PHYS 357 Pset 2. Due 11:59 PM Thursday Sep. 19

1. Solve for the eigenvalues of a 2x2 matrix A in terms of the trace and determinant of A. Use this expression to show that the trace is the sum of the eigenvalues.

   If you need a hint to get started, we know that

   $$\mathrm{A}v = \lambda v$$

   so

   $$\mathrm{A} - \lambda \mathrm{I} v = 0$$

   For that to be true for non-zero v, $\mathrm{A} - \lambda\mathrm{I}$ must be singular, so its determinant must be zero.

2. Explicitly show using the definition of matrix multiplication that $(\mathrm{AB})^\dagger = \mathrm{B}^\dagger \mathrm{A}^\dagger$

3. Townsend 2.8

4. A) Working in the $z-$basis, express the projection operators $|+y\rangle\langle+y|$ and $|-y\rangle\langle-y|$ as 2x2 matrices.

   B) Show that the $+y$ projection matrix times an arbitrary vector (a, b) outputs a vector that is proportional to $|+y\rangle$ (*i.e.* it comes out as (c, ic) for some value c). Show the same for the $-y$.

5. A) For an arbitrary state $|+n\rangle, |-n\rangle$, write down the 2x2 projection operators in the $z-$basis. As a reminder, you can look at Townsend problem 1.3 for the state in an arbitrary direction.

   B) Show that the sum of these two matrices is the identity matrix. We expect this because the $|+n\rangle$ component of a state plus the $|-n\rangle$ component must give us the state we started with.

6. A) Work out the angular momentum operators J_x, J_y in the $z-$basis. Verify that they are Hermitian. If you want to do this on a computer, that's fine, but include the (very short!) code you used to generate them, and comment what you are doing.

   B) Work out the angular momentum operators J_x, J_y, J_z in the $|\pm y\rangle$ basis. Again, verify that they are Hermitian.

7. Work our the $\pi/2$ rotation matrix about the $y-$axis in the $|\pm z\rangle$-basis. Do this two ways - first by writing down what this matrix has to do to the $|+x\rangle$ and $|+z\rangle$ states. Then by combining the matrices that turn a state represented in the $\pm z$-basis into the $\pm y$-basis, the matrix that rotates about its own axis (the rotation about $|+n\rangle$ represented in the $|\pm n\rangle$ basis can't depend on $|n\rangle$), and the matrix that converts states in the $|\pm y\rangle$ back into the $|\pm z\rangle$ basis. Show that these matrices are the same, possibly up to an overall phase factor.