Tutorial Problem for CMB Power Spectrum/MCMC Due Wednesday Nov 13th.

## Problem 1:

Let's extract cosmological parameters! While you have examples of how to do much of this from class notes, please write all your own code. Note that some of the chain calculations can take a while (hours) on a laptop, so for chains you want to leave going long enough to converge, you may want to leave your computers plugged in overnight. For related reasons, I won't re-run your MCMC codes, so please post your chains along with your codes to github.

For simplicity, you can use the text version of the power spectrum on Lambda (https://lambda.gsfc.nasa.gov/) Please pull the WMAP-9year spectrum. You'll be better off using the unbinned version (the binned averages over many l, which you would need to account for). On Lambda, you can click on the "Data" tab, then on "WMAP", etc. to get to the 9-year spectra.

You'll also need to be able to calculate model power spectra as a function of input parameters. You can get the source code for CAMB from Antony Lewis's github page: https://github.com/cmbant. There's a short tutorial online at https://camb.readthedocs.io/en/latest/CAMBdemo.html as well.

- a) Show that you can get a sensible spectrum by setting the basic 6 cosmological parameters. In our case, we'll use  $\Omega_b h^2$ ,  $\Omega_c h^2$ ,  $n_s$ ,  $A_s$ ,  $\tau$ , h. You can pull sensible values from either WMAP9 cosmology papers or Planck 2018 cosmology to get you started. Does your curve more-or-less go through the WMAP power spectrum points?
- b) Find 1-d curvatures for these parameters while keeping the others fixed. Get error estimates from these, and run a short (few hundred sample, typically) chain. Look at the chain, and confirm that it looks like you have many independent samples, even if you haven't converged. If this isn't working well, you may wish to rescale the step size you take until your acceptance rate seems sensible.
- c) Using the chain from part b), estimate a parameter covariance matrix. Run a few short chains using various scalings of this step size what scaling produces the shortest correlation length? What is the acceptance fraction at that scaling? With that scaling, run a chain to reasonable convergence. What are your parameter mean values, and error bars? One caveat  $\tau$  may go negative, so you can introduce a cutoff in your chain code where steps with negative  $\tau$  values are all rejected.
- d) Of course, the sky contains things other than the CMB. One such thing is point sources, which have a spectrum that is flat in  $C_l$ , or proportional to  $l^2$  in  $C_l$ . Add a non-negative point source component to your chain (i.e.  $C_l \to C_l + a_{src}l^2$ ) and re-run. What are your new error bars? What is your limit on any possible point source contribution? Which parameters are affected the most?
- bonus) Of course, treating the errors in the power spectrum as uncorrelated Gaussians is wrong. Can you call the full WMAP likelihood (also available on Lambda) and incorporate that

into a chain? You should see most parameters look similar, but  $\tau$  will be quite different. The *ctypes* module in python provides a relatively easy way of linking to external libraries. By way of warning, I failed in my first attempt from python, but I suspect that was due to heterogeneity of compilers on MacOS doing Fortran name-mangling differently.