

CSE110A: Compilers

Topics:

- *Start of Module 4 - Optimizations*

Module 4: Optimizations

Discussion

- What are compiler optimizations?
- Why do we want compiler optimizations?

Discussion

- What are compiler optimizations?
 - automated program transforms designed to make code more optimal
 - optimal can mean different things
 - code optimized for one system might be different for code optimized for a different system
 - we can optimize for speed, for energy efficiency, or for code size. What else?
- Why do we want the compiler to help us optimize?
 - So we can write more maintainable/portable code
 - So we don't have to worry about learning nuanced details about every possible system

Discussion

- What are some compiler optimizations you know about?

Discussion

- What are some compiler optimizations you know about?

```
for (int i = 0; i < 10; i++) {  
    x = x + 1;  
}
```

loop unrolling

```
for (int i = 0; i < 10; i++) {  
    x = x + 1;  
    i++;  
    x = x + 1;  
}
```

Discussion

- What are some compiler optimizations you know about?

```
for (int i = 0; i < 10; i++) {  
    x = x + 1;  
}
```

loop unrolling

```
for (int i = 0; i < 10; i++) {  
    x = x + 1;  
    i++;  
    x = x + 1;  
}
```

```
int foo() {  
    int i,j,k;  
    i = 10;  
    j = i;  
    k = j;  
    return k;  
}
```

constant propagation

```
int foo() {  
    int i,j,k;  
    return 10;  
}
```

Discussion

- What are some compiler optimizations you know about?

```
for (int i = 0; i < 10; i++) {  
    x = x + 1;  
}
```

loop unrolling

```
for (int i = 0; i < 10; i++) {  
    x = x + 1;  
    i++;  
    x = x + 1;  
}
```

What does this save us?

Discussion

- What are some compiler optimizations you know about?

```
for (int i = 0; i < 10; i++) {  
    x = x + 1;  
}
```

loop unrolling

```
for (int i = 0; i < 10; i++) {  
    x = x + 1;  
    i++;  
    x = x + 1;  
}
```

What does this save us?

optimizations at one stage can enable optimizations at another stage:

```
for (int i = 0; i < 10; i+=2) {  
    x = x + 2;  
}
```

provides a bigger window for local analysis

Discussion

- What are some compiler optimizations you know about?

let's do a few more

Function inlining

```
int add(int x, int y) {  
    return x + y;  
}  
  
int foo(int x, int y, int z) {  
    return add(x,y);  
}
```

```
int foo(int x, int y, int z) {  
    return x + y;  
}
```

What does this save us?
code size? speed? the ability to debug? local regions to optimize more?

Discussion

- What are some compiler optimizations you know about?

There are many more! This is an active area of research and development

For a rough metric:

`git effort` shows activities on different files and directories

clang C++/C parser: 3.5K commits

clang AST: 8.7K commits

LLVM transforms/optimizations: 30K commits

The LLVM project has been under active development since **2000**, and Clang since about **2007–2008**.

The transformation part of the code base
has the most activity by far

Discussion

- How do you enable compiler optimizations?

Discussion

- How do you enable compiler optimizations?
 - most C/C++ compilers
 - optimizing for speed
 - -O0, -O1, -O2, -O3
 - optimizing for size
 - -Os, -Oz
 - relax some constraints (especially around floating point):
 - -Ofast
 - We can use Godbolt to test various optimizations.

<https://stackoverflow.com/questions/15548023/clang-optimization-levels>

Discussion

STABILIZER: Statistically Sound Performance Evaluation

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2013 research paper

"the performance impact of -O3 over -O2 optimizations is indistinguishable from random noise."

Discussion

- What are some of the biggest improvements you've seen from compiler optimizations?

Discussion

- What are some of the biggest improvements you've seen from compiler optimizations?
- compiler optimizations are great at well-structured, regular loops and arrays

Discussion

- What kind of transforms on your code is the compiler allowed to do?

Discussion

- What kind of transforms on your code is the compiler allowed to do?
- Matrix-add
- Many-add example
- Why did we get such a dramatic increase?

Discussion

- Extreme example

```
void foo(int * arr, int n)
{
    int i, j;
    for (i = 0; i < n - 1; i++)
        for (j = 0; j < n - i - 1; j++)
            if (arr[j] > arr[j + 1]) {
                tmp = arr[j];
                arr[j] = arr[j + 1];
                arr[j + 1] = tmp;
            }
}
```

```
int p(int arr[], int start, int end)
{
    int pivot = arr[start];

    int count = 0;
    for (int i = start + 1; i <= end; i++) {
        if (arr[i] <= pivot)
            count++;
    }

    int pivotIndex = start + count;
    swap(arr[pivotIndex], arr[start]);

    int i = start, j = end;
    while (i < pivotIndex && j > pivotIndex) {

        while (arr[i] <= pivot) {
            i++;
        }

        while (arr[j] > pivot) {
            j--;
        }

        if (i < pivotIndex && j > pivotIndex) {
            swap(arr[i++], arr[j--]);
        }
    }

    return pivotIndex;
}

void foo(int *arr, int n)
{
    if (start >= end)
        return;

    int p = p(arr, m, n);

    foo(arr, start, p - 1);

    foo(arr, p + 1, end);
}
```

is this transform legal?

Discussion

- Extreme example

bubble sort

```
void foo(int * arr, int n)
{
    int i, j;
    for (i = 0; i < n - 1; i++)
        for (j = 0; j < n - i - 1; j++)
            if (arr[j] > arr[j + 1]) {
                tmp = arr[j];
                arr[j] = arr[j + 1];
                arr[j + 1] = tmp;
            }
}
```

Yes this transform
would be legal!

Could any compiler figure it out?
currently unlikely..

This is a technique called
“super optimizing” and it is
getting more and more interest

```
int p(int arr[], int start, int end)
{
    int pivot = arr[start];

    int count = 0;
    for (int i = start + 1; i <= end; i++) {
        if (arr[i] <= pivot)
            count++;
    }

    int pivotIndex = start + count;
    swap(arr[pivotIndex], arr[start]);

    int i = start, j = end;
    while (i < pivotIndex && j > pivotIndex) {
        while (arr[i] <= pivot) {
            i++;
        }
        while (arr[j] > pivot) {
            j--;
        }
        if (i < pivotIndex && j > pivotIndex) {
            swap(arr[i++], arr[j--]);
        }
    }

    return pivotIndex;
}

void foo(int *arr, int n)
{
    if (start >= end)
        return;

    int p = p(arr, m, n);

    foo(arr, start, p - 1);

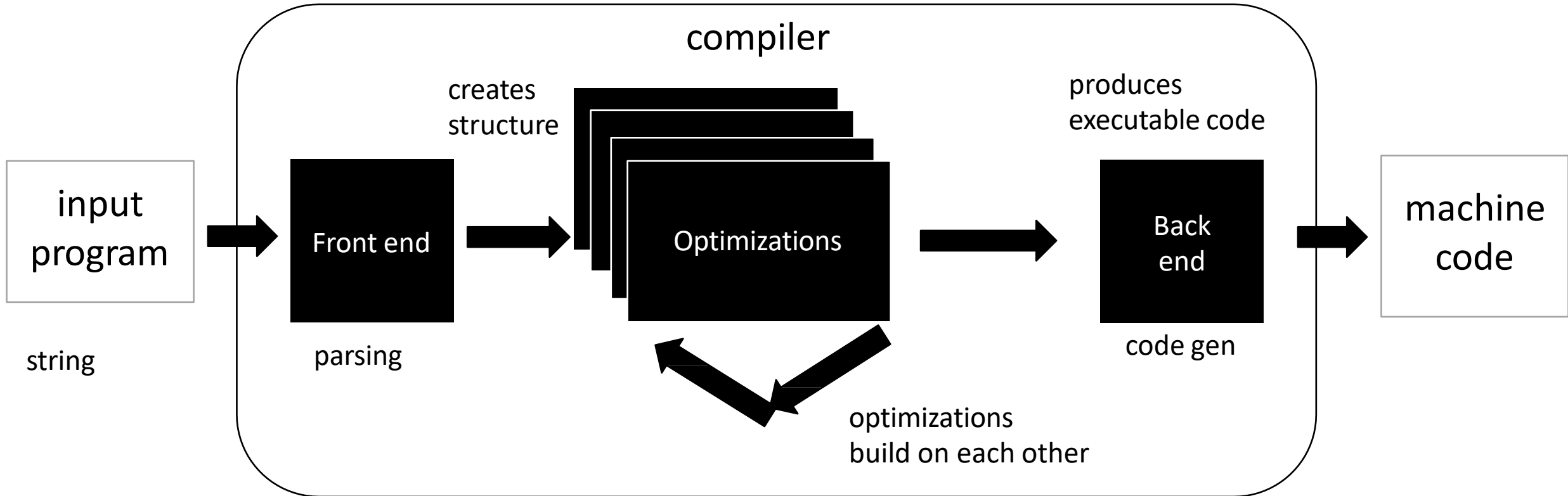
    foo(arr, p + 1, end);
}
```

quick sort

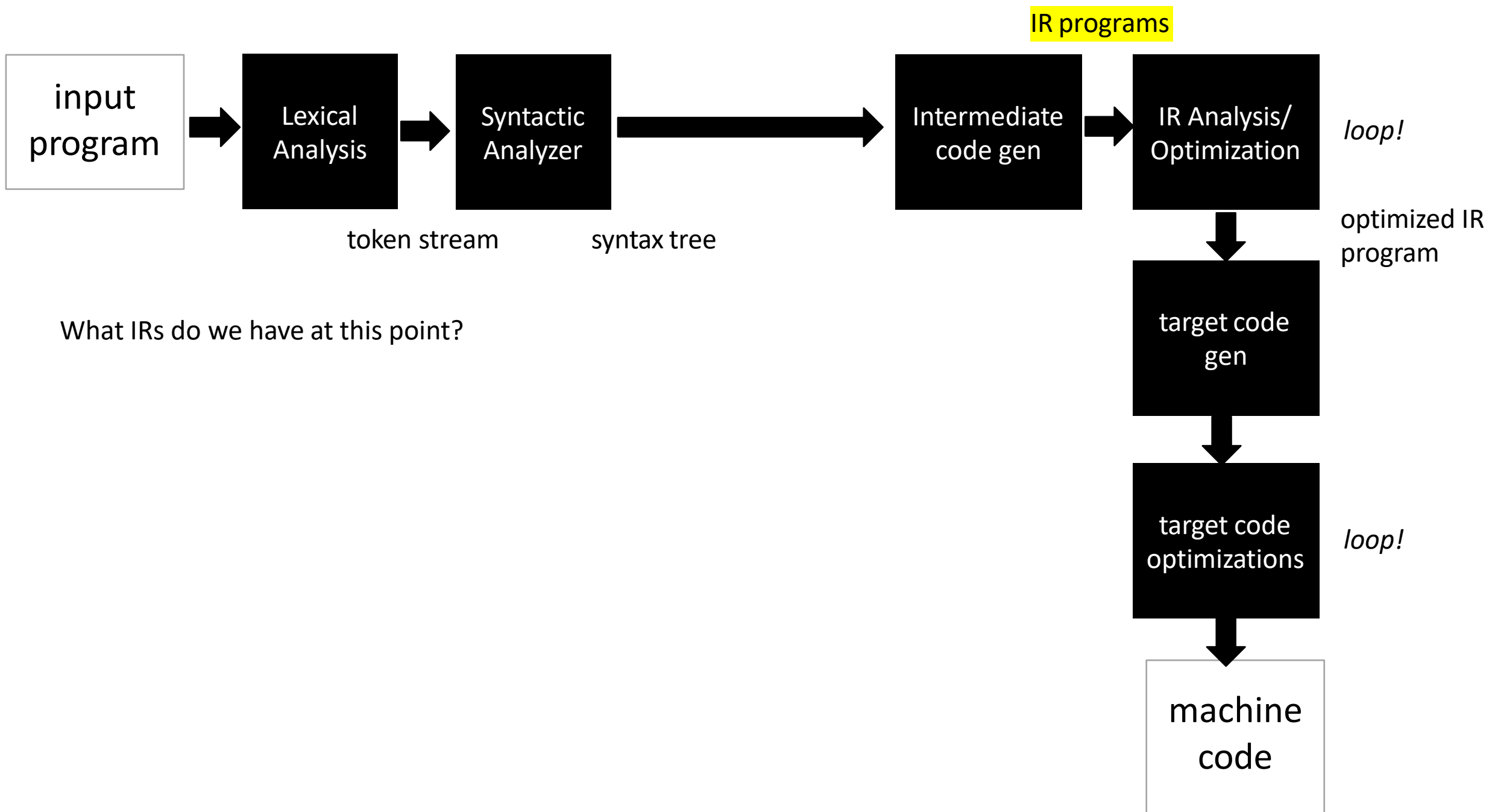
is this transform legal?

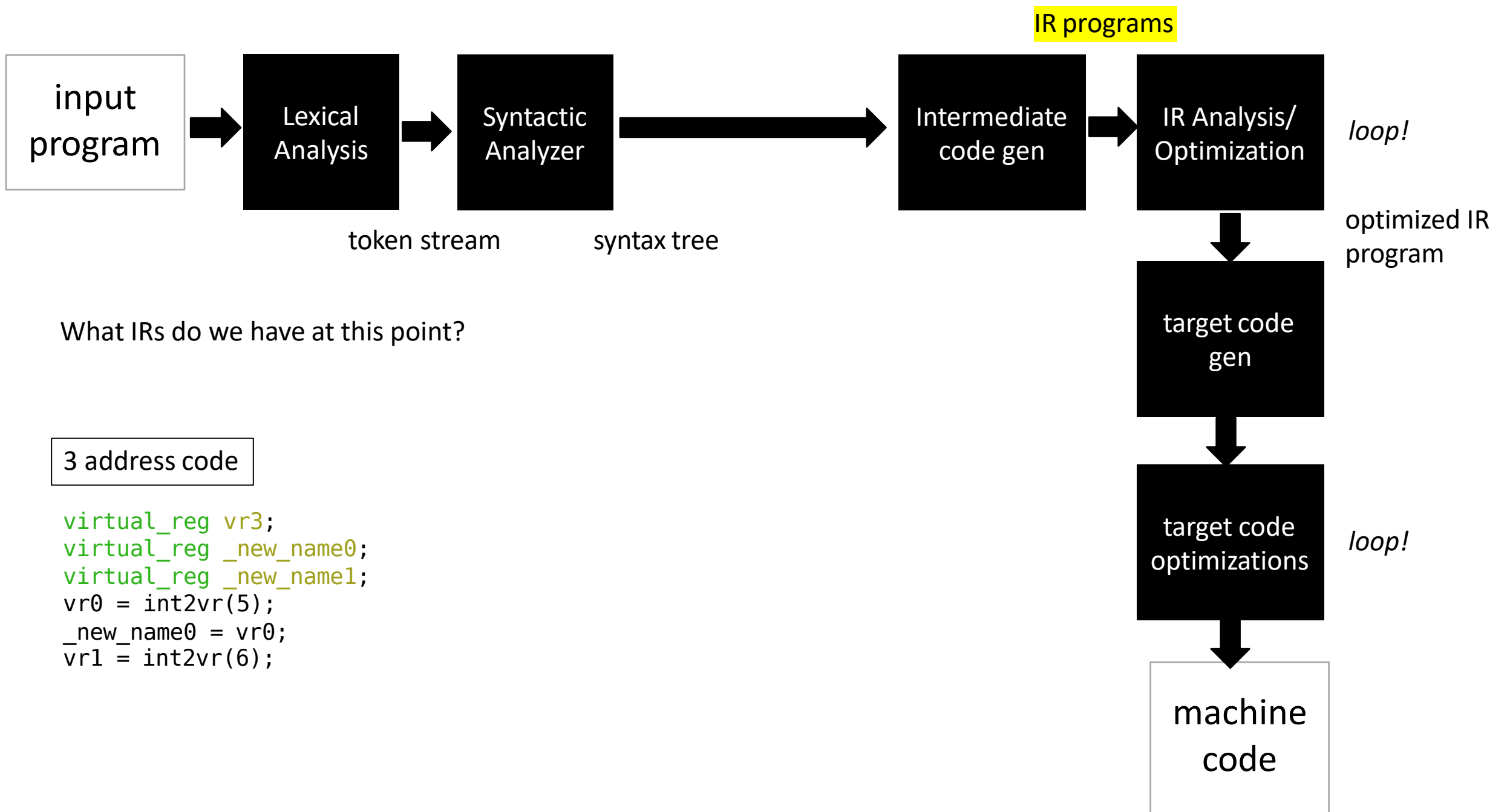
Moving on

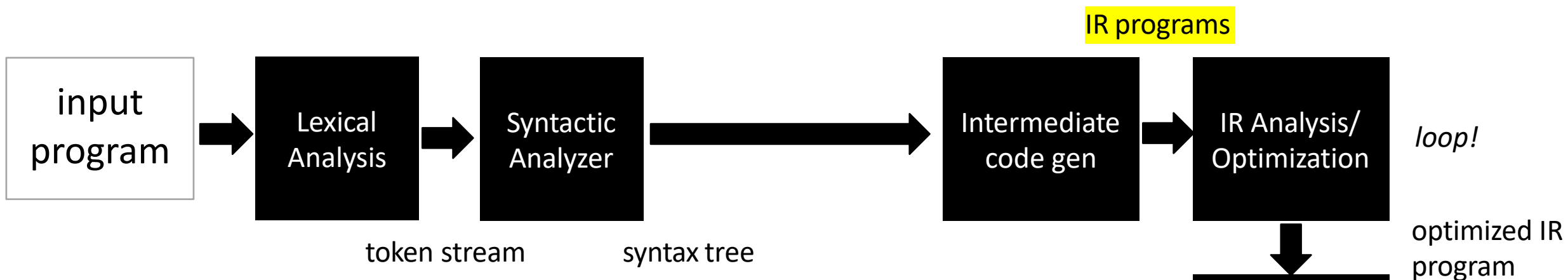
Zooming out again: Compiler Architecture



IRs and type inference are at the boundary of parsing and optimizations





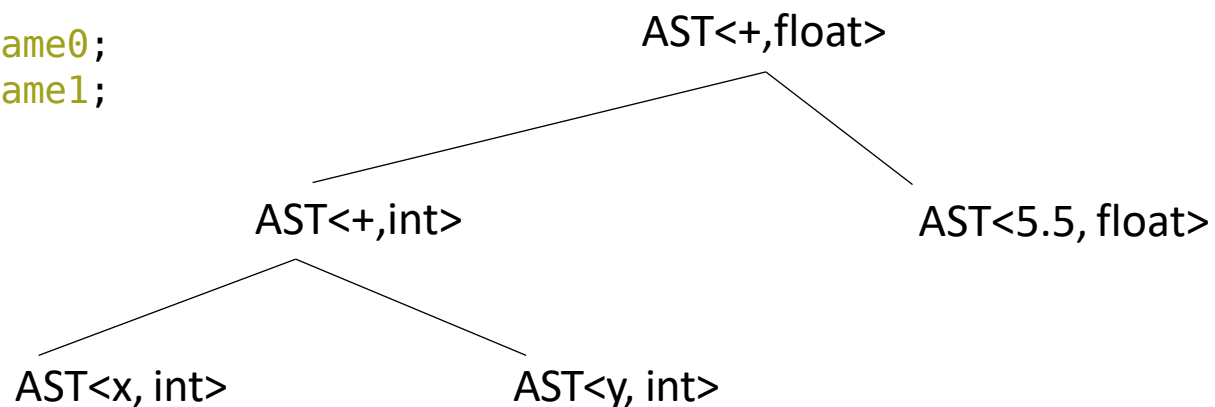


What IRs do we have at this point?

3 address code

```
virtual_reg vr3;  
virtual_reg _new_name0;  
virtual_reg _new_name1;  
vr0 = int2vr(5);  
_new_name0 = vr0;  
vr1 = int2vr(6);
```

AST



machine
code

Implicit parse tree

```
if_else_statement := IF LPAR expr RPAR statement ELSE statement
```

```
if (program0) {  
    program1  
}  
else {  
    program2  
}
```

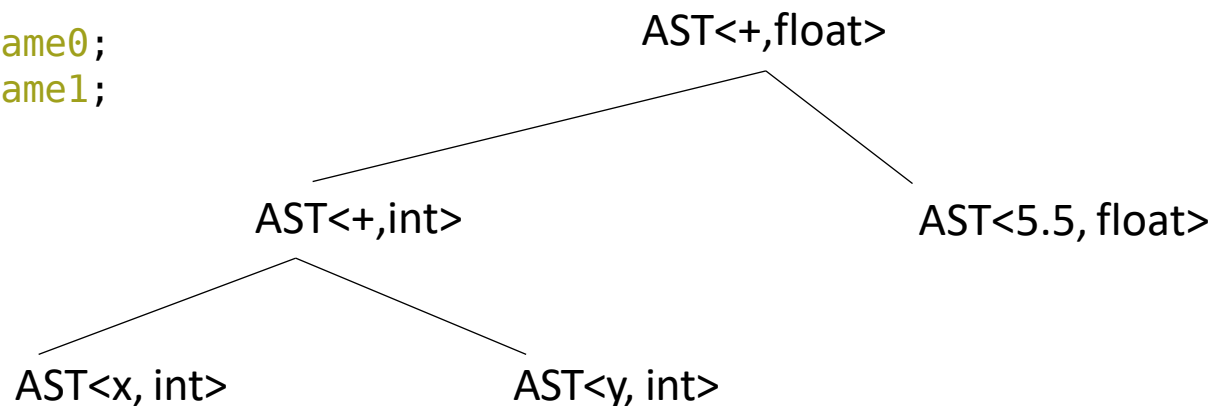
***We have several structures to utilize
to analyze and optimize programs!***

What IRs do we have at this point?

3 address code

```
virtual_reg vr3;  
virtual_reg _new_name0;  
virtual_reg _new_name1;  
vr0 = int2vr(5);  
_new_name0 = vr0;  
vr1 = int2vr(6);
```

AST



IR programs

IR Analysis/
Optimization

loop!

optimized IR
program

target code
gen

target code
optimizations

loop!

machine
code

Optimization categories

- Machine-independent - these optimizations should work well across many different systems
 - Examples?
- Machine dependent - these optimizations start to optimize the code for a given system
 - Examples?

Optimization categories

- Machine-independent - these optimizations should work well across many different systems
 - Examples?
 - All the examples we looked at before seem like they will help across many systems
- Machine dependent - these optimizations start to optimize the code for a given system
 - Examples?
 - loop chunking for cache line size and vectorization.
 - instruction re-orderings to take advantage of processor pipelines.
 - fused multiply-and-add instructions

Optimization categories

- **Machine-independent** - these optimizations should work well across many different systems
 - Examples?
 - All the examples we looked at before seem like they will help across many systems
- In this module we will be looking at machine-independent optimizations.
- What are the pros of machine-independent optimizations?

Optimization categories

Next category level is how much code we need to reason about for the optimization.

- **Local optimizations:** examine a "basic block", i.e. a small region of code with no control flow.
 - Examples?
- **Regional optimizations:** several basic blocks with simple control flow.
 - Examples?
- **Global optimization:** optimizes across an entire function

Optimization categories

- **Local optimizations:** examine a "basic block", i.e. a small region of code with no control flow.
- **Regional optimizations:** several basic blocks with simple control flow
- **Global optimization:** optimizes across an entire function
- **IDFA Inter-Procedural Data Flow Analysis :** Optimizes across functions

Discussion:

- What are the pros and cons of each?
- Going across functions is less common, why?

Optimization categories

- **Local optimizations:** examine a "basic block", i.e. a small region of code with no control flow.
- **Regional optimizations:** several basic blocks with simple control flow
- **Global optimization:** optimizes across an entire function

For this module:

- We will look at two optimizations in detail:
- A local optimization: Local value numbering
- A regional optimization: Loop unrolling
- We will implement both as homework
- We will discuss several other optimizations and analysis

Basic blocks

IR Program structure

- A sequence of 3 address instructions
- Programs can be split into **Basic Blocks**:
 - A **sequence of 3 address instructions** such that: there is a **single entry, single exit**
- *Important property*: an instruction in a basic block can assume that all preceding instructions will execute

Single Basic Block

```
Label_x:  
op1;  
op2;  
op3;  
br label_z;
```

IR Program structure

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Single Basic Block

```
Label_x:  
op1;  
op2;  
op3;  
br label_z;
```

Two Basic Blocks

```
Label_x:  
op1;  
op2;  
op3;  
  
Label_y:  
op4;  
op5;
```

IR Program structure

How might they appear in a high-level language? What are some examples?

- A sequence of 3 address instructions
- Programs can be split into **Basic Blocks**:
 - A **sequence of 3 address instructions** such that: there is a **single entry, single exit**

- *Important property*: an instruction in a basic block can assume that all preceding instructions will execute

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IR Program structure

- A sequence of 3 address instructions
- Programs can be split into **Basic Blocks**:
 - A **sequence of 3 address instructions** such that: there is a **single entry, single exit**
- *Important property*: an instruction in a basic block can assume that all preceding instructions will execute

How might they appear in a high-level language?

How many basic blocks?

```
...  
if (x) {  
    ...  
}  
else {  
    ...  
}  
...
```

Two Basic Blocks

Single Basic Block

```
Label_x:  
op1;  
op2;  
op3;  
br label_z;
```

```
Label_x:  
op1;  
op2;  
op3;  
  
Label_y:  
op4;  
op5;
```

Converting 3 address code into basic blocks

- Let's try an example: test 4 in HW 4:

Converting 3 address code into basic blocks

- Simple algorithm:
 - keep a list of basic blocks
 - a basic block is a list of instructions
- Iterate over the 3 address instructions
- if you see a branch or a label, finalize the current basic block and start a new one.

Converting 3 address code into basic blocks

pseudo code

```
basic_blocks = []
bb = []
for instr in program:
    if instr.type is in [branch, label]:
        bb.append(instr)
        basic_blocks.append(bb)
        bb = []
    else:
        bb.append(instr)
```


Optimization levels

- **Local optimizations:**
 - Optimizes an individual basic block
- **Regional optimizations:**
 - Combines several basic blocks
- **Global optimizations:**
 - operates across an entire procedure
 - what about across procedures?

Optimization levels

```
Label_0:  
x = a + b;  
y = a + b;
```

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Optimization levels

- **Local optimizations:**

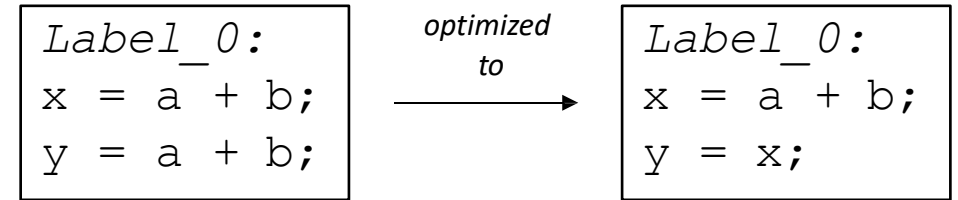
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Optimization levels

- **Local optimizations:**

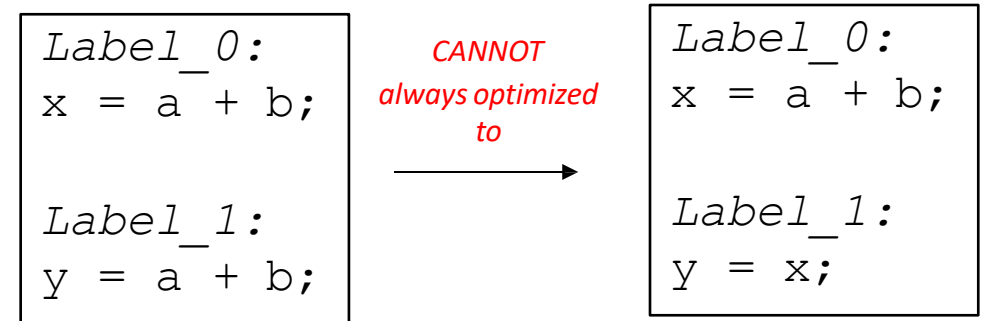
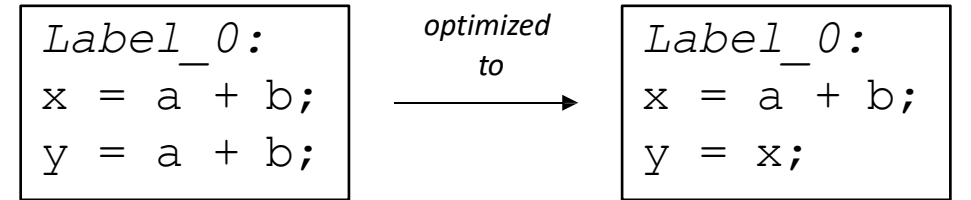
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Optimization levels

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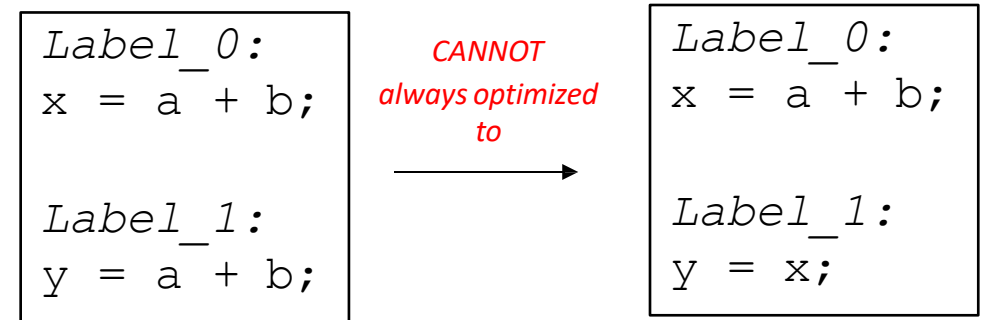
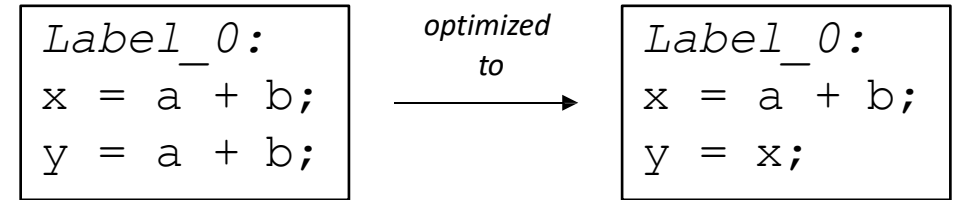
- Optimizes an individual basic block

- **Regional optimizations:**

- Combines several basic blocks

- **Global optimizations:**

- operates across an entire procedure
- what about across procedures?



*code could skip Label_0,
leaving x undefined!*

```
br Label_1;  
  
Label_0:  
x = a + b;  
  
Label_1:  
y = a + b;
```

Regional Optimization

```
...  
if (x) {  
    ...  
}  
else {  
    x = a + b;  
}  
y = a + b;  
...
```

*we cannot replace:
y = a + b.
with
y = x;*

Regional Optimization: Data Flow Analysis

```
...  
if (x) {  
    ...  
}  
else {  
    x = a + b;  
}  
y = a + b;  
...
```

we cannot replace:
y = a + b.
with
y = x;

```
x = a + b;  
if (x) {  
    ...  
}  
else {  
    ...  
}  
y = a + b;  
...
```

Can `x = a + b` be hoisted
out of the if then else?

*In that case, we can check if a
and b are not redefined, then*

*y = a + b;
can be replaced with
y = x;*

This requires regional analysis and optimizations

Local value numbering

- A local optimization over 3 address code
- Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)
- Can be extended to a regional optimization using flow analysis

Local value numbering

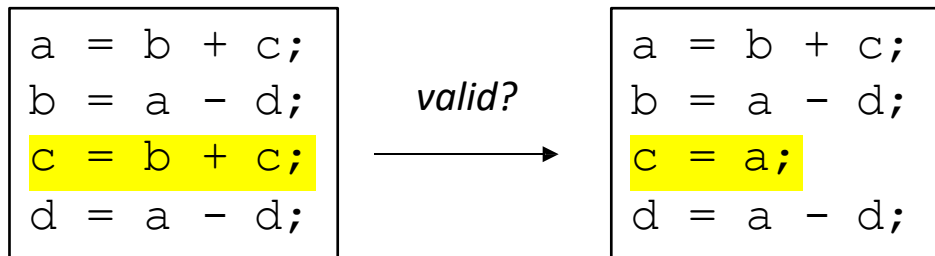
- A local optimization over 3 address code
- Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)
- Can be extended to a regional optimization using flow analysis

a = b + c;
b = a - d;
c = b + c;
d = a - d;

We are going to use notation similar to Class IR
applicable to any Intermediate representation.

Local value numbering

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- Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)
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Local value numbering

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- Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)
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```
a = b + c;  
b = a - d;  
c = b + c;  
d = a - d;
```

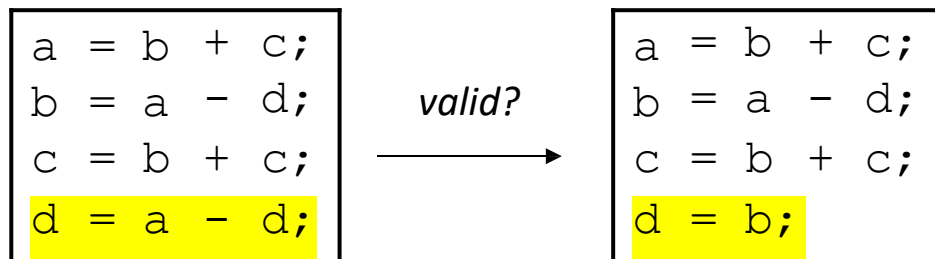
valid?

```
a = b + c;  
b = a - d;  
c = a;  
d = a - d;
```

No! Because b is redefined

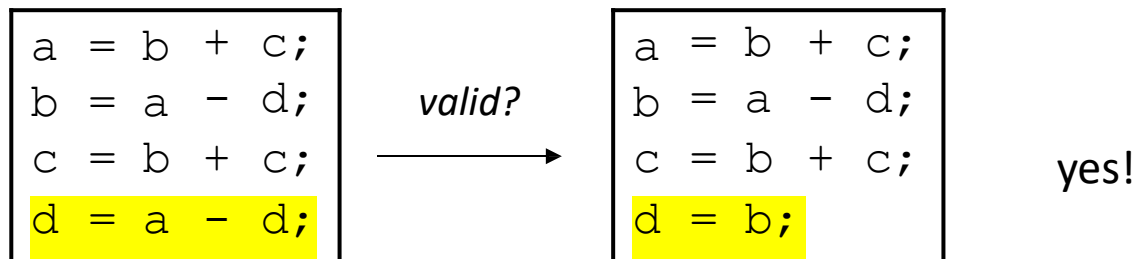
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Local value numbering (LVN)

LVN Algorithm:

- Provide a number to each variable. Update the number each time the variable is updated.
- Keep a global counter; increment with new variables or assignments

a	=	b	+	c	;
b	=	a	-	d	;
c	=	b	+	c	;
d	=	a	-	d	;

Global_counter = 0

Local value numbering

Algorithm:

- Provide a number to each variable. Update the number each time the variable is updated.
- Keep a global counter; increment with new variables or assignments

a	=	b0	+	c1	;
b	=	a	-	d	;
c	=	b	+	c	;
d	=	a	-	d	;

Global_counter = 2

**b and c have not been
seen, give them a number**

Local value numbering

Algorithm:

- Provide a number to each variable. Update the number each time the variable is updated.
- Keep a global counter; increment with new variables or assignments

a2	=	b0	+	c1;
b	=	a2	-	d3;
c	=	b	+	c;
d	=	a	-	d;

Global_counter = 4

a2 have been updated use
the last defined number.
i.e. a on RHS, becomes a2
and so on.

Local value numbering

Algorithm:

- Provide a number to each variable. Update the number each time the variable is updated (assigned to).
- Keep a global counter; increment with new variables or assignments

a2	=	b0	+	c1;
b4	=	a2	-	d3;
c5	=	b4	+	c1;
d6	=	a2	-	d3;

Global_counter = 7

So ... for LHS variable always increment with updated number, for right hand side use, always appended last number version

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. **Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.**
- At each step, check to see if the rhs has already been computed.

a2	=	b0	+	c1;
b4	=	a2	-	d3;
c5	=	b4	+	c1;
d6	=	a2	-	d3;

If we remember to declare every one of these variable names, the IR is still valid !!!

Local value numbering

Algorithm: Now that variables are numbered

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→

a2	=	b0	+	c1;
b4	=	a2	-	d3;
c5	=	b4	+	c1;
d6	=	a2	-	d3;

H = {
}

Local value numbering

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a2	=	b0	+	c1;
b4	=	a2	-	d3;
c5	=	b4	+	c1;
d6	=	a2	-	d3;

H = {
 "b0 + c1" : "a2",
}

Local value numbering

Algorithm: Now that variables are numbered

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a2 = b0 + c1;
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H = {
 "b0 + c1" : "a2",
 "a2 - d3" : "b4",
}

Local value numbering

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d6	=	a2	-	d3;

H = {
 "b0 + c1" : "a2",
 "a2 - d3" : "b4",
}

***mismatch due to
re-numbering!!!***

*i.e. it is no longer just
c = b + c*

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.
- At each step, check to see if the rhs has already been computed.

→

a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;

H = {
 "b0 + c1" : "a2",
 "a2 - d3" : "b4",
 "b4 + c1" : "c5",
}

Add new entry in
Hash table with new
Numbering.

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.
- At each step, check to see if the rhs has already been computed.

→

a2	=	b0	+	c1;
b4	=	a2	-	d3;
c5	=	b4	+	c1;
d6	=	a2	-	d3;

H = {
 "b0 + c1" : "a2",
 "a2 - d3" : "b4",
 "b4 + c1" : "c5",
}

Do a look up and ...

Local value numbering

Algorithm: Now that variables are numbered

- Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.
- At each step, check to see if the rhs has already been computed.

→

a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = b4;

H = {
 "b0 + c1" : "a2",
 "a2 - d3" : "b4",
 "b4 + c1" : "c5",
}

...match!

What else can we do?

What else can we do?

Consider this snippet:

```
a2 = c1 - b0;  
f4 = d3 * a2;  
c5 = b0 - c1;  
d6 = a2 * d3;
```

Commutative operations

What is the definition of commutative?

Commutative operations

What is the definition of commutative?

$$x \text{ OP } y == y \text{ OP } x$$

What operators are commutative? Which ones are not?

Adding commutativity to local value numbering

- For commutative operators (e.g. $+$ $*$), the analysis should consider a deterministic order of operands.
- You can use variable numbers or lexicographical order

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

→

a2	=	c1	-	b0;
f4	=	d3	*	a2;
c5	=	b0	-	c1;
d6	=	a2	*	d3;

H = {
}

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

cannot re-order because - is not commutative

→

a2	=	c1	-	b0;
f4	=	d3	*	a2;
c5	=	b0	-	c1;
d6	=	a2	*	d3;

H = {
 "c1 - b0" : "a2",
}

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

→

a2	=	c1	-	b0;
f4	=	d3	*	a2;
c5	=	b0	-	c1;
d6	=	a2	*	d3;

H = {
 "c1 - b0" : "a2",
}

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

→

a2	=	c1	-	b0;
f4	=	d3	*	a2;
c5	=	b0	-	c1;
d6	=	a2	*	d3;

re-ordered because a2 < d3 lexicographically

```
H = {  
    "c1 - b0" : "a2",  
    "a2 * d3" : "f4",  
}
```

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

→

a2	=	c1	-	b0;
f4	=	d3	*	a2;
c5	=	b0	-	c1;
d6	=	a2	*	d3;

H = {
 "c1 - b0" : "a2",
 "a2 * d3" : "f4",
}

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

→

a2	=	c1	-	b0;
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d6	=	a2	*	d3;

H = {
 "c1 - b0" : "a2",
 "a2 * d3" : "f4",
 "b0 - c1" : "c5",
}

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

→

a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;

H = {
 "c1 - b0" : "a2",
 "a2 * d3" : "f4",
 "b0 - c1" : "c5",
}

Local value numbering: commutative operations

Algorithm optimization:

- for commutative operations, re-order operands into a deterministic order

And achieves more optimization opportunities ...

→

a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = f4;

```
H = {  
    "c1 - b0" : "a2",  
    "a2 * d3" : "f4",  
    "b0 - c1" : "c5",  
}
```


Optimizing over wider regions

- Local value numbering operated over just one basic block.
- We want optimizations that operate over several basic blocks (a region), or across an entire procedure (global)
- For this, we need Control Flow Graphs and Flow Analysis
 - We may have time to discuss this later in the module

Next:

- More optimizations!