CMB

- Cosmic microwave background contains snapshot of universe at about 400,000 years.
- Useful because physics is linear, so we can link observations and theory very easily (unlike nonlinear physics in galaxies etc.)
- Key information contained in power spectrum.

CMB PS

- We've met power spectra so far in Fourier transforms.
- Different on a spherical sky. Normal modes become *spherical harmonics*.
- $T(\theta,\phi)=\sum_l\sum_m Y_l^m(\theta,\phi)a_{lm}$. a_{lm} are coefficients (like F(k)) and Y_l^m are orthogonal basis functions, analogous to sine waves.
- $Y_l^m(\theta,\phi) \propto P_l^m(\cos(\theta))\exp(i\phi)$ where P_l^m are associated Legendre polynomials. l,m are integers, $|m|\leq l$
- Usual normalization is such that $\iint Y_l^m Y_{l'}^{m'} d^2\Omega=\delta(l-l')\delta(m-m')$

Associated Legendre "Polynomials"

- May not surprise you to hear that P_l^m can be found with recurrence relations.

- $(l-m+1)P_{l+1}^m(x)=(2l+1)xP_l^m(x)-(l+m)P_{l-1}^m(x)$, with $|m|\leq l$

- Can get started with $P_{l+1}^{l+1}=-(2l+1)(1-x^2)^{1/2}P_l^l(x), P_0^0(x)=1$

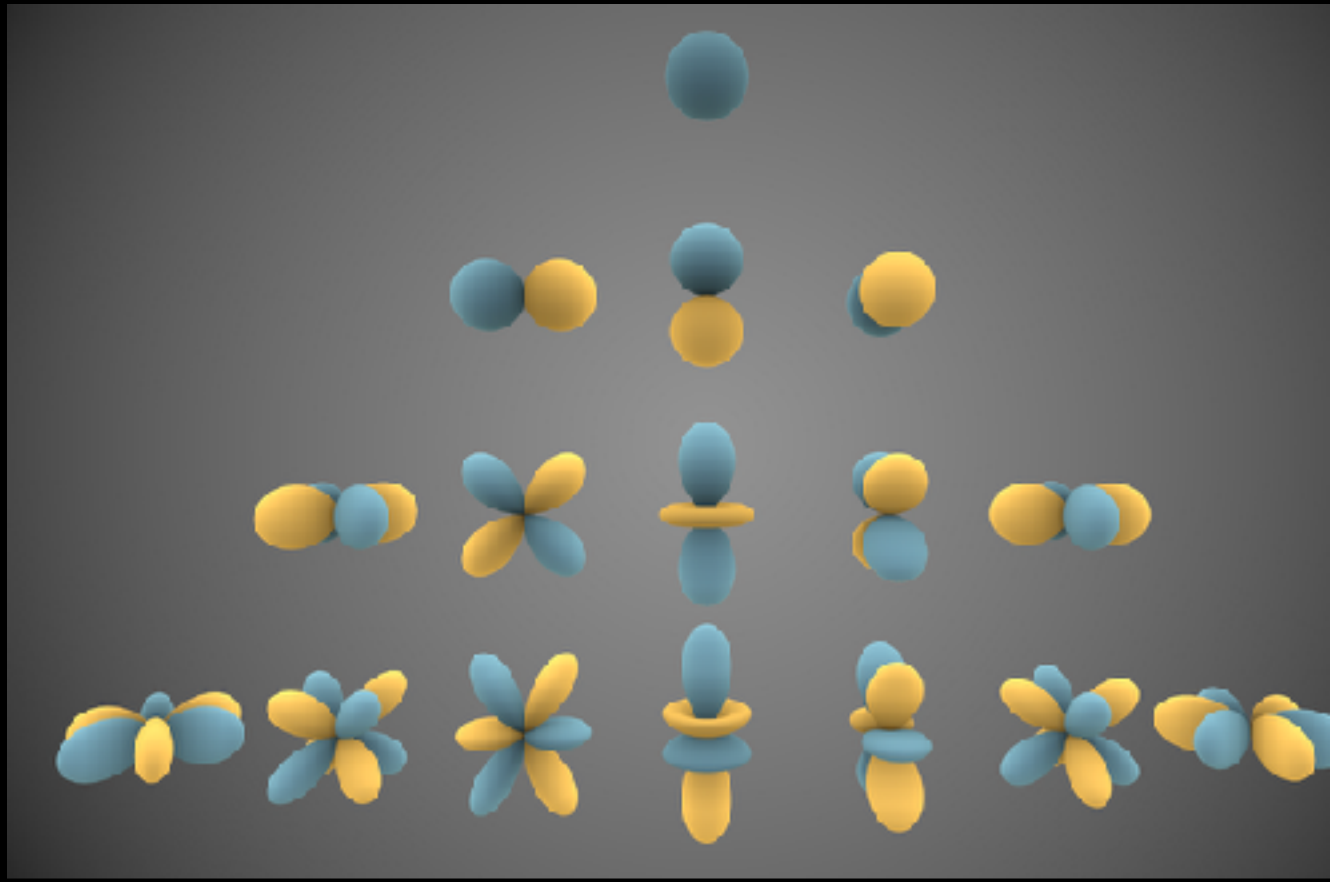
- $P_1^1=-(2*0+1)(1-x^2)^{1/2}P_0^0=-(1-x^2)^{1/2}$.

- If m even, these are polynomials (but not if m odd)

- $P_l^0=l^{th}$ Legendre polynomial

Spherical Harmonics

- With ability to calculate P_l^m, we could calculate first few spherical harmonics.
- What is angular dependence of m=0 modes? Can a_{l0} be complex?
- Y_l^{-m} is rotation of Y_l^m. Like FT, for real sky, $Y_l^{-m}=Y_l^{m*}$. So, usually only specify non-negative a_{lm}.
- Healpix is standard astro package to work with spherical harmonics
- healpy.map2alm converts map of sky to a_{lm}.
- healpy.alm2map converts a_{lm}s to map



(from Wikipedia)

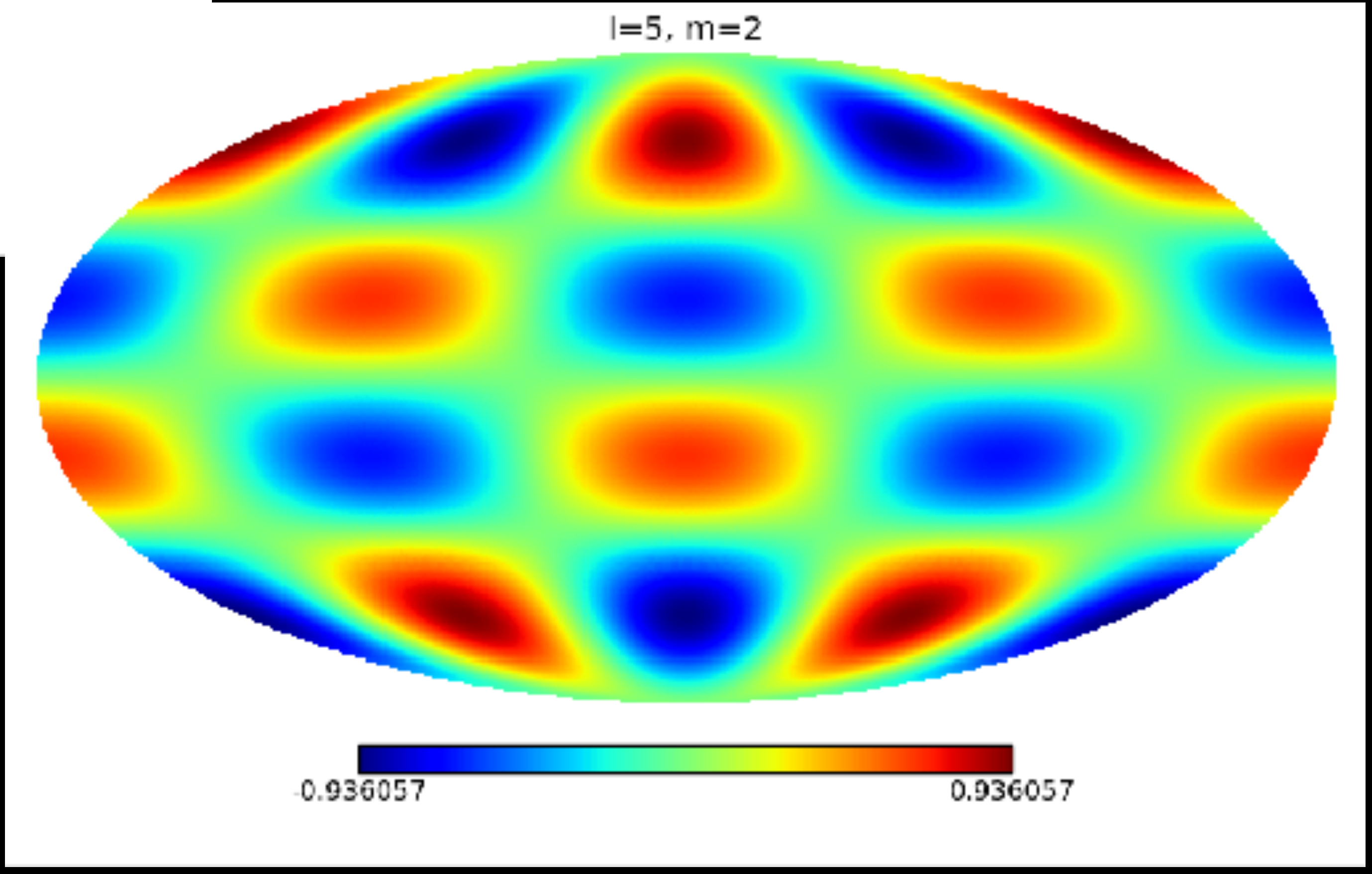
Healpy to Plot Modes

```
import numpy as np
from matplotlib import pyplot as plt
import healpy
#let's make a plot of individual Ylm's
l=5
m=2

nlm=(l+1)*(l+2)/2  #where does this come from?

#healpix indexing is such that all the m=0 modes are
#first, m=1 modes are next, etc. use this bit to brute-force
#find where in healpix ordering our requested mode will
icur=0
for mm in range(m):
    icur=icur+(l+1-mm)
icur=icur+(l-m)

alm=np.zeros(nlm,dtype='complex')
alm[icur]=1.0
nside=256
map=healpy.alm2map(alm,nside)
plt.ion()
healpy.mollview(map)
```



Warning!
alm2map(map2alm(x))!=x
Unlike Fourier transforms

CMB PS Ctd

- CMB should be Gaussian, with all information contained in power spectrum.
- $<a_{lm}^2>=C_l$, so variance independent of m.
- How would you measure C_l given a noiseless map of the full sky?
- If C_l were constant up to some l_{max}, what would standard deviation of the sky be?

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- If C_l were constant up to some l_{max}, what would standard deviation of the sky be?
  - at each l, have 2l+1 modes. sum(2l+1)=l(l+1)+l=l(l+2)
  - but, spread over sky. Approximately $l^2/2\pi$

CMB PS again

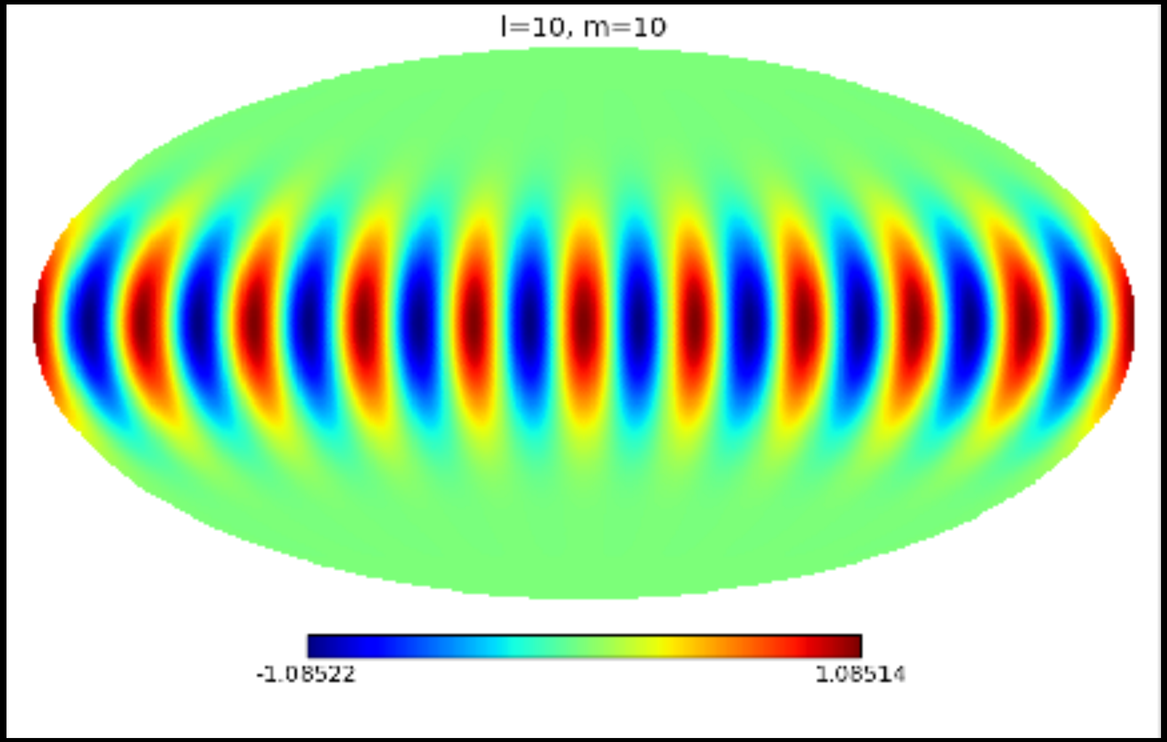
- So, CMB PS usually reported not as C_l, but as $l(l+1)Cl/2\pi$, often confusingly called $\mathscr{C}_l$, but sometimes D_l or B_l.

- But, sqrt($l(l+1)C_l/2\pi$) tells you about what fluctuations look like for broad band in power spectrum.

- Many experiments don't see full sky, but instead observe small patches.

- On small patches, flat sky approximation becomes good, $F(k)$ proportional to C_l, with k scaled to match l for patch size.

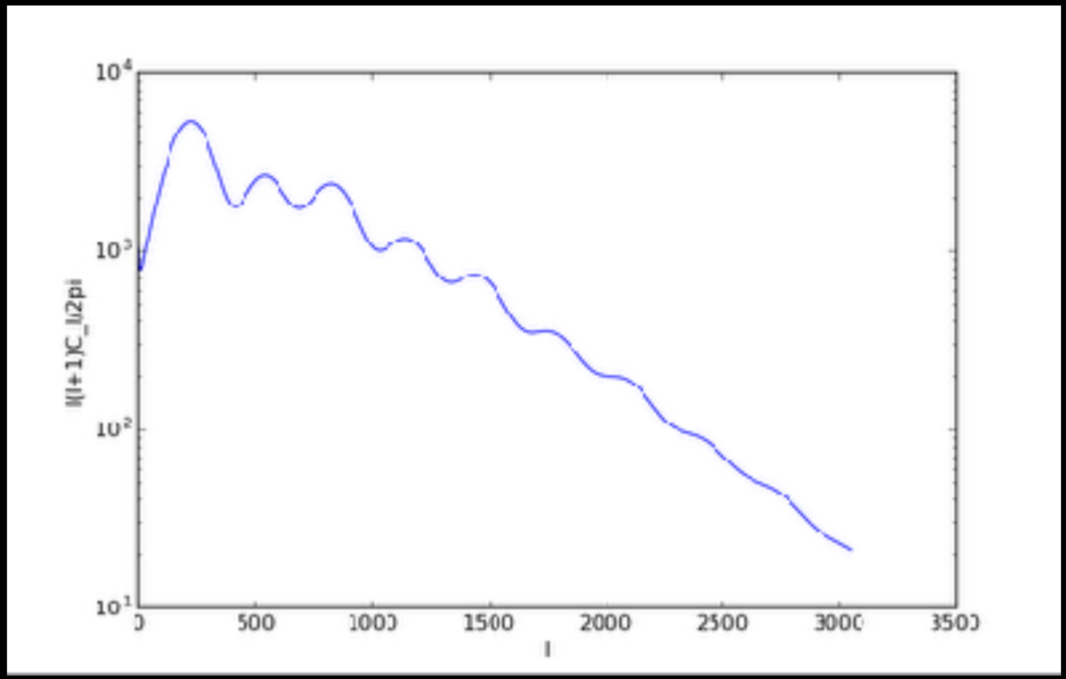
Scaling of k with l

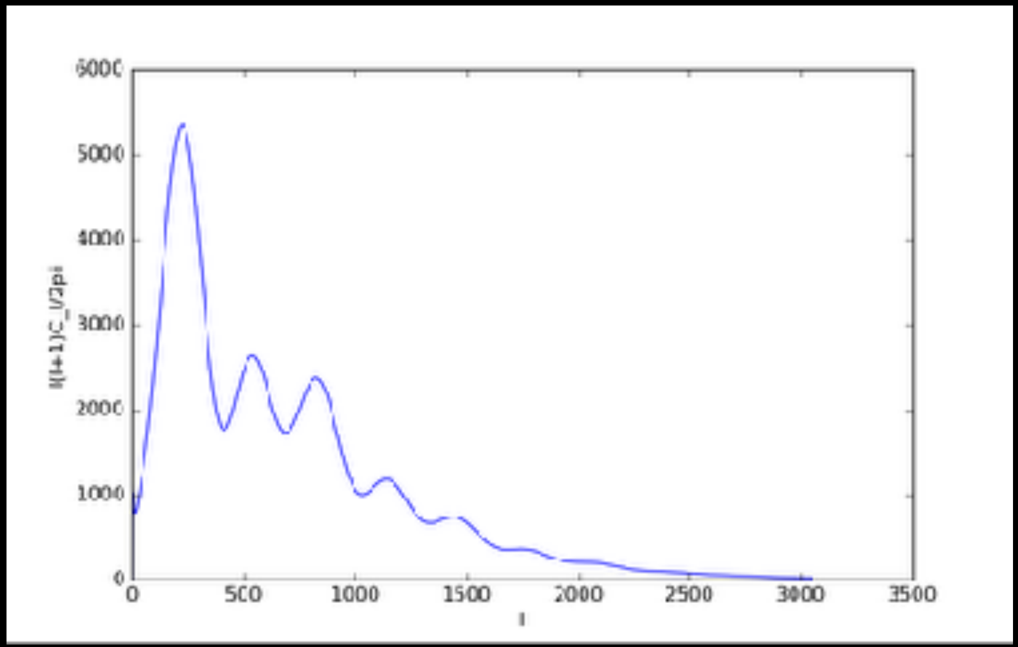


- Y_l^l has l periods across equator.
- Box that is x radians across will have one period if $l=2\pi/x$
- But, k=1 will have one period as well. So, $l \sim 2\pi k/x$
- By counting modes, can get scaling of power spectrum amplitude as well.
- What happened to our resolution in l by going to a small patch of sky?

CMB Predicted PS

- Standard code to model CMB power spectra is CAMB (available from Antony Lewis's github: https://github.com/cmbant)
- Top: log scale. Bottom: linear scale.





Sensitivity Needed

- Power spectrum max at $l \sim 200$, where $l(l+1)C_l/2\pi = 5300$ μK^2.
- What are typical fluctuation amplitudes?
- How many independent patches would I need to measure?
- In 1 second, Planck has a sensitivity of ~10 μK. How long would I need to measure full sky to SNR ~1?

Sensitivity Needed

- Power spectrum max at $l\sim200$, where $l(l+1)C_l/2\pi=5300\ \mu K^2$.
- What are typical fluctuation amplitudes?
  - sqrt(5300)~70 μK
- How many independent patches would I need to measure?
  - probably want to Nyquist sample, so $200^2*4=1.6e5$
- In 1 second, Planck has a sensitivity of ~10 μK. How long would I need to measure full sky to SNR ~1?
  - 1 Patch takes $(10/70)^2\sim0.02$s. Full sky takes 1.6e5 times longer, or about an hour.

Sensitivity Needed @l=2500

- Power spectrum at l~2500, where $l(l+1)C_l/2\pi = 70\ \mu K^2$.
- What are typical fluctuation amplitudes?
    - sqrt(70)~8.5 μK
- How many independent patches would I need to measure?
    - probably want to Nyquist sample, so $2500^2 * 4 = 2.5e7$
- In 1 second, Planck has a sensitivity of ~10 μK. How long would I need to measure full sky to SNR ~1?
    - 1 Patch takes $(10/8.5)^2 \sim 1.4$. Full sky takes 2.5e7 times longer, or about 3.5e7 seconds, or just over a year. High resolution much harder!

Detector Sensitivity

- If I have a detector that averages n photons per second, and observes for t seconds, what is the fractional uncertainty on n?
- I get a total of nt photons, which are (probably) Poisson distributed. Variance in nt, so $\sigma=\sqrt{(nt)}$.
- Fractional error is then $\delta n/n = 1/\sqrt{(nt)}$.
- If I want more accuracy, I can either observe longer, or increase photon rate. Larger telescopes are good!

How low can we go

- Is there a limit to how high I can push photon rate *n*?
- At some point, photons start to overlap. What *n* would I need for optical wavelengths?
- If wavelength ~500 nm, need C/λ~6e14 photons/s.
- Could pack in (1m/500 nm)^2 detectors/m^2=4e12. So, would need 4e12*6e14=2.4e27 photons/m^2/s.
- Energy/photon=hv, or 4e-19 joules. Total power 4e-19*2.4e27= 1GW/m^2. Camera will melt first.

What Happens When Saturate?

- In limit of many overlapping photons, best I can do is measure electric field continuously.
- How often do I get a new electric field measurement?
- If I have signal up to some ν_{max}, correlation length goes like $1/\nu_{max}$. We call this the *bandwidth* B.
- # of independent samples is Bt, so fractional error is just $1/\sqrt{n\_samples}$.
- Usually refer to temperature (instea of count rate), which gives $\delta T/T=1/\sqrt{(Bt)}$ (radiometer equation)

Shot noise vs. continuous

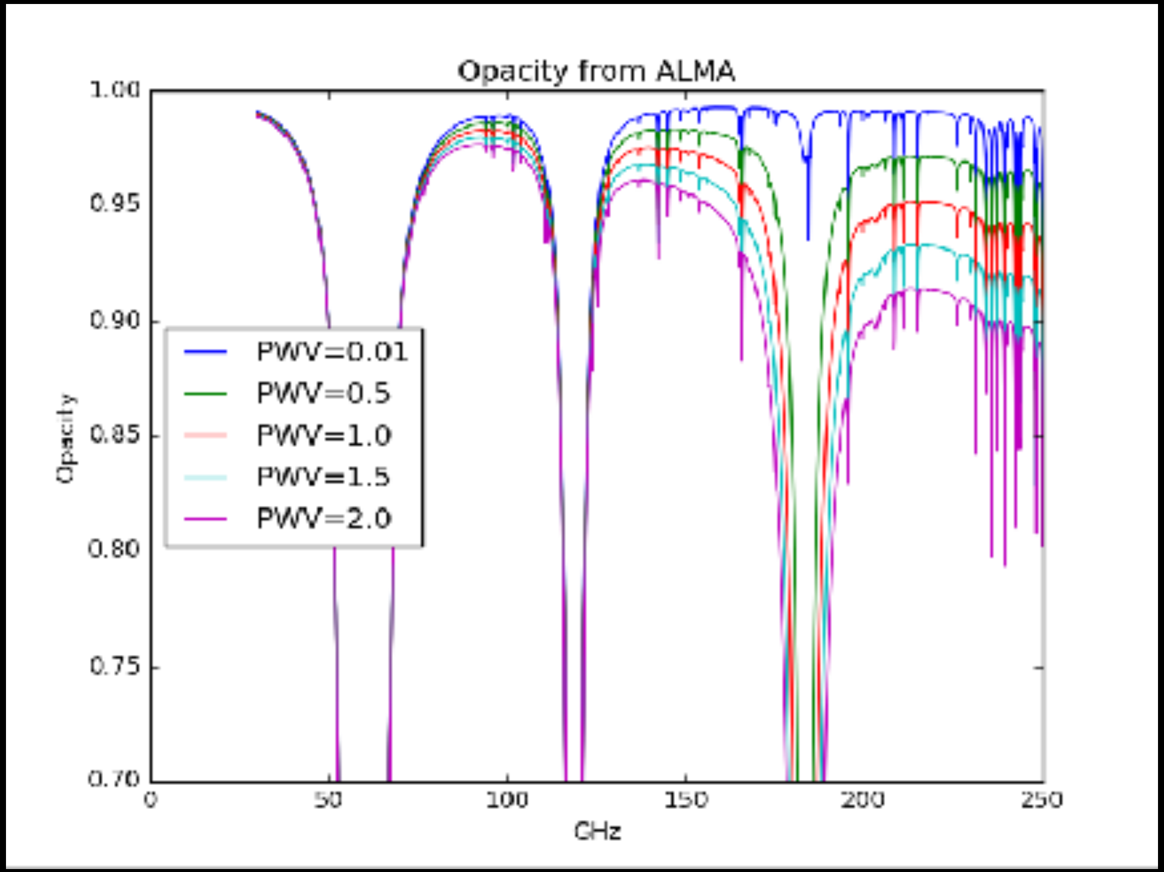
- As I crank up intensity, fractional sensitivity increases until I saturate at continuous limit.
- Shot noise absolute error: $\delta n = n/\sqrt{nt} \sim n^{1/2}$. If I want to measure a start (fixed n) and add noise, my error on star goes like sqrt(noise)
- Continuous: $\delta T = T/\sqrt{(Bt)} \sim T$. If I increase noise power, my error scales linearly (not as sqrt).

Comparison to Black Body

- Where will we transition?
- If staring at black-body radiation, $B_\nu=2h\nu^3/c^2(\exp(x)-1)$, where $x=h\nu/kT$.
- Photon occupation number is $1/(\exp(x)-1)$, so far to left of BB peak, we will be in continous, and far to right shot noise.
- For CMB, $x=1$ at $\nu=50$ GHz. Radio always in continuous limit.

Ground-based CMB

- At typical CMB frequencies, O_2 and H_20 block some wavelengths.
- Oxygen lines can't do much about. Water can be avoided -> go high, and dry. South Pole, Chilean Atacama best places so far. (Tibet, Greenland…)
- Plot at right shows opacity from ALMA site as function of precipitable water vapor. What frequencies would you use?



Ground-based Sensitivity

- Let’s pick 90 GHz window. Have 30 GHz window.
- For PWV=1mm (decent day in Chile), opacity is 0.025.
- Call temperature 270K - emission equivalent to 0.025*270=7K + 3K CMB ~ 10K noise signal.
- For 1sec, $\delta T = 10K/\mathrm{sqrt}(30e9) = 6e{-5} = 60\ \mu K$.
- However… x=1.6, pushes noise to~80 μK.
- Other sources of noise contribute, plus CMB not in full continuous limit. Typical ground-based limit more like 300-500 μK.