Problem Set for Radio. Due Wednesday November 16 at 11:59 PM

Problem 1 - Synthesized Beams: Let's make UV plots and calculate synthesized beams for the VLA. In the radio directory, look at vla_[a,d]_array.txt. The first three columns are the (x,y,z) coordinates of the dishes in light-nanoseconds. The coordinate system is relative to the center of the Earth, where +z is in the direction from the center of the Earth to the North pole. You may look at my code for inspiration, but please write your own.
a) Plot the 2D positions of the antennas in NS/EW in metres. You'll want to convert the 3D coordinates into 2D coordinates using the local zenith (the latitude of the VLA is 34.1 degrees). The unit vector for east is always (0,1,0), and you can get north by taking a suitable cross product. If all has gone well, you should see the E/W and N/S spreads are similar, and the vertical scatter (which you should also report) is much, much smaller. Please report your vertical RMS scatter for both A and D-arrays.
b) Make a UV plot for the two configurations for a source directly overhead. You may assume you are observing in L-band (1.4 GHz). Please make sure your axes are labelled in wavelengths, not metres.
c) Plot the dirty beam for the UV coverage you generated in part b). Normalization can be tricky, so report the values you get for the beam FWHM and compare to the published VLA values.
d) Repeat the UV plots, but now assuming you observe a source for 8 hours, starting 4 hours before the source transits overhead, to 4 hours after. Do this for a source that crosses directly overhead, a source on the equator, and a source at the north celestial pole.
e) Plots the synthesized beams for the cases in d).

Problem 2 - w Term: a) Estimate the field of view of the VLA at 1.4 GHz and at 8 GHz.
b) We'll now estimate the effects of w by looking at a pair of sources. For a source directly overhead, calculate the *difference* in distance for each baseline for the overhead source and a source 30 arcminutes to the south. For this part, you can assume the path lenth difference is given by the dot product of the source angle and the 2D UV coordinates of the antennas.
c) Now repeat but using the full 3D antenna positions. What is the RMS *difference* between these path length differences and the differences you calculated in b)? To be clear, for a single baseline, you would calculate the UV coordanate for the baseline in a coordinate system pointed at source

1. in b), you calculate the difference in path length between source 1 and source 2 assuming you keep the same UV coordinates. In c) you use the full 3D antenna positions (so there is no such thing as a UV coordinate system any more), but you still report the path length difference for the baseline to source 1 and source 2. If the difference between these two differences is small compared to a wavelength, we can ignore the w term.
d) Now that you are set up to do this, report the RMS *phase* difference for a source 1 FWHM from the pointing center at both 1.4 and 8 GHz when the pointing center is 1) directly overhead, and 2) at the equator. For which of these cases will you need to worry about the w term for both A and D-arrays?

Problem 3 - Frequency Smearing:
Radio telescopes typically record many, many more frequencies than at other wavelengths. For this problem, assume we're working at 1.4 GHz, and take the UV coverage for the VLA for a source directly overhead. Now take a second source separated by 1 FWHM to the south. As we change observing frequency, the UV coordinates in wavelengths will change. By how much do we need to shift the frequency to make the RMS path length difference change by $\lambda/2\pi$ for both A and D-arrays? This is called bandwith smearing, and especially at high resolution, can become a major driver of how one observes. Without enough frequency channels, you can lose much of your field of views.

Problem 4 - Confusion Limit:
The usual rule of thumb is that you hit the confusion limit at one visible source for every 30 beams. We'll also start with FIRST having 100 sources per square degree above 1 mJy at 1.4 GHz.
a) Assuming Euclidean counts, show that the number of sources brighter than some flux limit $N(S) > S \propto S^{-3/2}$. The easiest way to do this is assume all sources have the same brightness, and are uniformly distributed in space. Then calculate the volume in which you would observe a source. For the rest of the problem, we'll assume counts are Euclidean even though this isn't really true.
b) For GBT, FAST (effective diameter 300m), and VLA in A and D-arrays, what is the confusion level, using the source-per-30 beams rule?
c) How long would each telecope be able to integrate before hitting the confusion limit? You can assume a T_{sys} of 25 degrees, bandwidth of 500 MHz, and an aperture efficiency of 70%.

d) Repeat parts b) and c) but for an observing frequency of 8 GHz and a bandwidth of 2 GHz. For simplicity, assume all sources have a spectral index of -0.8, so a source at 8 GHz has a flux $(8/1.4)^{-0.8}$ times its flux at 1.4 GHz.