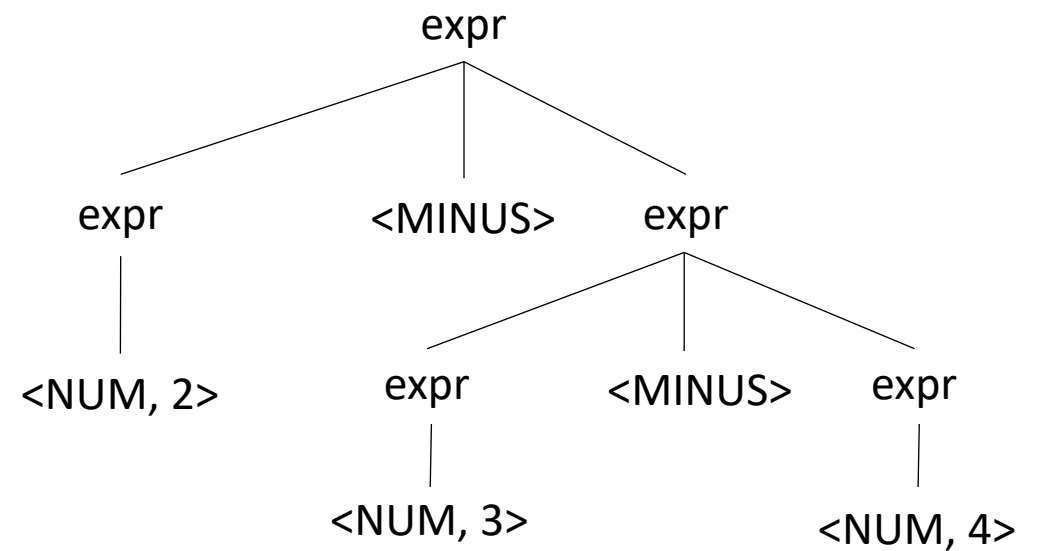
| Operator | Name | Productions |
|----------|------|----------------------------|
| - | expr | : expr MINUS expr NUM |



Associativity for a single operator

input: 2-3-4

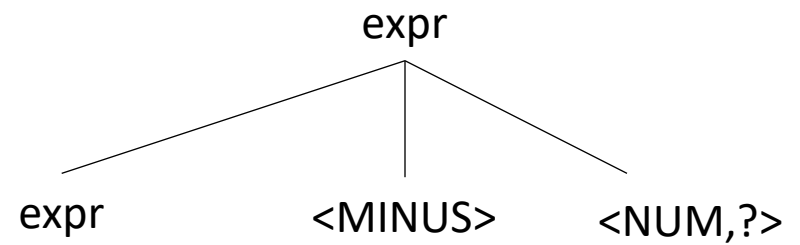
| Operator | Name | Productions |
|----------|------|---------------------------|
| - | expr | : expr MINUS NUM NUM |



This grammar no longer allows this.

Associativity for a single operator

input: 2-3-4

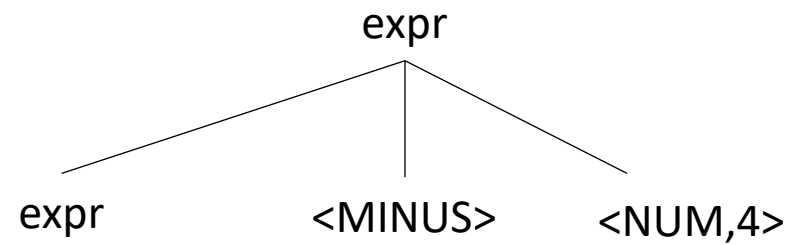


| Operator | Name | Productions |
|----------|------|---------------------------|
| - | expr | : expr MINUS NUM NUM |

So let's start over

Associativity for a single operator

input: 2-3-4

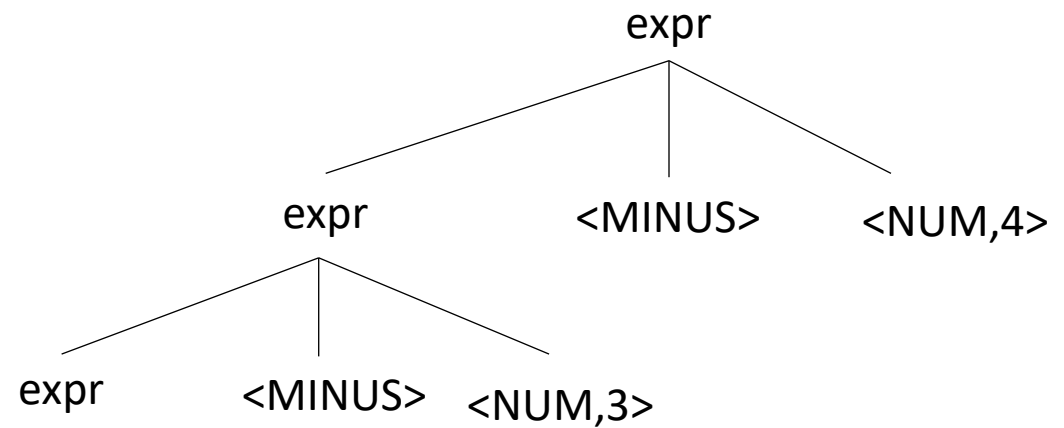


| Operator | Name | Productions |
|----------|------|---------------------------|
| - | expr | : expr MINUS NUM NUM |

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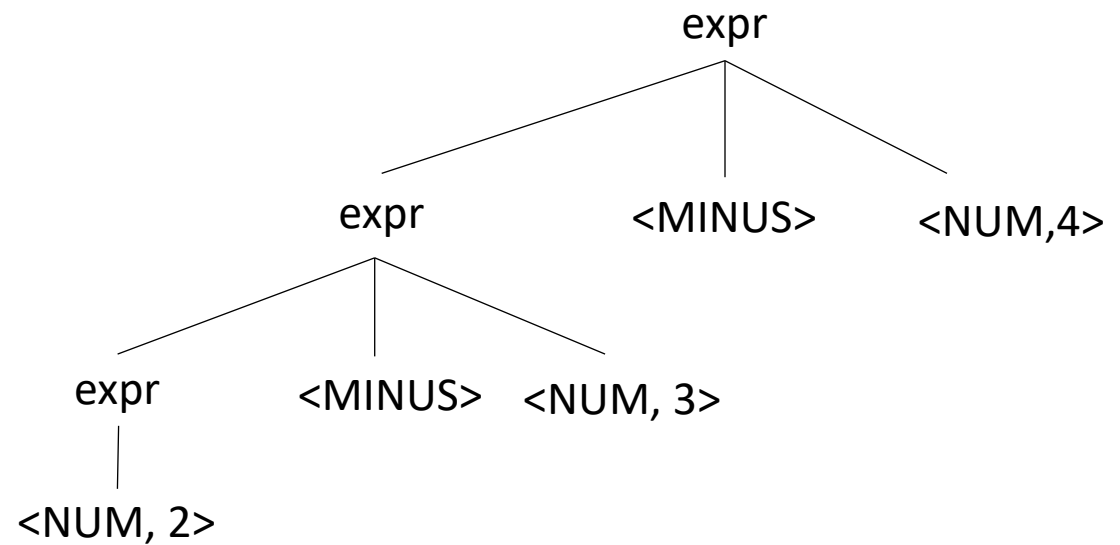
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Associativity for a single operator

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| Operator | Name | Productions |
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| - | expr | : expr MINUS NUM NUM |



This grammar makes the MINUS operator left associative and avoids parsing ambiguity!

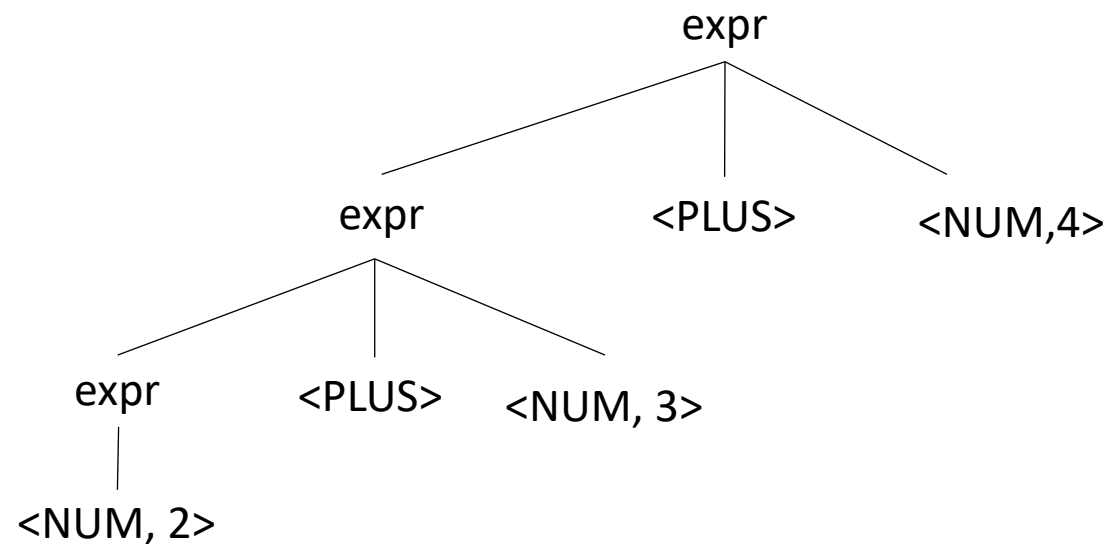
Should you have associativity when its not required?

Benefits?

Drawbacks?

| Operator | Name | Productions |
|----------|------|--------------------------|
| + | expr | : expr PLUS NUM NUM |

input: 2+3+4



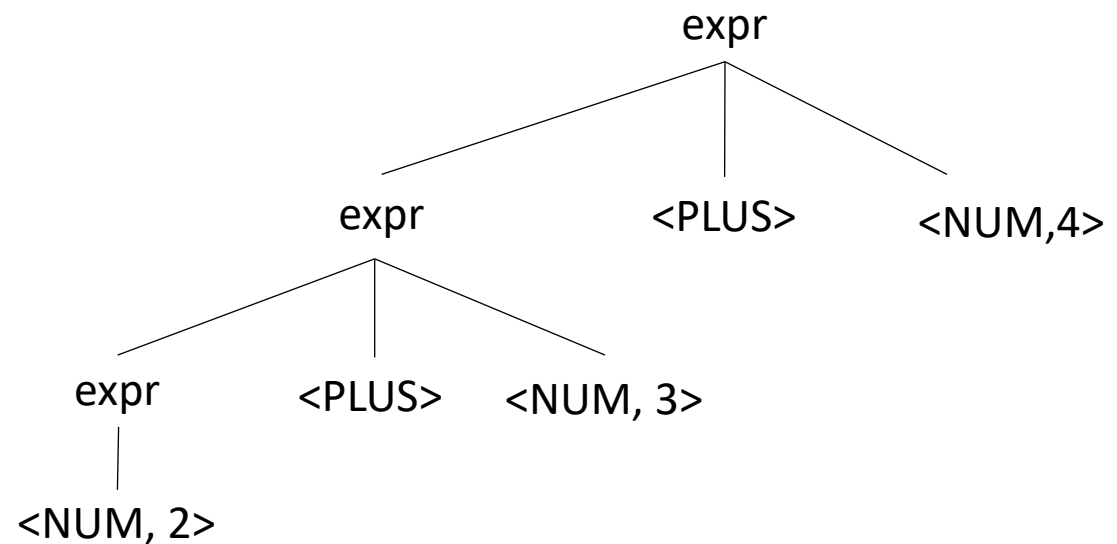
Should you have associativity when its not required?

Benefits?

Drawbacks?

| Operator | Name | Productions |
|----------|------|--------------------------|
| + | expr | : expr PLUS NUM NUM |

input: 2+3+4



Good design principle is to avoid ambiguous grammars, even when strictly not required too.

Helps with debugging, etc. etc.

Many tools will warn if it detects ambiguity

Let's make a richer expression grammar

*Let's do operators $[+, *, -, /, ^]$ and $()$*

| Operator | Name | Productions |
|----------|------|-------------|
| | | |
| | | |
| | | |
| | | |

Tokens:

NUM = "[0-9]+"

PLUS = "\+"

TIMES = "*"

LP = "\("

RP = "\)"

MINUS = "\-"

DIV = "\/"

CARROT = "\^"

Let's make a richer expression grammar

*Let's do operators $[+, *, -, /, ^]$ and $()$*

| Operator | Name | Productions |
|----------|--------|--|
| $+, -$ | expr | $: \text{expr PLUS term}$ $ \text{expr MINUS term}$ $ \text{term}$ |
| $*, /$ | term | $: \text{term TIMES pow}$ $ \text{term DIV pow}$ $ \text{pow}$ |
| $^$ | pow | $: \text{factor CARROT pow}$ $ \text{factor}$ |
| $()$ | factor | $: \text{LPAR expr RPAR}$ $ \text{NUM}$ |

Tokens:

NUM = "[0-9]+"

PLUS = "\+"

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MINUS = "\-"

DIV = "\/"

CARROT = "\^"

What associativities does C have?

- https://en.cppreference.com/w/c/language/operator_precedence

Next time: algorithms for syntactic analysis

- Top down parsing
 - oracle parsing
 - removing left recursion
 - constructing lookahead sets