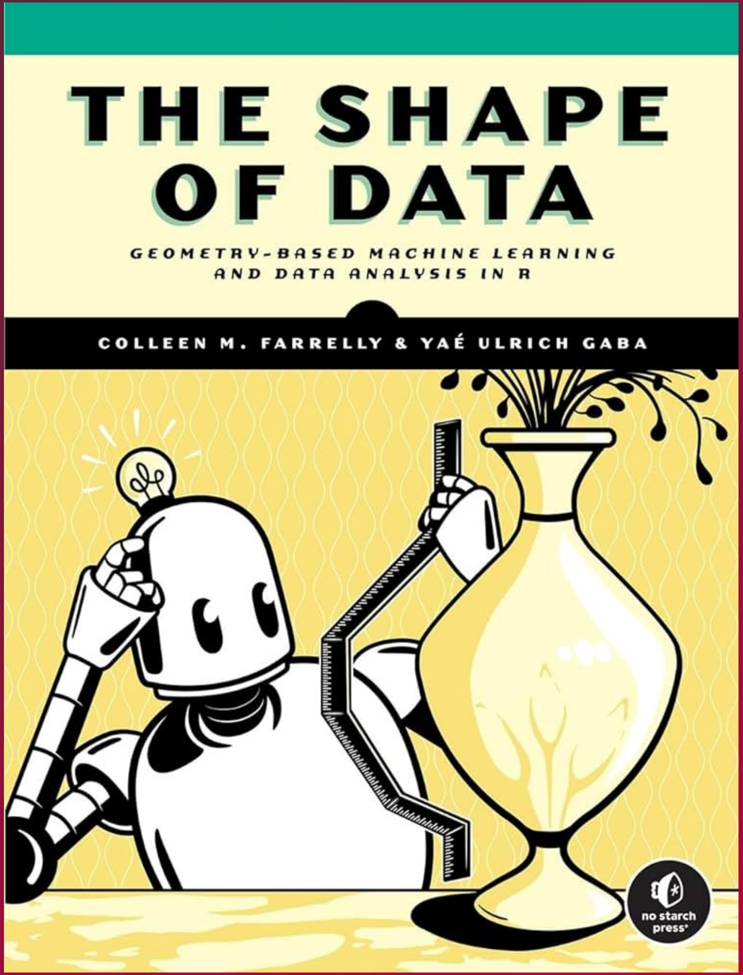
Topology, Data Science, Machine Leaning, Deep Learning, Image Processing….. *an more*

D. Sierra-Porta, UTB, June 2024

ENO-CANCOA 24
Cartagena de Indias, COL



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USD Quotes and Orders for South American Customers

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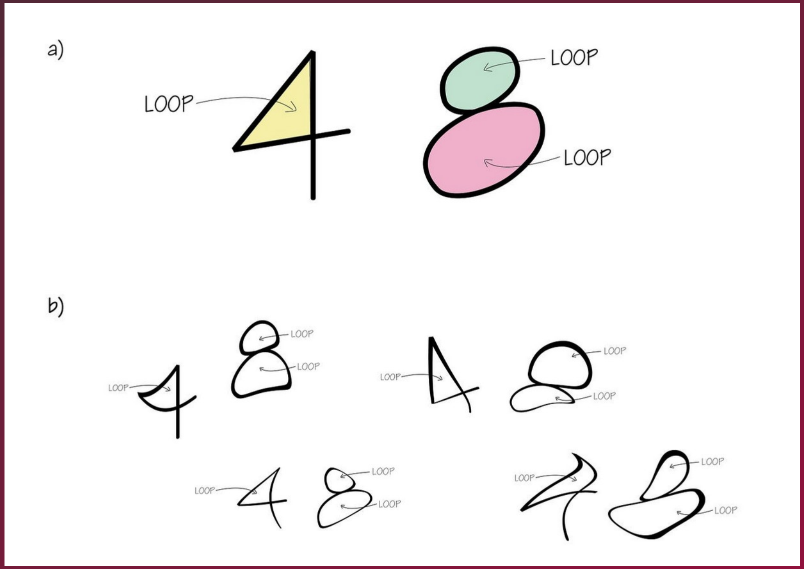
In applied mathematics, **<u>topological data analysis (TDA)</u>** is an approach to the analysis of datasets using techniques from topology. Extraction of information from datasets that are high-dimensional, incomplete and noisy is generally challenging.

<u>Significance Statement</u>



Information such as the geometric structure and texture of image data can greatly support the inference of the physical state of an observed Earth system, for example, in remote sensing to determine whether wildfires are active or to identify local climate zones. Persistent homology is a branch of topological data analysis that allows one to extract such information in an interpretable way—unlike black-box methods like deep neural networks.

Time Series Data

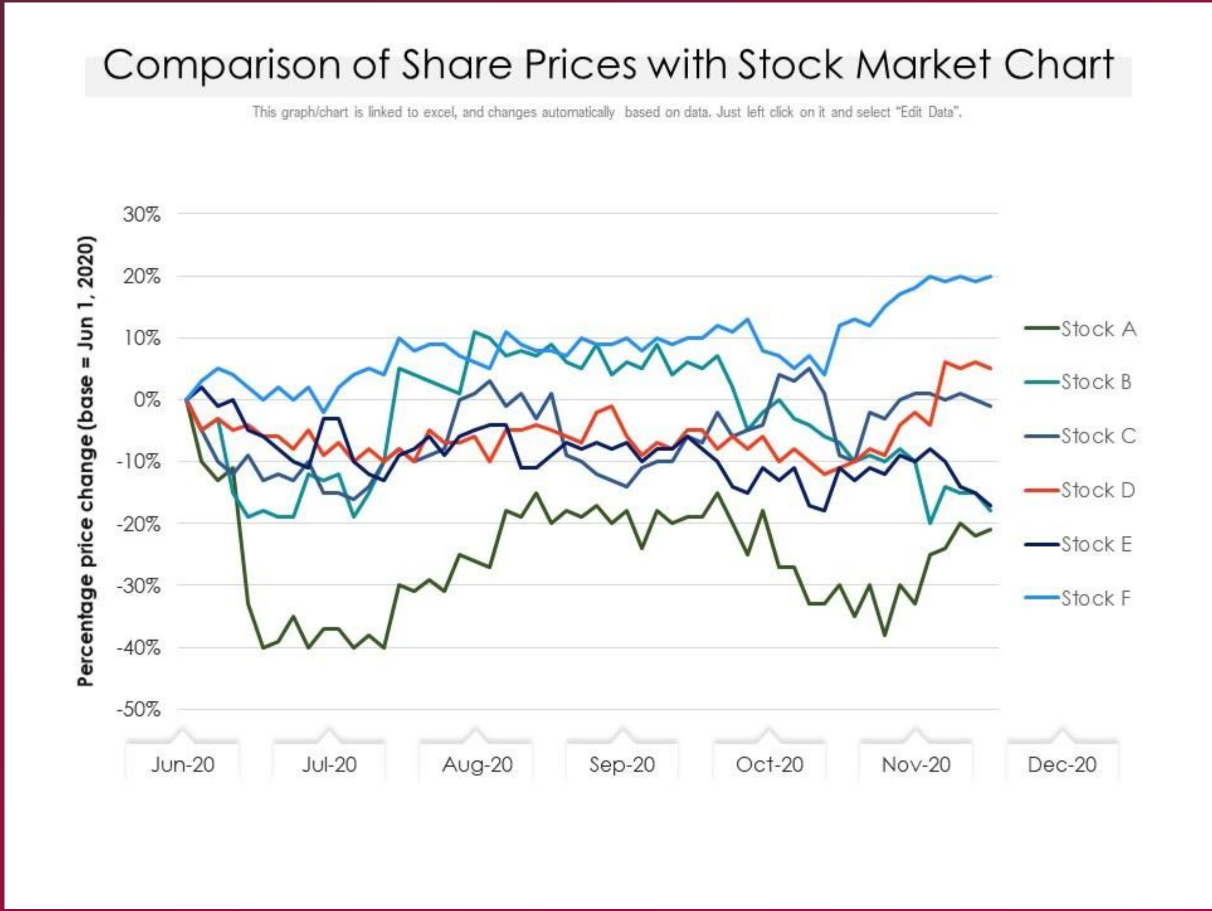
- **Comparing trends over time across different regions/products**
  - Comparing how multiple stock- markets perform across similar time periods
  - Comparing product sentiment over time across different types of bread
- **Forecasting future sales or population**
  - Forecasting population growth
  - Forecasting how much impact an epidemic might have in region
- **Detecting future changes in system behavior**
  - Price spikes in grain
  - Stock market crashes
  - Public health crisis leveling out

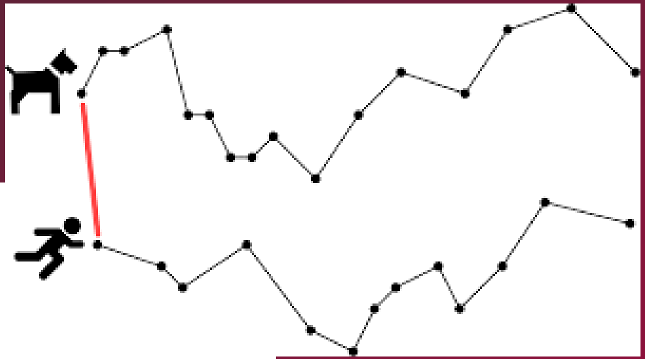
Comparing Time Series

- Stationary assumption
- Measurement at same time points
- Autocorrelation assumpitions
- No outliers
- Fréchet distance

Fréchet distance

In mathematics, the Fréchet distance is a measure of similarity between curves that takes into account the location and ordering of the points along the curves. It is named after Maurice Fréchet. Wikipedia

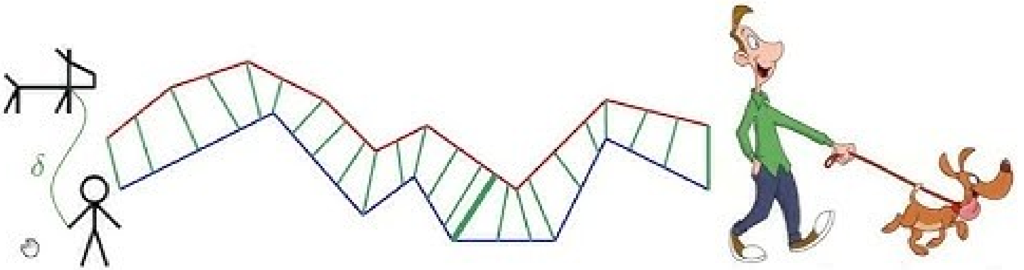


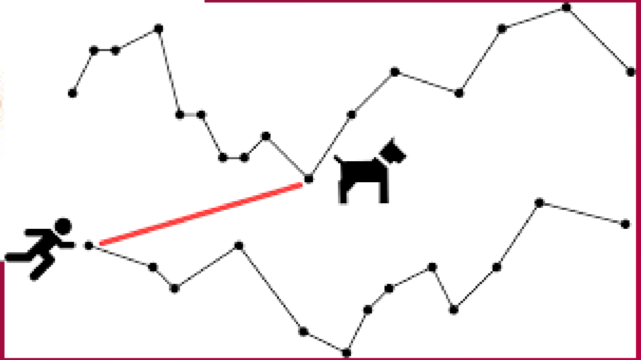


The (continuous) Fréchet distance

Two polygonal curves, A and B in $\mathbb{R}^{d \times m}$.
A **man** and a **dog** connected by a leash of length δ,
walking along the curves A and B, respectively, no backtracking.

- Fréchet distance: the **minimum** δ that is sufficient for traversing the curves.





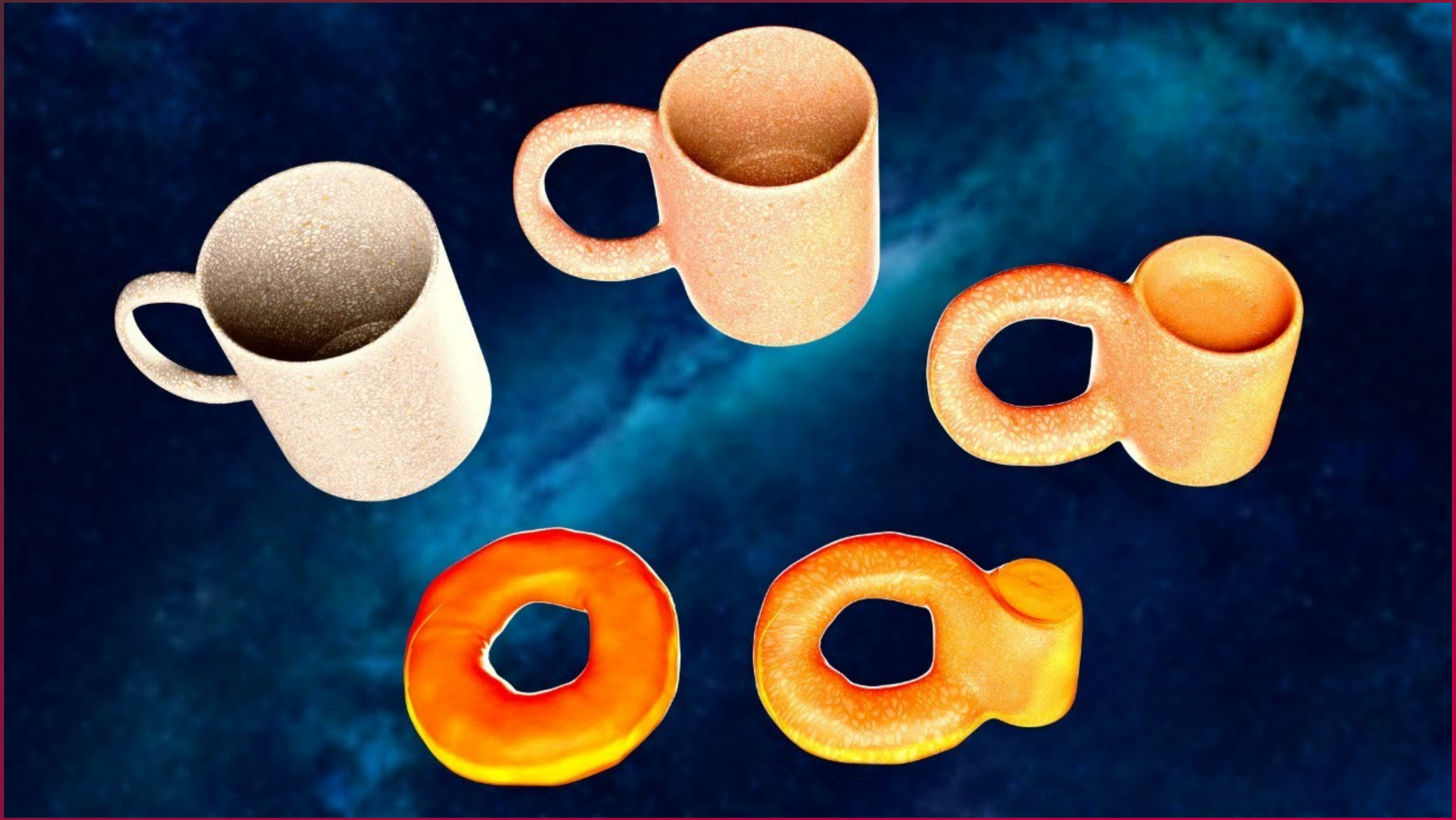
Time Series as Dynamic System

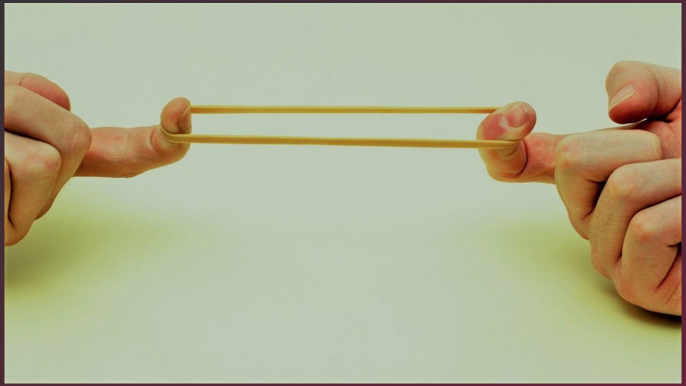
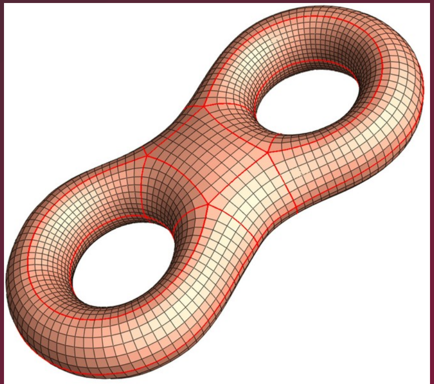
- Sampling of dynamic system at different or similar intervals of time
- Comparison of the systems themselves useful in comparison tasks
- Allows for the use of a field called topology to model the components of the system underlying the time series data

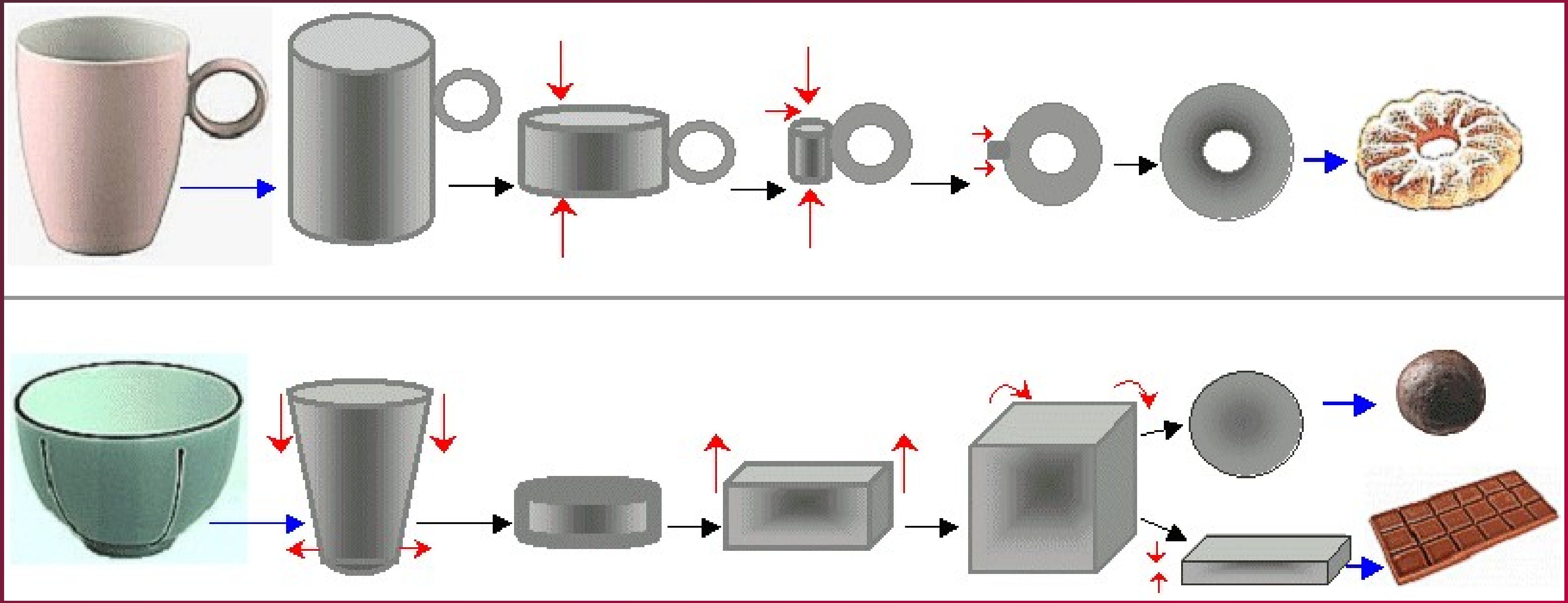
- What is the difference between a doughnut and a cup of coffee?
- How are a balloon and a cup of soup similar?

Topology.....?

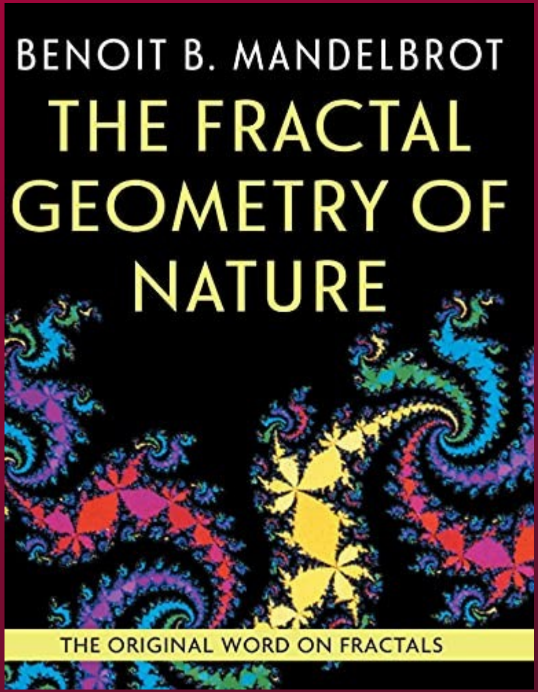
- <u>**Global properties of a space**</u>
  - *Many invariants (hole-counting)*
  - *Not concerned with distances or curvature*
- <u>**Donuts and coffee mug example**</u>
  - *Solid ring with a hole in the middle*





Topology, as a geometry, is extended to the definition of fractal structures by characterising equivalences in different levels of detail. A group of distinct topological fractals are defined by taking the equipotence and multiplicity properties of sets.





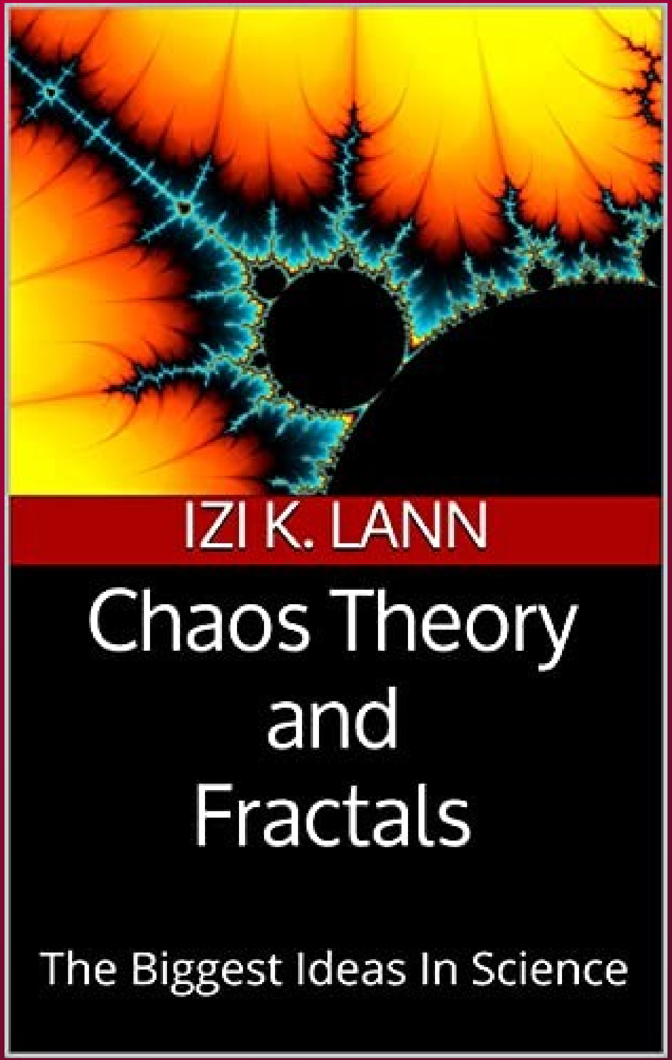


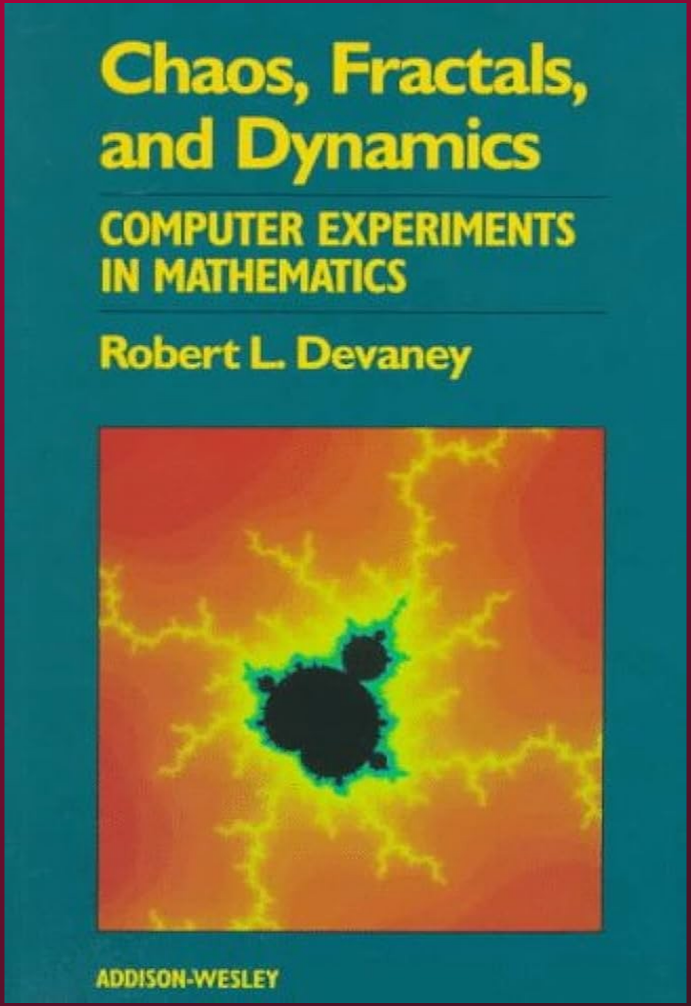









IZI K. LANN
Chaos Theory
and
Fractals
The Biggest Ideas In Science


Chaos, Fractals,
and Dynamics
COMPUTER EXPERIMENTS
IN MATHEMATICS
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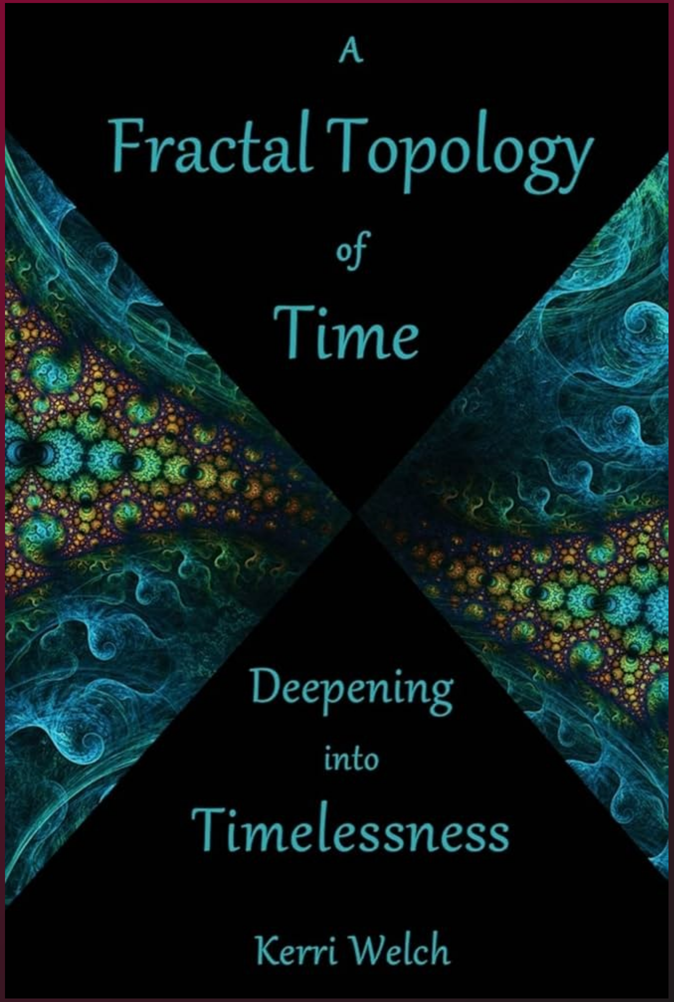
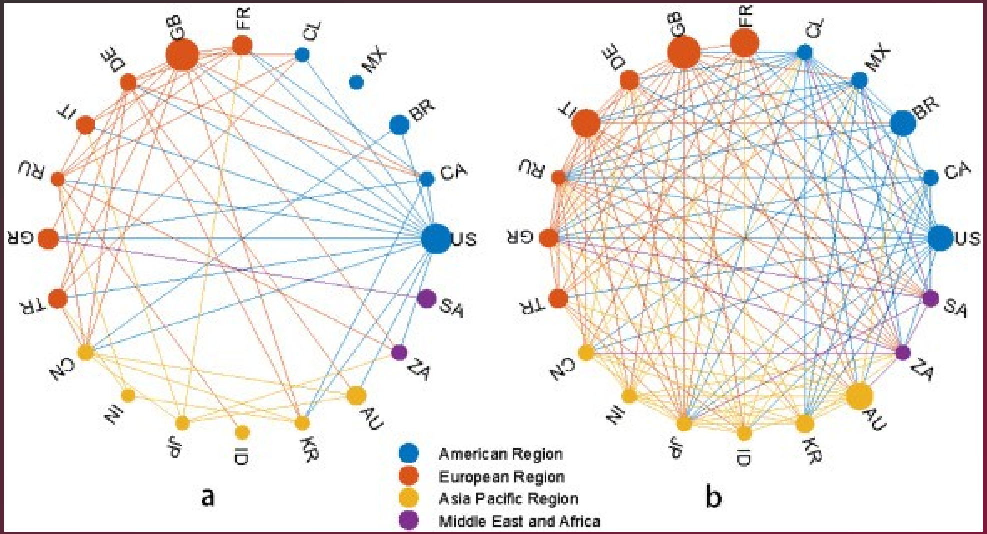
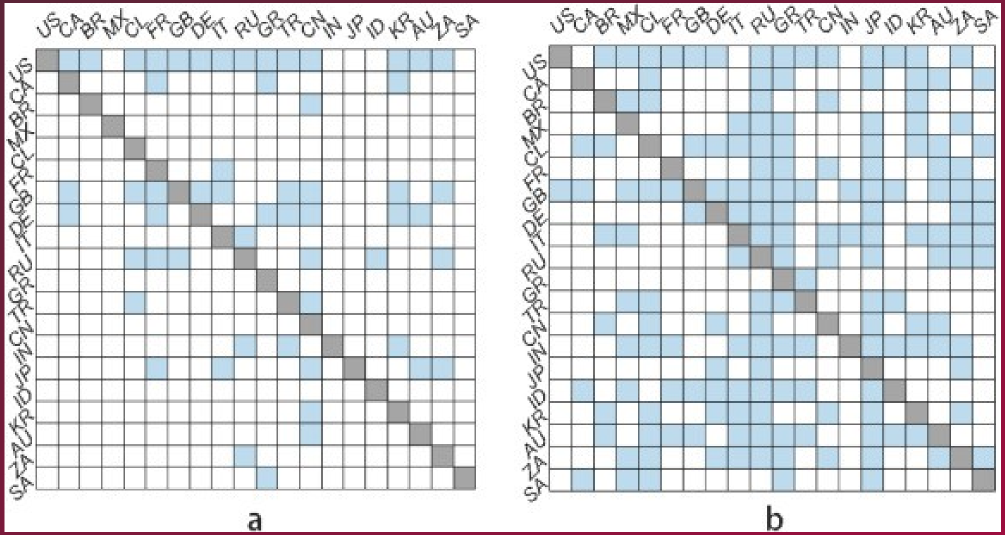

A
Fractal Topology
of
Time
Deepening
into
Timelessness
Kerri Welch

Some applications on time series….

Stock markets...





ELSEVIER
Physica A: Statistical Mechanics and its Applications
Volume 566, 15 March 2021, 125613

A study of systemic risk of global stock markets under COVID-19 based on complex financial networks

Yujie Lai [a], Yibo Hu [b]

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Highlights

- The small-world phenomenon is significant during the fluctuation period.
- Network centrality is enhanced under COVID-19.
- The global financial network is at high risk.
- This network can be employed for measuring and warning the systemic risk.

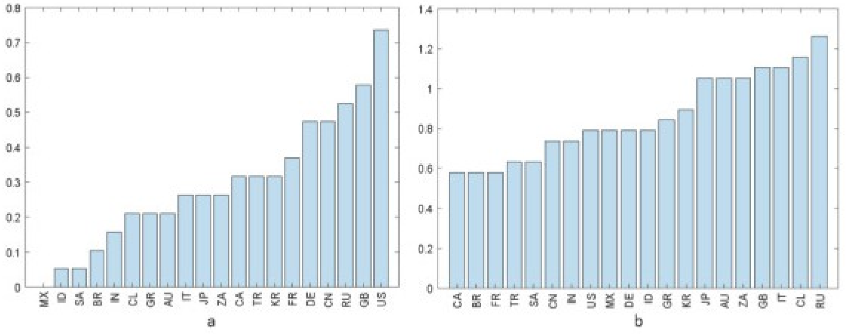

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Fig. 3. Degree centrality comparison of the stabilization period (a) and the fluctuation period (b).

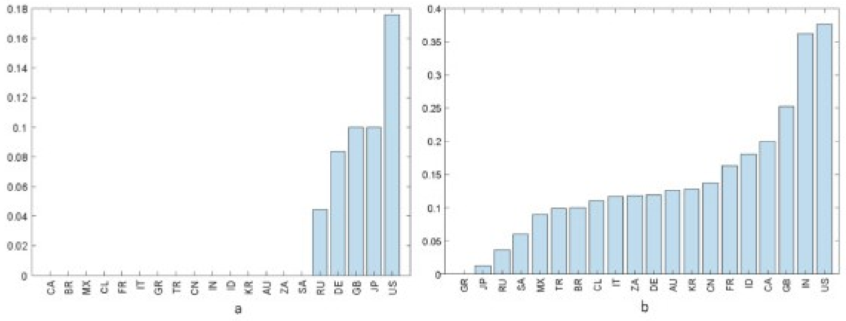

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Fig. 4. Clustering coefficients during (a) the stabilization period and (b) the fluctuation period.

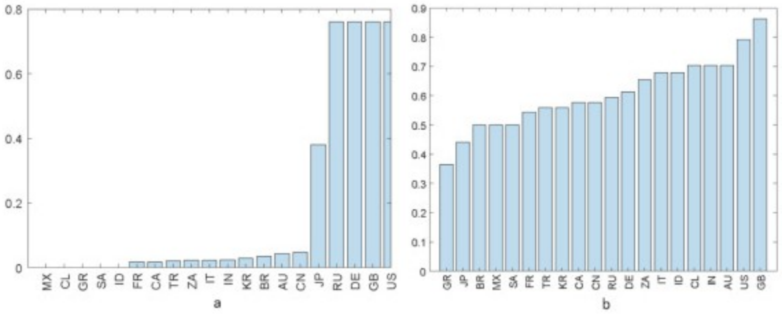

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Fig. 5. Comparison of the closeness centrality in (a) the stabilization period and (b) the fluctuation period.

Figure 4. A precrisis minimum spanning tree map of 181 stocks on the PSX network (8 March 2007 to 2 May 2008).

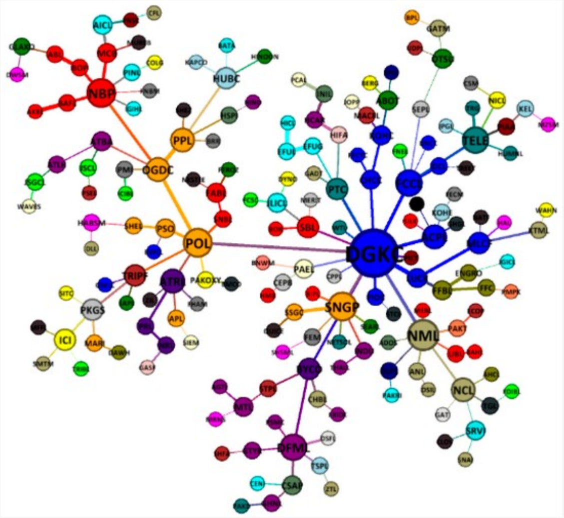
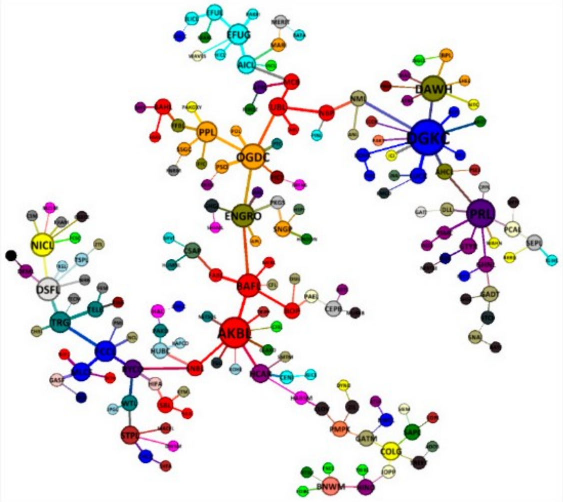


Figure 5. A crisis period minimum spanning tree map of 181 stocks on the PSX network (5 May 2008 to 30 June 2009).



Open Access Article

Structural Change and Dynamics of Pakistan Stock Market during Crisis: A Complex Network Perspective

by Bilal Ahmed Memon * and Hongxing Yao

School of Finance and Economics, Jiangsu University, Zhenjiang 212013, China

* Author to whom correspondence should be addressed.

Entropy **2019**, *21*(3), 248; https://doi.org/10.3390/e21030248

Submission received: 1 February 2019 / Revised: 1 March 2019 / Accepted: 3 March 2019 / Published: 5 March 2019

(This article belongs to the Section Complexity)

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Abstract

We studied the cross-correlations in the daily closing prices of 181 stocks listed on the Pakistan stock exchange (PSX) covering a time period of 2007–2017 to compute the threshold networks and minimum spanning trees. In addition to the full sample analysis, our study uses three subsamples to examine the structural change and topological evolution before, during, and after the global financial crisis of 2008. We also apply Shannon entropy on the overall sample to measure the volatility of individual stocks. Our results find substantial clustering and a crisis-like less stable overall market structure, given the external and internal events of terrorism, political, financial, and economic crisis for Pakistan. The subsample results further reveal hierarchal scale-free structures and a reconfigured metastable market structure during a postcrisis period. In addition, time varying topological measures confirm the evidence of the presence of several star-like structures, the shrinkage of tree length due to crisis-related shocks, and an expansion in the recovery phase. Finally, changes of the central node of minimum spanning trees (MSTs), the volatile stock recognition using Shannon entropy, and the topology of threshold networks will help local and international investors of Pakistan Stock Exchange limited (PSX) to manage their portfolios or regulators to monitor the important nodes to achieve stability and to predict an upcoming crisis.

Figure 6. A postcrisis minimum spanning tree map of 181 stocks on the PSX network (1 July 2009 to 19 August 2010).

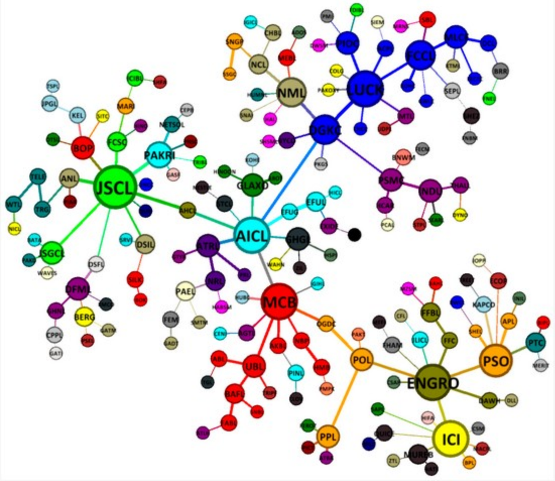
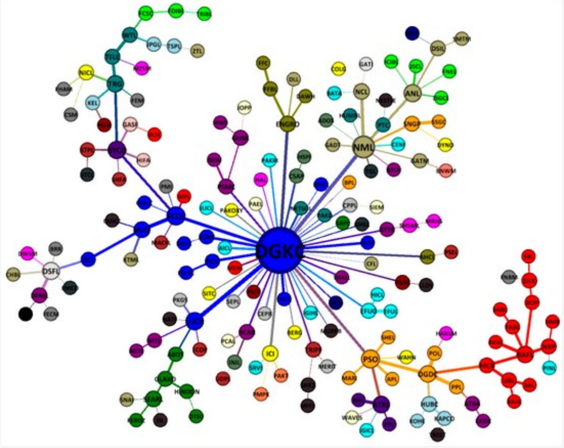


Figure 7. An overall-period star-like minimum spanning tree map of 181 stocks on the PSX (3 January 2007 to 29 December 2017).

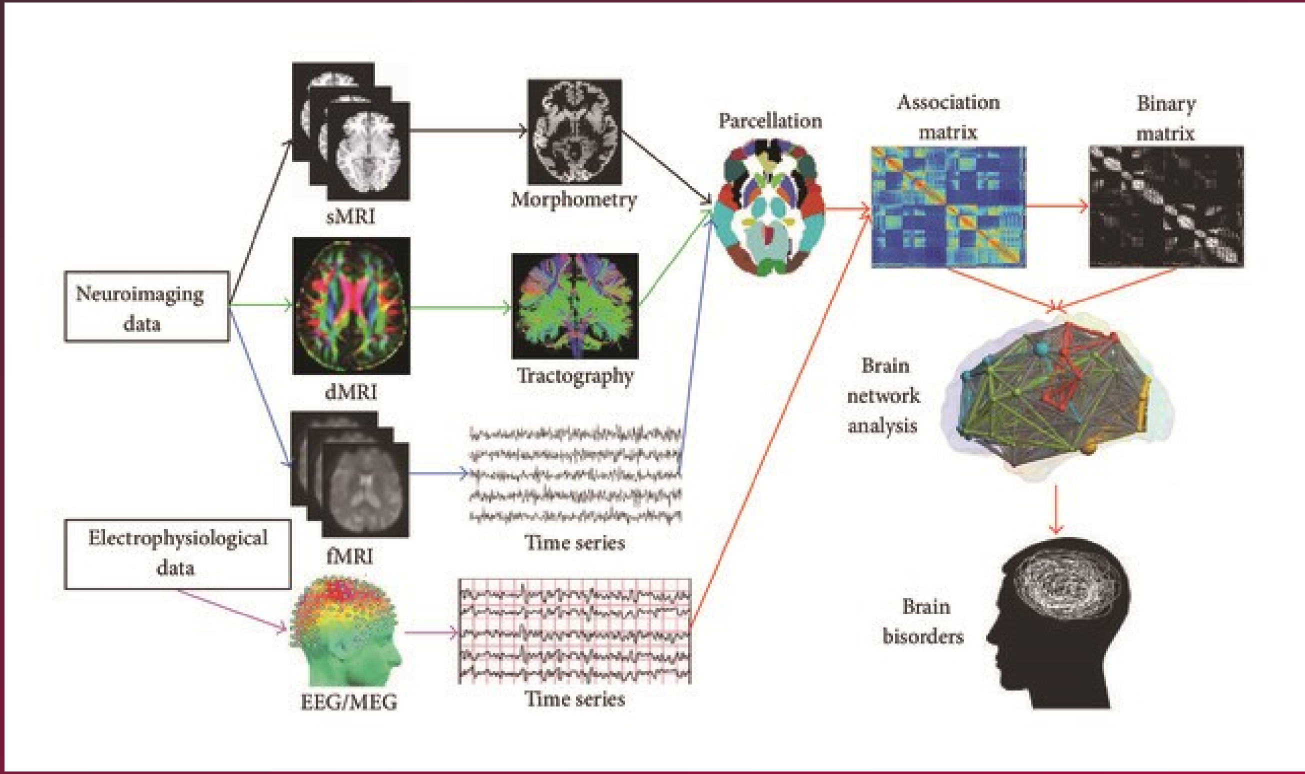


4.5. Scale-Free Structure of MSTs

Table 3. A list of the top five most and least volatile stocks of the Pakistan stock exchange based on the Shannon Entropy results.

Rank	Node	Sector	Entropy with bins 0.01	Entropy with bins 0.05
List of top five stocks with the highest Shannon entropy scores				
1	ICIBL	Inv. Banks/Inv. Cos./Securities Cos.	4.634	2.533
2	TSPL	Power Generation and Distribution	4.607	2.525
3	CSM	Modarabas	4.318	2.503
4	MZSM	Sugar and Allied Industries	4.245	2.226
5	SPLC	Leasing	4.209	2.324
List of top five stocks with the lowest Shannon entropy scores				
1	PSEL	Miscellaneous	1.694	0.887
2	GATI	Synthetic and Rayon	2.025	0.948
3	KAPCO	Power Generation and Distribution	2.415	1.111
4	CFL	Textile Spinning	2.421	1.389
5	SHEZ	Food and Personal Care Products	2.484	1.073

NeuroScience… Parkinson / brain disorders



Complexity

Forward Series

Review Article | Open Access

Complex Brain Network Analysis and Its Applications to Brain Disorders: A Survey

Jin Liu, Min Li, Yi Pan, Wei Lan, Ruiqing Zheng, Fang-Xiang Wu, Jianxin Wang

First published: 22 October 2017 | https://doi.org/10.1155/2017/8362741 | Citations: 84

Academic Editor: Manlio De Domenico

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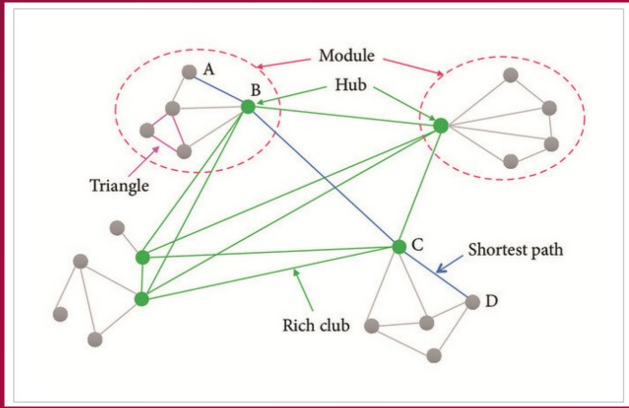
Abstract

It is well known that most brain disorders are complex diseases, such as Alzheimer's disease (AD) and schizophrenia (SCZ). In general, brain regions and their interactions can be modeled as complex brain network, which describe highly efficient information transmission in a brain. Therefore, complex brain network analysis plays an important role in the study of complex brain diseases. With the development of noninvasive neuroimaging and electrophysiological techniques, experimental data can be produced for constructing complex brain networks. In recent years, researchers have found that brain networks constructed by using neuroimaging data and electrophysiological data have many important topological properties, such as small-world property, modularity, and rich club. More importantly, many brain disorders have been found to be associated with the abnormal topological structures of brain networks. These findings provide not only a new perspective to explore the pathological mechanisms of brain disorders, but also guidance for early diagnosis and treatment of brain disorders. The purpose of this survey is to provide a comprehensive overview for complex brain network analysis and its applications to brain disorders.



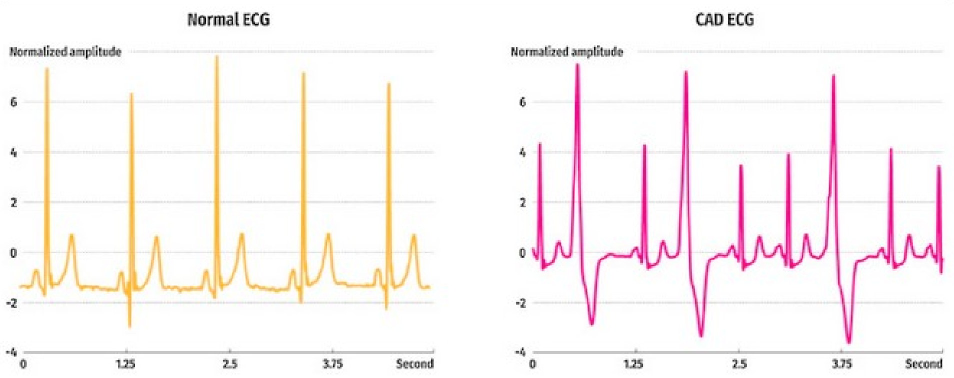


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Fig. 2. A sample normal and CAD 5-s ECG segment.

Biocybernetics and Biomedical Engineering

Volume 38, Issue 2, 2018, Pages 373-384

Review Article

Entropies for automated detection of coronary artery disease using ECG signals: A review

Udyavara Rajendra Acharya [a b c], Yuki Hagiwara [a], Joel En Wei Koh [a], Shu Lih Oh [a], Jen Hong Tan [a], Muhammad Adam [a], Ru San Tan [d e]

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Electro-Cardiograms disease….

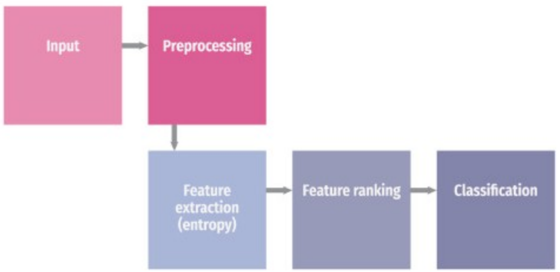

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Fig. 3. A generalized block diagram of a CADS.

3.2.1. Shannon entropy (E_{sh}) [37]

The E_{sh} is the fundamental entropy formula of information theory. It is a measure of the probability distribution of information obtained from the input ECG segment. Eq. (1) shows the formula to compute E_{sh}.

$$E_{sh} = -\sum_{n=1}^{x} E_n \log_2 E_n \quad (1)$$

where E_n is the probability of occurrence of the feature value, p_n being an element of the feature P that takes values $\{p_1, \ldots, p_x\}$.

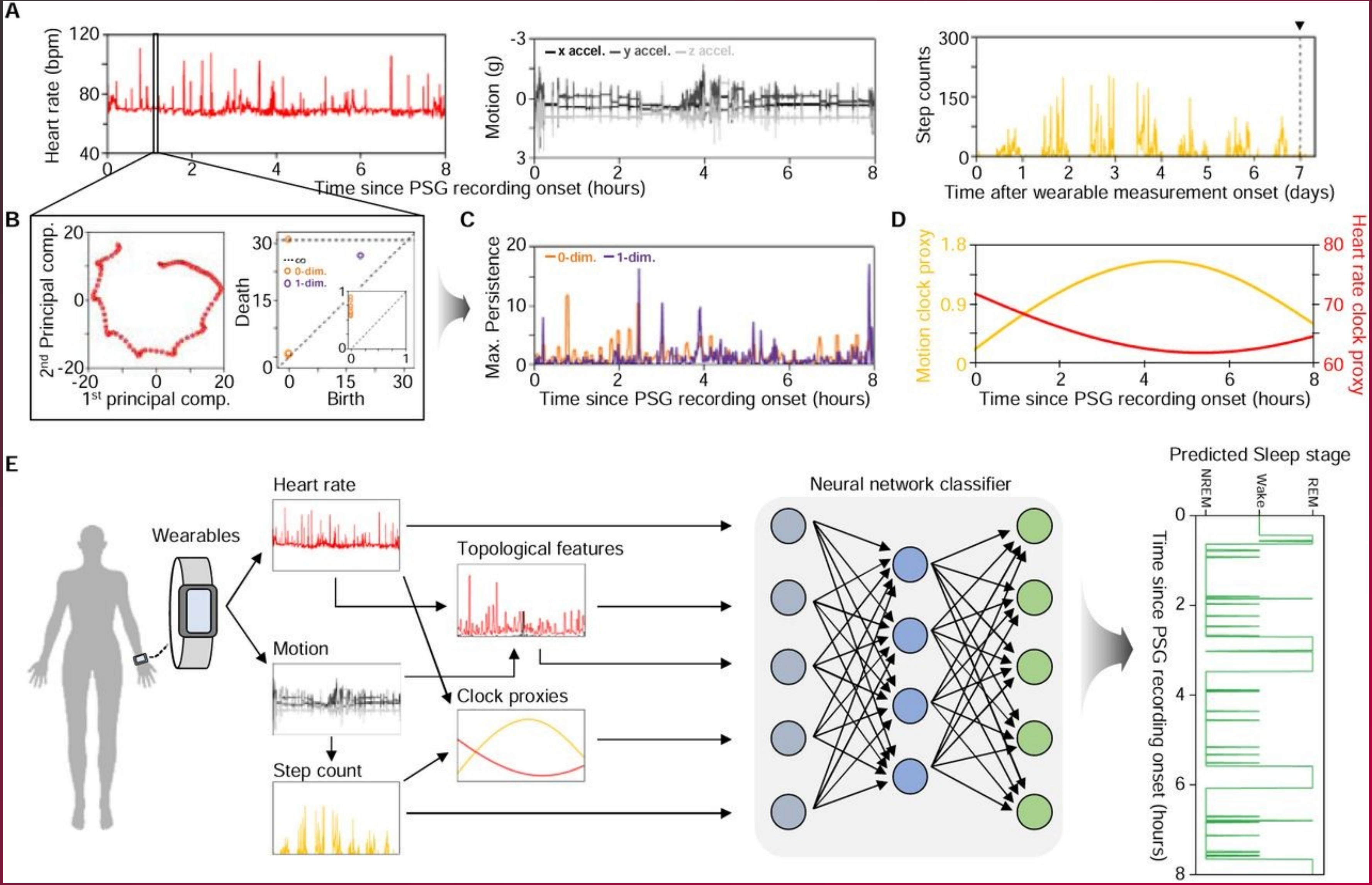
The following entropies – Tsallis and Renyi entropies are generalized form of E_{sh} and these entropies can be simplified to E_{sh} when α=1. The formulae are given in Eqs. (2), (3) respectively.

3.2.2. Tsallis entropy (E_{ts}) [38]

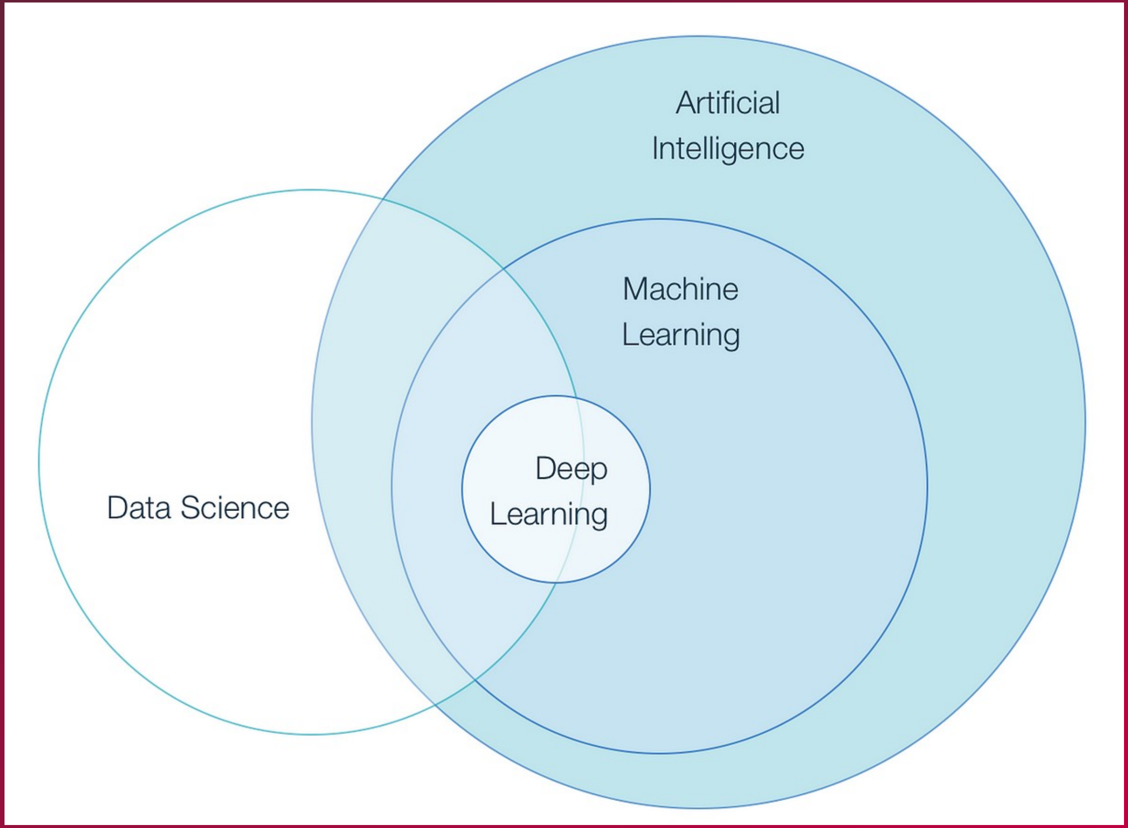
$$E_{ts} = \frac{1}{\alpha-1}\left(1 - \sum_{n=1}^{x} E_n^{\alpha}\right), \text{for} \alpha \neq 1 \quad (2)$$

3.2.3. Renyi entropy (E_r) [39]

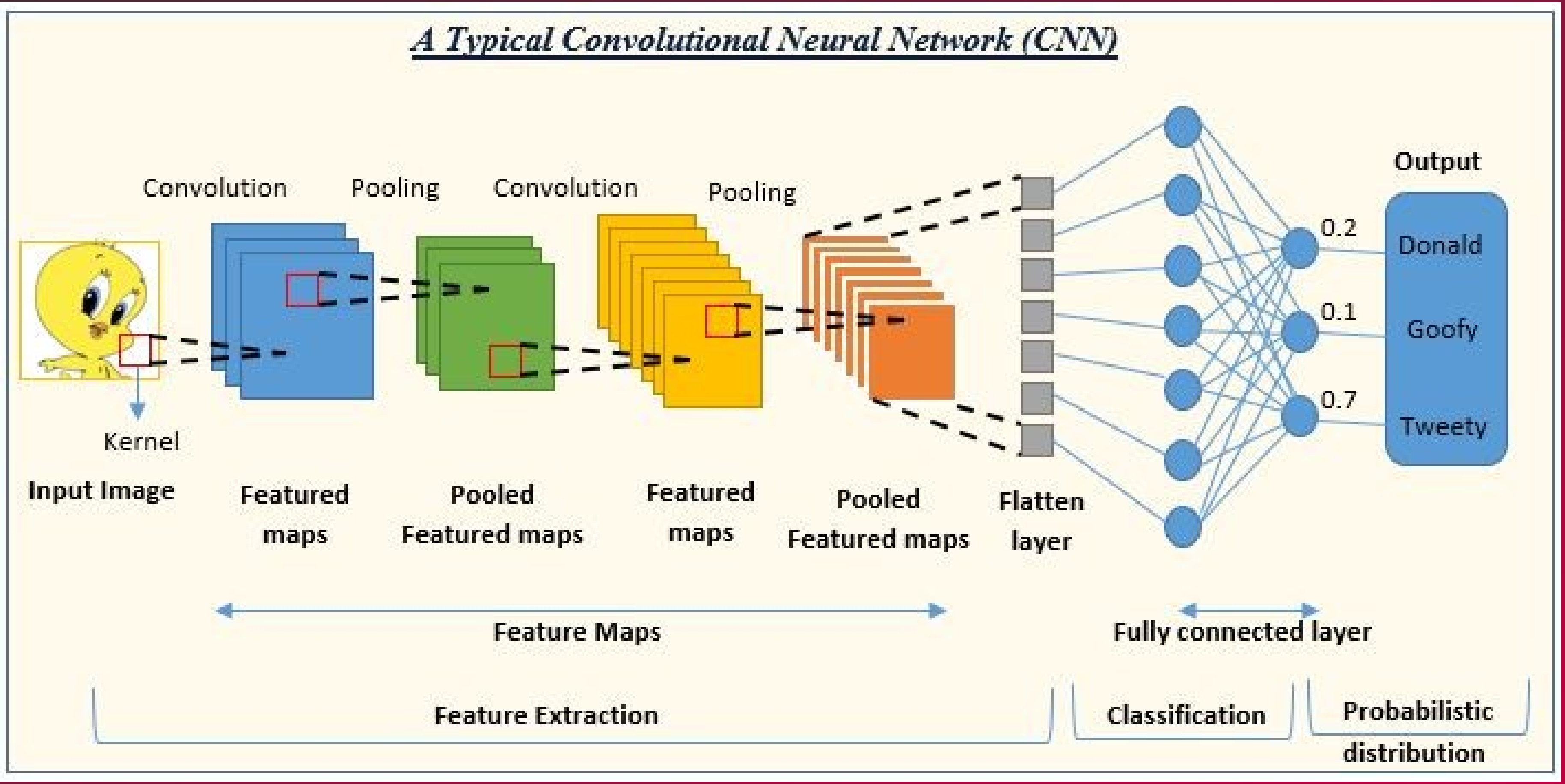
$$E_r = \frac{1}{\alpha-1}\log\left(\sum_{n=1}^{x} E_n^{\alpha}\right), \text{for} \alpha \neq 1 \quad (3)$$

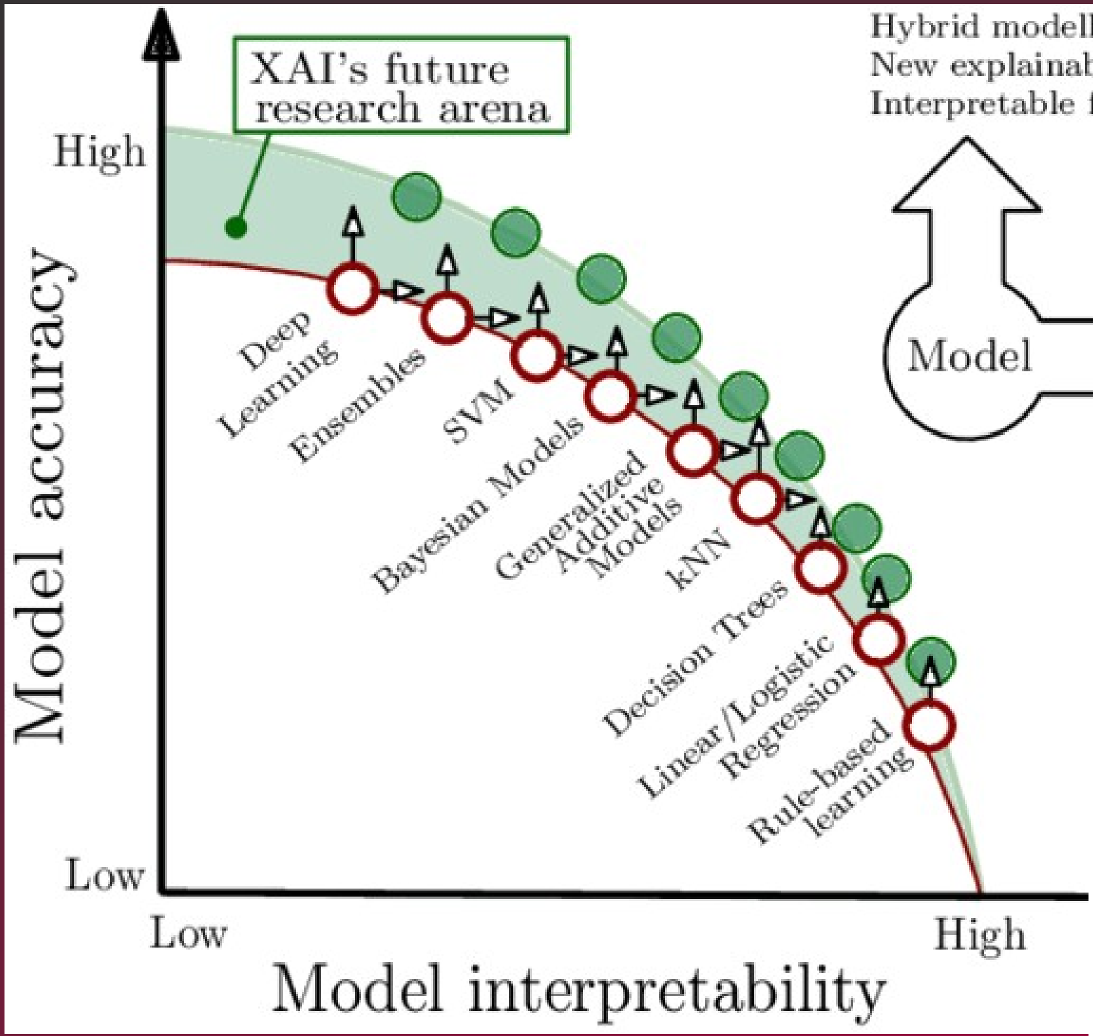

A
Heart rate (bpm)
Motion (g)
x accel. y accel. z accel.
Step counts
Time since PSG recording onset (hours)
Time after wearable measurement onset (days)
B
2nd Principal comp.
1st principal comp.
Death
Birth
∞
0-dim.
1-dim.
C
Max. Persistence
0-dim. 1-dim.
Time since PSG recording onset (hours)
D
Motion clock proxy
Heart rate clock proxy
Time since PSG recording onset (hours)
E
Wearables
Heart rate
Motion
Step count
Topological features
Clock proxies
Neural network classifier
Predicted Sleep stage
NREM
Wake
REM
Time since PSG recording onset (hours)

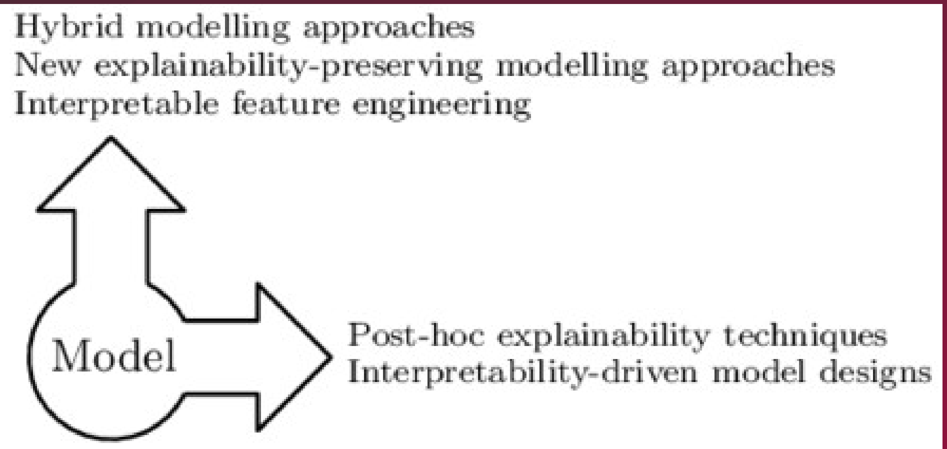
Some applications on image processing and classification….

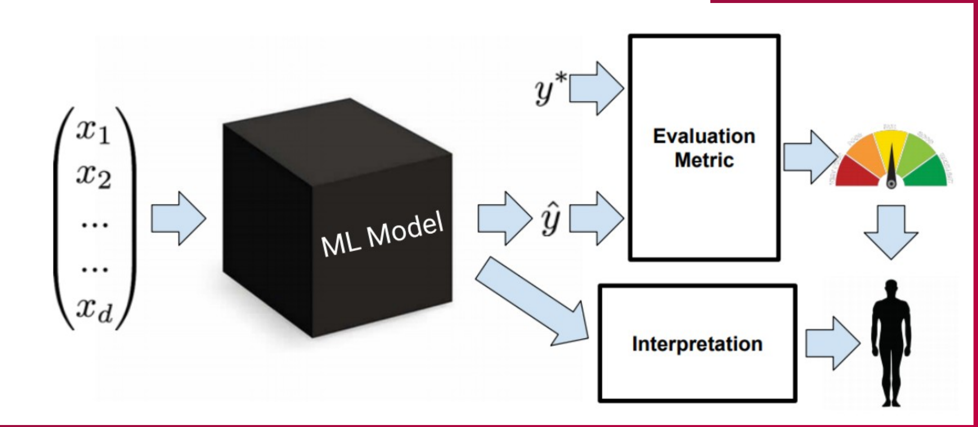


Machine learning (ML) methods such as convolutional neural networks (CNNs) are now the dominant technique for many such tasks, where they operate as black boxes




XAI's future research arena
High
Low
Model accuracy
Deep Learning
Ensembles
SVM
Bayesian Models
Generalized Additive Models
kNN
Decision Trees
Linear/Logistic Regression
Rule-based learning
Low
High
Model interpretability


Hybrid modelling approaches
New explainability-preserving modelling approaches
Interpretable feature engineering
Model
Post-hoc explainability techniques
Interpretability-driven model designs


x_1
x_2
x_d
ML Model
y^*
$\hat{y}$
Evaluation Metric
Interpretation

Advantages of TDA for image analysis tasks

- Transparency: All the internal steps of the algorithm are known and well-understood, and the method has a high degree of theoretical interpretation, giving it far more transparency than most ML methods.
- Known failure modes: No technique is perfect, and there will always be situations that cause errors and incorrect results. To use a method in practice, it is important to understand in what situations it struggles and what sorts of errors can result. Because persistent homology is a deterministic method, we can both theoretically predict these failure modes and interpret experimental results in terms of the original feature space.
- No need for large, labeled datasets: As a deterministic algorithm, persistent homology does not require large, reliably labeled datasets. Instead, a small set of representative examples can be used to explore the different patterns that emerge in the transformed data. TDA is often used in combination with a simple ML model, and the number of labeled samples to obtain good performance is smaller than would be required to train a CNN or similar tool without TDA. This is a huge advantage for environmental science datasets, which are frequently large and detailed but almost entirely unlabeled.
- Environmentally friendly: Many CNNs for image analysis tasks are known to have a surprisingly high carbon footprint due to the extensive computational resources required for model training. TDA is more in line with the Green AI movement, as it enables context-driven numerical results without the environmental impact inherent in training a deep neural network.

Next, we discuss the key abilities that persistent homology brings to image analysis tasks. These fall into three general categories: the incorporation of spatial context into a deterministic algorithm, the detection of texture and contrast, and invariance under certain transformations. The categories are discussed further below:

- Incorporating spatial context: Many deterministic algorithms, as well as fully connected neural networks, struggle to incorporate the spatial context inherent in satellite data. Integrating this spatial context is precisely what motivated the development of CNNs, but CNNs are costly to train and challenging to make explainable. Persistent homology naturally incorporates spatial context, so patterns that are evident in this spatial context can be incorporated without resorting to CNNs or other spatially informed neural network architectures.

- Detection of texture and contrast: Persistent homology excels at detecting contrast differences—regions (small or large) that differ from the surrounding average, which gives a representation of the texture present in an image. This focus on texture is useful in analyzing satellite weather imagery, as texture is frequently a key distinguishing factor, even more than a cloud being a particular shape or size.

- Invariance to homeomorphisms: The notion of not wanting to be constrained by a particular geometry brings us to the final advantage: invariance under a common class of transformations called homeomorphisms.

Combining TDA with simple ML algorithms

- For some image analysis tasks, TDA methods can be used as a stand-alone tool, but for the majority of tasks, one would first use TDA to extract topological features and then afterward add a simple machine learning algorithm. TDA can thus be viewed as a transparent means to construct new, physically meaningful, and interpretable features that may reduce the need for black-box machine learning algorithms. Using TDA in this way can support the goals of creating ethical, responsible, and trustworthy artificial intelligence approaches for environmental science outlined in, since transparency is a key requirement for ML approaches to be used in tasks that affect life-and-death decision-making, such as severe weather forecasting.
-
- Two different ways to extract desired information from imagery: (a) the pure ML approach, in which image information is extracted by using a complex ML model, typically a deep neural network, and (b) the TDA approach, in which image information is extracted by using TDA followed by a simpler ML model/method. The latter can lead to more transparent and computationally efficient approaches.

Feature extraction...

Taruma's Features

Contraste de Taruma:

Mide la diferencia de intensidad entre píxeles vecinos en una imagen. Cuanto mayor es el contraste, más marcadas son las transiciones entre las regiones claras y oscuras en la imagen.Se calcula como la media de los cuadrados de los gradientes de la imagen.

$$\text{Taruma Contrast} = \sum_{i,j=0}^{N_g-1} (i-j)^2 \cdot P(i,j)$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.

$$\text{Taruma Directionality} = \sum_{i,j=0}^{N_g-1} (i-j)^2 \cdot P(i,j) \cdot \cos(\theta_{i,j})$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.
- $\theta_{i,j}$ es el ángulo entre los píxeles con niveles de gris i y j.

Directionality de Taruma:

Indica la dirección predominante de las transiciones de intensidad en una imagen. Se calcula como la desviación estándar de los ángulos de los gradientes de la imagen. Una dirección más alta sugiere que las transiciones son más uniformes en una dirección específica.

Taruma's Features

Coarseness de Taruma:

Representa la textura gruesa o fina de una imagen. Una coarseness alta indica una textura más gruesa, con cambios abruptos en la intensidad. Se calcula como la varianza de la imagen suavizada con un filtro de desenfoque.

$$\text{Taruma Coarseness} = \sum_{i,j=0}^{N_g-1} |i-j| \cdot P(i,j)$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.

$$\text{Taruma Linelikeness} = \sum_{i,j=0}^{N_g-1} (i-j)^2 \cdot P(i,j)$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.

Linelikeness de Taruma:

Mide la presencia y la prominencia de líneas o bordes en una imagen. Cuanto mayor es el valor, más líneas distintas se encuentran en la imagen. Se calcula como la media de la imagen después de aplicar el detector de bordes de Canny.

Taruma’s Features

Regularity de Taruma:

Describe la uniformidad de la textura en una imagen. Una regularidad alta indica una textura más uniforme sin cambios bruscos. Se calcula como la desviación estándar de la imagen suavizada con un filtro de desenfoque.

$$\text{Taruma Regularity} = \sum_{i,j=0}^{N_g-1} \frac{P(i,j)}{1+(i-j)^2}$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.

$$\text{Taruma Roughness} = \sum_{i,j=0}^{N_g-1} \frac{P(i,j)\cdot(i-j)^2}{1+(i-j)^2}$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.

Roughness de Taruma:

Representa la irregularidad o rugosidad de una imagen. Una roughness alta indica una superficie más desigual o textura más rugosa. Se calcula como la desviación estándar de la intensidad de los píxeles en la imagen.

Haralick's Features

Homogeneidad de Haralick:

Mide la proximidad de la distribución de intensidad de los píxeles al máximo valor posible. Cuanto mayor es el valor, más uniforme es la textura de la imagen.

$$\text{Homogeneity} = \sum_{i=0}^{N_g-1} \sum_{j=0}^{N_g-1} \frac{P(i,j)}{1+(i-j)^2}$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.

$$\text{Entropy} = -\sum_{i=0}^{N_g-1} \sum_{j=0}^{N_g-1} P(i,j) \log_2(P(i,j))$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.

Entropía de Haralick:

Describe la desorden en la distribución de intensidad de los píxeles. Un valor alto indica una textura más compleja y aleatoria.

Haralick's Features

Correlación de Haralick:

Describe la correlación lineal de la distribución de intensidad de los píxeles. Un valor alto sugiere una imagen más homogénea, donde los píxeles tienen intensidades similares.

$$\text{Correlación} = \sum_{i=0}^{N_g-1} \sum_{j=0}^{N_g-1} \frac{(i-\mu)(j-\mu)P(i,j)}{\sigma^2}$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.
- μ es la media de la distribución de los niveles de gris.
- σ^2 es la varianza de la distribución de los niveles de gris.

$$\text{Energía} = \sum_{i=0}^{N_g-1} \sum_{j=0}^{N_g-1} P(i,j)^2$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.

Energía de Haralick:

Representa la uniformidad de la distribución de intensidad de los píxeles. Un valor alto indica una imagen más uniforme, donde la mayoría de los píxeles tienen la misma intensidad.

Another important Features

Dimensión de Correlación:

- La dimensión de correlación es una medida de la autocorrelación espacial de una imagen.
- Se calcula utilizando métodos como la función de correlación de pares o la función de estructura espacial.
- Esta dimensión indica cuánto se correlacionan los píxeles a diferentes distancias dentro de la imagen.
- Una imagen con una mayor dimensión de correlación tiene más autocorrelación espacial y estructuras repetitivas.

$$\text{Correlación} = \sum_{i=0}^{N_g-1} \sum_{j=0}^{N_g-1} \frac{(i-\mu)(j-\mu)P(i,j)}{\sigma^2}$$

Donde:

- N_g es el número de niveles de gris en la imagen.
- $P(i,j)$ es la probabilidad conjunta de que un píxel con nivel de gris i y otro píxel con nivel de gris j aparezcan en la imagen, según la matriz GLCM.
- μ es la media de la distribución de los niveles de gris en la imagen.
- σ^2 es la varianza de la distribución de los niveles de gris en la imagen.

Another important Features

Dimensión Fractal con Box Counting:

- La dimensión fractal es una medida de la complejidad geométrica y la irregularidad de una estructura.
- El método de Box Counting es una técnica común para estimar la dimensión fractal de una imagen.
- Consiste en dividir la imagen en cuadrados de diferentes tamaños y contar cuántos cuadrados son necesarios para cubrir la imagen.
- La dimensión fractal se calcula como el límite de la relación entre el logaritmo del número de cuadrados necesarios y el logaritmo del tamaño del cuadrado cuando el tamaño del cuadrado tiende a cero.
- Una imagen con una dimensión fractal mayor indica una mayor irregularidad y detalle en su estructura, mientras que una dimensión fractal menor sugiere una estructura más suave y regular.

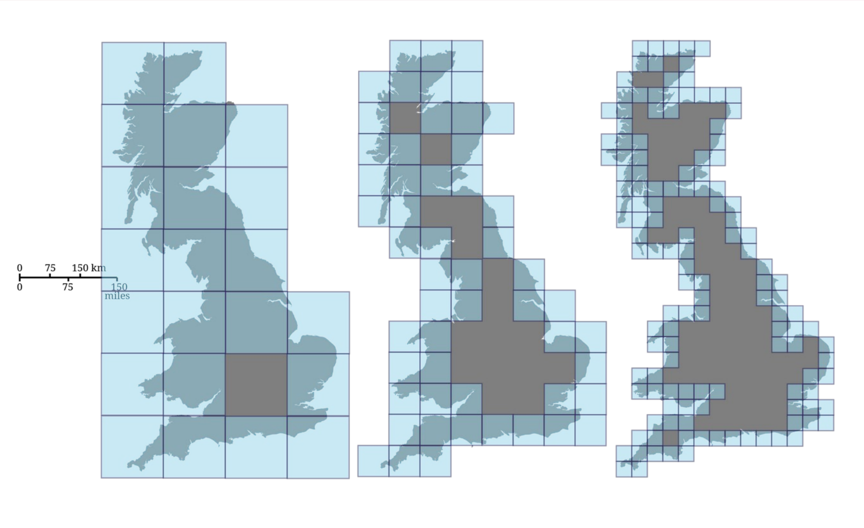
La fórmula básica para calcular la dimensión fractal D utilizando el método de Box Counting es:

$$D = \lim_{\epsilon \to 0} \frac{\log(N(\epsilon))}{\log(\frac{1}{\epsilon})}$$

Donde:

- $N(\epsilon)$ es el número mínimo de cuadros necesarios para cubrir completamente el objeto en una escala ϵ.
- ϵ es el tamaño del cuadro.

Para calcular $N(\epsilon)$, primero dividimos la imagen en una cuadrícula de cuadros de tamaño ϵ. Luego, contamos el número de cuadros que contienen al menos un punto del objeto. A medida que ϵ se hace más pequeño, el número de cuadros necesarios para cubrir el objeto aumenta, y al tomar el límite cuando ϵ tiende a cero, podemos estimar la dimensión fractal D.



Vamos al asunto con Notebook...

Muchas gracias por las atenciones….

D. Sierra-Porta, UTB