Why are Metals Shiny?

- Because they conduct.
- So why can we see through sea water?
- Well, sometimes we can’t. Radio doesn’t make it through.

Plasma Frequency

- Say I have a block of plasma, and move all electrons by x. What is electric field?
- Surface charge is $en_e x$. From Gauss's law, $E=en_e x/2\varepsilon_0$, where factor of 2 is from 2-sided cylinder.
- Of course, have protons have charge, so total field is $en_e x/\varepsilon_o$.
- $F=qE=-e^2 n_e x/\varepsilon_0$. also $F=ma=mx''$, so $x''=-(e^2 n_e/m_e\varepsilon_o)x$
- Plasma will oscillate with angular frequency $\omega_p=\sqrt{(e^2 n_e/m_e\varepsilon_o)}$.

Dispersion Relation

- If I send an EM wave into plasma, plasma can adjust if low frequency, not if high frequency.
- Dispersion relation is $\omega^2=\omega_p^2+k^2c^2$.
- Phase velocity is $\omega/k=c(1+\omega_p^2/k^2c^2)^{1/2}$, $>c$
- Group velocity $d\omega/dk=kc^2/(\omega_p^2+k^2c^2)^{1/2}$, $<c$

Low Frequency

- Recall wave equation is $\exp(i(kx-\omega t))$. If $\omega<\omega_p$, $k=(\omega_p^2-\omega^2)^{1/2} i/c$ and spatial part of wave equation is $\exp(-kx)$
- Wave decays exponentially with decay length $c/(\omega_p^2-\omega^2)^{1/2}$
- Limit is c/ω_p, the plasma skin depth.
- What is skin depth of typical conductor?
- Other effects (like collisions) become important in real media, e.g. submarines can communicate underwater at very low (few kHz) frequencies.

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- What is skin depth of typical conductor?
    - copper atomic wt. is 63.5, density is 9. If 1 e^- per atom, then ω_p=1.6e16, and skin depth is ~200 Å.
    - NB - sea water has ions instead of electrons, and only ~2% vs. H_20, so simple plasma model would put frequency at ~1e12, skin depth of 0.3mm.
- Other effects become important in real media, e.g. submarines can communicate underwater at very low (few kHz) frequencies.

High Frequency

- Group velocity $d\omega/dk = kc^2/(\omega_p^2 + k^2c^2)^{1/2}$
- For large ω, $kc >> \omega_p$, and $v_g \sim kc^2/kc(1+ \omega_p^2/k^2c^2)^{1/2} = c(1-\omega_p^2/2k^2c^2)$
- But, $kc \sim \omega$, so $v_g/c = 1-\omega_p^2/2\omega^2$.
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- If I had 1 e/cm3 for 1 pc, how long would a 1 MHz signal be delayed relative to high frequency?
    - ωp~56kHz/vp~9 kHz. Fractional velocity change is 0.5*(9 kHz/1 MHz)^2=4e-5. Light travel time is 3.26 years, so total delay is 4e-5*3.26=4150 seconds.
    - Origin of pulsar delay law that dt=4150*DM/v^2 where v in MHz and DM defined to be column density of electrons relative to 1/cm^3 for 1 pc.

Waves in Plasma/Why Metals Reflect

- If free space wave hits plasma, if frequency below plasma frequency wave can't propogate, gets reflected by plasma.

- If above v_p, wave can go through, with group velocity $<c$, phase velocity $>c$.

- Metals have v_p~2e15 Hz ~10 eV. Optical photons reflect, but hard UV/x-rays will go through.

- If I want to build an X-ray telescope, I can't use traditional mirrors.

Refraction

- If I have a wave propagating at some angle going between two different media, what happens?
- Wave phase has to be continuous across boundary.
- Wave will bend so that phase velocity along boundary is same on both sides.
- Phase velocity vs. free space is inverse of index of refraction, n.
- Snell's law is phase continuity constraint: $\sin(\theta_1)n_1=\sin(\theta_2)n_2$.

Total Internal Reflection

- $\sin(\theta_2)$ can't be larger than 1. So, $\sin(\theta_1)<n_2/n_1$.

- If $n_2<n_1$, then there's a critical angle at which we can no longer go through. Instead, wave reflects.

- This is called total internal reflection.

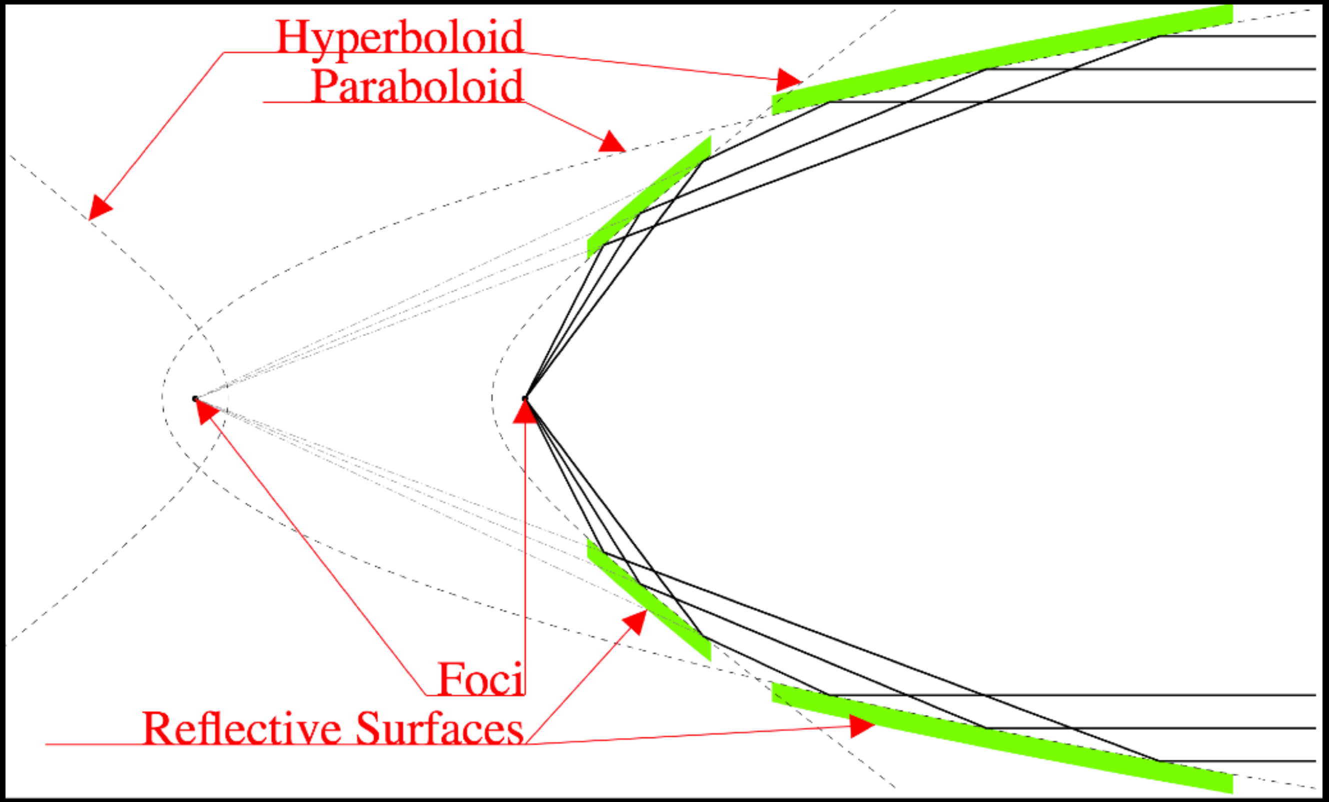
- Only happens if we go from medium with low phase velocity to medium with high phase velocity.

Grazing Incidence

- What is index of refraction of metals at 1 keV?
- phase velocity=$c(1+\omega_p^2/k^2c^2)^{1/2}$,~$c(1+v_p^2/v^2)^{1/2}$,$v^p/c$~$1+0.5(v_p/v)^2$.
- Index of refraction ~$1-0.5(v_p/v)^2$.
- At 1 kHz, n~$1-0.5(10/1000)^2$=1-5e-5 Critical angle is asin(1-5e-5)~89.5 degrees.
- So, if X-ray comes in within half degree of parallel, it will reflect. This is *grazing incidence*.

X-ray Telescope Design

- Turns out FOV is tiny for single grazing incidence mirror.
- Solution is to have multiple element to correct coma.
- Few different designs possible (Wolter types 1-3)
- To get collecting area, have to stack many elements.



Top: Wolter type 1 diagram
Bottom: Chandra primary mirror