PHYS 357 Pset 1. Due 11:59 PM Thursday Sep. 12

1. Starting with the expressions for the $|\pm Z\rangle$, $|\pm X\rangle$, and $|\pm Y\rangle$ states, show that the amplitude for a state to be in the opposite direction along the same axis is zero (*i.e.* $\langle +X| -X\rangle = 0$ *etc.*) for all three directions. Similarly, show that the *magnitude* of the amplitude for any state to be in any state along a different axis is $\frac{1}{\sqrt{2}}$ (*i.e.* $|\langle +Y| \pm X\rangle| = \frac{1}{\sqrt{2}}$ *etc.*). Feel free to express this as compactly as possible - I don't need 30 separate answers. Also, you are allowed to rely on the fact that $\langle \pm Z| \pm Z\rangle = 1$ and $\langle \mp Z| \pm Z\rangle = 0$.

2. Townsend problem 1.3

3. Townsend problem 1.6

4. a) If I have a unit vector pointing in the (θ, ϕ) direction, what are θ and ϕ for the unit vector pointing in the opposite direction? By opposite, if I expressed the vector in Cartesian coordinates $v = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$ then the vector in the opposite direction is $-v_x\hat{x} - v_y\hat{y} - v_z\hat{z}$.
   b) If your original unit vector gives you the $|+n\rangle$ state from problem 1.3, show that when you plug in the opposite direction, you get the $|-n\rangle$ state from problem 1.6.
   c) Show that a state (again as defined in problem 1.3) in the (θ', ϕ') direction can be expressed as the sum of the $|+n\rangle$ and $|-n\rangle$ states in the (θ, ϕ) direction (note the lack of primes on on these states), *i.e.*$|n'\rangle = c_+ |+n\rangle + c_- |-n\rangle$. You should get a 2x2 matrix equation for c_+ and c_-, feel free to leave your answer in this form.
   d) What is the determinant of the matrix from part c)? As a reminder, the determinant of a 2x2 matrix is $ad - bc$. You should get something with magnitude 1. That tells us that we can always solve the system of equations in part c), which in turn tells us that we can pick any arbitrary $|+n\rangle$ and $|n'\rangle$, and express $|n'\rangle$ as a sum of $|+n\rangle$ and $|-n\rangle$.

5. Townsend problem 1.13.

6. Townsend problem 1.15

7. Townsend problem 1.9