Order-of-Magnitude

How big was the all-time largest armored car robbery?

Have a picture

How would you go about guessing?

- upper limit - how much cash can you fit inside?
- are there other constraints?

Be Prepared

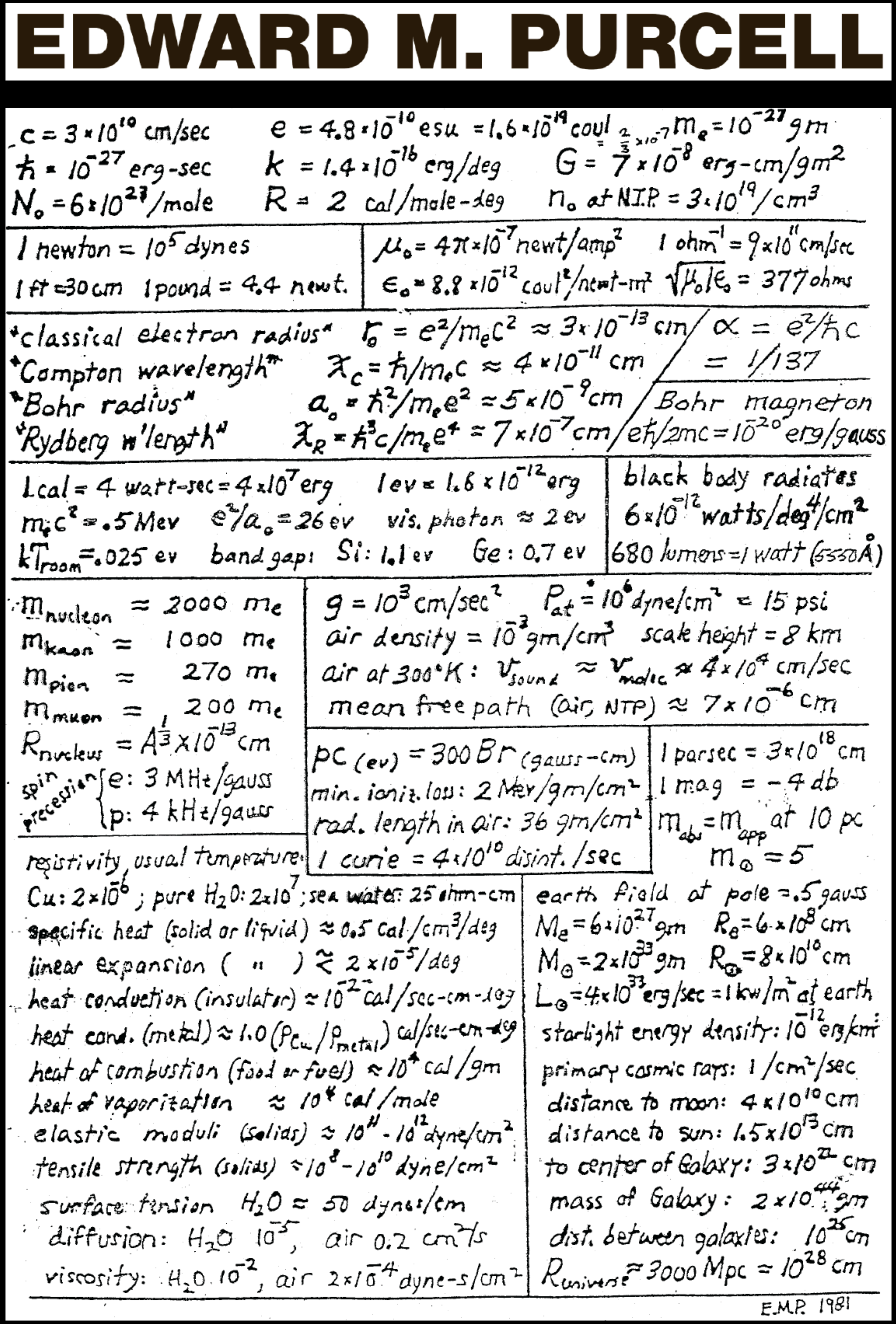
- You will get asked many random questions.
  Lectures, talks, prelim exams…
- You may have nutty idea for research. Is it worth pursuing?
- It's *really* useful to get sensible answers quickly, without much math.
- This is a skill that can be practiced.
- When faced with new problem, think about essential physics. Put in what you know, make reasonable guesses for what you don't.
- Dimensionless integrals "usually" order unity. Getting them right is hard! And often pointless!

Overview

- Today - do a few practice problems together.
- Volunteer will present answer. We'll critique it.
- #Write up your answer (plus one other) and submit to mycourses AS PDF!
- Next week - you will get randomly assigned problem at start of class. Will have 30 minutes to work on solution.
- In groups of 3, each person presents problem/their solution. The other 2 comment on/evaluate the solution.
- Evaluate based on how well physics captured (50%) and how well calculated (50%)

Facts

- You do have to know *something*.
- Purcell made sheet of useful numbers.
- Many things can be estimated from this!
- It can be very handy to have these sorts of numbers at your fingertips.



EDWARD M. PURCELL

$c = 3 \times 10^{10}$ cm/sec $e = 4.8 \times 10^{-10}$ esu $= 1.6 \times 10^{-19}$ coul $m_e = 10^{-27}$ gm

$\hbar = 10^{-27}$ erg-sec $k = 1.4 \times 10^{-16}$ erg/deg $G = \frac{2}{3} \times 10^{-7} = 7 \times 10^{-8}$ erg-cm/gm^2

$N_0 = 6 \times 10^{23}$/mole $R = 2$ cal/mole-deg n_0 at N.T.P. $= 3 \times 10^{19}/cm^3$

1 newton $= 10^5$ dynes

1 ft = 30 cm 1 pound = 4.4 newt.

$\mu_0 = 4\pi \times 10^{-7}$ newt/amp^2 1 ohm^{-1} $= 9 \times 10^{11}$ cm/sec

$\epsilon_0 = 8.8 \times 10^{-12}$ coul^2/newt-m^2 $\sqrt{\mu_0/\epsilon_0} = 377$ ohms

"classical electron radius" $r_0 = e^2/m_e c^2 \approx 3 \times 10^{-13}$ cm

"Compton wavelength" $\lambda_c = \hbar/m_e c \approx 4 \times 10^{-11}$ cm

"Bohr radius" $a_0 = \hbar^2/m_e e^2 \approx 5 \times 10^{-9}$ cm

"Rydberg w'length" $\lambda_R = \hbar^3 c/m_e e^4 \approx 7 \times 10^{-7}$ cm

$\alpha = e^2/\hbar c = 1/137$

Bohr magneton $e\hbar/2mc = 10^{-20}$ erg/gauss

1 cal = 4 watt-sec $= 4 \times 10^7$ erg 1 ev $= 1.6 \times 10^{-12}$ erg

$m_e c^2 = .5$ Mev $e^2/a_0 = 26$ ev vis. photon ≈ 2 ev

$kT_{room} = .025$ ev band gap: Si: 1.1 ev Ge: 0.7 ev

black body radiates 6×10^{-12} watts/deg^4/cm^2

680 lumens = 1 watt (5550 Å)

$m_{nucleon} \approx 2000\ m_e$

$m_{kaon} \approx 1000\ m_e$

$m_{pion} \approx 270\ m_e$

$m_{muon} = 200\ m_e$

$R_{nucleus} = A^{\frac{1}{3}} \times 10^{-13}$ cm

spin precession { e: 3 MHz/gauss; p: 4 kHz/gauss

$g = 10^3$ cm/sec^2 $P_{at} \doteq 10^6$ dyne/cm^2 ≈ 15 psi

air density $= 10^{-3}$ gm/cm^3 scale height = 8 km

air at 300°K: $v_{sound} \approx v_{molec} \approx 4 \times 10^4$ cm/sec

mean free path (air, NTP) $\approx 7 \times 10^{-6}$ cm

$pc_{(ev)} = 300\, Br_{(gauss-cm)}$

min. ioniz. loss: 2 Mev/gm/cm^2

rad. length in air: 36 gm/cm^2

1 curie $= 4 \times 10^{10}$ disint./sec

1 parsec $= 3 \times 10^{18}$ cm

1 mag = −4 db

$m_{abs} = m_{app}$ at 10 pc

$m_\odot = 5$

resistivity, usual temperature: Cu: 2×10^{-6}; pure H_2O: 2×10^7; sea water: 25 ohm-cm

specific heat (solid or liquid) ≈ 0.5 cal/cm^3/deg

linear expansion (") $\gtrsim 2 \times 10^{-5}$/deg

heat conduction (insulator) $\approx 10^{-2}$ cal/sec-cm-deg

heat cond. (metal) $\approx 1.0\,(\rho_{Cu}/\rho_{metal})$ cal/sec-cm-deg

heat of combustion (food or fuel) $\approx 10^4$ cal/gm

heat of vaporization $\approx 10^4$ cal/mole

elastic moduli (solids) $\approx 10^{11} - 10^{12}$ dyne/cm^2

tensile strength (solids) $\approx 10^8 - 10^{10}$ dyne/cm^2

surface tension $H_2O \approx 50$ dynes/cm

diffusion: H_2O 10^{-5}, air 0.2 cm^2/s

viscosity: H_2O 10^{-2}, air 2×10^{-4} dyne-s/cm^2

earth field at pole = .5 gauss

$M_e = 6 \times 10^{27}$ gm $R_e = 6 \times 10^8$ cm

$M_\odot = 2 \times 10^{33}$ gm $R_\odot = 8 \times 10^{10}$ cm

$L_\odot = 4 \times 10^{33}$ erg/sec = 1 kw/m^2 at earth

starlight energy density: 10^{-12} erg/cm^3

primary cosmic rays: 1/cm^2/sec

distance to moon: 4×10^{10} cm

distance to sun: 1.5×10^{13} cm

to center of Galaxy: 3×10^{22} cm

mass of Galaxy: 2×10^{44} gm

dist. between galaxies: 10^{25} cm

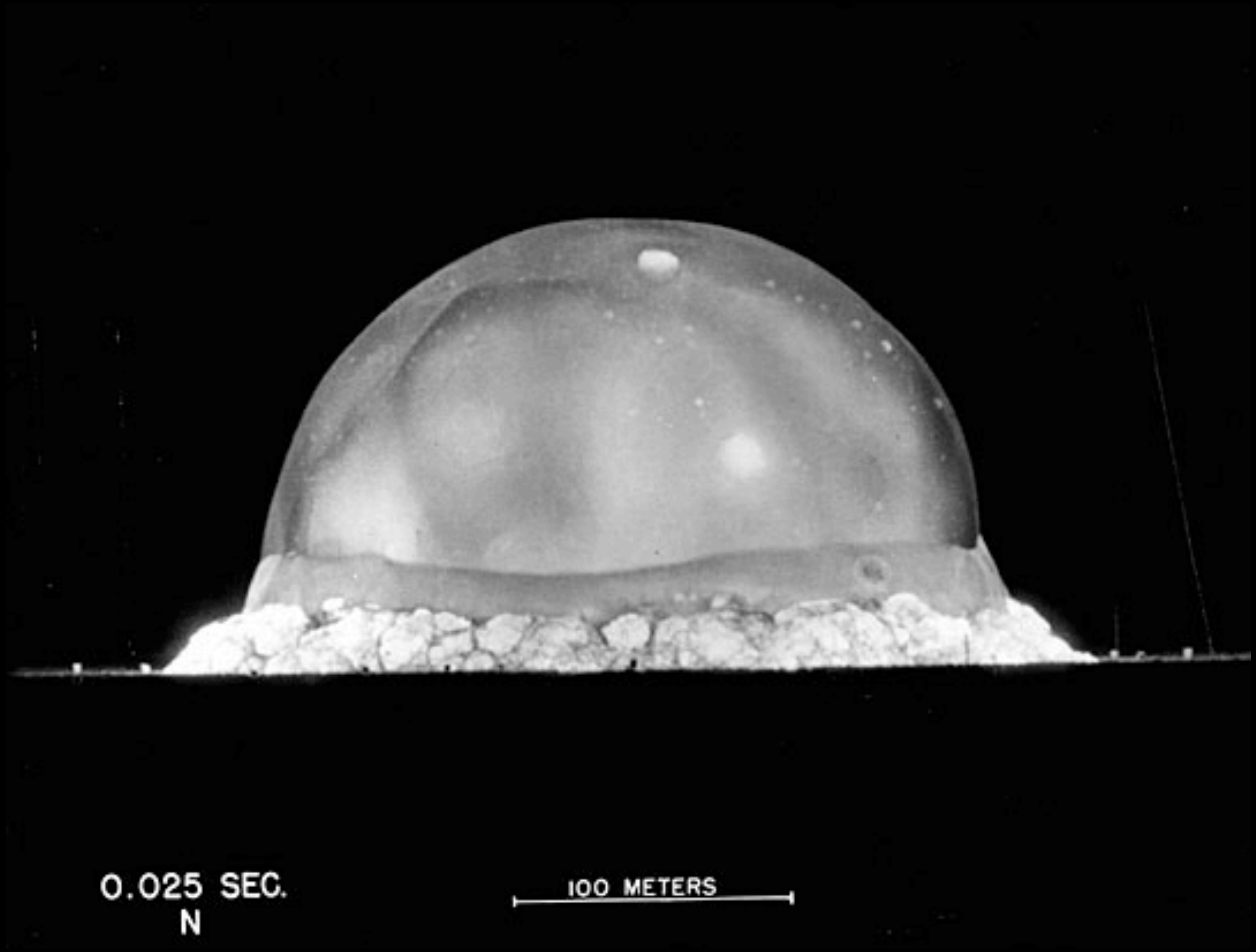
$R_{universe} \approx 3000$ Mpc $= 10^{28}$ cm

E.M.P. 1981

If you Don’t Know a Fact

- Can you figure it out from what you do know?
- Density of the sun?

How big was the first atomic bomb?



Blast Radius vs. Time

- What can solution depend on?
- Energy of blast. Density of matter. Time since blast.
- Units: kg m^2/s^2. kg/m^3. s.
- Get rid of kg: $E/\rho = m^5/s^2$. multiply by t^2 to get rid of s^2.
- Leaves: $r^5 \sim Et^2/\rho$.

Dimensional Analysis

- There’s (often) only one way to combine relevant quantities to have desired units.
- Up to an unknown multiplicative factor, answer has to scale like that combination.
- Getting the factor is hard!
- It’s “usually” “close” to one.

Plug in numbers

- density of air: 1 kg/m^3.
- from picture: $r \sim 150m$ at $t=0.025s$.
- $E=r^5 rho/t^2$. = 7e10*1*1600=1e14 Joules.

Convert to kilotons

- How much energy is in a ton of TNT?
- How many calories per gram of food?

1 gram of fat is ~9 kCal ~ 40 kJ. Explosives self-oxidize, so 1g TNT ~1 kCal.
1e14 J/4 kJ/g /1e6 g/ton = 25,000 tons. Very close!

Matrix/Buckingham Pi

- Multiplying constants means summing powers of units.
- Can write this as matrix equation - exponents of units of quantities we have go into matrix A. Exponents of units of quantity we desire goes in to b.
- All that is left is to solve Ax=b. x gives the powers of the quantities that give the right units for b.
- If A is well-behaved, we have only one way to get correct units.
- If A is singular, we have a degeneracy/dimensionless quantity. You may have to think harder - e.g., what power of α makes sense?

Let's practice some more problems

Kursk Explosion

- Russian submarine Kursk exploded in 2000 in 108m of water.
- Seismographs picked up 1.45 Hz oscillation.
- How big was the explosion?

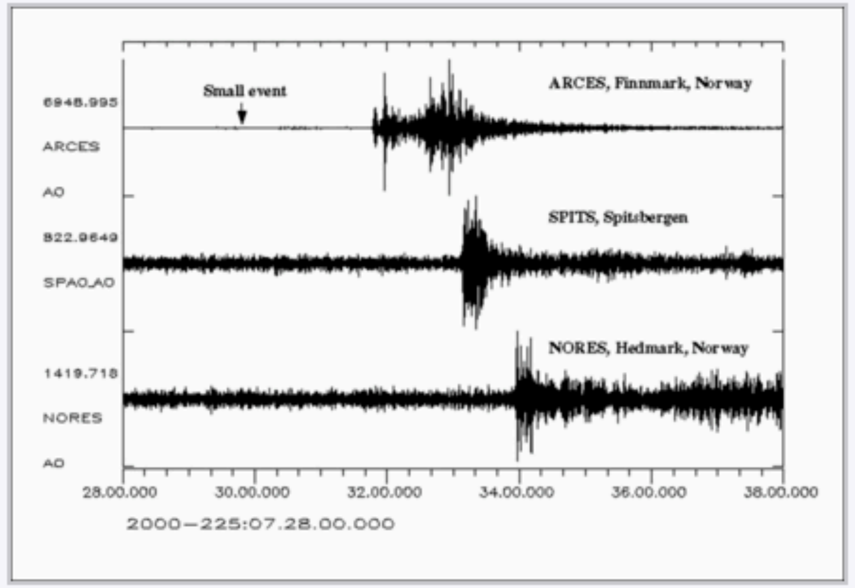
Kursk 2

- What quantities are relevant?
- P, rho, f.
- How do we turn them into an energy?
- NB - we could turn them into a radius as well. P=E/V, so turn r into V and get E.
- P= E/m^3. rho= kg/m^3. f=s^{-1}. P/rho=m^2/s^2. R~f^{-1}(P/rho)$^{1/2}$.
- En=4R^3 P ~4$f^{-3}P^{5/2}$ rho$^{-3/2}$.
- rho~1e3. 10 m=1 atmosphere, so 100m ~10^6 Pa.
- 4*1.45^{-3}*$10^{2.5*6}$*$10^{-1.5*3}$~$10^{10.5}$. or ~10 tons.

Wikipedia Answer

Initial seismic event detected [edit]

At 11:29:34 (07:29:34 GMT), seismic detectors at the Norwegian seismic array (NORSAR) and in other locations around the world recorded a seismic event of magnitude 1.5 on the Richter scale.[15] The location was fixed at coordinates 69°38′N 37°19′E, north-east of Murmansk, approximately 250 km (160 mi) from Norway, and 80 km (50 mi) from the Kola Peninsula.[16]



Norwegian Seismic Array seismic readings at three locations of the explosions on the submarine *Kursk* on 12 August 2000.

Secondary event [edit]

At 11:31:48,[15] 2 minutes and 14 seconds after the first, a second event, measuring 4.2 on the Richter scale, or 250 times larger than the first,[14] was registered on seismographs across northern Europe[17] and was detected as far away as Alaska.[9] The second explosion was equivalent to 2–3 tons of TNT.[5]

The seismic data showed that the explosion occurred at the same depth as the sea bed.[15] The seismic event, triangulated at 69°36.99′N 37°34.50′E, showed that in a little over 2 minutes the boat had moved about 400 m (1,300 ft) from the site of the initial explosion. It was enough time for the submarine to sink to a depth of 108 m (354 ft) and remain on the sea floor for a short period.[15]

Practice Questions

- How many atoms from Einstein’s last breath are in you right now?
- What’s the magnitude of the vector sum of the angular momentum in the solar system?