Homework 1

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Problem 2.1

Suppose you flip four fair coins:

- Make a list of all possible outcomes.
- Make a list of all the different macro states and their probablilities.
- Compute the multiplicity of the macrostates using the the Choose function $\binom{N}{n}$

a

List of all possible outcomes:

- HHHH
- HHHT
- HHTH
- HTHH
- HTHH
- THHH
- HHTT
- HTHT
- HTTH
- THHT
- TTHH
- HTTT
- THTT
- TTHT
- TTTH
- TTTT

b

List of all different macro states and their probabilities:

- Four heads: 1 outcome has a probability of 1/16 chance.
- Three heads: 4 outcomes has a probability of 4/16 = 1/4 chance.
- Two heads: 6 outcomes has a probability of 6/16 = 3/8 chance.
- One head: 4 outcomes has a probability of 4/16 = 1/4 chance.
- No heads: 1 outcome has a probability of 1/16 chance.

c

Calculating the probability of each macrostate is as follows:

$$\binom{4}{n} = \frac{24}{n! \times (24-n)\,!} \tag{1}$$

Where n is the number of coins that landed on heads. So for this

- Four heads:

$$\begin{aligned}\binom{4}{4} &= \frac{24}{4! \times (4-4)\,!} \\ &= \frac{24}{4! \times (0)\,!} \\ &= 1\end{aligned}$$

- Three heads:

$$\begin{aligned}\binom{4}{3} &= \frac{24}{3! \times (4-3)\,!} \\ &= \frac{24}{3! \times (1)\,!} \\ &= 4\end{aligned}$$

- Two heads:

$$\begin{aligned}\binom{4}{2} &= \frac{24}{2! \times (4-2)\,!} \\ &= \frac{24}{2! \times (2)\,!} \\ &= 6\end{aligned}$$

- One head:

$$\begin{aligned}\binom{4}{1} &= \frac{24}{1! \times (4-1)\,!} \\ &= \frac{24}{1! \times (3)\,!} \\ &= 4\end{aligned}$$

- No heads:

$$\begin{aligned}\binom{4}{0} &= \frac{24}{0! \times (4-0)\,!}\\ &= \frac{24}{0! \times (4)\,!}\\ &= 1\end{aligned}$$

Problem 2.2

Suppose you flip 20 coins:

a

How many possible outcomes (microstates) are there? Coins can be considered a binary state, i.e. an N number of coins is: 2^N. Therefore ther are $2^{20} = 1048576$ possible outcomes (microstates) when flipping 20 coins.

b

What is the probability for getting the flip sequence of: HTHHTTTHTHHHTH-HHHTHT in exactly that order? It can be assumed that the probabily of a state of a binary state like a coin is $1/2^N$ where N is the how many coins are being flipped. The question only asks for the microstate and not the macrostate of 12 head flips and 8 tail flips. Therefore the probability of getting the flip sequence of HTHHTTTHTHHHTHHHHTHT in exactly that order is $(1/2)^2 0 = 1/1048576$.

c

What is the probability of getting 12 heads and 8 tails in any order? In this case the question asks in more explicit terms: what is the probability of the macrostate having 12 heads and 8 tails in the outcome, or what is $\Omega(12)/2^{20}$.

$$\begin{aligned}\Omega(12)/2^{20} &= \frac{\binom{20}{12}}{2^{20}}\\ &= \frac{20!}{12! \times (8)\,!} \times \frac{1}{2^{20}}\\ &= 125970 \times \frac{1}{2^{20}}\\ &= \frac{62985}{524288} \text{ or } 12.0134\%\end{aligned}$$

Problem 2.3

Suppose you flip 50 coins:

a

How many possible outcomes are there? There are $2^{50} = 1125899906842624$ possible outcomes when flipping 50 coins.

b

How many ways are there of getting exactly 25 heads and 25 tails? There are $\binom{50}{25} = 2118760$ ways of getting exactly 25 heads and 25 tails.

c

What is the probability of getting exactly 25 heads and 25 tails? For the following excercises, refer to the method as outlined in **Problem 2.2b** as these caculations are identical except in inputs. The probability of getting exactly 25 heads and 25 tails is $\frac{\binom{50}{25}}{2^{50}} = 11.228\%$.

d

What is the probability of getting exactly 30 heads and 20 tails? The probability of getting exactly 30 heads and 20 tails is $\binom{50}{30} * \left(\frac{1}{2}\right)^{30} * \left(\frac{1}{2}\right)^{20} = 4.19\%$.

e

What is the probability of getting exactly 40 heads and 10 tails? The probability of getting exactly 40 heads and 10 tails is $\binom{50}{40} * \left(\frac{1}{2}\right)^{40} * \left(\frac{1}{2}\right)^{10} = 9.12 \times 10^{-6}\%$.

f

What is the probability of getting exactly 50 heads and 0 tails? The probability of getting exactly 50 heads and 0 tails is $\binom{50}{50} * \left(\frac{1}{2}\right)^{50} = 8.88 \times 10^{-16}\%$.

g

Make a plot of the probability of getting n heads, as a function of n. Using a simple python program:

```python
import matplotlib.pyplot as plt
import numpy as np
from math import comb

# Define the number of coins
n = 50

# Create an array of possible number of heads
heads = np.arange(0, n+1)

# Calculate the probability of getting n heads
prob = [comb(n, h) * (1/2)**h * (1/2)**(n-h) for h in heads]

# Create the plot
plt.plot(heads, prob)
plt.xlabel('Number of Heads')
plt.ylabel('Probability')
plt.title(f'Probability of Getting n Heads for {n} Coins')
plt.show()
```

This generates the following plot:

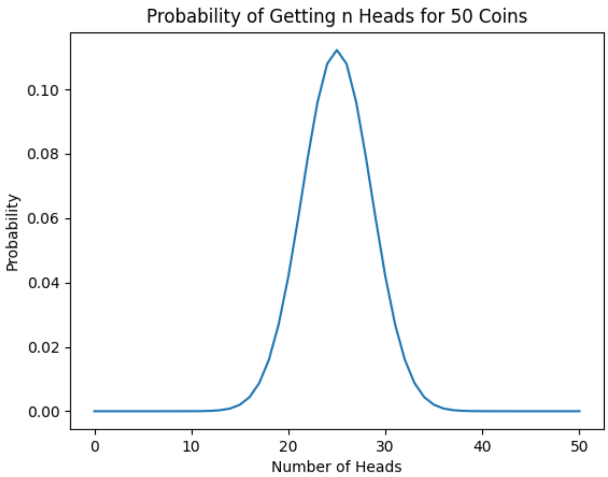

Figure 1: Probability of Getting n Heads for 50 Coins