Any Questions?

- Everyone be ready to say 1-2 minutes on Wednesday about what they're doing, send me 1-paragraph abstract.

- Will send that out, then everyone rank on a scale from 1-5 how comfortable they would be reviewing that proposal (1=very comfortable, 5=new research project)

Aberration

- If I move source off-axis, what happens?

- Light ray along center will stay in phase. Off-center changes, though.

- This will lead to out-of-phase errors, or smearing out of signal.

Phase Errors

```python
import numpy as np
from matplotlib import pyplot as plt
#figure out how far away an off-axis source focuses, and what phase errors look like
D_dish=0.5
f_ratio=10
f=D_dish*f_ratio;
th_deg=0.5;th=th_deg*np.pi/180 #are we in focus at a distance of th_deg from center?

rdish=D_dish/2.0
a=0.25/f #equation for parabola
lamda=500e-9

x=np.linspace(-rdish,rdish,20)
d=np.linspace(0.95,1.05,100001)*f

dtot=np.zeros([len(d),len(x)])

xf=-d*np.sin(th)    #x/y coordinates of possible focal points
yf=d*np.cos(th)

for  i in range(len(x)):
    x0=x[i]
    y0=a*x0**2
    d1=-(np.cos(th)*y0+np.sin(th)*x0) #distance from infinity to dish
    d2=np.sqrt((yf-y0)**2+(xf-x0)**2) #distance from dish to focus

    dtot[:,i]=d1+d2

plt.ion();
plt.clf();
mystd=np.std(dtot,axis=1)
ii=np.argmin(mystd)
print 'min scatter at angle ',th_deg, ' degrees is ',mystd[ii]
print 'in wavelengths that is ',mystd[ii]/lamda
print 'distance from dish center of focus is ',d[ii]
plt.clf();plt.semilogy(d,mystd)
```

- As we go further off-axis, errors get bigger

- As dish gets bigger, errors get bigger

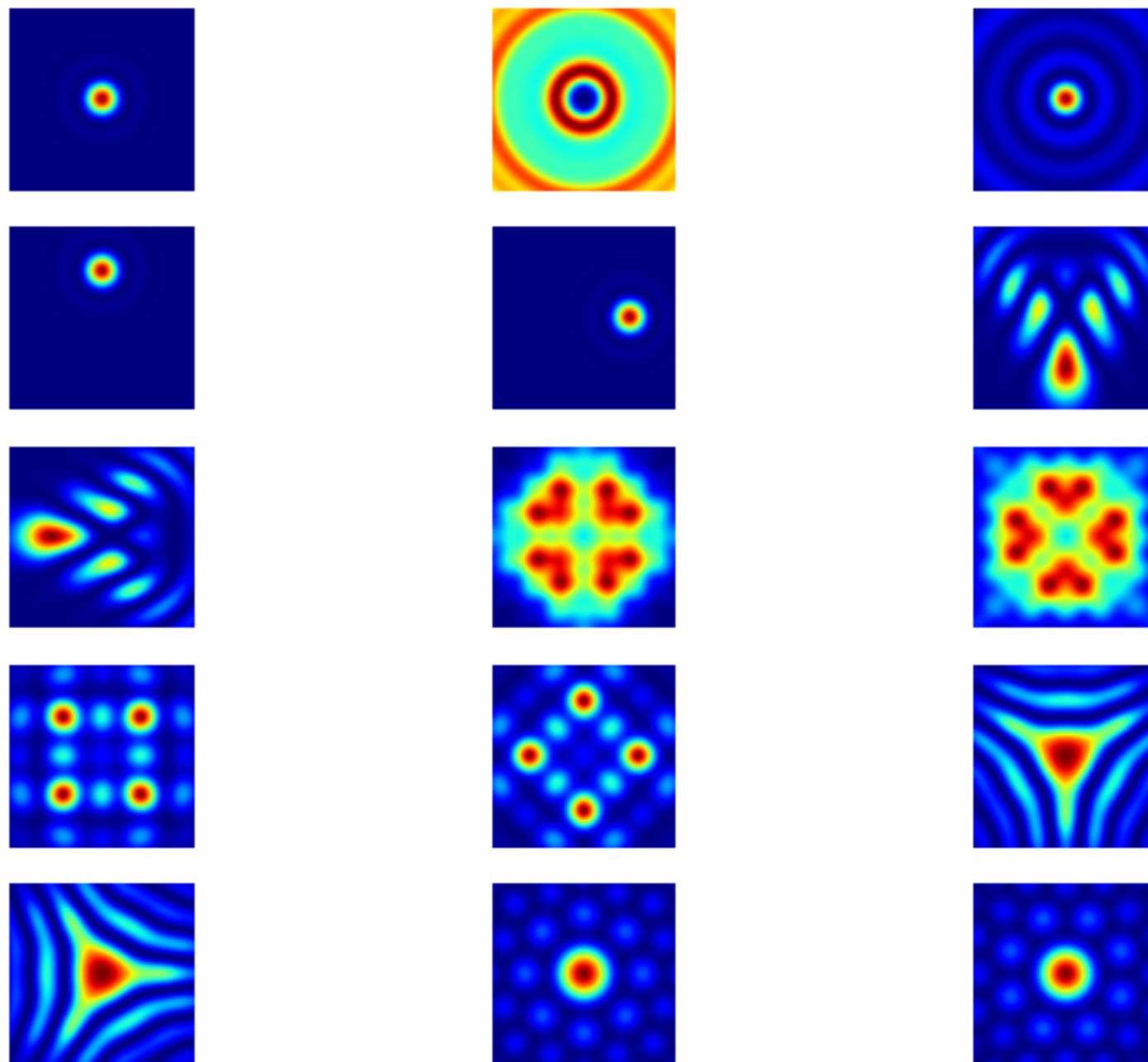- As focal length gets bigger, errors get smaller.

Zernike Polynomials

- From optics, wavefront errors are usually low-order phase deviations

- We can decompose into orthogonal functions across a circular aperture

- Angular part is sines/cosines

- Radial part is polynomials.

- This defines the Zernike polynomials

First Few Zernikes
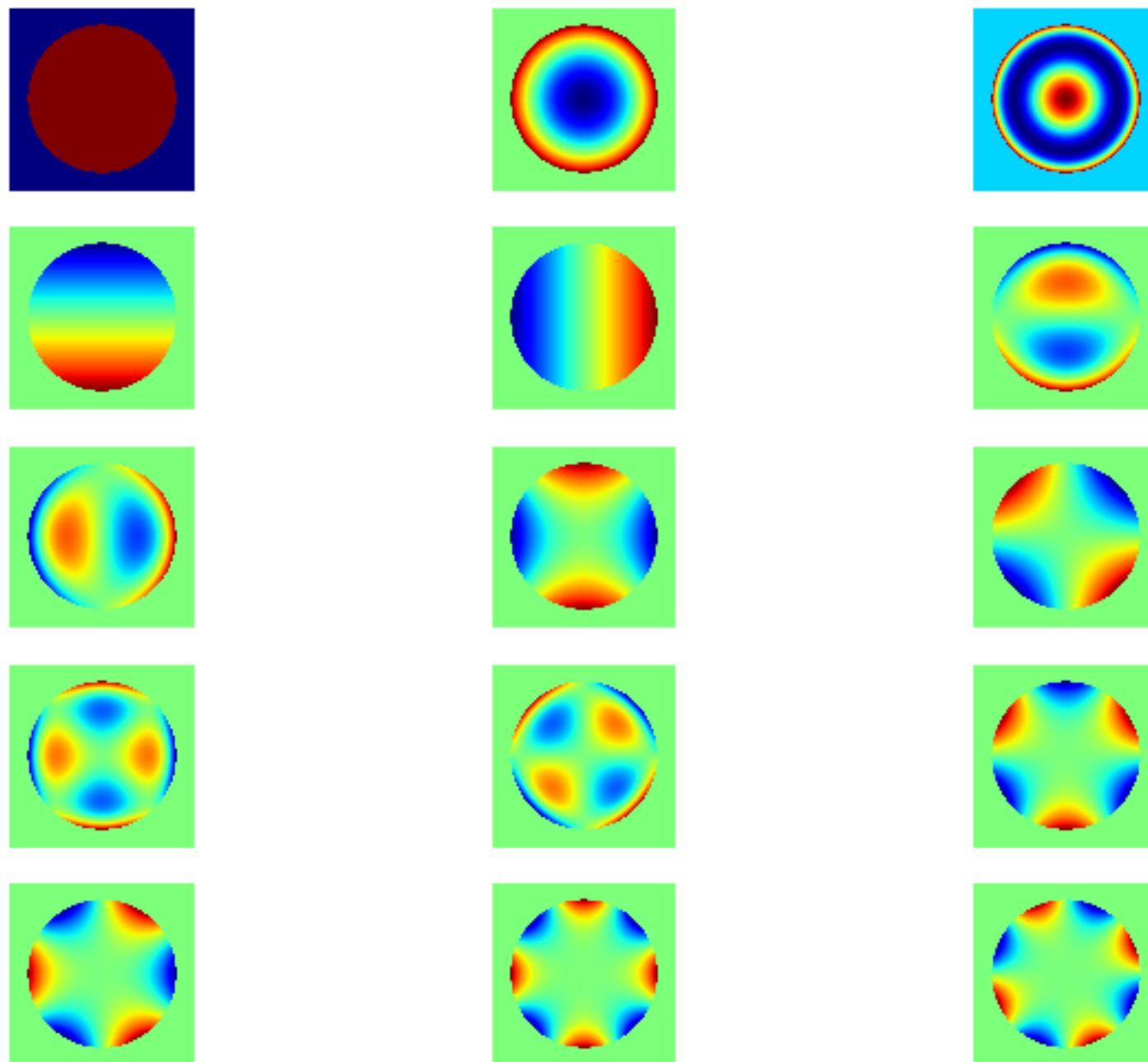
And Aberrations

Names

	OSA/ANSI index (j)	Noll index (j)	Radial degree (n)	Azimuthal degree (m)	Z_j	Classical name
Z_0^0	0	1	0	0	1	Piston (see, Wigner semicircle distribution)
Z_1^{-1}	1	3	1	−1	$2\rho\sin\theta$	Tilt (Y-Tilt, vertical tilt)
Z_1^1	2	2	1	+1	$2\rho\cos\theta$	Tip (X-Tilt, horizontal tilt)
Z_2^{-2}	3	5	2	−2	$\sqrt{6}\rho^2\sin 2\theta$	Oblique astigmatism
Z_2^0	4	4	2	0	$\sqrt{3}(2\rho^2-1)$	Defocus (longitudinal position)
Z_2^2	5	6	2	+2	$\sqrt{6}\rho^2\cos 2\theta$	Vertical astigmatism
Z_3^{-3}	6	9	3	−3	$\sqrt{8}\rho^3\sin 3\theta$	Vertical trefoil
Z_3^{-1}	7	7	3	−1	$\sqrt{8}(3\rho^3-2\rho)\sin\theta$	Vertical coma
Z_3^1	8	8	3	+1	$\sqrt{8}(3\rho^3-2\rho)\cos\theta$	Horizontal coma
Z_3^3	9	10	3	+3	$\sqrt{8}\rho^3\cos 3\theta$	Oblique trefoil
Z_4^{-4}	10	15	4	−4	$\sqrt{10}\rho^4\sin 4\theta$	Oblique quadrafoil
Z_4^{-2}	11	13	4	−2	$\sqrt{10}(4\rho^4-3\rho^2)\sin 2\theta$	Oblique secondary astigmatism
Z_4^0	12	11	4	0	$\sqrt{5}(6\rho^4-6\rho^2+1)$	Primary spherical
Z_4^2	13	12	4	+2	$\sqrt{10}(4\rho^4-3\rho^2)\cos 2\theta$	Vertical secondary astigmatism
Z_4^4	14	14	4	+4	$\sqrt{10}\rho^4\cos 4\theta$	Vertical quadrafoil

More Elements

- One can often cancel out various aberrations with more optical elements.

- Can be used to make faster optics - shorter focal length, which means smaller telescope dome

- Also important for larger FOV. Aberrations get worse as one goes off-axis.

- Cassegrain, Schmit-Cassegrain, Ritchy-Chretien, Crossed-Dragone all varieties of multi-element optics.

Resolution of Telescope

- Resolution important - noise in images usually from background. Larger dish = more photons, but also smaller area from which noise comes.

- HST has 2.4m primary. Optical wavelengths ~500 nm. Resolution ~$1.22\lambda/D = 0.05$".

- Larger telescope - better resolution, right? Well…
                (watch wikipedia movie)

Seeing.

- Stars twinkle. Phase is changing across mirror.

- Twinkle time ~5 ms.

- Atmosphere height ~10 km (scale height)

- Wind velocities ~10 m/s

- What does this tell us about ground-based resolution?

- Isoplanatic patch - phase constant across region. Typical size ~wind velocity times coherence time ~10 cm
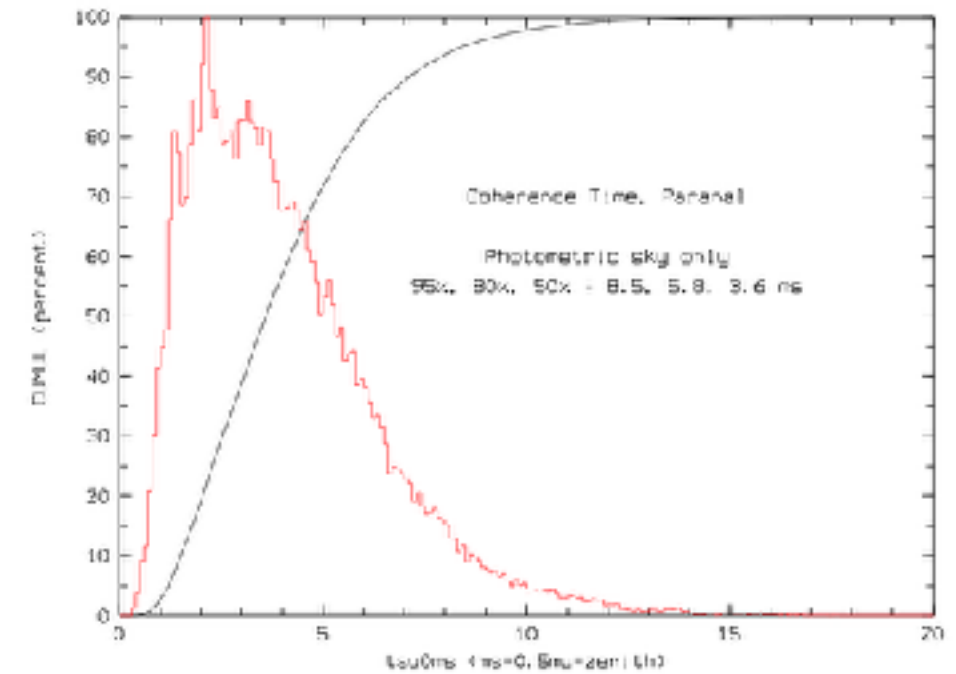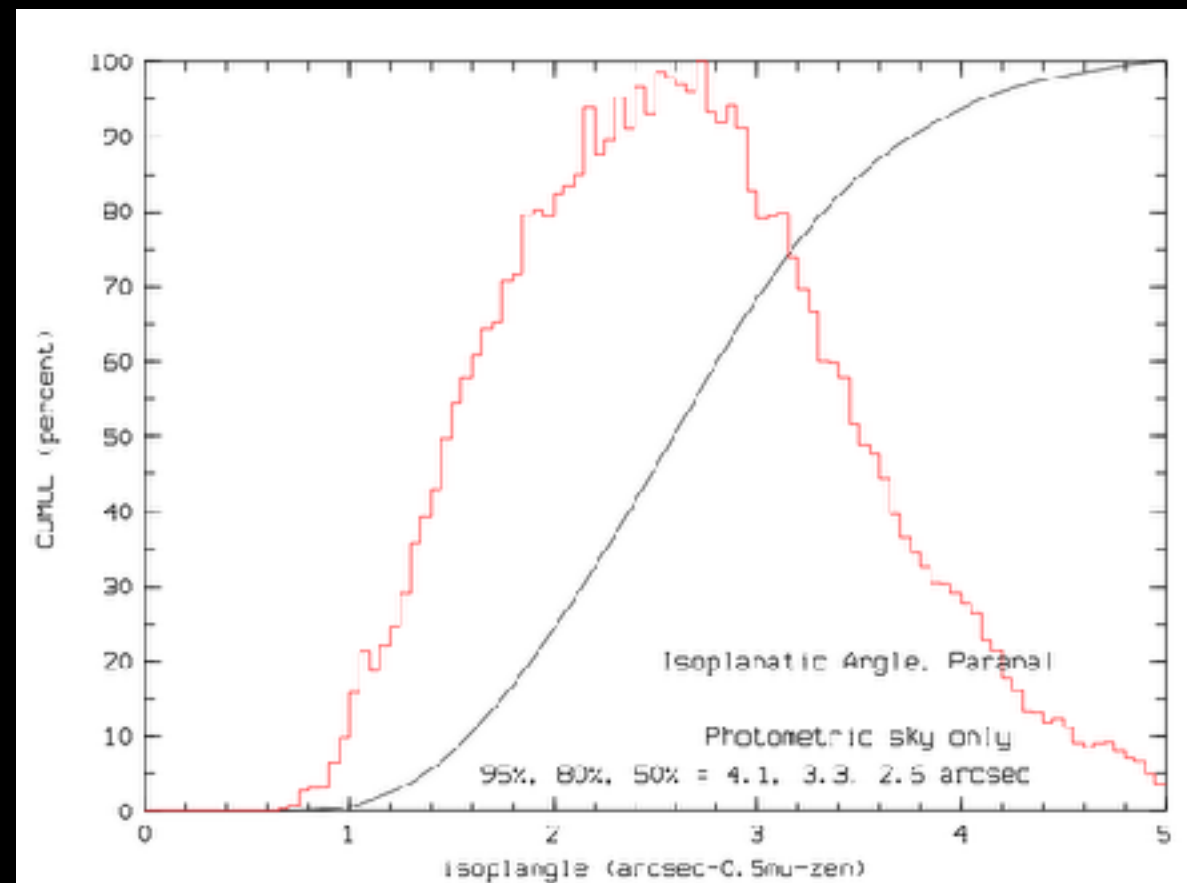
- Ground-based resolution ~1".

Figure 6. Statistics of the coherence time τ_0 at Cerro Paranal for the year 2000 (ms at 0.5 μm at zenith).

Effects of Seeing on SNR

- How does sensitivity scale with telescope diameter when diffraction limited?

- Sensitivity set either by source shot noise (bright) or sky shot noise (faint).

- For faint sources, as diameter increases, source photons go up as collecting area. Sky photons do what?

Effects of Seeing on SNR

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  - Photons per solid angle goes up as collecting area, but beam size decreases as collecting area. Net is wash (just like radio astronomy with a temperature background)

Seeing Ctd.

- Get D^2 more photons, but constant noise. t_{obs} goes like D^{-4} for fixed SNR.

- If seeing-limited, get same image size, so sky rate goes like collecting area as well. Get D^2 more photons, D^2 more noise photons, for noise like D. t_{obs} goes like D^{-2}.

- This is big part of why small telescopes in space are valuable.

- Bright sources - noise comes from source, not sky, so in all cases goes like D^{-2} or collecting area (just like in radio astronomy). No need to observe bright sources from space.

Adaptive Optics

- What would it take to correct for phase errors?

- Atmosphere puts in phase delay as function of position. We can remove by deforming mirror.

- Need to measure high SNR phase delay over each isoplanatic patch for each correlation time. Larger telescope collecting area doesn't help!

- Need guide star to measure deviations. Tiny fraction of sky has bright enough sources, so use lasers.

- With AO, can get image sizes to 0.03-0.06"
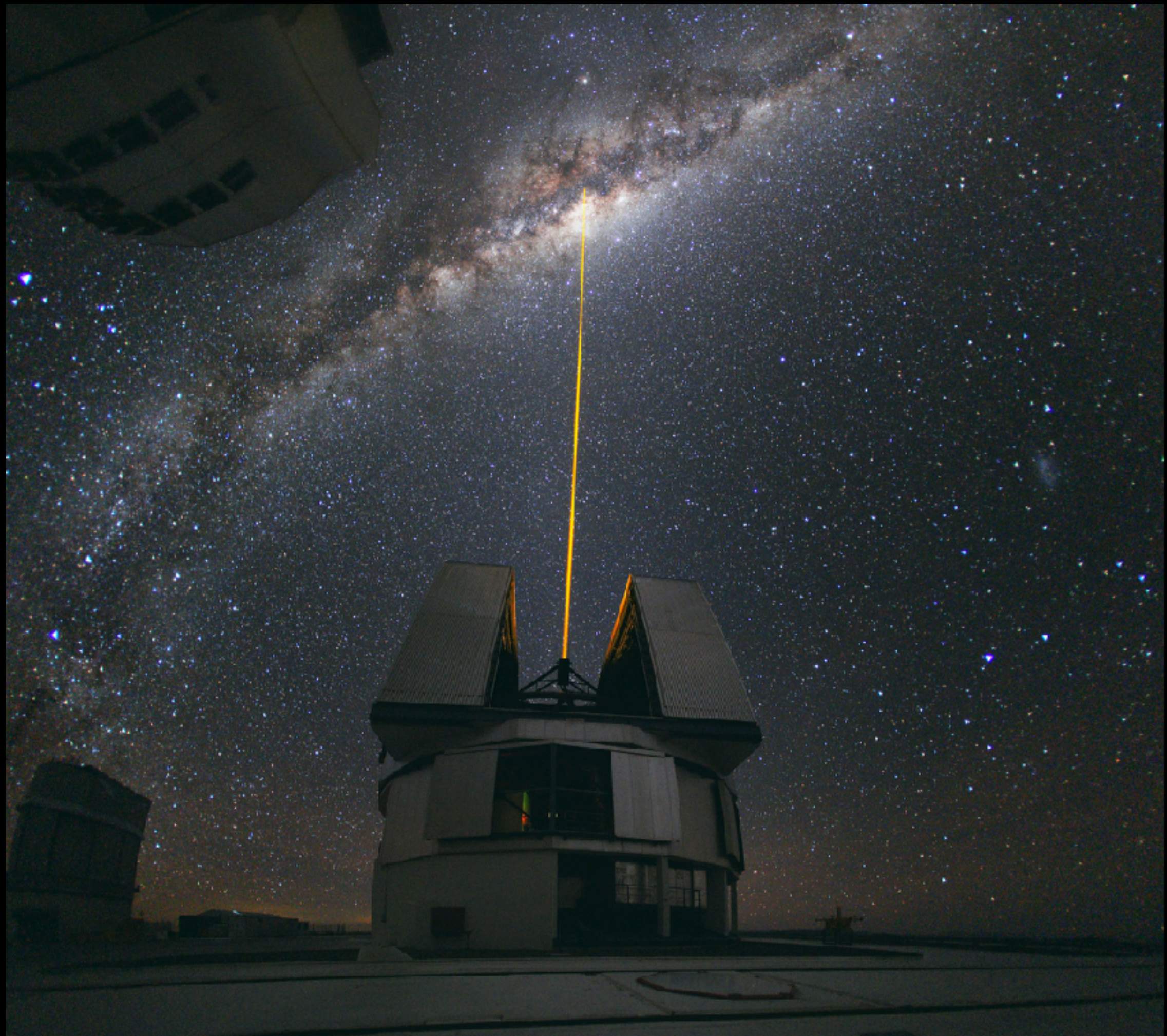
Magnitudes

- Historically, stellar brightness was based on how bright things seemed to eye.

- Greeks set up the magnitude system. Not surprisingly, not that convenient…

- Magnitude now defined to be -2.5 $\log_{10}$(Flux)+M_0.

- What is reference point M_0? Well, usally Vega…

- Alternatively, can use AB magnitudes, which are defined in Jy. 0 magnitude AB=3631 Jy.

AB-Vega Conversion

Conversions among magnitude systems:

Conversion from AB magnitudes to Johnson magnitudes:

The following formulae convert between the AB magnitude systems and those based on Alpha Lyra:

```
  V  =    V(AB) + 0.044        (+/- 0.004)
  B  =    B(AB) + 0.163        (+/- 0.004)
 Bj  =   Bj(AB) + 0.139        (+/- INDEF)
  R  =    R(AB) - 0.055        (+/- INDEF)
  I  =    I(AB) - 0.309        (+/- INDEF)
  g  =    g(AB) + 0.013        (+/- 0.002)
  r  =    r(AB) + 0.226        (+/- 0.003)
  i  =    i(AB) + 0.296        (+/- 0.005)
 u' =   u'(AB) + 0.0
 g' =   g'(AB) + 0.0
 r' =   r'(AB) + 0.0
 i' =   i'(AB) + 0.0
 z' =   z'(AB) + 0.0
 Rc =   Rc(AB) - 0.117         (+/- 0.006)
 Ic =   Ic(AB) - 0.342         (+/- 0.008)
```

Source: Frei & Gunn 1995

Conversion from STMAG magnitudes to Johnson magnitudes:
See the WFPC2 Photometry Cookbook

Photon Flux

- How many photons per square meter per second for a 25th magnitude AB V-band source?

- 0 mag=3631 Jy. $25=-2.5*\log_{10}(F)$, $\log_{10}(F)=-10$, $F=10^{-10}$ vs. 0 mag, so $10^{-10}*3631=3.63e-7$ Jy

- Bandwidth ~1000 Å, $\nu=c/5500$ Å$=5.5e14$ Hz, 20% BW=1e14 Hz. Flux = $10^{-26}*3.63e-7*10^{14}=3.63e-19$ W/m^2.

- Photon energy $h\nu=5.5e14*6.67e-34=3.7e-19$, or about 1 photon per square meter per second.

Noise Sources

- When looking at observing times, several sources of noise.

- Read noise, certain fixed noise just from reading out camera.

- Dark current from thermal generation of charge in CCD

- Thermal emission from telescope optics (concern in mid-IR, less so in optical)

- Shot noise from source

- Shot noise from sky background.

- If everything is working well, you'll be limited by source/sky shot noise.

Sky Brightness

- Sky brightness is measured in mag/ square arcsecond - worst unit I have ever met.

- Equivalent to 1 star of x magnitudes every arcsecond.

- Varies quite a bit depending on site, moon phase, wavelength…

- Right: distribution of V-band sky brightness from Gemini on Mauna Key
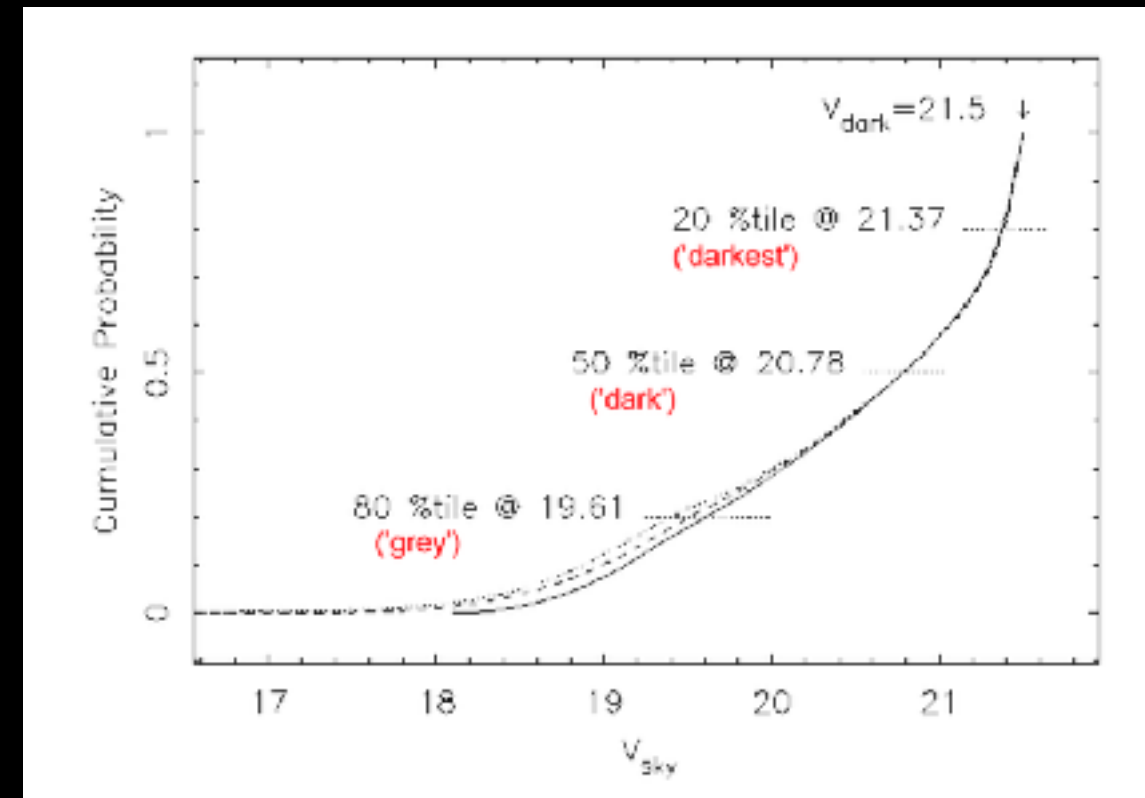
Observing Time

- How long would it take to measure a 25th magnitude star to 5σ in V-band with Gemini?

- 8m aperture=50m^2. 1 photon/m^2/s=50 photons/s.

- Efficiency coming through atmosphere, telescope, say 50%?

- Sky brightness: dark times typically 21.5 mag/arcsec^2, bright 19.6 mag/arcsec^2 in V-band. Gets worse at longer wavelengths. V-band HST more like 23.5, so much better from space.

Observing Time

```python
import numpy as np

area=1.2**2*np.pi
eff=0.8 #fraction of photons that make it through
sky_brighness=[23.5]
seeing=[0.05]
ab=[10,25]
t_obs=[1,100,10000]
m0=25.0 #magnitude of 1 photon per square meter per second

snr_thresh=5.0
for mag in ab:
    my_rate=10**(-0.4*(mag-m0))*eff*area
    for sky in sky_brightness:
        sky_rate_raw=10**(-0.4*(sky-m0))
        for fwhm in seeing:
            my_area=fwhm**2 #we'll go with this for the psf area
            my_sky_rate=ny_area*sky_rate_raw*area*eff
            # nunber of good photons is my_rate*t
            #noise is sqrt((my_rate+sky_rate)*t)
            #to get to 5 sigma, takes my_rate*sqrt(t)/sqrt(my_rate+sky_rate)=snr
            #or t=(snr*sqrt(my_rate+sky_rate)/my_rate)**2
            t_obs=(snr_thresh*np.sqrt(my_rate+my_sky_rate)/my_rate)**2
            print 'for sky/seeing/mag ',sky,fwhm,mag,t,' rates are ',ny_sky_rate,my_rate,
```

```python
import numpy as np

area=50
eff=0.5 #fraction of photons that make it through
sky_brightness=[19.61, 20.78, 21.37]
seeing=[0.6,0.85,1.1]
ab=[10,25]
t_obs=[1,100,10000]
m0=25.0 #magnitude of 1 photon per square meter per second

snr_thresh=5.0

for mag in ab:
    my_rate=10**(-0.4*(mag-m0))*eff*area
    for sky in sky_brightness:
        sky_rate_raw=10**(-0.4*(sky-m0))
        for fwhm in seeing:
            my_area=fwhm**2 #we'll go with this for the psf area
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            # number of good photons is my_rate*t
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            #or t=(snr*sqrt(my_rate+sky_rate)/my_rate)**2
            t_obs=(snr_thresh*np.sqrt(my_rate+my_sky_rate)/my_rate)**2
            print 'for sky/seeing/mag ',sky,fwhm,mag,t,' rates are ',my_sky_rate,my_rate,' a
```

Observing time outputs

```
[>>> execfile("gemini_tobs.py")
for sky/seeing/mag  19.61 0.6 10 10000   rates are  1288.96910935 25000000.0  and t_obs for  5.0  sigma is  1.00005155876e-06
for sky/seeing/mag  19.61 0.85 10 10000  rates are  2586.88939306 25000000.0  and t_obs for  5.0  sigma is  1.00010347558e-06
for sky/seeing/mag  19.61 1.1 10 10000   rates are  4332.3683953 25000000.0  and t_obs for  5.0  sigma is  1.00017329474e-06
for sky/seeing/mag  20.78 0.6 10 10000   rates are  438.775641093 25000000.0  and t_obs for  5.0  sigma is  1.00001755103e-06
for sky/seeing/mag  20.78 0.85 10 10000  rates are  880.598335249 25000000.0  and t_obs for  5.0  sigma is  1.00003522393e-06
for sky/seeing/mag  20.78 1.1 10 10000   rates are  1474.77368256 25000000.0  and t_obs for  5.0  sigma is  1.00005899095e-06
for sky/seeing/mag  21.37 0.6 10 10000   rates are  254.825279622 25000000.0  and t_obs for  5.0  sigma is  1.000010019301e-06
for sky/seeing/mag  21.37 0.85 10 10000  rates are  511.420179241 25000000.0  and t_obs for  5.0  sigma is  1.00002045681e-06
for sky/seeing/mag  21.37 1.1 10 10000   rates are  856.496078729 25000000.0  and t_obs for  5.0  sigma is  1.00003425984e-06
for sky/seeing/mag  19.61 0.6 25 10000   rates are  1288.96910935 25.0  and t_obs for  5.0  sigma is  52.5587643738
for sky/seeing/mag  19.61 0.85 25 10000  rates are  2586.88939306 25.0  and t_obs for  5.0  sigma is  104.475575723
for sky/seeing/mag  19.61 1.1 25 10000   rates are  4332.3683953 25.0  and t_obs for  5.0  sigma is  174.294735812
for sky/seeing/mag  20.78 0.6 25 10000   rates are  438.775641093 25.0  and t_obs for  5.0  sigma is  18.5510256437
for sky/seeing/mag  20.78 0.85 25 10000  rates are  880.598335249 25.0  and t_obs for  5.0  sigma is  36.22393341
for sky/seeing/mag  20.78 1.1 25 10000   rates are  1474.77368256 25.0  and t_obs for  5.0  sigma is  59.9909473025
for sky/seeing/mag  21.37 0.6 25 10000   rates are  254.825279622 25.0  and t_obs for  5.0  sigma is  11.1930111849
for sky/seeing/mag  21.37 0.85 25 10000  rates are  511.420179241 25.0  and t_obs for  5.0  sigma is  21.4568071697
for sky/seeing/mag  21.37 1.1 25 10000   rates are  856.496078729 25.0  and t_obs for  5.0  sigma is  35.2598431492
[>>> execfile("hst_tobs.py")
for sky/seeing/mag  19.61 0.05 10 10000  rates are  1.29581308308 3619114.73694  and t_obs for  5.0  sigma is  6.90776910056e-06
for sky/seeing/mag  20.78 0.05 10 10000  rates are  0.441105385802 3619114.73694  and t_obs for  5.0  sigma is  6.90776746919e-06
for sky/seeing/mag  21.37 0.05 10 10000  rates are  0.256178312451 3619114.73694  and t_obs for  5.0  sigma is  6.90776711622e-06
for sky/seeing/mag  19.61 0.05 25 10000  rates are  1.29581308308 3.61911473694  and t_obs for  5.0  sigma is  9.38107157089
for sky/seeing/mag  20.78 0.05 25 10000  rates are  0.441105385802 3.61911473694  and t_obs for  5.0  sigma is  7.7496998857
for sky/seeing/mag  21.37 0.05 25 10000  rates are  0.256178312451 3.61911473694  and t_obs for  5.0  sigma is  7.39673150568
```

- Large light bucket on ground much faster for bright source.

- Much smaller space telescope wins on faint sources due to better resolution, fainter sky background.

Combining Images

- Signal coming into camera is true sky convolved with instrumental point-spread function (PSF), which possibly varies across focal plane.

- CCD camera integrates photons inside pixels.

- Output is sky convolved with PSD and pixel response, sampled at pixels.

- Smaller pixels means better sampling of PSF, minimizing information loss.

- Larger pixels means larger FOV, faster surveying of areas.

- If pixels are larger than Nyquist, image is *undersampled*, and we've lost some information.

- Seeing changes, resolution a function of frequency, so data can easily be undersampled.

Hubble Deep Field

- Old HST camera had one CCD with very small pixels to avoid undersampling, 3 with large pixels for survey speed.

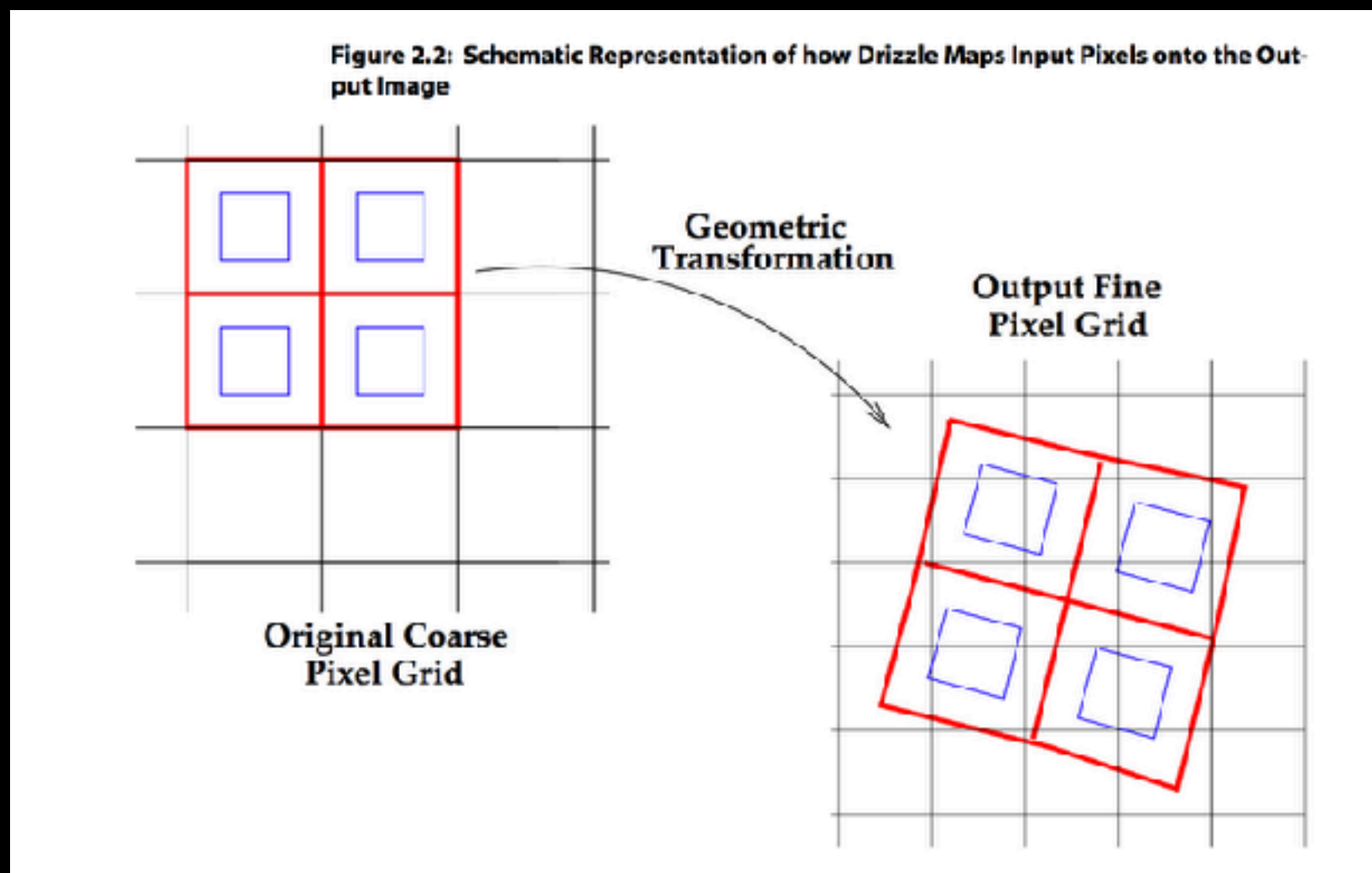- You could decide if you wanted smaller, better-sampled region.

Drizzling

- In space, PSF is constant. Can I reconstruct well-sampled data from undersampled?

- If I took 4 images spaced by 0.5 pixels, I would get a better reconstruction by interlacing the images.

- Of course, difficult to perfectly align images. What can I do if they aren't aligned?

- Could make finely pixellized sky. Take each image pixel and grid onto fine sky. What would be the downside?

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- Could make finely pixellized sky. Take each image pixel and grid onto fine sky. What would be the downside?

  - I've spread out each pixel by its area, so I have convolved my image by pixel size. Could be sqrt(2) worse resolution. We went to space for fine resolution, don't give it away for no good reason!

Drizzling 2.

- Alternative - "shrink" each pixel before spreading it out, keeps convolution smaller.

- Can set how much to shrink, how fine the pixel reconstruction is.

- Noise gets correlated, need to keep track…

Figure 2.2: Schematic Representation of how Drizzle Maps Input Pixels onto the Output Image

Geometric Transformation

Output Fine Pixel Grid

Original Coarse Pixel Grid

PSFs Change

- Of course, ground-based PSFs change. How would you combine images then?

- Say comparing image A&B - can convolve A with B's PSF and vice-versa, can now combine/compare directly (used e.g. in transient searches).

- Would you be pleased with this technique?

Least-Squares

- A better thing to do: write down χ^2 and minimize.

- $A^T N^{-1} A m = A^T N^{-1} d$. In this case, m would be finely sampled, non-smoothed map, d coarse maps, and A the mapping that takes the fine map into the variable-PSF/varying-pointing maps.

- If A contains full PSF, what could go wrong?

Least-Squares

- A better thing to do: write down χ^2 and minimize.

- $A^T N^{-1} A m = A^T N^{-1} d$. In this case, m would be finely sampled, non-smoothed map, d coarse maps, and A the mapping that takes the fine map into the variable-PSF/varying-pointing maps.

- If A contains full PSF, what could go wrong?

  - We could in effect try to reconstruct scales smaller than PSF, which have not been measured. Amounts to dividing FT by PSF FT, which could go to zero, causing solution to explode.

Better Way

- Rather than try to reconstruct the true sky, we should reconstruct the true sky convolved with some "small" beam so that PSF transform doesn't go to zero.

- PSF convolution in A then becomes taking worse PSFs to the ideal - as long as some data gets that resolution problem is well defined.

- Effectively, large scales get info from everyone, small scales preferentially weight good-seeing data.