

Order-of-Magnitude

How big was the all-time
largest armored car robbery?

Have a picture



How would you go about guessing?

- upper limit - how much cash can you fit inside?
- are there other constraints?

Be Prepared

- You will get asked many random questions.
Lectures, talks, prelim exams...
- You may have nutty idea for research. Is it worth pursuing?
- It's *really* useful to get sensible answers quickly, without much math.
- This is a skill that can be practiced.
- When faced with new problem, think about essential physics. Put in what you know, make reasonable guesses for what you don't.
- Dimensionless integrals “usually” order unity. Getting them right is hard! And often pointless!

Overview

- Today - do a few practice problems together.
- Do one on your own. TA will solve as well, and will present his answer.
- We'll critique it.
- Write up your answer (plus one other) and submit to mycourses AS PDF!
- Next week - you will get randomly assigned problem at start of class. Will have 30 minutes to work on solution.
- In groups of 3, each person presents problem/their solution. The other 2 comment on/evaluate the solution.

Facts

- You do have to know *something*.
- Purcell made sheet of useful numbers.
- Many things can be estimated from this!
- It can be very handy to have these sorts of numbers at your fingertips.

EDWARD M. PURCELL

$c = 3 \times 10^10 \text{ cm/sec}$	$e = 4.8 \times 10^{-10} \text{ esu} = 1.6 \times 10^{19} \text{ coul}$	$m_e = 10^{-27} \text{ gm}$
$\hbar = 10^{-27} \text{ erg-sec}$	$k = 1.4 \times 10^{16} \text{ erg/deg}$	$G = 7 \times 10^{-8} \text{ erg-cm/gm}^2$
$N_A = 6 \times 10^{23}/\text{mole}$	$R = 2 \text{ cal/mole-deg}$	$n_0 \text{ at N.I.R.} = 3 \times 10^{19}/\text{cm}^3$

1 newton = 10^5 dynes	$\mu_0 = 4\pi \times 10^{-7} \text{ newt/amp}^2$	$1 \text{ ohm}^{-1} = 9 \times 10^9 \text{ cm/sec}$
1 ft = 30 cm 1 pound = 4.4 newt.	$E_0 = 8.8 \times 10^{12} \text{ coul}^2/\text{newt-m}^2$	$\sqrt{\mu_0/E_0} = 377 \text{ ohms}$

"classical electron radius" $r_0 = e^2/m_e c^2 \approx 3 \times 10^{-13} \text{ cm}$	$\alpha = e^2/\hbar c$
"Compton wavelength" $\lambda_c = \hbar/m_e c \approx 4 \times 10^{-11} \text{ cm}$	$= 1/137$
"Bohr radius" $a_0 = \hbar^2/m_e e^2 \approx 5 \times 10^{-9} \text{ cm}$	Bohr magneton
"Rydberg w/length" $\lambda_R = \hbar^2 c/m_e e^4 \approx 7 \times 10^{-7} \text{ cm}$	$e\hbar/2mc = 10^{20} \text{ erg/gauss}$

$L_{\text{cal}} = 4 \text{ watt-sec} = 4 \times 10^7 \text{ erg}$	$1 \text{ ev} = 1.6 \times 10^{-12} \text{ erg}$	black body radiates
$m_e c^2 = .5 \text{ Mev}$	$\epsilon^2/a_0 = 26 \text{ ev}$	$6 \times 10^{-12} \text{ watts/deg}^4/\text{cm}^2$
$kT_{\text{room}} = .025 \text{ ev}$	vis. photon $\approx 2 \text{ ev}$	680 lumens = 1 watt (650 Å)
band gap: Si: 1.1 ev	Ge: 0.7 ev	

$m_{\text{nucleon}} = 2000 m_e$	$g = 10^3 \text{ cm/sec}^2$	$P_{\text{at}} = 10^6 \text{ dyne/cm}^2 \approx 15 \text{ psi}$
$m_{\text{kaon}} = 1000 m_e$	$\text{air density} = 10^{-3} \text{ gm/cm}^3$	scale height = 8 km
$m_{\text{pion}} = 270 m_e$	air at 300°K: $v_{\text{sound}} \approx v_{\text{molc}} \approx 4 \times 10^4 \text{ cm/sec}$	
$m_{\text{muon}} = 200 m_e$	mean free path (air, NTP) $\approx 7 \times 10^{-6} \text{ cm}$	
$R_{\text{nucleus}} = A^{1/3} \times 10^{-13} \text{ cm}$	$PC(\text{ev}) = 300 B R (\text{gauss-cm})$	$1 \text{ parsec} = 3 \times 10^{18} \text{ cm}$

spin recession {e: 3 MHz/gauss	min. ioniz. loss: 2 Mev/gm/cm^2	1 mag = -4 db
{p: 4 kHz/gauss	rad. length in air: 36 gm/cm^2	$M_{\text{abs}} = M_{\text{app}}$ at 10 pc
resistivity, usual temperature	1 curie = $4 \times 10^{10} \text{ disint./sec}$	$M_\odot = 5$
Cu: 2×10^6 ; pure H ₂ O: 2×10^7 ; sea water: 25 ohm-cm	earth field at pole = .5 gauss	

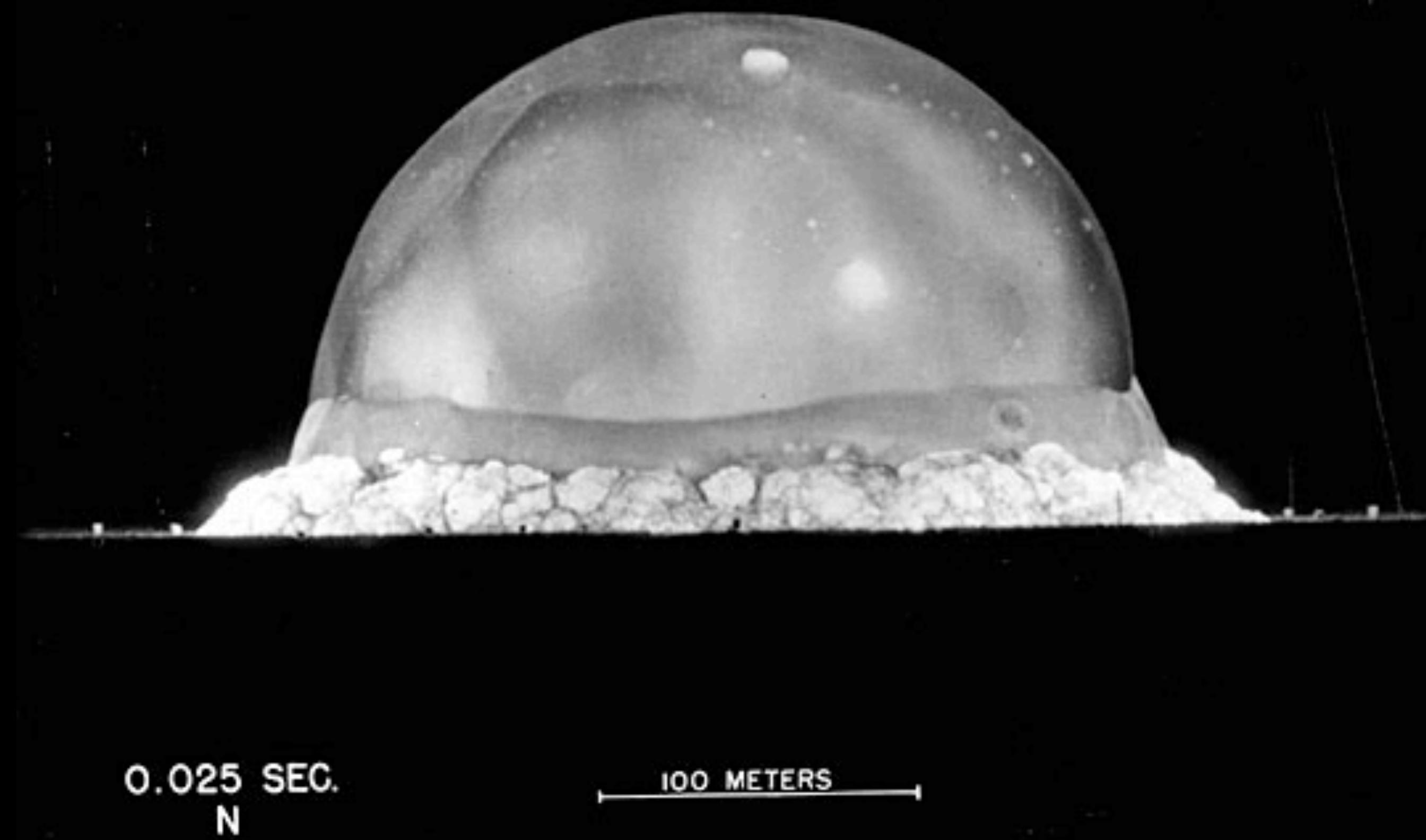
specific heat (solid or liquid) $\approx 0.5 \text{ cal/cm}^3/\text{deg}$	$M_e = 6 \times 10^{27} \text{ gm}$	$R_e = 6 \times 10^8 \text{ cm}$
linear expansion ("") $\approx 2 \times 10^{-5}/\text{deg}$	$M_\odot = 2 \times 10^{33} \text{ gm}$	$R_\odot = 8 \times 10^{10} \text{ cm}$
heat conduction (insulator) $\approx 10^{-2} \text{ cal/sec-cm-deg}$	$L_\odot = 4 \times 10^{33} \text{ erg/sec} = 1 \text{ kw/m}^2$ at earth	starlight energy density: $10^{-12} \text{ erg/km}^2$
heat cond. (metal) $\approx 1.0 (P_{\text{Cu}}/P_{\text{metal}}) \text{ cal/sec-cm-deg}$	primary cosmic rays: $1/\text{cm}^2/\text{sec}$	distance to moon: $4 \times 10^{10} \text{ cm}$
heat of combustion (food or fuel) $\approx 10^4 \text{ cal/gm}$	distance to sun: $1.5 \times 10^{13} \text{ cm}$	distance to center of Galaxy: $3 \times 10^{22} \text{ cm}$
heat of vaporization $\approx 10^4 \text{ cal/mole}$	mass of Galaxy: $2 \times 10^{44} \text{ gm}$	dist. between galaxies: 10^{25} cm
elastic moduli (solids) $\approx 10^8 - 10^{12} \text{ dyne/cm}^2$	$R_{\text{universe}} \approx 3000 \text{ Mpc} = 10^{28} \text{ cm}$	
tensile strength (solids) $\approx 10^8 - 10^{10} \text{ dyne/cm}^2$		
surface tension H ₂ O = 50 dynes/cm		
diffusion: H ₂ O 10^{-5} , air $0.2 \text{ cm}^2/\text{s}$		
viscosity: H ₂ O 10^{-2} , air $2 \times 10^{-4} \text{ dyne-s/cm}^2$		

E.M.P. 1981

If you Don't Know a Fact

- Can you figure it out from what you do know?
- Density of the sun?

How big was the first atomic bomb?



Blast Radius vs. Time

- What can solution depend on?
- Energy of blast. Density of matter. Time since blast.
- Units: $\text{kg m}^2/\text{s}^2$. kg/m^3 . s.
- Get rid of kg: $E/\rho = \text{m}^5/\text{s}^2$. multiply by t^2 to get rid of s^2 .
- Leaves: $r^5 \sim Et^2/\rho$.

Dimensional Analysis

- There's (often) only one way to combine relevant quantities to have desired units.
- Up to an unknown multiplicative factor, answer has to scale like that combination.
- Getting the factor is hard!
- It's “usually” “close” to one.

Plug in numbers

- density of air: 1 kg/m^3 .
- from picture: $r \sim 150\text{m}$ at $t=0.025\text{s}$.
- $E = r^5 \rho / t^2 = 7e10 * 1 * 1600 = 1e14 \text{ Joules}$.

Convert to kilotons

- How much energy is in a ton of TNT?
- How many calories per gram of food?

1 gram of fat is ~9 kCal ~ 40 kJ. Explosives self-oxidize, so 1g TNT ~1 kCal.
 $1e14 \text{ J} / 4 \text{ kJ/g} / 1e6 \text{ g/ton} = 25,000 \text{ tons}$. Very close!

Matrix/Buckingham Pi

- Multiplying constants means summing powers of units.
- Can write this as matrix equation - exponents of units of quantities we have go into matrix A. Exponents of units of quantity we desire goes in to b.
- All that is left is to solve $Ax=b$. x gives the powers of the quantities that give the right units for b.
- If A is well-behaved, we have only one way to get correct units.
- If A is singular, we have a degeneracy/dimensionless quantity. You may have to think harder - e.g., what power of α makes sense?

Let's practice some more problems

Kursk Explosion

- Russian submarine Kursk exploded in 2000 in 108m of water.
- Seismographs picked up 1.45 Hz oscillation.
- How big was the explosion?

Kursk 2

- What quantities are relevant?
- P, rho, f.
- How do we turn them into an energy?
- NB - we could turn them into a radius as well. $P=E/V$, so turn r into V and get E.
- $P = E/m^3$. $\rho = \text{kg}/m^3$. $f = s^{-1}$. $P/\rho = m^2/s^2$. $R \sim f^{-1}(P/\rho)^{1/2}$.
- $E_n = 4R^3$ $P \sim 4f^{-3}P^{5/2} \rho^{-3/2}$.
- $\rho \sim 1\text{e}3$. 10 m=1 atmosphere, so 100m $\sim 10^6$ Pa.
- $4 * 1.45^{-3} * 10^{2.5 * 6} * 10^{-1.5 * 3} \sim 10^{10.5}$. or ~ 10 tons.

Wikipedia Answer

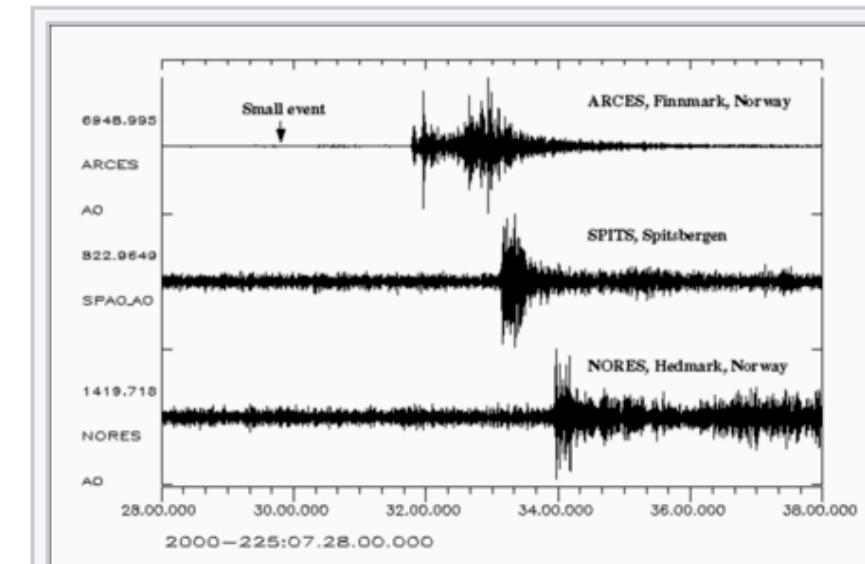
Initial seismic event detected [edit]

At 11:29:34 (07:29:34 GMT), seismic detectors at the Norwegian seismic array (NORSAR) and in other locations around the world recorded a seismic event of magnitude 1.5 on the Richter scale.^[15] The location was fixed at coordinates 69°38'N 37°19'E, north-east of Murmansk, approximately 250 km (160 mi) from Norway, and 80 km (50 mi) from the Kola Peninsula.^[16]

Secondary event [edit]

At 11:31:48,^[15] 2 minutes and 14 seconds after the first, a second event, measuring 4.2 on the Richter scale, or 250 times larger than the first,^[14] was registered on seismographs across northern Europe^[17] and was detected as far away as Alaska.^[9] The second explosion was equivalent to 2–3 tons of TNT.^[5]

The seismic data showed that the explosion occurred at the same depth as the sea bed.^[15] The seismic event, triangulated at 69°36.99'N 37°34.50'E, showed that in a little over 2 minutes the boat had moved about 400 m (1,300 ft) from the site of the initial explosion. It was enough time for the submarine to sink to a depth of 108 m (354 ft) and remain on the sea floor for a short period.^[15]



Norwegian Seismic Array
seismic readings at three
locations of the explosions on the
submarine *Kursk* on 12 August
2000.

Practice Questions

- How many atoms from Einstein's last breath are in you right now?
- What's the magnitude of the vector sum of the angular momentum in the solar system?