Phys 512 Problem Set 4

Due on github Tuesday October 8 at 11:50 PM. You may discuss problems, but everyone must write their own code. This problem set can take a while to do, so I suggest not leaving it until the last minute. It is also likely the deadline will be extended, but we will discuss in class.

We will use the power spectrum of the Cosmic Microwave Background (CMB) to constrain the basic cosmological parameters of the universe. The parameters we will measure are the Hubble constant, the density of regular baryonic matter, the density of dark matter, the amplitude and tilt of the initial power spectrum of fluctuations set in the very early universe, and the Thomson scattering optical depth between us and the CMB. In this excercise, we will only use intensity data, which does a poor job constraining the optical depth. IMPORTANT: this problem set relies on using the CAMB module, which in turn relies on having compilers installed on your system.

For the data, we will use data from the Planck satellite, which I have helpfully downloaded for you. Look for

`COM_PowerSpect_CMB-TT-full_R3.01.txt`

in the mcmc directory. This gives the variance of the sky as a function of angular scale, and the uncertainty of the variance. The columns are 1) multipole (starting with the l=2 quadrupole), 2) the variance of the sky at that multipole, 3) the 1σ lower uncertainty, and 4) the 1σ upper uncertainty. To make your lives easy, assume the errors are Gaussian and uncorrelated, and that the error on each point is the average of the upper and lower errors. This is one of several simplifications we'll make so our answers won't be exactly correct, but they will be very close.

You'll also need to be able to calculate model power spectra as a function of input parameters. You can get the source code for CAMB from Antony Lewis's github page: https://github.com/cmbant. There's a short tutorial online at https://camb.readthedocs.io/en/latest/CAMBdemo.html as well. Note that CAMB returns the power spectrum starting with the monopole, so you may need to manually remove the first two entries. You might want to try e.g. "pip3 install camb", which worked for me (but you may have to install a fortran compiler first. gfortran is open source and freely available). If you have troubles getting this installed, check in with the TAs early.

To help you out, I have posted a sample script that calculates the power spectrum from CAMB, reads in the Planck data, and plots them on top of each other for one guess for the cosmological parameters, and prints the value of χ^2 for that guess. The ordering of the parameters (where $h = H_0/100$) is:
0) Hubble constant H_0
1) Baryon density $\Omega_b h^2$
2) Dark matter density $\Omega_c h^2$
3) Optical depth τ 4) Primordial amplitude of the spectrum A_s 5) Primordial tilt of the spectrum n_s

1) Based on the χ^2 value, are the parameters dialed into my test script an acceptable fit? What do you get for χ^2 for parameters equal to [69, 0.022, 0.12, 0.06, 2.1e-9, 0.95], which are closer to their currently-accepted values? (I get a value around 3270, please give a precise one.) Would you consider these values an acceptable fit? Note - the mean and variance of χ^2 are n and $2n$, respectively, where n is the number of degrees of freedom.

2) Use Newton's method or Levenberg-Marquardt to find the best-fit parameters, using numerical derivatives. Your code should report your best-fit parameters and their errors in planck_fit_params.txt. Please write your own fitter/numerical-derivative-taker rather than stealing one. Note - you will want to keep track of the curvature matrix at the best-fit values for the next problem.

Bonus: The CMB is some of the best evidence we have for dark matter. What are the best-fit parameters with the dark-matter density set to zero? How does this χ^2 compare to the standard value? Note - getting this to converge can be tricky, so you might want to slowly step down the dark matter density to avoid crashes. If you get this to work, print the parameters/errors in planck_fit_params_nodm.txt

3) Use your curvature matrix from part 2) to generate random realizations of what parameter errors. What is the mean difference in χ^2 relative to the best-fit value? You should find typical $\delta\chi^2$ values of a few. If you get this part right, the next part will be much, much easier.

4) Estimate the parameter values and uncertainties using an MCMC sampler you write yourself. I strongly suggest you draw your trial steps from the curvature matrix you generated in Problem 2 and tested in Problem 3. Save your chain (including the χ^2 value for each sample in the first column) in planck_chain.txt. Explain why you think your chains are converged (if you indeed think they have converged). What is your estimate on the mean value of the dark energy Ω_Λ and its uncertainty? Note that we have (for good reasons) assumed the universe is spatially flat, so $\Omega_b + \Omega_c + \Omega_\Lambda = 1$. Make sure to remember that your chain is reporting $\Omega_b h^2$ and $\Omega_c h^2$.

5) Polarization data (we won't directly use the raw data here) give a much better constraint on reionization, with $\tau = 0.0540 \pm 0.0074$. Run a new chain where you include this constraint (saved to planck_chain_tauprior.txt), and compare those results to what you get from importance sampling your chain from Problem 4. I would encourage you to re-estimate the parameter covariance matrix (possibly via importance sampling) before running the new chain.

Bonus 2: What are your 5σ-equivalent error bars for the parameters? Please estimate these using whatever MCMC tricks you need to use, and recall that none of the parameters can be negative. By 5σ, I mean the limits on parameters so that the integral of the probility outside that region is equivalent to the probability outside of 5σ for a Gaussian (e.g. 1σ errors would contain 68% of the probability, *etc.).*