any questions?

Mapping Speed

- How long would GBT take to map half of sky to 200 µJy RMS, with T_{sys}=25K, 400 MHz of bandwidth at 600 MHz?
  - Beam size ~1.22λ/D, 50cm wavelength, ~20' beam.
  - need 1.5e5 beams. $t_{obs}=(T/dt)^2/B$, dt=400µK, t=10s/beam, 1.5e6 s total~20 days.
- CHIME sensitivity after one day? Similar gain, 50K Tsys, 90 degree by 1 degree strip. 4 minutes to cross strip. dT=50/sqrt(240*400e6)=160 µK, or 80 µJy. Equivalent to ~100 days of GBT(!).

Will This be Our Noise?

- Maybe. If we're lucky. But probably not…
- 1/f noise always exists in amplifiers.
- If integrate longer than 1/f knee, noise gets *worse* with longer integrations.
- 1/f modulates T_{sys}, so often dominates after ~ms timescales.
- Usual solution - switched receiver. Use two feedhorns going into one amplifier, switch back & forth between them. Can also switch against loads.
- How does this affect our noise?

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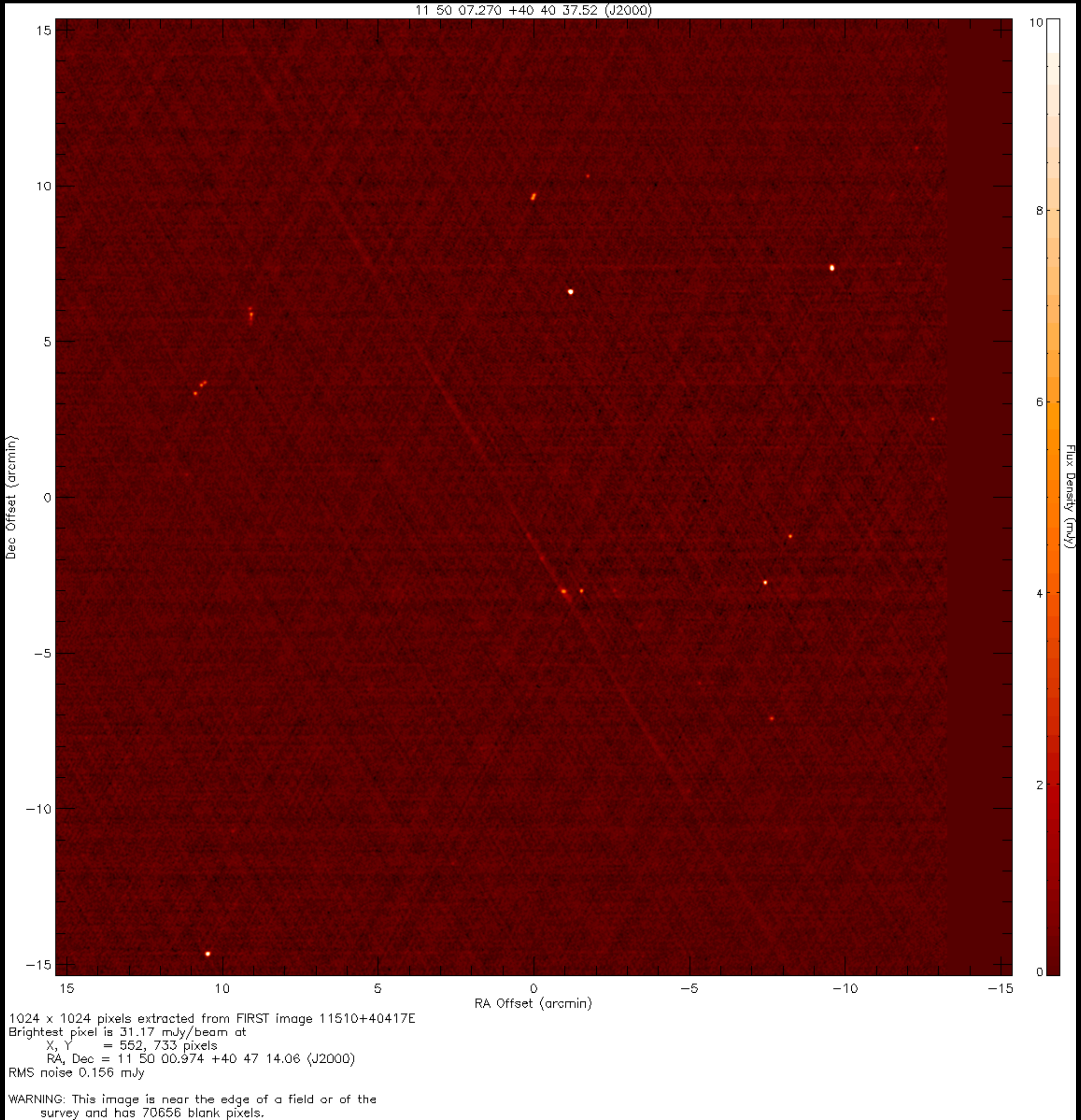
    - spend half time on-source, half time off-source. Then difference the two. Half as much time double the variance, subtracting two same-variance #'s doubles again, so σ goes up by 2, t_{obs} goes up by 4.

Random Square Degree from FIRST



Lots of Sources

- Our telescope will read out true sky convolved with telescope beam.
- If I point at source, but second source falls in beam, I will measure combined flux.
- Alternatively, convolve map of sky with telescope beam, what is median scatter?
- Unwanted sources contaminate signal, called *confusion noise*. Does not integrate down with time.
- How does importance of confusion noise scale with time?
- How does confusion noise scale with resolution?

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- Unwanted sources contaminate signal, called *confusion noise*. Does not integrate down with time.
- How does importance of confusion noise scale with time?
    - gets worse, since SNR on faint sources goes up.
- How does confusion noise scale with resolution?
    - gets better, since odds of second source in-beam go down.

Quick GBT Confusion Limit

- FIRST survey has ~100 sources/square degree to 1 mJy
- What is GBT beam size at 1.4 GHz? 1.22λ/d ~10'
- How many sources per beam? $100*(10/60)^2=2\text{-}3$
- What is confusion limit? ~sqrt(2-3)*(>1 mJy) ~few mJy
- How long does it take to get there? 3 mJy=6 mK, BW=400, Tsys=25, dT/T=1/sqrt(Bt) so $t=(T/dT)^2/B=(25/0.006)^2/400e6=0.04$s.
- Say looking for 10 km/s spectral line - BW=1.4e9*(v/c)~50 kHz. $t=(T/dT)^2/50e3=6$ min. (in fact better, because confusing sources are likely spectrally smooth)

Receivers

- Radio waves come into detector.
- Something needs to measure them - usually an ADC (analog to digital converter)
- ADCs are noisy, and we may have signal loss en route through cables.
- So, usually amplify signal as soon as it comes in.
- Amplifiers usually quoted in logarithmic dB, 10 dB a factor of 10 in power. (sometimes electric field, so be careful)
- If I have 20K coming into 20 dB amplifier, how much comes out?
  - $20\text{dB}=10^{(20/10)}=100\text{x}$ increase, so at 2000K.

Receiver Noise

- If I add 10K noise before amplifier, what is my new power? If I add 10K after amplifier, what is new power?
- 3000K, 2010K respectively.
- If I had 1K signal, what is my output signal level?
- 100K, 100K.
- SNR? 100/3000 vs. 100/2010. Adding noise after amplification didn't make much difference, but before makes huge difference.
- So, huge emphasis on noise of first amplifier in a system, and in reducing noise upstream of that. Downstream matters much less.

Quantum Limit

- Heisenberg uncertainty principle says $\delta x \delta p > \hbar/2$
- But also, $\delta E \delta t > \hbar/2$ - if I take a measurement over a short time, energy uncertainty grows (thankfully, since this is why structure exists in the universe).
- If I coherently amplify a wave (i.e. keep phase the same), time uncertainty should be less than a radian, or $1/2\pi\nu$.
- Means $\delta E = k\delta T > h\nu/2$, or $\delta T > h\nu/2k$. This is quantum limit.
- 30 GHz, limit is ~1K. Good amplifiers get within factor of few. Other noise wins at low frequency, but amplifier often sets noise at high frequency.
- Why do bolometers not suffer from this?

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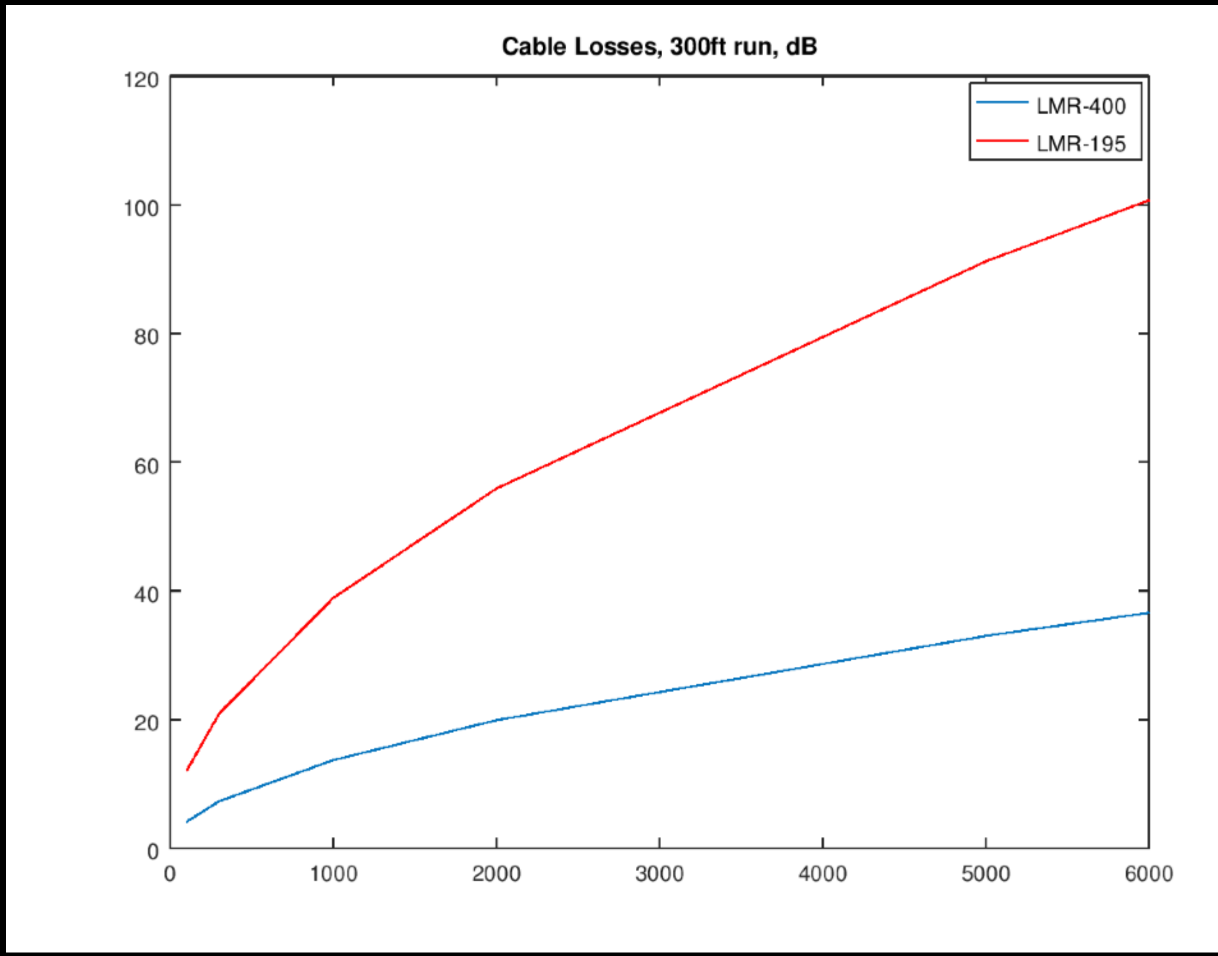
- 30 GHz, limit is ~1K. Good amplifiers get within factor of few. Other noise wins at low frequency, but amplifier often sets noise at high frequency.
- Why do bolometer not suffer from this?
    - Not trying to amplify phase, so arrival time uncertainty can be much larger.

Mixing

- We'd like to be able to observe at any frequency.
- ADCs will only work at some frequency. Usually (but not always) lower than frequencies we care about.
- Sending high frequencies along cables is also challenging. Cable loss grows quickly with frequency.
- Getting signal down from GBT at 5 GHz could lose 999,999,999 out of every billion photons(!)

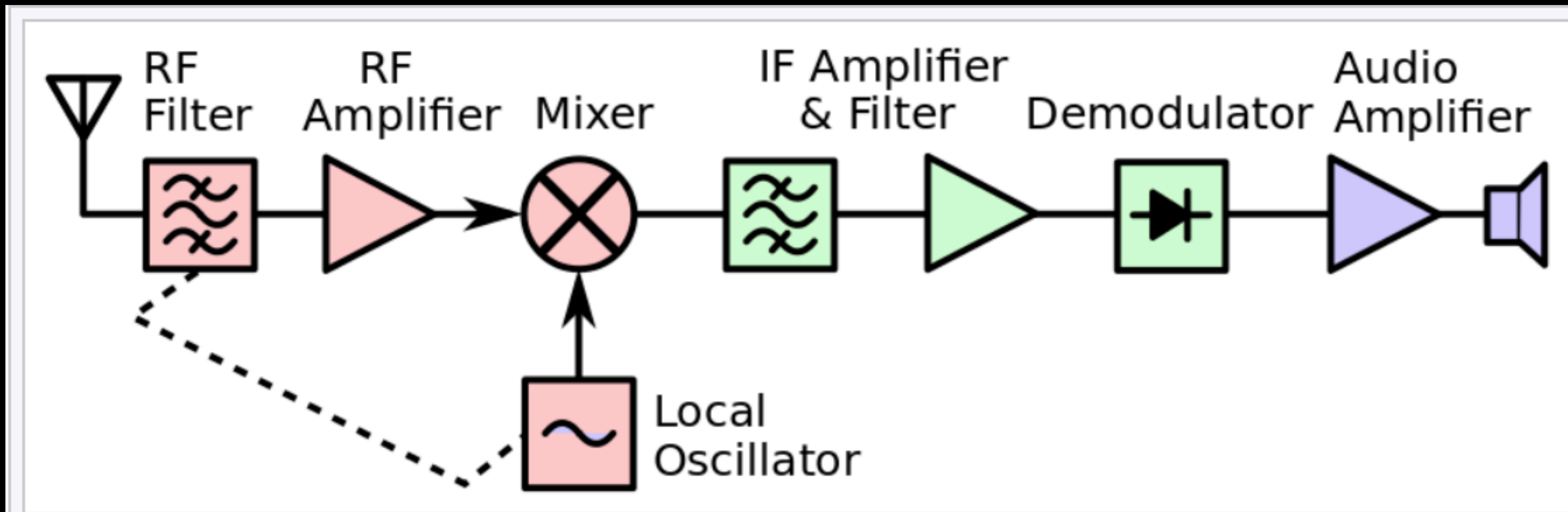


Cable losses from handy-dandy Times Microwave online calculator

Mixing V2

- Can we do something about this?
- Yes! If I have a nonlinear component, and put in two signals V_1,V_2, output will be $c_1(V_1+V_2)+c_2(V_1+V_2)^2+\ldots$
- Second term gets a V_1V_2 component. What does this look like if V_1 and V_2 are sine waves closely spaced in frequency?
- $V1=a\sin(2\pi\nu_1 t)$, $V2=b\sin(2\pi\nu_2 t)$. Angle summation formulas give $\sin(2\pi(\nu_1-\nu_2)t)+\sin(2\pi(\nu_1+\nu_2)t)$
- We can filter out the $\nu_1+\nu_2$ term with analog device, leaving $\nu_1-\nu_2$. We have shifted the signal to lower frequency.
- Process is called heterodyning (developed by Canadian Reginald Fessenden), and nonlinear device is called a mixer.
- Good mixers only put out one frequency combination, but others can put out various combinations of ν_1,ν_2, called intermodulation products. These are bad.
- NB - both $\nu+d\nu$ and $\nu-d\nu$ will generally get mixed in - these are the *sidebands*. Can make mixers that separate them out, can filter one out beforehand, or can just eat them.

Super Heterodyne Receiver



- Signal comes in. Gets amplified/filtered (often amplified before the filter. Why? half dB loss typical, equals 30K if warm)

- Separate signal from local oscillator gets piped into mixer, to shift to lower intermediate frequency (IF).

- IF much easier to move around. Usually goes into another thing that does the detecting, often gets mixed again.

CHIME

- Instead, we could sample directly if ADC fast enough.
- CHIME works 400-800 MHz. Normally need to run at 1600 Msamp/sec (why? Nyquist…)
- BUT - if we analog filter everything outside of band, then we could run at 800 Msamp/s. We would alias low-frequency power, but that is gone.
- This is called working in the second Nyquist zone.
- Other low-frequency telescopes (e.g. PAPER, HERA) directly sample electric field w/out any tricks at all.

Interferometry

- Resolution from single dishes not great. But we can do better!
- If I have two telescopes separated by distance d, observing source at angle θ at wavelength λ, what is phase difference in arriving electric field?
- Source in far field (probably) so path length is $d\sin(\theta)$. Phase difference is $2\pi d/\lambda\sin(\theta)$.
- If I measure electric fields and multiply together $\langle E_1 E^*_2 \rangle$, I will get $E_0^2\exp(2\pi i d/\lambda\sin(\theta))$
- Call d/λ b, the dish separation in wavelengths, and assume small angles on the sky, and $I=E_0^2$. Then we get $I\exp(2\pi i b\theta)$

Signal over Sky

- If I have many sources, I have to integrate over the (2D) sky brightness. Gives: $\langle E_1 E_2^* \rangle = \int I(\theta) \exp(2\pi i b \cdot \theta) d^2\theta$

- Have you seen this before?

- Yes! This is just one component of the FT of the sky, measured at wavevector *b*.

- This time-average is the fundamental output of a radio array, called a *visibility*.

- Resolution set by *baseline length*, not by dish size.

Interferometer

- An array of radio telescopes working together is called an interferometer. Every pair of antennas (called a *baseline*) gets multiplied together in the *correlator*, producing a visibility.

- Vector separation labelled by (u,v(,w)) where u is EW, v is NS (and w is vertical separation, generally much less important).

- UV coverage is sampling of points in UV plane - denser sampling means better understanding of sky.

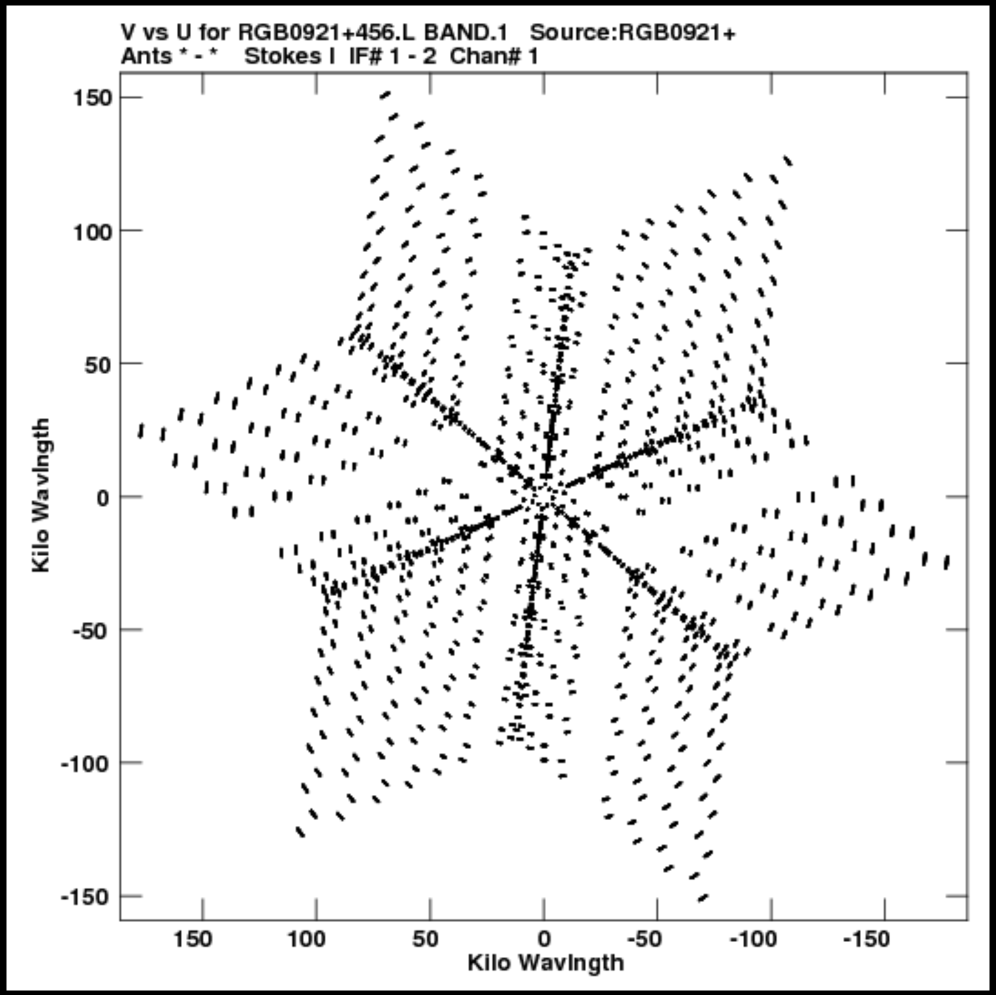
- Telescope PSF given by FT of UV coverage - which may not surprise you given FT optics result from Monday. Every antenna acts as a radiator.

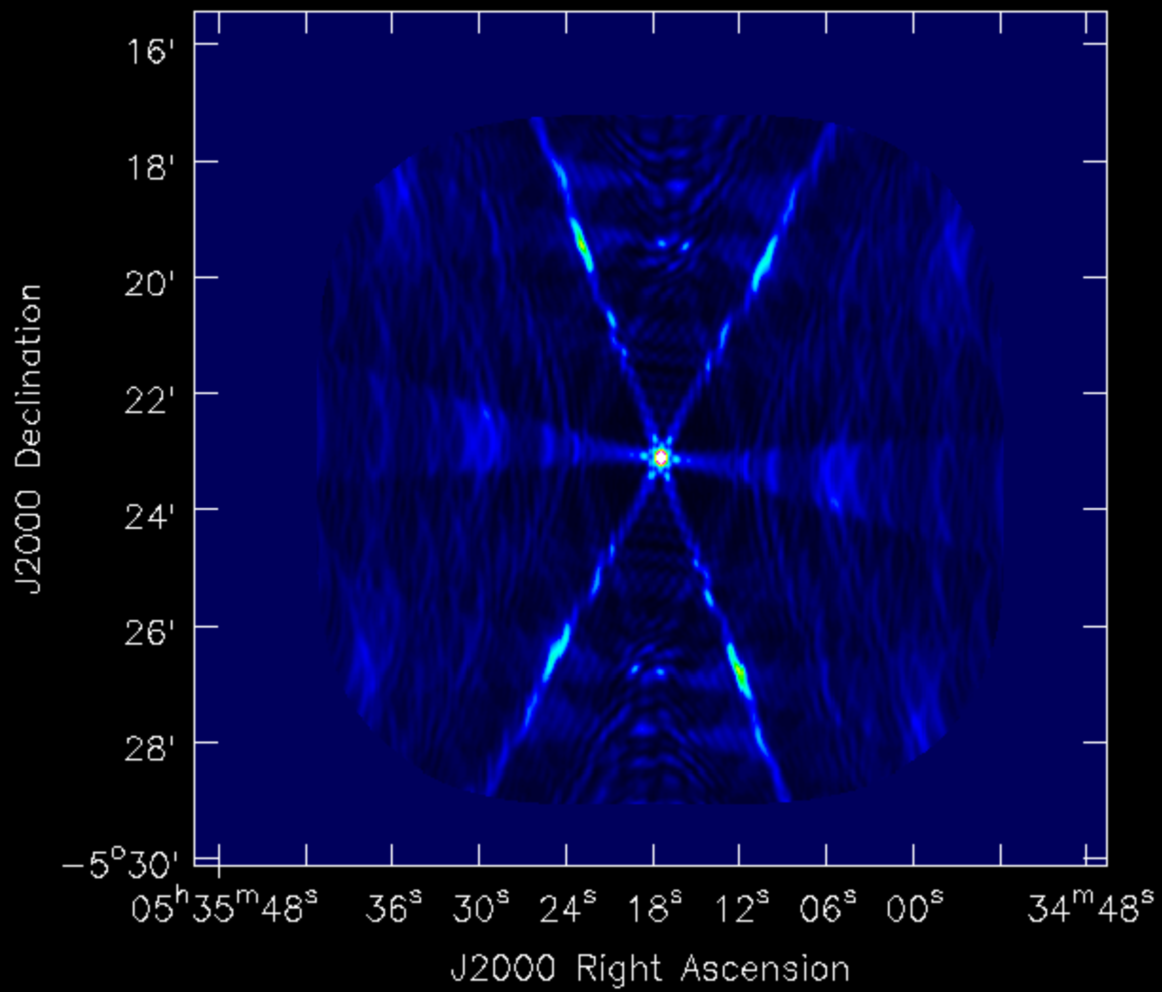
- Slightly more technically: far field is square of abs(beam pattern). That means in Fourier space, we take xcorr of illumination. When illumination is set of discrete points, xcorr is sum over xcorr of illumination points. Exactly the UV FT.

How many operations to correlate?

- Correlation used to be done by analog circuits. Almost uniformly no longer the case.
- VLA - 27 dual pol receivers, 8 GHz bandwidth (total - they split up). 54 inputs, npair=54*55/2~1500 * 8e9*8 ops/complex multiply+add = 100 TFlops.
- CHIME - 1024 dual pol, 400 MHz - 6.7 PFlops. Lots of crunching needed for large arrays.

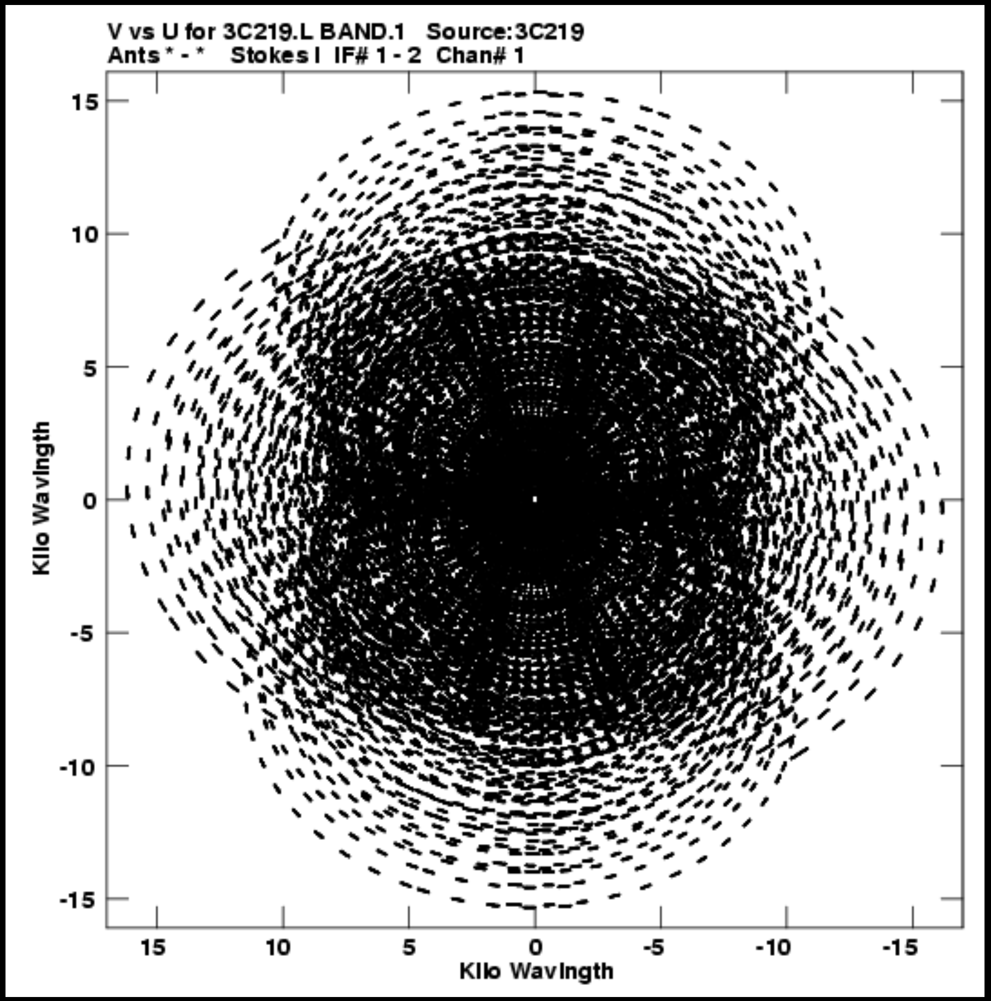
VLA UV Coverage





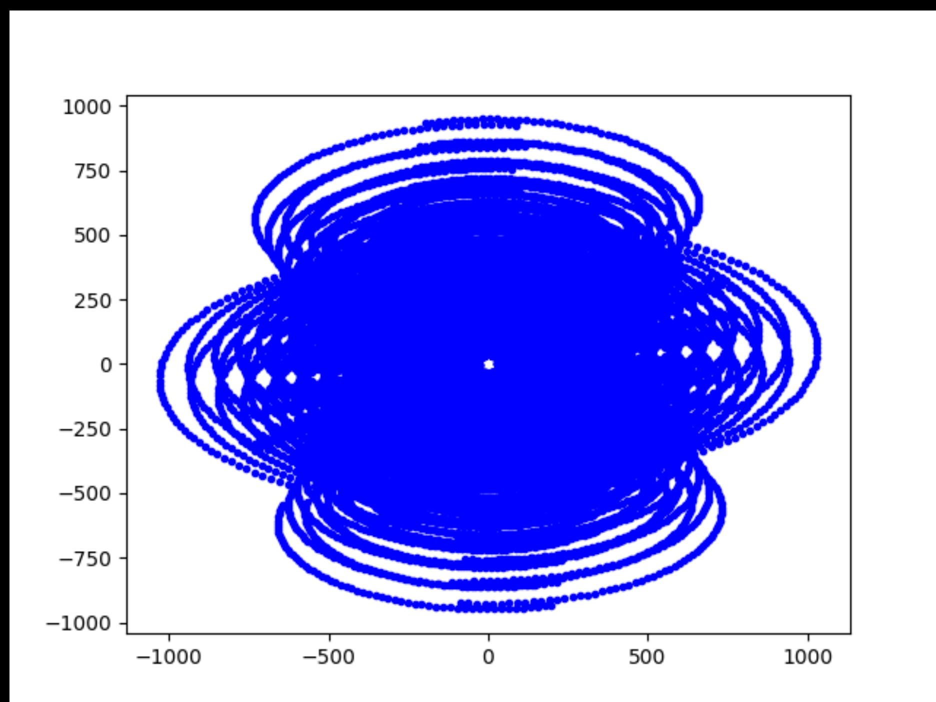
Left: Instantaneous UV coverage from VLA
Right: FT of coverage. Known as the “dirty beam”

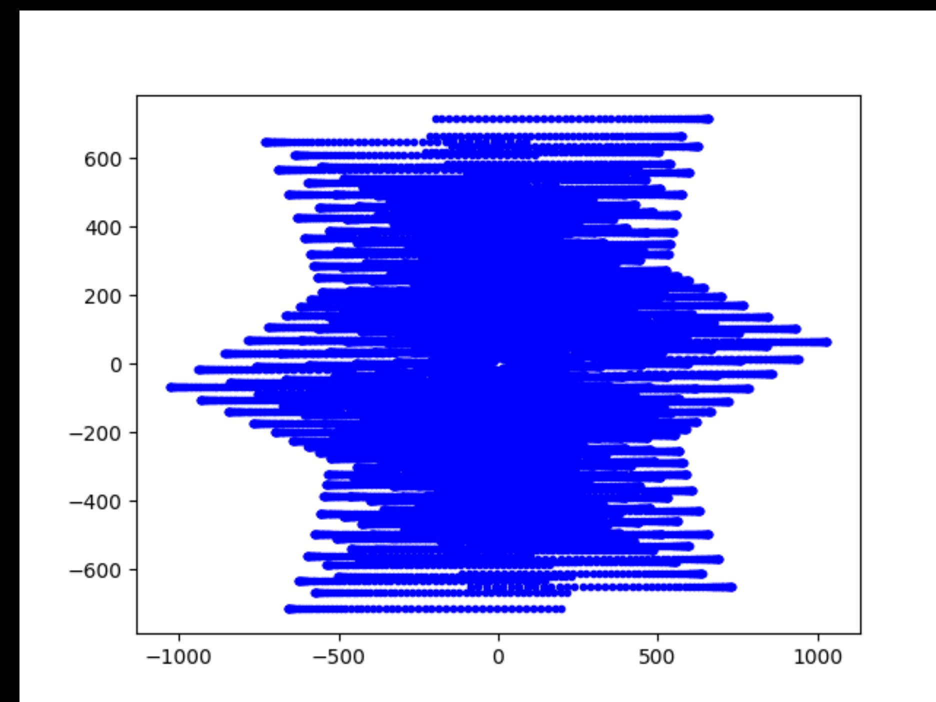
Rotation

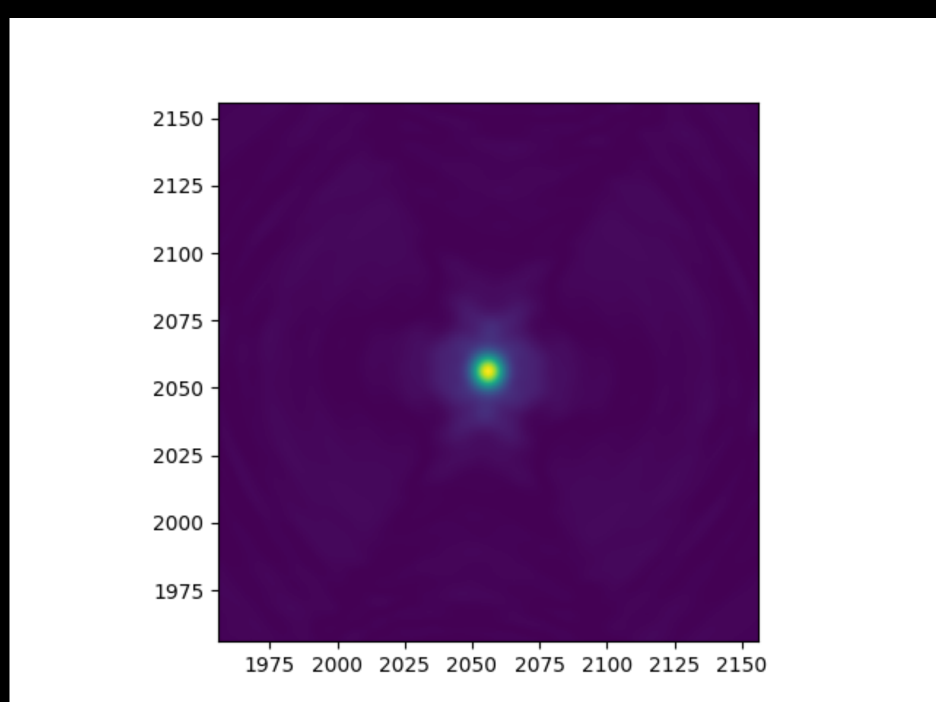


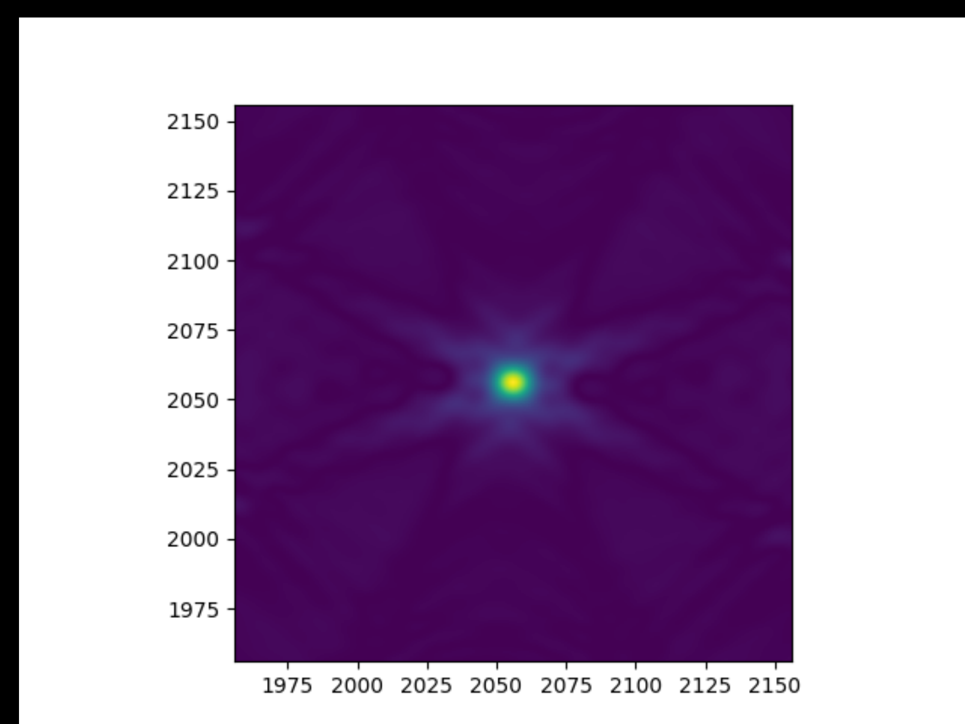
- We want as much UV coverage as possible, right?
- Instantaneous snapshots usually not great.
- However, we are on a rotating platform. Wait long enough, and baseline moves relative to source.
- UV coordinate set by baseline separation perpendicular to field center. Fills things out enormously.
- Multiple channels also samples UV plane radially.

UV, Beam after 8 hours
dec=30 (left), and equator (right)



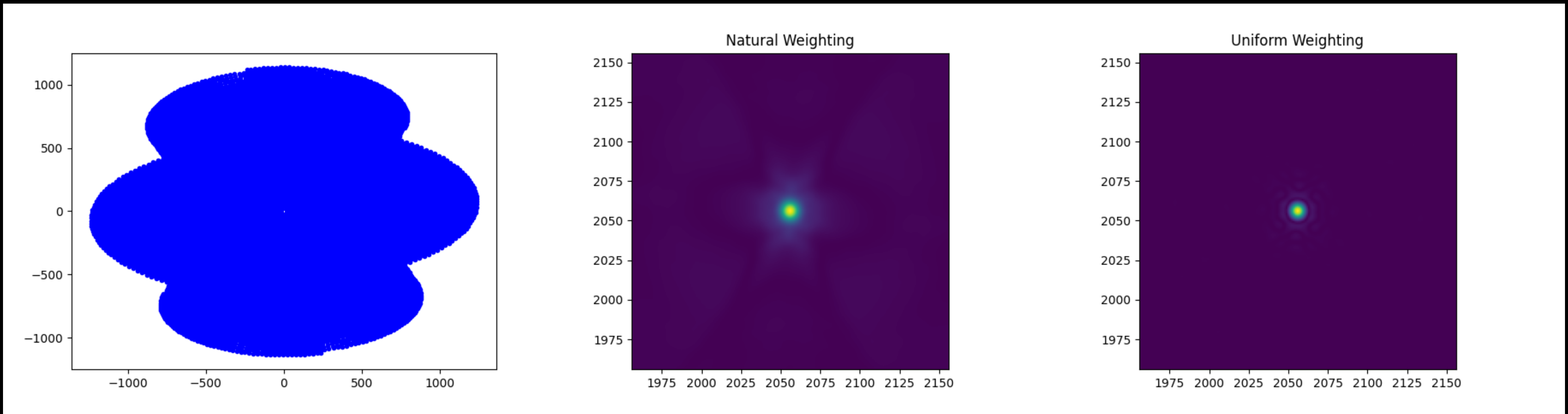






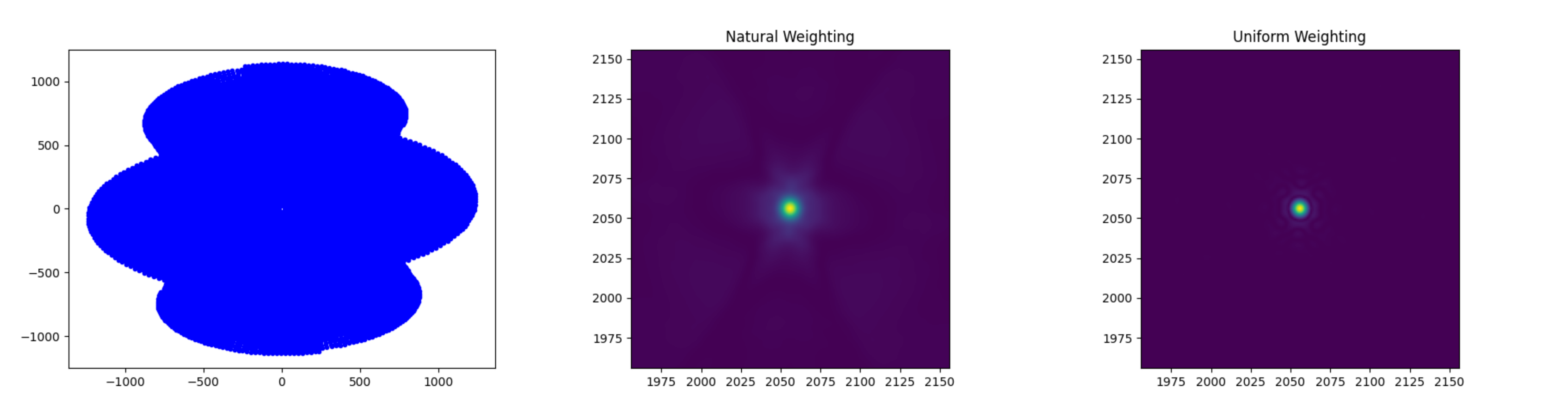
Frequency

- Finite width frequency also fills out UV coverage.
- UV coord is b/λ, so UV sampling *must* change with frequency. Interferometers intrinsically chromatic.
- Often good for imaging since UV coverage denser.



Weighting

- We are also free to weight our data.
- Most of the time weight each visibility with $1/\sigma^2$ - “natural” weighting, in least-squares sense.
- For imaging, we often want to weight each region of UV space equally. Can give better beam at price of SNR.
- Other schemes possible. Depends on what you want!



Primary Beam

- Dishes have a response on the sky, usually limits field of view
- In interferometer, each dish sees primary *beam times* sky, so visibility is $\int A(\theta)I(\theta)\exp(2\pi i u \cdot \theta)d^2\theta$ where A is the primary beam (in power)
- In UV space, we measure not the sky transform, but the sky transform convolved with the primary beam transform. PB sets UV-space resolution
- PB is electric field response squared, so in UV space, that's electric field response convolved with itself. But, electric field response is just aperture illumination, so PB transform is just dish+feed autocorrelation with itself in wavelengths.

Imaging

- If we build telescopes, we usually want to take pictures of things…
- If I measure FT of sky, I can just IFT to get map, right?
- Well… There are usually gaps in UV coverage. If baselines sampled more densely than dish diameter, this might work.
- Not possible for high-resolution. To make a map of sky, we must fill in UV plane with some guess.
- Unobserved parts of UV could be anything! The art of imaging is sensibly filling in those unobserved areas.
- Remember - we understand very well how to predict data/measure χ^2 given a map of the sky. It's only inverse problem that is ill-defined.

CLEAN

- One standard way to do this is CLEAN. Pretend the sky is full of point sources.

- Make a dirty map - direct FT of visibilities. Requires putting visibilities on a grid - how fine does that grid need to be?

- Look for brightest peak (or peaks).

- Subtract from data. Repeat.

- Experience has shown that subtracting fraction of peak brightness works better in practice - say 30-50% of peak.

- When should we stop?

CLEAN

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- Make a dirty map - direct FT of visibilities. Requires putting visibilities on a grid - how fine does that grid need to be?
    - UV resolution set by dish diameter in wavelengths. Need to have grid be significantly finer than that.
- Look for brightest peak (or peaks).
- Subtract from data. Repeat.
- Experience has shown that subtracting fraction of peak brightness works better in practice - say 30-50% of peak.
- When should we stop?
    - We know what noise in dirty map should be - weight is just sum of visibility weights, so stop when noise is (roughly) equal to that.

CLEAN CTD.

- Process of cleaning gives us a list of source fluxes/positions, plus dirty map that may be noise.
- Usual reported thing is to put sources into map, convolve with Gaussian with same size as main dirty beam, add in noise map.
- Significant freedom available in how we weight data. Often density in UV space higher at short baselines.
- Least-squares ("natural") weighting may not give "best" images since weight across UV plane uneven.
- For pretty pictures, you may want to use "uniform" weight, where each *area* of UV plane gets same weight.

Bayesian

- General imaging problem is usually under constrained. We have to "make up" data to make an image.
- Amongst all the possible maps that agree with the data, *you* need to decide which one you think makes sense.
- Which brings us to the Reverend Bayes. P(a|b)P(b)=P(b|a)P(a). P(m|d)=P(d|m)P(m)/P(d) for data d and map m.
- Drop P(d) because we already have the data. P(d|m) is straightforward to calculate. Effectively, mapping problem is deciding on P(m)
- There are many ways to do this - multiscale clean where you fit Gaussians instead of sources, maximum entropy… Do think about what you're looking at as it will guide choice of imaging.