PHYS 357 Pset 11. Due before the final.

This is an optional problem set where each question will be graded pass/fail. If you make a good-faith attempt, it will be deemed a "pass" for that question. This problem set can replace one from earlier in the term where you did worse. I suggest you try to do this early (in the coming week) but there will be no lateness penalty if submitted before the final.

1. Show that we can write the Hamiltonian for a simple harmonic oscillator as $\hbar\omega(\mathrm{N} + 1/2)$ where $\mathrm{N} = a^\dagger a$.

2. Consider a quadratic potential with a barrier in the middle - $V(x) = \frac{1}{2}m\omega^2 x^2 + V_0 \exp(-x^2/2\sigma^2)$. If E_0, x_0 are the ground-state energy and width of the SHO with no barrier, then let V_0 be much, much greater than E_0, and σ be much, much less than x_0.

   A) On top of a plot of the potential, sketch the wave function for the ground state and first excited state of our new potential, including the barrier. Try to figure out what these wave functions must look like, noting that the potential is symmetric, before doing any calculations (but feel free to check numerically after you've attempted to draw them). We'll call the lowest-energy state $|0\rangle$ and the first excited state $|1\rangle$.

   B) Plot the sum/difference of the two lowest-energy eigenstates. We'll call these two states $|L\rangle$ and $|R\rangle$.

   C) If the two lowest energies are E and $E + dE$, what is the Hamiltonian if we restrict ourselves to states $|0\rangle$ and $|1\rangle$?

   D) What is the Hamiltonian if we instead write our state in terms of $|L\rangle$ and $|R\rangle$?

   E) If we start with a particle in state $|L\rangle$, how long does it take for its state to turn into $|R\rangle$?

   You might want to go back and look at Townsend section 4.5 now, where he considers an ammonia molecule with the nitrogen on one side of the hydrogen atoms. After some amount of time, the nitrogen atom will end up on the other side of the hydrogen, because nitrogen "above" the hydrogens is not an energy eigenstate. When the book gets to 4.5, you don't have the tools to understand why the Hamiltonian has to look like part D), but now you do.

3. Townsend 7.1. We didn't work out the matrix elements of the SHO raising/lowering operators in class, so now is your chance to do so. Since $H\,|n\rangle = \hbar\omega(n + 1/2)\,|n\rangle$ and

$H = \hbar\omega(N + 1/2)$, you know what $N|n\rangle$ is. You also know that $N = a^\dagger a$, which is all you need to work out what $a|n\rangle$ and $a^\dagger|n\rangle$ are.

4. Townsend 7.4

5. Townsend 7.13

6. In a harmonic oscillator, the time-average of the kinetic and potential energies are equal. Use this fact to show that $\sigma(x)\sigma(p) = \hbar(n+1/2)$ for eigenstate $|n\rangle$ of the harmonic oscillator. As expected, the ground state is a minimum uncertainty state, but the excited states are not.