PHYS 357 Pset 9. Due 11:59 PM Thursday Nov. 21

1. Townsend 6.4.

2. Balancing a pencil. What's the longest can you balance a pencil on its tip? The Heisenberg uncertainty principle means I can't start with both $x = 0$ and $p = 0$. I'm after a ballpark answer here, so you can e.g. assume all the mass in the pencil is at the end away from the tip. You can use 10g and 20cm for the pencil weight/length. NB - a pencil balanced on its tip looks very much like a pendulum, but with a sign change in the equation of motion.

3. Townsend 6.6

4. Townsend 6.12

5. Repeat Townsend 6.4, but this time do it on a computer. We can approximate a free particle by having a long stretch of space with zero potential. Find the eigenvalues/eigenvectors of this free space (this is fastest using scipy.linalg.eigh_tridiagonal, but you could certainly use numpy.linalg.eigh if you wanted), and describe a Gaussian well away from the boundary region as the sum of these eigenmodes. Make a movie showing the evolution of the Gaussian as it spreads out. Does your time for the width to double agree with your calculation from 6.4? Now make the same movie, but using a boxcar initial wave function ($\Psi(x) = 1$ for $0 < x < 1$, and zero otherwise).

6. Bonus - repeat the previous problem, but now use the Fourier transforms built into numpy (numpy.fft.rfft and irfft will be easiest to use). The discrete Fourier transform is defined to be

$$\sum_{0}^{N-1} f(x) \exp(-2\pi i k x/N)$$

for integer x, k, and $0 \leq x, k < N$. You'll need to take care with numerical factors and tune the spacings of your points, but you'll see that you can handle these free-space questions much, much faster with FFTs than with direct matrix inversions.