lecture 4: matching and indexing

deep learning for vision

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outline

bag of words
codebooks
beyond codebooks
pyramid matching
nearest neighbor search
discussion

bag of words

image matching

- so far, we have a representation that is very robust in matching different views of the same object or scene—same instance—to be used *e.g.* for retrieval
- the same representation can be used in matching views of different instances of the same category—same class—to be used *e.g.* for classification or detection
- main differences

	instance	class
features	sparse	dense
descriptors	same	
vocabulary	fine	coarse
geometry	rigid	flexible

image matching

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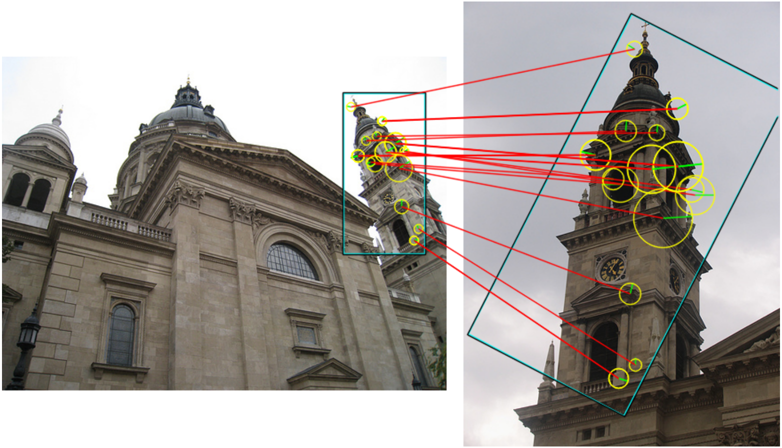
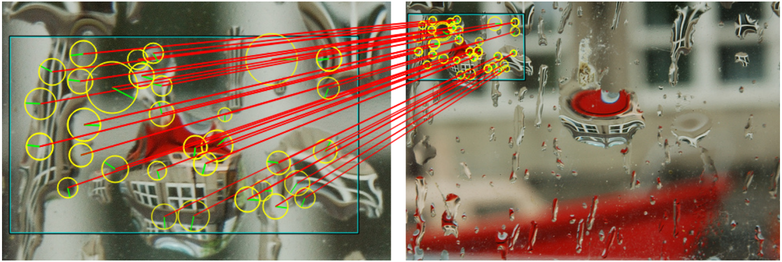
spatial matching—same instance



- now robust to scale, viewpoint, occlusion, clutter, lighting
- and very fast

spatial matching—same instance



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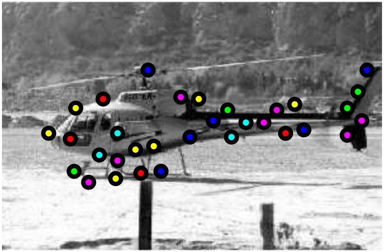
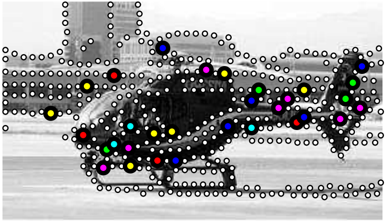
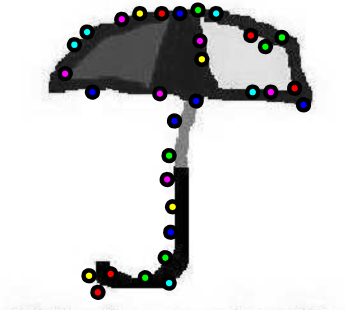


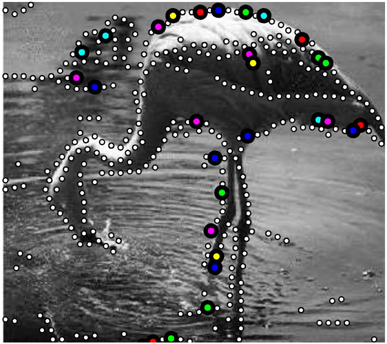


- solve for feature correspondence, flexible transformation and outliers on all possible correspondence pairs by joint optimization
- very expensive

Berg, Berg and Malik. CVPR 2005. Shape Matching and Object Recognition Using Low Distortion Correspondences.

spatial matching—same class





- solve for feature correspondence, flexible transformation and outliers on all possible correspondence pairs by joint optimization
- very expensive and error prone

Berg, Berg and Malik. CVPR 2005. Shape Matching and Object Recognition Using Low Distortion Correspondences.

geometry

- spatial matching on same instance is robust, but expensive
- we can
  - encode position, *e.g.* with dense features; easier to match, but we loose invariance
  - discard geometry altogether and use a global representation; even easier, we maintain invariance, but loose discriminative power
  - discard geometry as a first step, then bring it back
  - make it more efficient?
- rigid transformations won't work for classification, and matching is even more expensive
  - make it more flexible?
  - make it more efficient?
  - maintain invariance?

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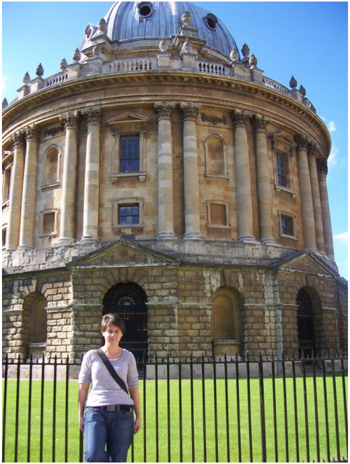
matching discriminative local features

[Lowe, ICCV 1999]



Lowe. ICCV 1999. Object recognition from local scale-invariant features.

matching discriminative local features

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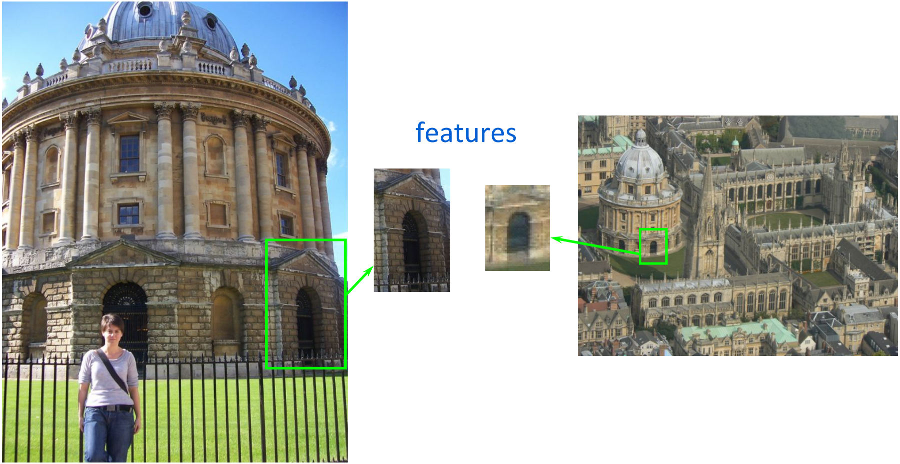

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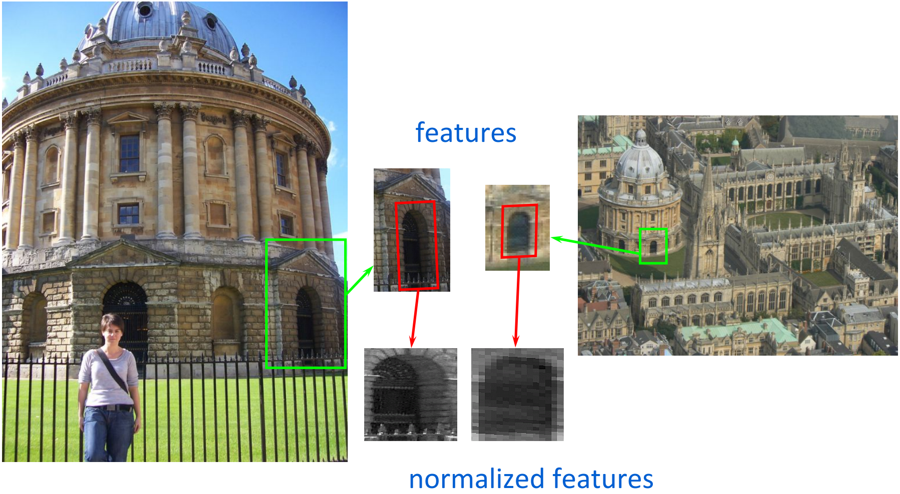


Lowe. ICCV 1999. Object recognition from local scale-invariant features.

appearance

- matching appearance via descriptors should be easier than geometry
- but
  - if we have positions *e.g.* with dense features, we know what to match (but we loose invariance)
  - otherwise, we need to find correspondences (expensive)
  - we can apply some pooling in image space or in descriptor space; more efficient; it may help or not
  - global pooling is the most efficient (but is not as discriminative)
  - local descriptors take up a lot of space; with pooling or not, we can compress them

forget about geometry: bag-of-words

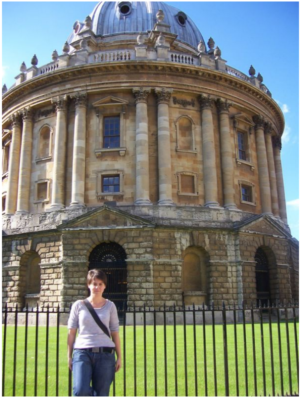
[Sivic and Zisserman 2003]

Object → Bag of ‘words’

- in fact, discarding geometry (bag) is one thing and quantizing descriptors (words) is another

Image credit: Fei-Fei, Fergus and Torralba. CVPR 2007 Tutorial. Recognizing and Learning Object Categories.

vector quantization → visual words



query

15

- query *vs.* dataset image

Sivic and Zisserman. ICCV 2003. Video Google: A Text Retrieval Approach to Object Matching in videos.

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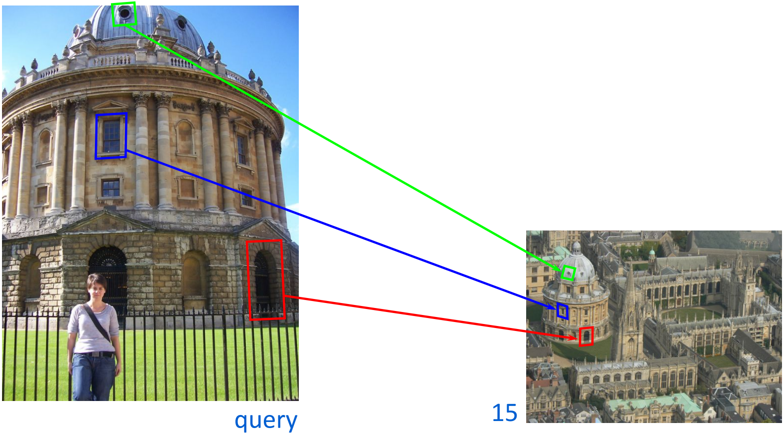

- pairwise descriptor matching

Sivic and Zisserman. ICCV 2003. Video Google: A Text Retrieval Approach to Object Matching in videos.

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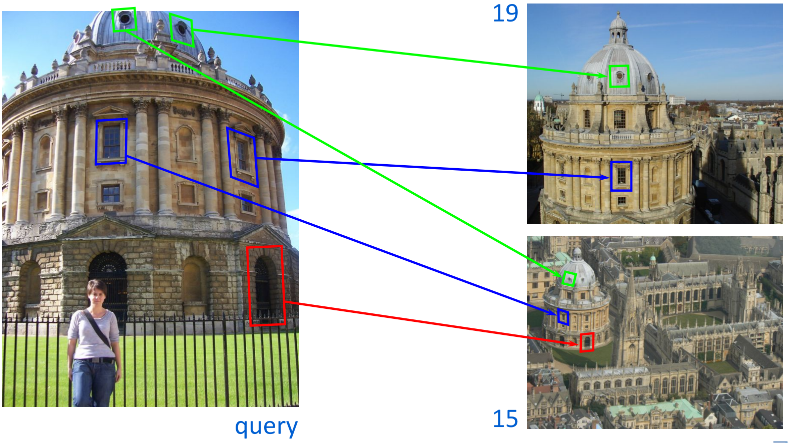

- pairwise descriptor matching for every dataset image

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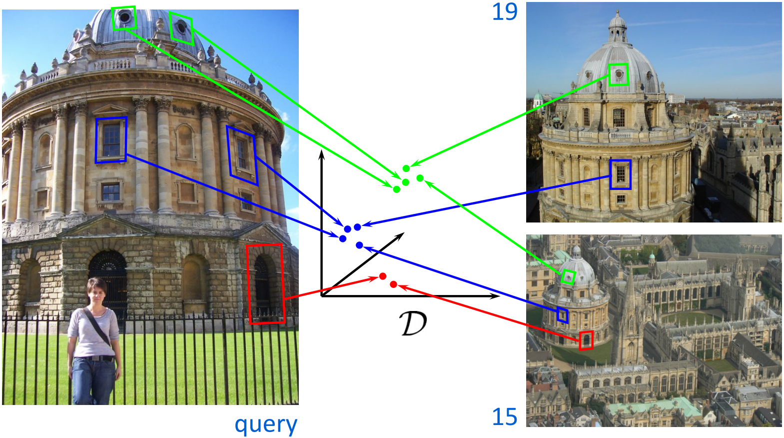
vector quantization → visual words



- similar descriptors should all be nearby in the descriptor space

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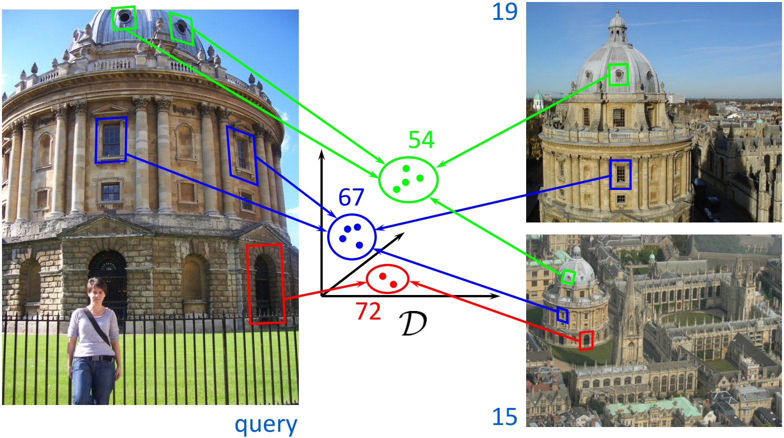

- let's quantize them into visual words

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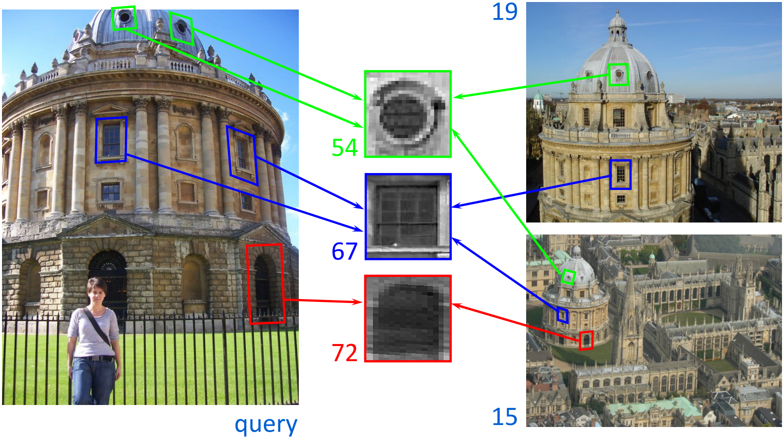

- now visual words act as a proxy; no pairwise matching needed

Sivic and Zisserman. ICCV 2003. Video Google: A Text Retrieval Approach to Object Matching in videos.

bag-of-words and "cosine" similarity

- each image is represented by a single vector $\mathbf{z} \in \mathbb{R}^k$, where k is the size of the codebook
- each element $z_i = w_i n_i$ where w_i fixed weight per visual word (*e.g.* inverse document frequency) and n_i the number of occurrences of this word in the image
- this vector then typically normalized, *e.g.* $\|\mathbf{z}\|_1 = 1$ or $\|\mathbf{z}\|_2 = 1$
- given two images represented by $\mathbf{z}, \mathbf{y}$, similarity is usually measured by dot product

$$s_{\text{BoW}}(\mathbf{z}, \mathbf{y}) := \mathbf{z}^\top \mathbf{y}$$

- with ℓ_2 normalization, this is equivalent to measuring Euclidean distance $\|\mathbf{z} - \mathbf{y}\|$ because $\|\mathbf{z} - \mathbf{y}\|^2 = 2(1 - \mathbf{z}^\top \mathbf{y})$

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bag-of-words for retrieval

- given a set of n images represented by matrix $Z \in \mathbb{R}^{k \times n}$ (each image as a column) and query image $\mathbf{q}$, we need a vector of similarities

$$\mathbf{s} = S_{\mathsf{BoW}}(Z, \mathbf{q}) := Z^\top \mathbf{q}$$

  and then sort $\mathbf{s}$ by descending order
- when $k \gg p$, where p is the number of features per image on average, Z and $\mathbf{q}$ are sparse
- rather than whether a word is contained in an image, ask which images contain a given word

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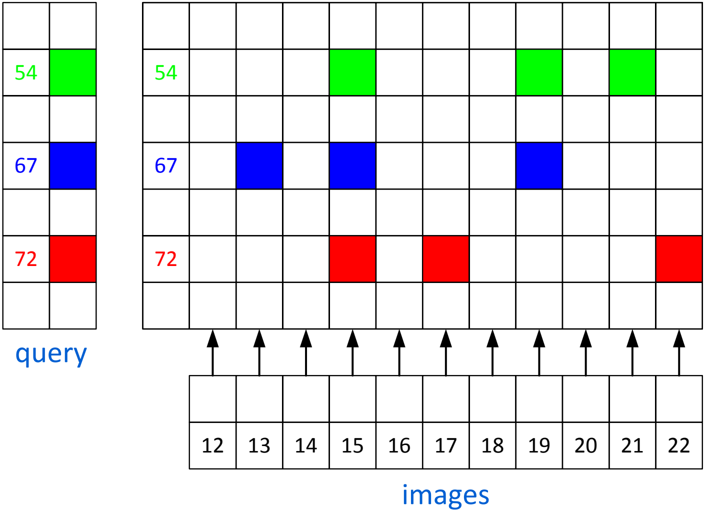
inverted file indexing



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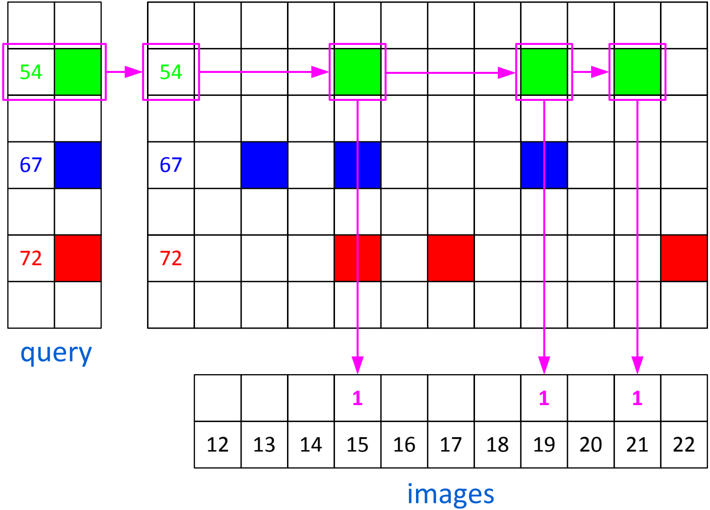

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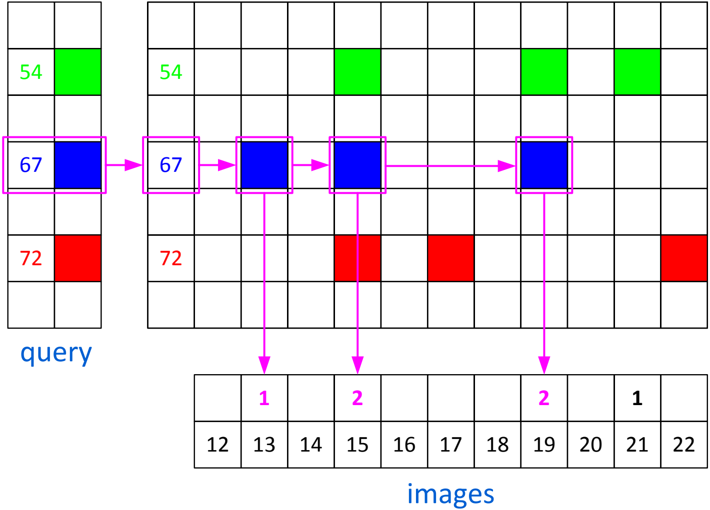

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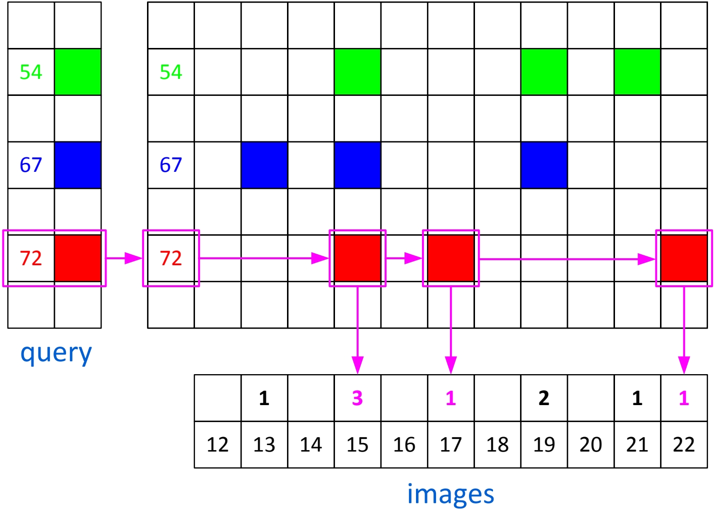

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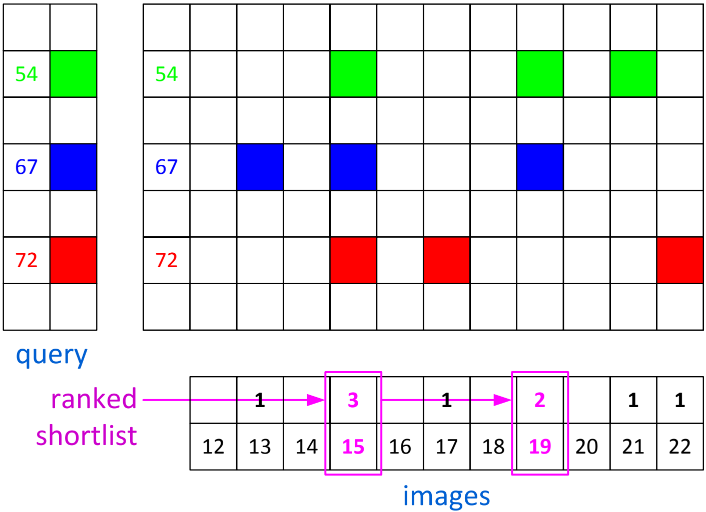

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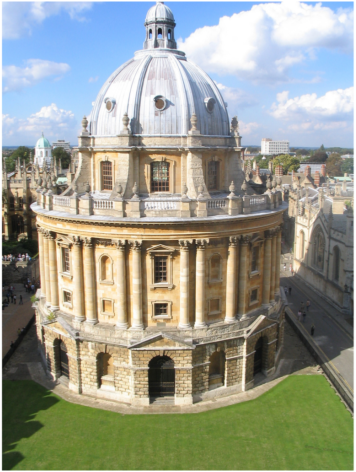
back to geometry: re-ranking

- dot product similarity is fast but quantized descriptors are not discriminative enough; performs poorly in the presence of distractors
- perform spatial matching only on top-ranking images, and re-ranking according to a score based on geometry, *e.g.* number of inliers
- but to save space, descriptors are not available: tentative correspondences are based on visual words, and there are too many (too features are in correspondence if they are assigned to the same visual word)

back to geometry: re-ranking

original images

Fischler and Bolles. CACM 1981. Random Sample Consensus: A Paradigm for Model Fitting With Applications to Image Analysis and Automated Cartography.

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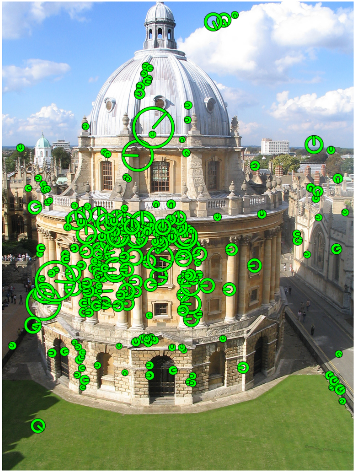

local features

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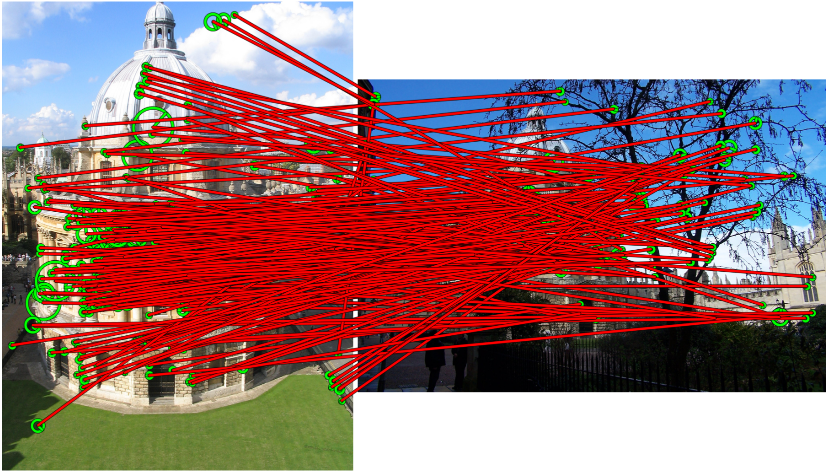

tentative correspondences: too many

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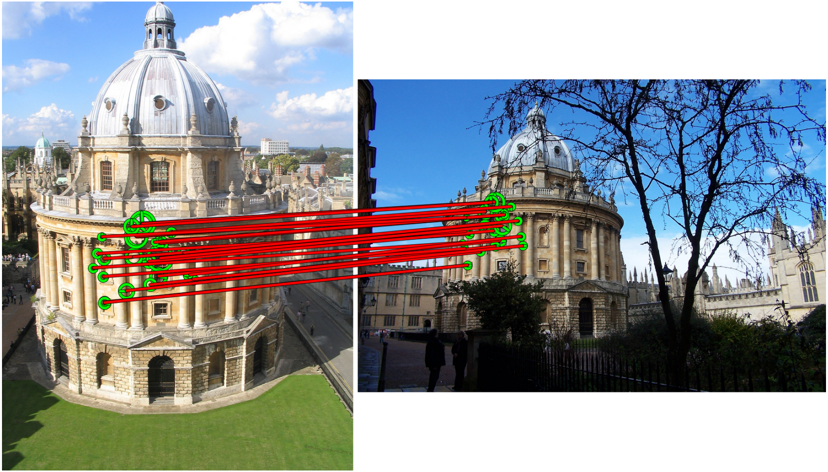

inliers: now more expensive to find

Fischler and Bolles. CACM 1981. Random Sample Consensus: A Paradigm for Model Fitting With Applications to Image Analysis and Automated Cartography.

bag of words for classification

- each image represented by $\mathbf{z} \in \mathbb{R}^k$; each element z_i the number of occurrences of visual word c_i in the image
- Naïve Bayes: chose maximum posterior probability of class C given image $\mathbf{z}$ assuming features are independent $\rightarrow$ linear classifier with parameters estimated by visual word statistics on training set
- support vector machine (SVM): images $\mathbf{z}, \mathbf{y}$ compared by kernel function $\kappa(\mathbf{z}, \mathbf{y})$; if $\kappa(\mathbf{z}, \mathbf{y}) = \mathbf{z}^\top \mathbf{y}$, this is again a linear classifier and is a standard choice at large scale

Csurka, Dance, Fan, Willamowski and Bray. SLCV 2004. Visual Categorization With Bags of Keypoints.

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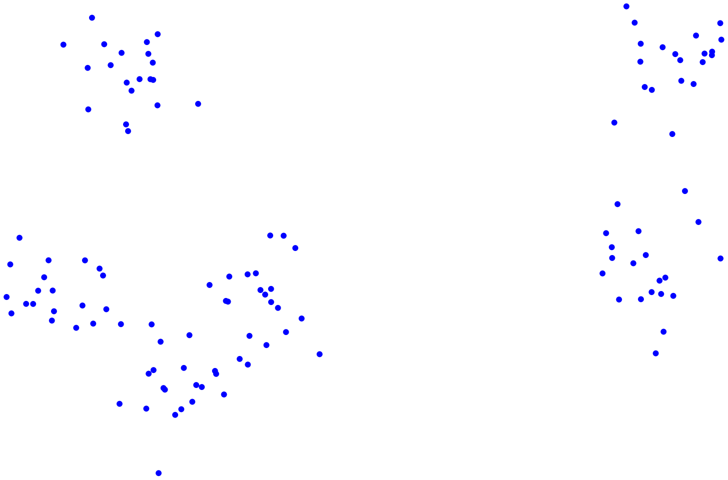
codebooks

vector quantization: k-means clustering

[MacQueen 1967]



dataset

MacQueen. SMSP 1967. Some Methods for Classification and Analysis of Multivariate Observations.

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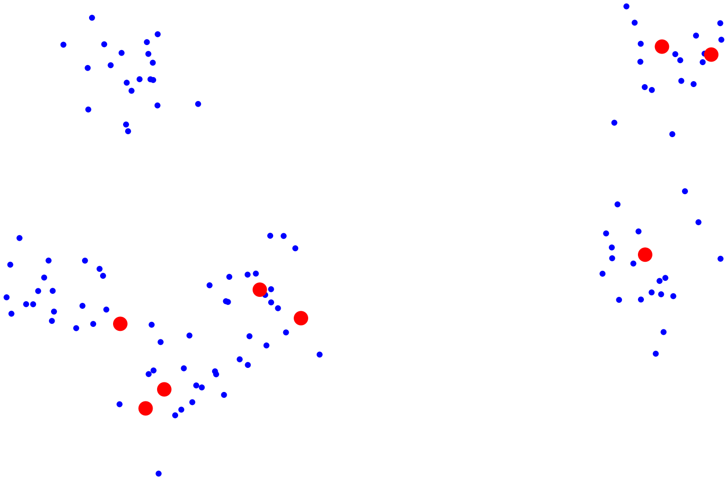

initial centroids

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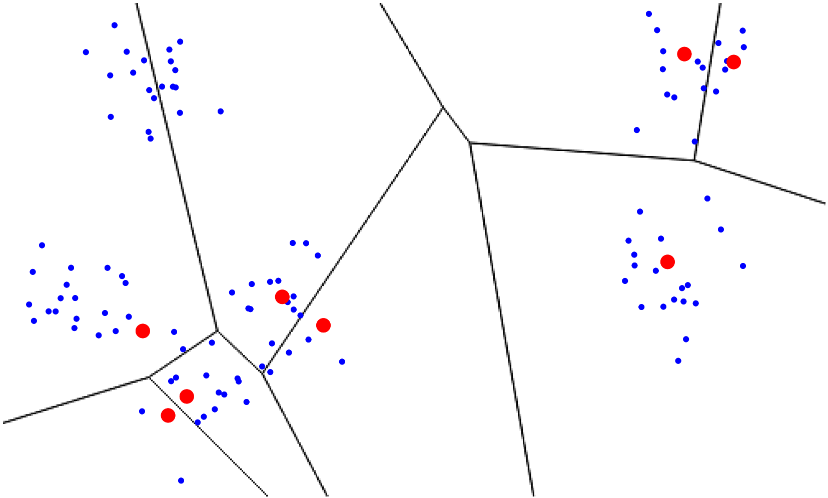

Voronoi cells

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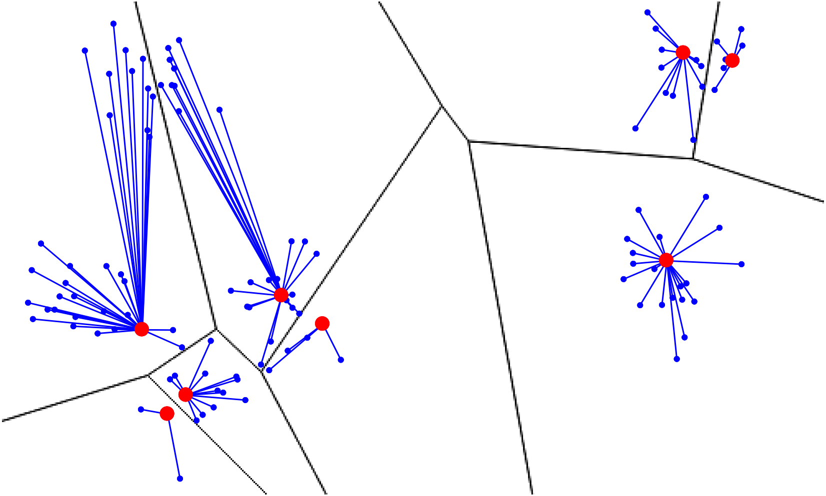

points assigned to nearest centroids

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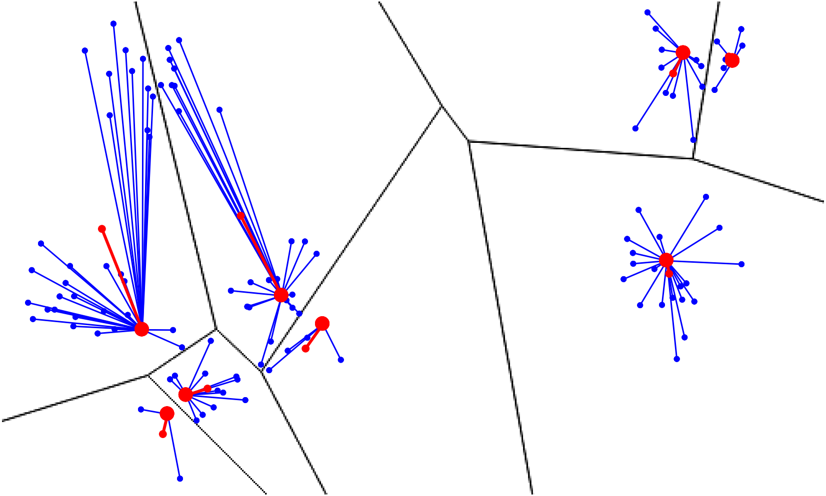

centroids move to mean per cell

MacQueen. SMSP 1967. Some Methods for Classification and Analysis of Multivariate Observations.

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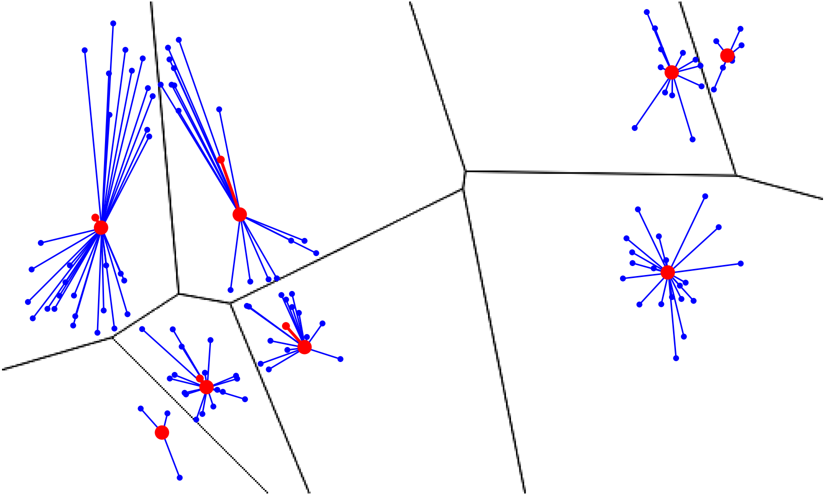

iterate until convergence

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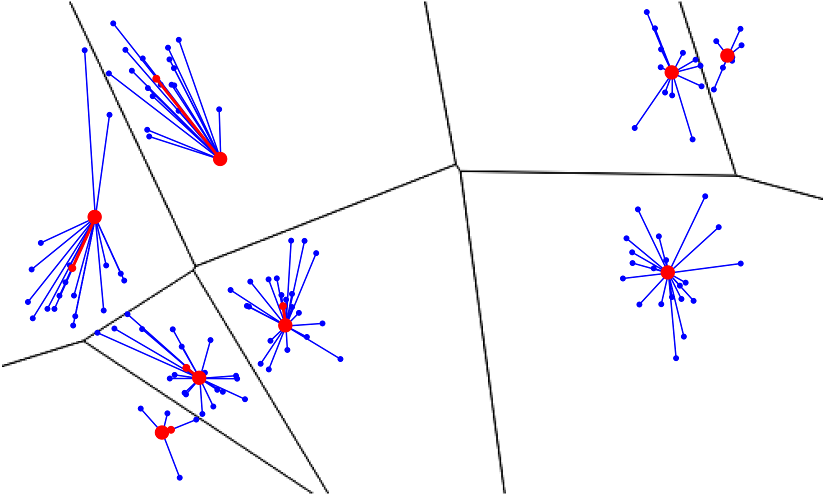

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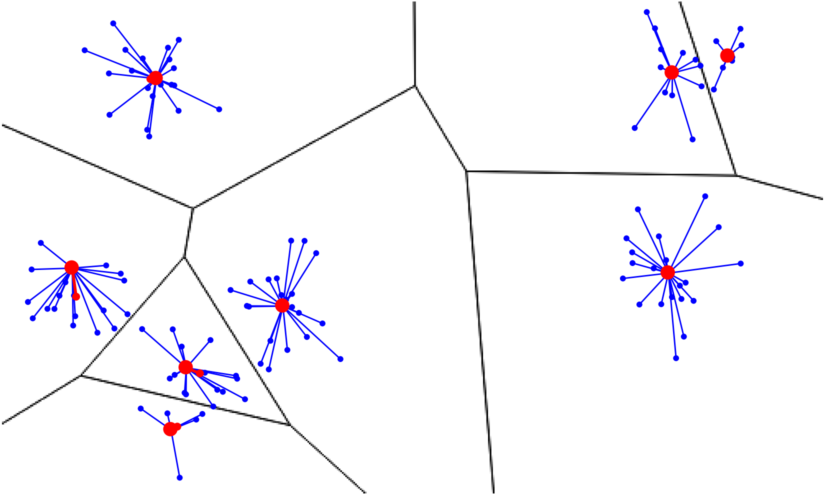

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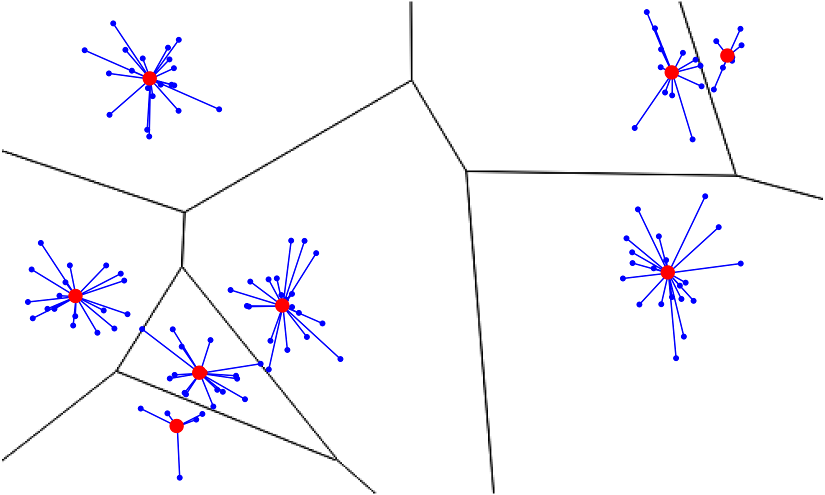

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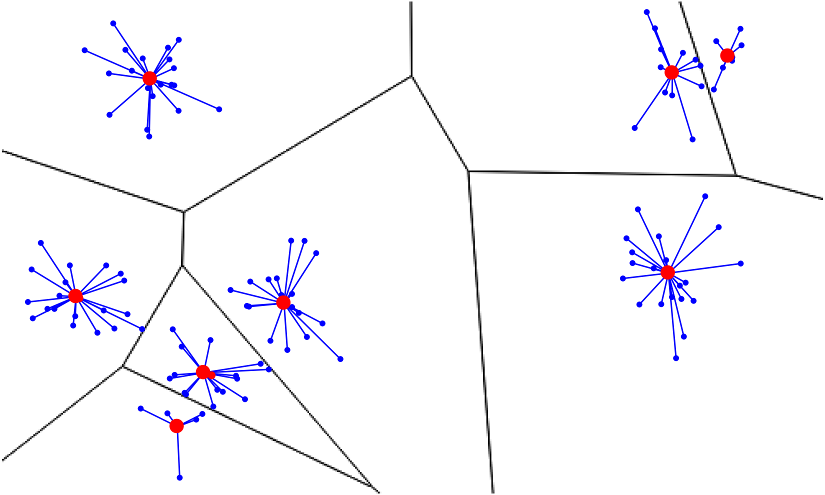

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vector quantization: k-means clustering

- objective: given dataset $X \subset \mathbb{R}^d$, find codebook $C \subset \mathbb{R}^d$, with $|C| = k$, and quantizer function $q : \mathbb{R}^d \to C$, minimizing distortion

$$E(C, q) := \sum_{x \in X} \|x - q(x)\|^2$$

- regardless of C, q should map vector x to its nearest centroid

$$q(x) = \arg\min_{c \in C} \|x - c\|$$

- algorithm: at each iteration, given the set $X_c = \{x \in X : q(x) = c\}$ of points assigned to centroid c, (assignment step), c moves to their mean (update step)

$$c \leftarrow \frac{1}{|X_c|} \sum_{x \in X_c} x$$

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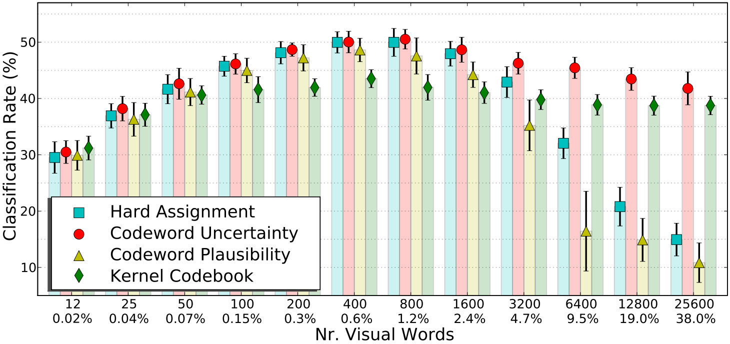
MacQueen. SMSP 1967. Some Methods for Classification and Analysis of Multivariate Observations.

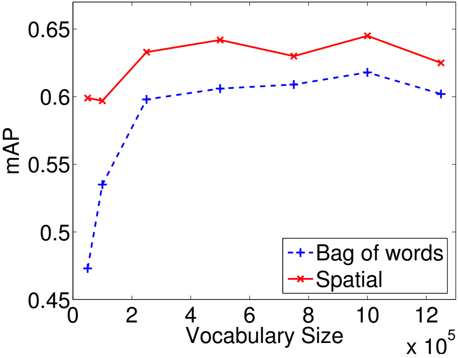
codebook size



- classification: thousands
- depends on a lot of factors *e.g.* the number of features in the image representation and size and variability of the dataset

van Gemert, Veenman, Smeulders and Geusebroek PAMI 2010. Visual Word Ambiguity.

codebook size

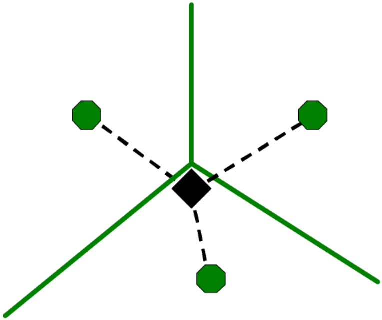


- instance retrieval: millions
- depends on a lot of factors *e.g.* the number of features in the image representation and size and variability of the dataset

Philbin, Chum, Isard, Sivic and Zisserman. CVPR 2007. Object Retrieval With Large Vocabularies and Fast Spatial Matching.

hierarchical k-means (HKM)

[Fukunaga and Narendra 1975]



- partition data into b clusters using k-means

Fukunaga and Narendra. ToC 1975. A Branch and Bound Algorithm for Computing K-Nearest Neighbors.

hierarchical k-means (HKM)

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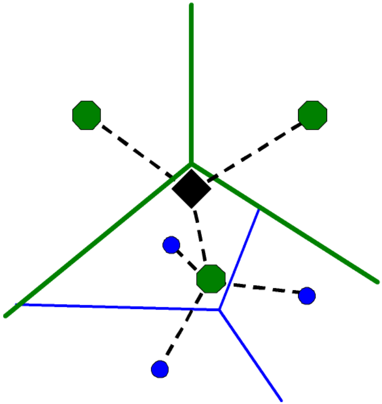
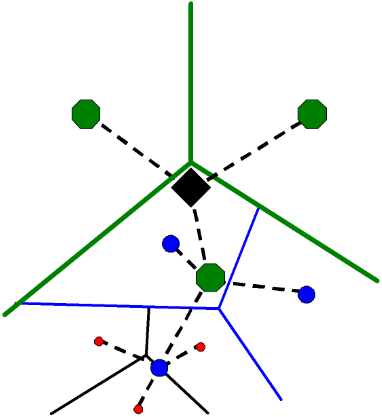


- within each cluster, partition data into b clusters

Fukunaga and Narendra. ToC 1975. A Branch and Bound Algorithm for Computing K-Nearest Neighbors.

hierarchical k-means (HKM)

[Fukunaga and Narendra 1975]



- and repeat; b is called the branching factor

Fukunaga and Narendra. ToC 1975. A Branch and Bound Algorithm for Computing K-Nearest Neighbors.

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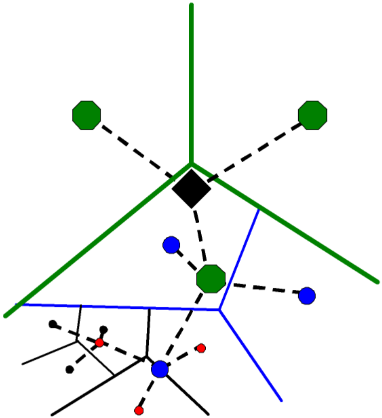
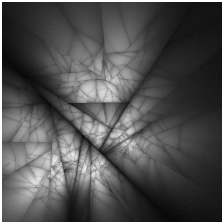


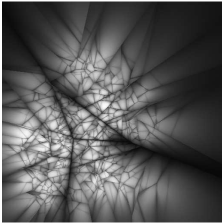
- at ℓ levels, there are b^ℓ total clusters

Fukunaga and Narendra. ToC 1975. A Branch and Bound Algorithm for Computing K-Nearest Neighbors.

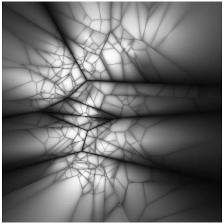
hierarchical k-means



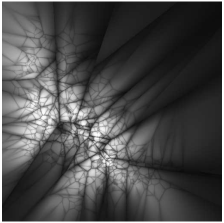
$b = 2$



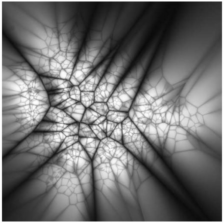
$b = 4$



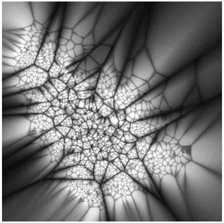
$b = 8$



$b = 16$



$b = 32$

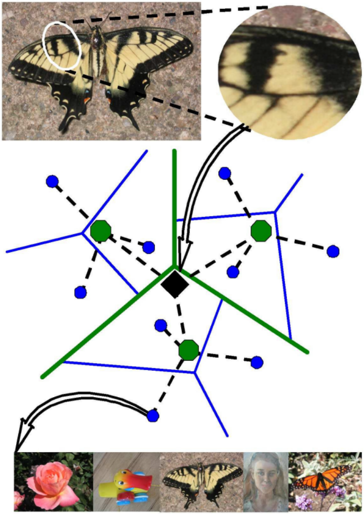


$b = 128$

- intensity: ratio of first to second neighbor distance

vocabulary tree

[Nister and Stewenius. CVPR 2006]



- apply k-means hierarchically and build a fine partition tree
- descriptors descend from root to leaves by finding nearest node at each level
- image represented by $x_i = w_i n_i$ as in BoW, but now there is one element per node including internal nodes
- dataset searched by inverted files at leaves

Nister and Stewenius. CVPR 2006. Scalable Recognition With a Vocabulary Tree.

vocabulary tree

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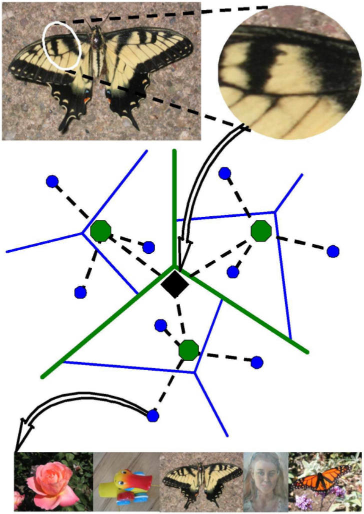

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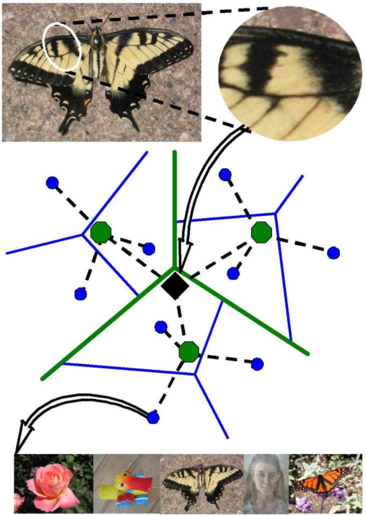

however:

- no principled way of defining w_i across levels
- distortion minimized only locally; points get assigned to leaves that are not globally nearest

Nister and Stewenius. CVPR 2006. Scalable Recognition With a Vocabulary Tree.

vocabulary tree

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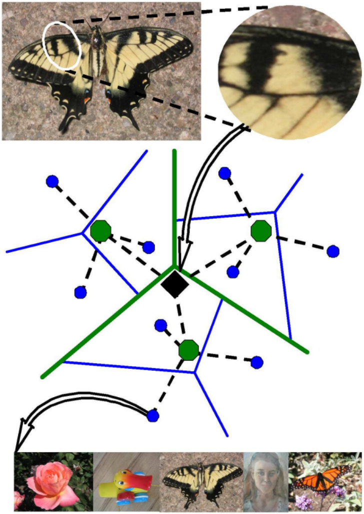

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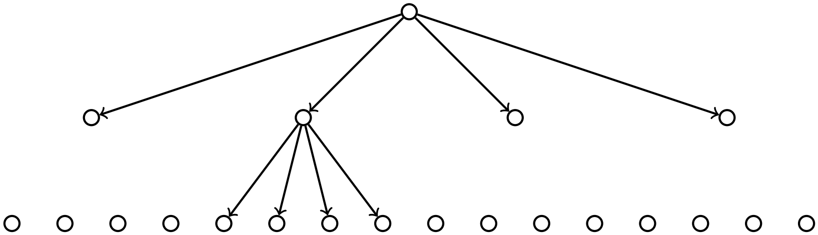
Nister and Stewenius. CVPR 2006. Scalable Recognition With a Vocabulary Tree.

approximate k-means (AKM)

[Philbin et al. 2007]



- with branching factor $b = 10$ and $\ell = 6$ levels, HKM yields $k = 10^6$ visual words; complexity is $O(nb\ell)$
- search through multiple randomized trees (comparison to HKM in color)

Philbin, Chum, Isard, Sivic and Zisserman. CVPR 2007. Object Retrieval With Large Vocabularies and Fast Spatial Matching.

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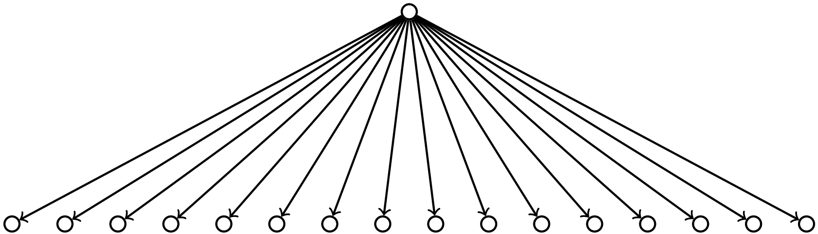

- flat k-means with *e.g.* $n = 10^7$ points and $k = 10^6$ centroids is prohibitive; complexity is $O(nk)$, because each assignment is $O(k)$
- search through multiple randomized trees (comparison to HKM in color)

Philbin, Chum, Isard, Sivic and Zisserman. CVPR 2007. Object Retrieval With Large Vocabularies and Fast Spatial Matching.

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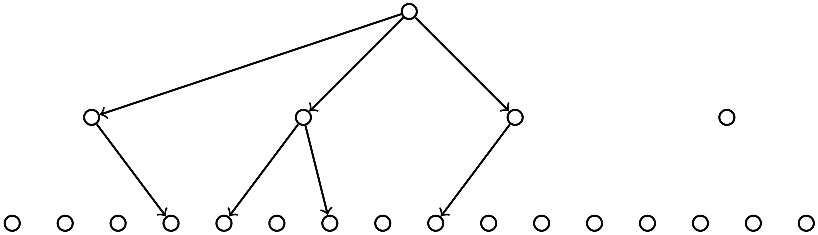

- approximate nearest neighbor search to find the nearest centroid: each assignment is now $O(\log k)$, and complexity drops to $O(n \log k)$
- search through multiple randomized trees (comparison to HKM in color)

Philbin, Chum, Isard, Sivic and Zisserman. CVPR 2007. Object Retrieval With Large Vocabularies and Fast Spatial Matching.

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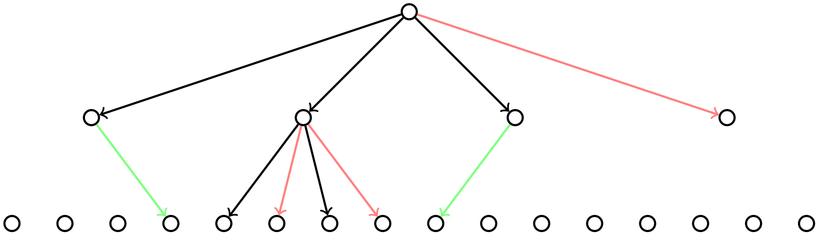

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Muja and Lowe. ICCV 2009. Fast Approximate Nearest Neighbors with Automatic Algorithm Configuration.

approximate k-means (AKM)

- if the sole purpose of the hierarchy is to accelerate assignment, both at learning and at search, it is better to use a flat vocabulary combined with a more principled nearest neighbor search method
- however, with appropriate node weighting, a hierarchical structure can help (see pyramid matching later on)

Philbin, Chum, Isard, Sivic and Zisserman. CVPR 2007. Object Retrieval With Large Vocabularies and Fast Spatial Matching.

pipeline, again

- given codebook $C = \{c_1, \dots, c_k\} \subset \mathbb{R}^d$
- given image with descriptors $x_i \in \mathbb{R}^d$ at positions $y_i \in \mathbb{R}^2$, $i = 1, \dots, n$ into $\mathbf{a}_i \in \mathbb{R}^k$
- encode each descriptor x_i into $\mathbf{a}_i \in \mathbb{R}^k$

$$\mathbf{a}_i := F(x_i; C) := (f(x_i, c_1; C), \dots, f(x_i, c_k; C))$$

- pool each spatial region $R_j, j = 1, \dots, m$ into $\mathbf{z}^j \in \mathbb{R}^k$

$$\mathbf{z}^j := g(\{\mathbf{a}_i : y_i \in R_j\})$$

- concatenate into $\mathbf{z} \in \mathbb{R}^{km}$

$$\mathbf{z} := (\mathbf{z}^1; \dots; \mathbf{z}^m)$$

- global pooling is just $m = 1$

Boureau, Bach, Lecun and Ponce. CVPR 2010. Learning Mid-Level Features for Recognition.

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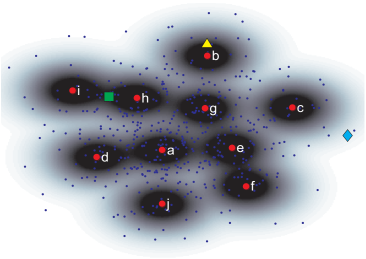
soft assignment

[van Gemert et al. 2008]



- △: ok; ■: ambiguous; ◆: not represented
- left: assigned to nearest neighbor; right: to all visual words with different weights
- top: total weight normalized to one; bottom: depends on distance

van Gemert, Geusebroek, Veenman and Smeulders. ECCV 2008. Kernel Codebooks for Scene Categorization.

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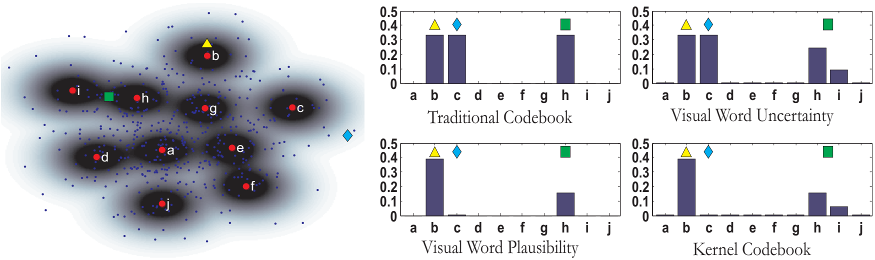

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soft assignment

- r-nearest neighbors of x in C: $\mathrm{NN}_C^r(x)$
- kernel function

$$h(x) = h_G(x; \sigma) := \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})(x) \propto \exp\left(-\frac{\|x\|^2}{2\sigma^2}\right)$$

- encoding descriptor x into visual word c

$f(x, c; C)$	visual word	
	nearest	all
fixed weight	$\mathbb{1}[c \in \mathrm{NN}_C^1(x)]$ "BoW"	$\frac{h(x-c)}{\sum_j h(x-c_j)}$ "uncertainty"
variable weight	$\mathbb{1}[c \in \mathrm{NN}_C^1(x)]h(x-c)$ "plausibility"	$h(x-c)$ "kernel"

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soft assignment

- on classification: best model is "uncertainty"

$$f(x, c; C) = \frac{h(x - c)}{\sum_j h(x - c_j)}$$

- it is better to contribute to visual words even if all are far away
- we shall see this is the $\mathrm{softmax}$ of negative distances $-\|x - c\|^2$
- it is also the responsibility of visual word c for descriptor x in a Gaussian mixture model with C as components

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van Gemert, Geusebroek, Veenman and Smeulders. ECCV 2008. Kernel Codebooks for Scene Categorization.

soft assignment

[Liu et al. 2011]

- on classification: it turns out, it is better to limit contributions to r nearest neighbors

$$f(x, c; C) = \mathbb{1}[c \in \mathrm{NN}_C^r(x)] \frac{h(x - c)}{\sum_j h(x - c_j)}$$

- this is attributed to respecting the manifold structure of the data, and it superior to more expensive *sparse coding* that have been proposed in the meantime

Liu, Wang and Liu. ICCV 2011. In Defense of Soft-assignment Coding.

soft assignment

[Philbin et al. 2008]

- on retrieval: "kernel" is followed on r nearest neighbors

$$f(x, c; C) = \mathbb{1}[c \in \mathrm{NN}_C^r(x)]h(x - c)$$

- it is better to discard descriptors if they are not well represented
- r should be small: this applies to dataset images and increases the required index space and query time (including spatial matching) by r

Philbin, Chum, Sivic, Isard and Zisserman. CVPR 2008. Lost in Quantization: Improving Particular Object Retrieval in Large Scale Image Databases.

soft assignment

[Philbin et al. 2008]

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multiple assignment

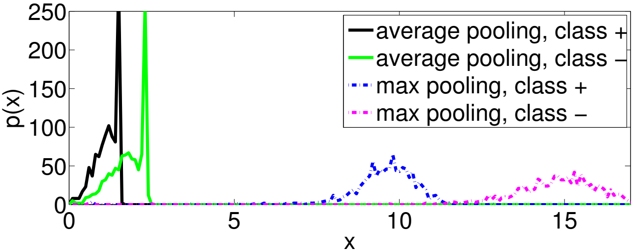
[Jégou et al. 2010]

- on retrieval: same as before, but now applies only to query images
- $f(x, c; C)$ further limited to visual words at distance $\leq \alpha d_1$ from x, where d_1 is the distance of $\mathrm{NN}_C^1(x)$
- index space maintained as in standard hard assignment, but query time is still increased by r

Jegou, Douze and Schmid. IJCV 2010. Improving Bag-of-Features for Large Scale Image Search.

max pooling vs. average pooling

[Boureau et al. 2010]



- on classification: $\max$-pooling superior to average pooling

$$g_{\max}(A) = \left(\max_{\mathbf{a}\in A} a_1, \ldots, \max_{\mathbf{a}\in A} a_k\right) \qquad g_{\mathsf{avg}}(A) = \frac{1}{|A|}\sum_{\mathbf{a}\in A} \mathbf{a}$$

- with $\max$-pooling, SVM with linear and nonlinear kernel perform nearly the same

Boureau, Bach, Lecun and Ponce. CVPR 2010. Learning Mid-Level Features for Recognition.

burstiness

[Jégou et al. 2009]

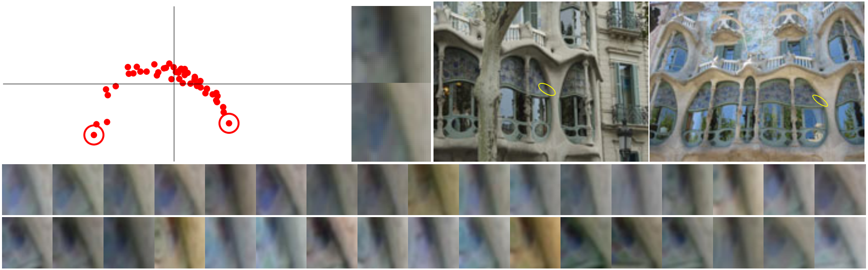
- burstiness: descriptors appear more frequently than a statistically independent model predicts; it hurts performance because bursty features dominate the image similarity
- on retrieval: the situation is more complex here; max-pooling would be like keeping only one representative per cell, average pooling like keeping all, but none is the best choice

Jegou, Douze and Schmid. CVPR 2009. On the burstiness of visual elements.

beyond codebooks

learning cell shapes

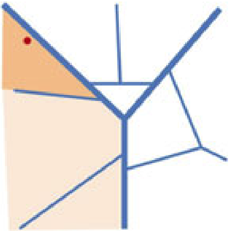
[Mikulik et al. 2010]



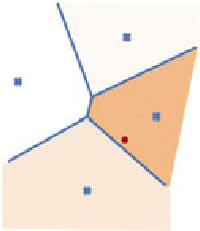
- on retrieval: matched across images in an entire dataset, features are connected into feature tracks
- feature tracks have curved shape in descriptor space, contrary to the Gaussian assumption—an example of manifold structure
- even if such structure cannot be captured by k-means, cells can still be connected via feature tracks $\rightarrow$ vocabulary of 16M words

Mikulik, Perdoch, Chum and Matas. ECCV 2010. Learning a Fine Vocabulary.

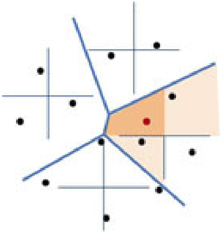
learning cell shapes



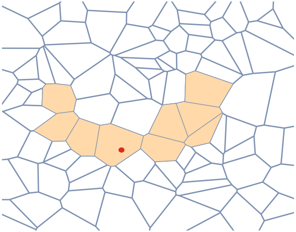
HKM



soft assignment



Hamming



learned

Mikulik, Perdoch, Chum and Matas. ECCV 2010. Learning a Fine Vocabulary.

descriptor matching

- on retrieval: given two images with descriptors $X, Y \subset \mathbb{R}^d$, and recalling $X_c = \{x \in X : q(x) = c\}$, bag-of-words similarity on C is

$$\begin{aligned} s_{\mathsf{BoW}}(X,Y) &\propto \sum_{c \in C} w_c |X_c||Y_c| \\ &= \sum_{c \in C} w_c \sum_{x \in X_c} \sum_{y \in Y_c} 1 \end{aligned}$$

- if descriptors are available in some form (more space), it is better to use a more general function of the form

$$K(X,Y) := \gamma(X)\gamma(Y) \sum_{c \in C} w_c M(X_c, Y_c)$$

where M is a within-cell matching function and $\gamma(X)$ serves for normalization

Tolias, Avrithis and Jegou. ICCV 2013. To Aggregate or not to Aggregate: Selective Match Kernels for Image Search.

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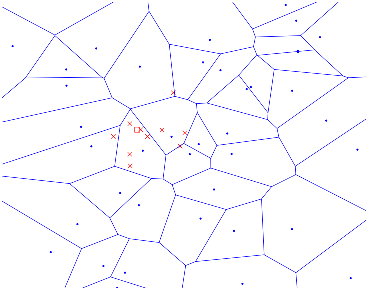
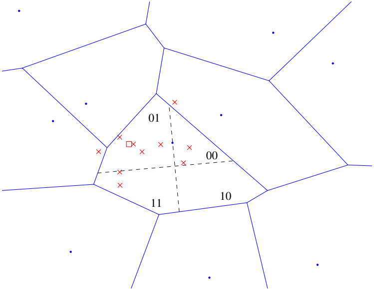
Tolias, Avrithis and Jegou. ICCV 2013. To Aggregate or not to Aggregate: Selective Match Kernels for Image Search.

Hamming embedding (HE)

[Jégou et al. 2008]



fine vocabulary



Hamming embedding

- each descriptor x is binarized into $b(x) \in \{0,1\}^d$
- pairs within cells are kept only if Hamming distance is at most τ

$$M_{\mathsf{HE}}(X_c, Y_c) := \sum_{x \in X_c} \sum_{y \in Y_c} \mathbb{1}[d_{\mathsf{H}}(b(x), b(y)) \leq \tau]$$

Jegou, Douze and Schmid. ECCV 2008. Hamming Embedding and Weak Geometric Consistency for Large Scale Image Search.

aggregated selective match kernel (ASMK)

[Tolias et al. 2013]

- borrow from HE the idea that descriptor pairs are selected by a nonlinear function

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- borrow from VLAD the idea that residuals are pooled per cell

$$M_{\mathsf{VLAD}}(X_c, Y_c) := V(X_c)^\top V(Y_c) = \sum_{x \in X_c} \sum_{y \in Y_c} r(x)^\top r(y)$$

- combine pooling within cells with selectivity between cells

$$M_{\mathsf{ASMK}}(X_c, Y_c) := \sigma_\alpha(\hat{V}(X_c)^\top \hat{V}(Y_c))$$

where $\hat{x} := x/\|x\|$ and σ_α a nonlinear function

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aggregated selective match kernel (ASMK)

- apart from saving space, pooling and normalizing per cell helps fight burstiness
- still, unlike VLAD, due to the nonlinearity we cannot have a low dimensional embedding
- it is targeting large vocabularies, which, together with compressed descriptors (as in HE), takes up a lot of space

Jegou, Douze and Schmid. CVPR 2009. On the burstiness of visual elements.

efficient match kernels (EMK)

[Bo and Sminchisescu. NIPS 2009]

- on classification: given two images with descriptors $X, Y \subset \mathbb{R}^d$, bag-of-words similarity on C is

$$s_{\mathsf{BoW}}(X,Y) \propto \sum_{c \in C} |X_c||Y_c| = \sum_{x \in X} \sum_{y \in Y} \mathbb{1}[q(x) = q(y)]$$

- use a continuous function $\kappa(x, y)$ instead, with no codebook

$$K(X,Y) := \gamma(X)\gamma(Y) \sum_{x \in X} \sum_{y \in Y} \kappa(x,y)$$

- derive an approximate finite-dimensional feature map ϕ such that $\kappa(x,y) = \phi(x)^\top \phi(y)$, and

$$K(X,Y) = \left(\gamma(X) \sum_{x \in X} \phi(x)\right) \left(\gamma(Y) \sum_{y \in Y} \phi(y)\right) = \Phi(X)^\top \Phi(Y)$$

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efficient match kernels (EMK)

- given a function $K(X, Y)$ on sets X, Y in the form of a pairwise sum of nonlinear functions $\kappa(x, y)$ of the elements $x \in X$, $y \in Y$, we can decompose it into an inner product of $\Phi(X)$, $\Phi(Y)$
- this can be done by
  - encoding $x \mapsto \phi(x)$
  - pooling $X \mapsto \Phi(X) = \gamma(X) \sum_{x \in X} \phi(x)$
- this is always possible for positive-definite functions κ but ϕ may be infinite-dimensional; in nonlinear SVM, it is only implicit through κ
- here, we are interested in an explicit, low-dimensional feature map ϕ, which can be designed or learned

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pyramid matching

histogram intersection

[Swain and Ballard 1991]

- the sum $\sum_{x \in X_c} \sum_{y \in Y_c} 1$ appearing in $s_{\text{BoW}}(X, Y)$ implies an all-all matching; it is often preferable to have an one-one matching instead

- given two histograms x, y of b bins, their histogram intersection is

$$\kappa_{\text{HI}}(x, y) = \sum_{i=1}^{b} \min(x_i, y_i)$$

- this is related to ℓ_1 distance by

$$\|x - y\|_1 = \|x\|_1 + \|y\|_1 - 2\kappa_{\text{HI}}(x, y)$$

Swain and Ballard. IJCV 1991. Color Indexing.

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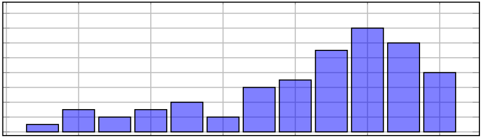

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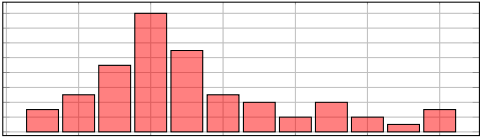

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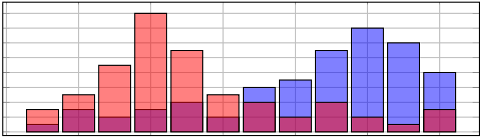

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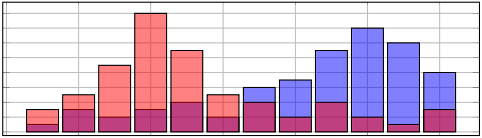

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pyramid match kernel (PMK)

[Grauman and Darrell 2005]

- given the descriptors of two images as point sets X, Y in $\mathbb{R}^d$
- a weighted sum of histogram intersections at different levels approximates their optimal pairwise matching

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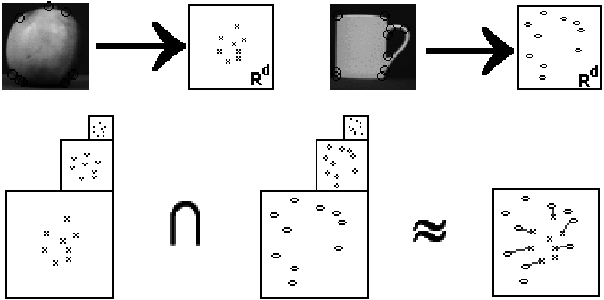

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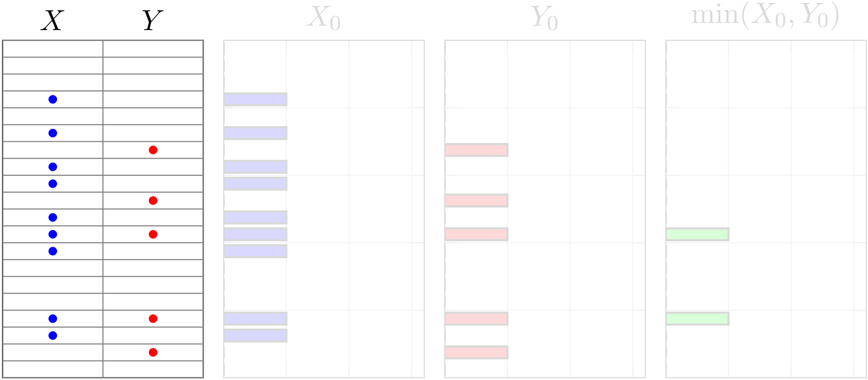

- 1d point sets X, Y on grid of size 1

Grauman and Darrell. ICCV 2005. The Pyramid Match Kernel: Discriminative Classification With Sets of Image Features.

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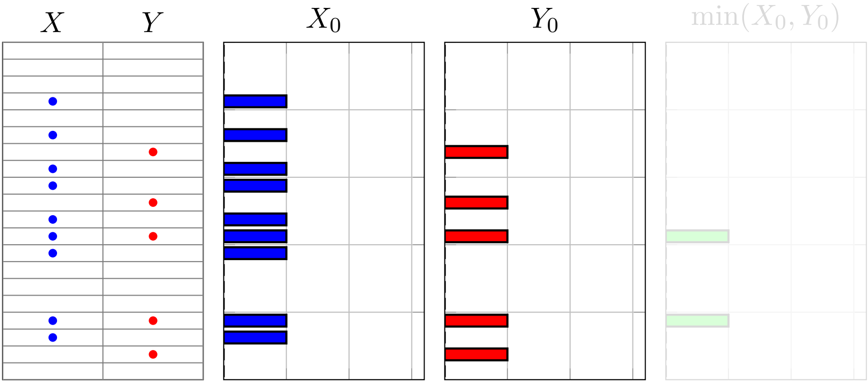

- 1d point sets X, Y on grid of size 1 - level 0 histograms

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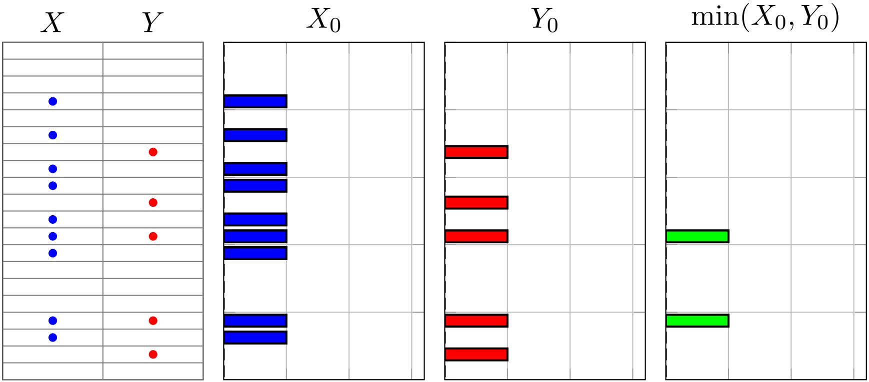

- 1d point sets X, Y on grid of size 1 - level 0 histograms - intersection

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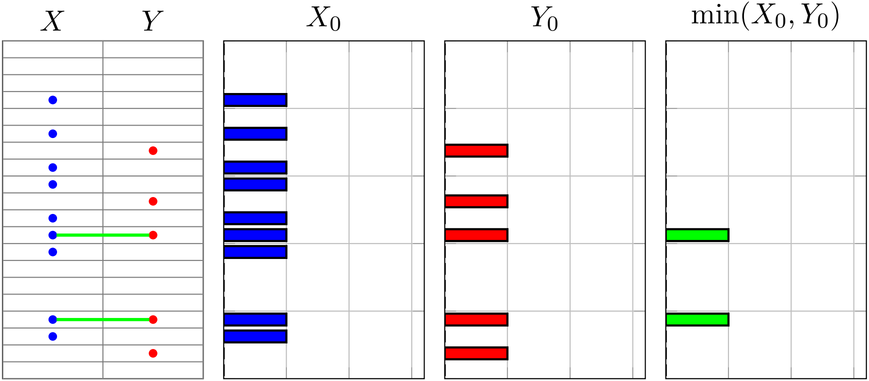

- 1d point sets X, Y on grid of size 1 - level 0 histograms - intersection
- $(2$ matches weighted by $1)$
- total score 2×1

Grauman and Darrell. ICCV 2005. The Pyramid Match Kernel: Discriminative Classification With Sets of Image Features.

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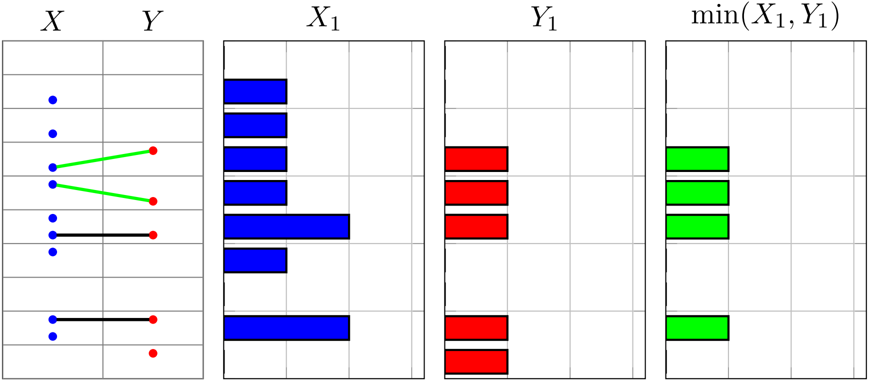

- 1d point sets X, Y on grid of size 2 - level 1 histograms - intersection
- $(2$ matches weighted by $1) + (2$ weighted by $\frac{1}{2})$
- total score $2 \times 1 + 2 \times \frac{1}{2}$

Grauman and Darrell. ICCV 2005. The Pyramid Match Kernel: Discriminative Classification With Sets of Image Features.

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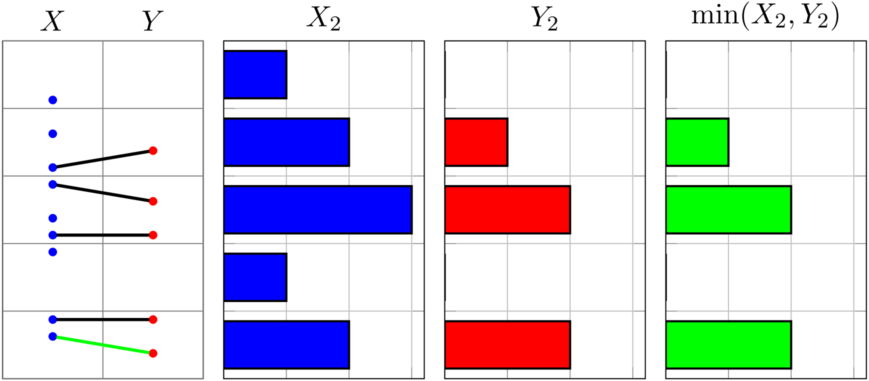

- 1d point sets X, Y on grid of size 4 - level 2 histograms - intersection
- $(2$ matches weighted by $1) + (2$ weighted by $\frac{1}{2}) + (1$ weighted by $\frac{1}{4})$
- total score $2 \times 1 + 2 \times \frac{1}{2} + 1 \times \frac{1}{4}$

pyramid match kernel (PMK)

- given a set $X = \{x_1, \dots, x_n\} \subset \mathbb{R}^d$, where distances of elements range in $[1, D]$
- let X_i be a histogram of X in $\mathbb{R}^d$ on a regular grid of side length 2^i
- i ranges from -1, where each bin has at most one element, to $L = \lceil \log_2 D \rceil$, where X is contained in a single bin
- given two images with descriptors $X, Y \subset \mathbb{R}^d$, their pyramid match is

$$K_\Delta(X,Y) = \gamma(X)\gamma(Y) \sum_{i=0}^{L} \frac{1}{2^i} (\kappa_{\mathsf{HI}}(X_i, Y_i) - \kappa_{\mathsf{HI}}(X_{i-1}, Y_{i-1}))$$

$$= \gamma(X)\gamma(Y) \left(\frac{1}{2^L} \kappa_{\mathsf{HI}}(X_L, Y_L) + \sum_{i=0}^{L-1} \frac{1}{2^{i+1}} \kappa_{\mathsf{HI}}(X_i, Y_i) \right)$$

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PMK is a positive-definite kernel

- κ_Δ can be written as a weighted sum of κ_{HI} terms, with nonnegative coefficients
- κ_{HI} can be written as a sum of $\min$ terms
- $\min$ can be written as a dot product:

x	$\phi(x)$							
3	1	1	1	0	0	0	0	0
5	1	1	1	1	1	0	0	0
$\min(x, y) = 3$	1	1	1	0	0	0	0	0

- therefore, so can κ_Δ
- but what other function does κ_Δ approximate itself?

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PMK as an embedding

[Indyk and Thaper 2003]

- there is an explicit embedding for κ_{HI}, therefore also for κ_Δ
- if $|X| \leq |Y|$ and $\pi : X \to Y$ is one-to-one, then $K_\Delta(X, Y)$ approximates the optimal pairwise matching

$$\max_\pi \sum_{x \in X} \|x - \pi(x)\|_1^{-1}$$

- this was first shown on the earth mover's distance

$$\min_\pi \sum_{x \in X} \|x - \pi(x)\|_1$$

- but PMK is a similarity measure; it allows partial matching and does not penalize clutter, expect for the normalization

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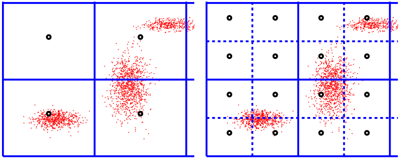
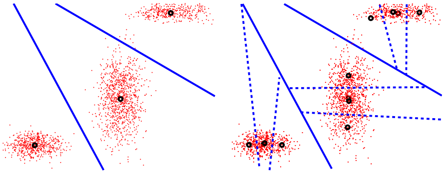
PMK and vocabulary tree

[Grauman and Darrell 2007]



uniform bins



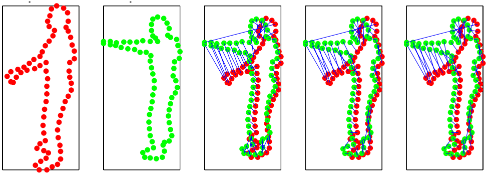
vocabulary-guided bins

- replace regular grid with hierarchical vocabulary cells
- compared to vocabulary tree, there is a principle in assigning cell weights
- still, its approximation quality suffers at high dimensions

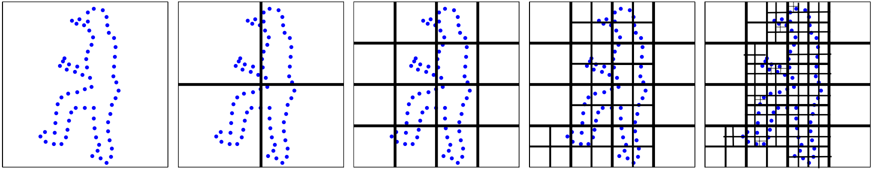
Grauman and Darrell. NIPS 2007. Approximate Correspondences in High Dimensions.

PMK and spatial matching

[Grauman and Darrell 2004]



optimal matching



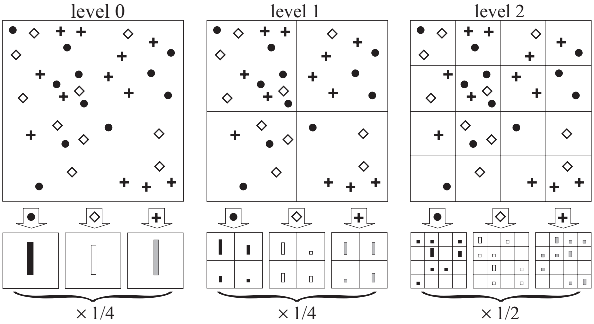
representation

- same idea, applied to image 2d coordinate space for spatial matching
- matching cost is only based on point coordinates; no appearance

Grauman and Darrell. CVPR 2004. Fast Contour Matching Using Approximate Earth Mover's Distance.

spatial pyramid matching (SPM)

[Lazebnik et al. 2006]



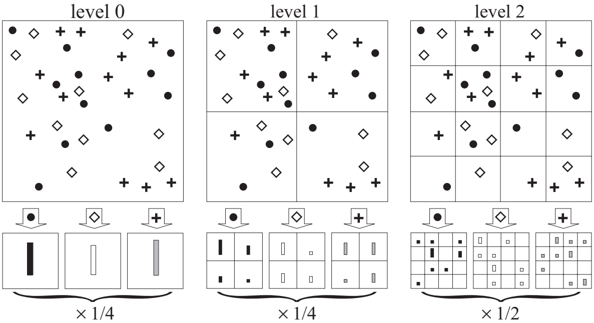
- if $X^{(j)}, Y^{(j)}$ are the feature coordinates of images X, Y with descriptors assigned to visual word j,

$$K_{\mathsf{SP}}(X, Y) = \sum_{j=1}^{k} K_{\Delta}(X^{(j)}, Y^{(j)})$$

Lazebnik, Schmid and Ponce. CVPR 2006. Beyond Bags of Features: Spatial Pyramid Matching for Recognizing Natural Scene categories.

spatial pyramid matching (SPM)

[Lazebnik et al. 2006]



- coupled with BoW, it is a set of joint appearance-geometry histograms
- robust to deformation but not invariant to transformations; applied to global scene classification

Lazebnik, Schmid and Ponce. CVPR 2006. Beyond Bags of Features: Spatial Pyramid Matching for Recognizing Natural Scene categories.

Hough pyramid matching (HPM)

[Tolias and Avrithis 2011]

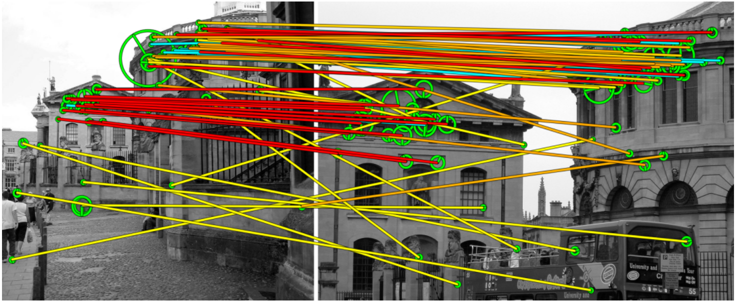
fast spatial matching

- work with a single set of correspondences instead of two sets of features
- determine a transformation hypothesis by a pair of features and then use histograms to collect votes in the transformation space

Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

Hough pyramid matching (HPM)

[Tolias and Avrithis 2011]



Hough pyramid matching

- work with a single set of correspondences instead of two sets of features
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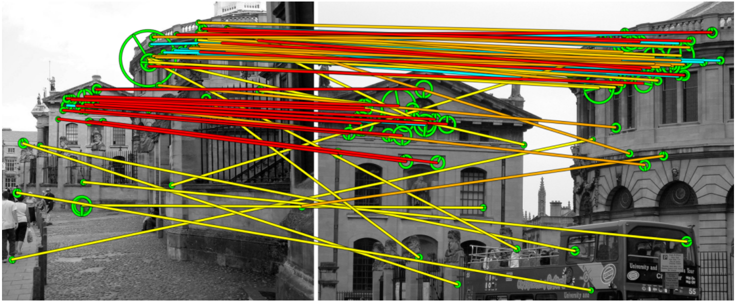
Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

Hough pyramid matching (HPM)

[Tolias and Avrithis 2011]



Hough pyramid matching

- work with a single set of correspondences instead of two sets of features
- determine a transformation hypothesis by a pair of features and then use histograms to collect votes in the transformation space

Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

Hough pyramid matching (HPM)

- a local feature p in image P has position $\mathbf{t}(p)$, scale $s(p)$ and orientation $\theta(p)$ given by matrix $R(p) \in \mathbb{R}^{2\times 2}$

$$F(p) = \begin{pmatrix} s(p)R(p) & \mathbf{t}(p) \\ \mathbf{0}^\top & 1 \end{pmatrix}$$

- a correspondence $c = (p, q)$ is a pair of features $p \in P, q \in Q$ of two images P, Q and determines relative similarity transformation from p to q

$$F(c) = F(q)F(p)^{-1} = \begin{pmatrix} s(c)R(c) & \mathbf{t}(c) \\ \mathbf{0}^\top & 1 \end{pmatrix}$$

with translation $\mathbf{t}(c) = \mathbf{t}(q) - s(c)R(c)\mathbf{t}(p)$, relative scale $s(c) = s(q)/s(p)$ and rotation $R(c) = R(q)R(p)^{-1}$ or $\theta(c) = \theta(q) - \theta(p)$

Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

Hough pyramid matching (HPM)

- the 4-dof relative transformation represented by 4d vector

$$f(c) = (\mathbf{t}(c), s(c), \theta(c))$$

- to enforce one-to-one mapping, two correspondences $c = (p, q)$, $c' = (p', q')$ are conflicting if they refer to the same feature on either image, *i.e.* $p = p'$ or $q = q'$

Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

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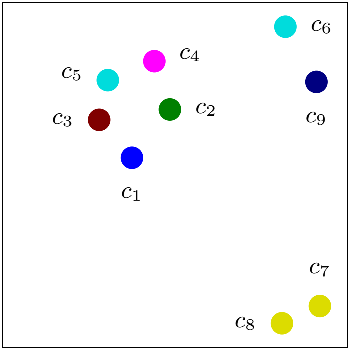
Hough pyramid matching (HPM)

correspondences

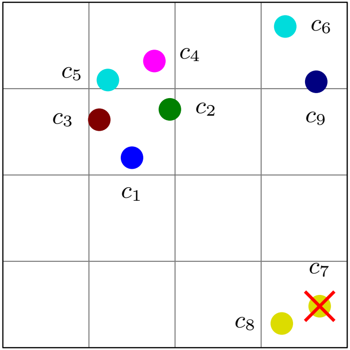


	p	q	similarity score
c_1			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_1)$
c_2			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_2)$
c_3			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_3)$
c_4			$(1 + \frac{1}{2}3 + \frac{1}{4}2)w(c_4)$
c_5			$(1 + \frac{1}{2}3 + \frac{1}{4}2)w(c_5)$
c_6			0
c_7			0
c_8			$(\frac{1}{4}6)w(c_8)$
c_9			$(\frac{1}{4}6)w(c_9)$

- correspondence c contributes by $w(c)$, based *e.g.* on visual word
- conflicting correspondences in the same bin b are erased
- in a bin b with n_b correspondences, each groups with $[n_b - 1]_+$ others
- level 0 weight 1

Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

Hough pyramid matching (HPM)



	p	q	similarity score
c_1			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_1)$
c_2			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_2)$
c_3			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_3)$
c_4			$(1 + \frac{1}{2}3 + \frac{1}{4}2)w(c_4)$
c_5			$(1 + \frac{1}{2}3 + \frac{1}{4}2)w(c_5)$
c_6			0
c_7			0
c_8			$(\frac{1}{4}6)w(c_8)$
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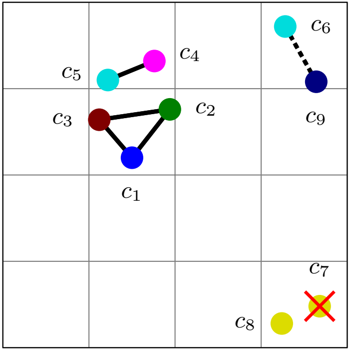
Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

Hough pyramid matching (HPM)

pyramid level 0



	p	q	similarity score
c_1			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_1)$
c_2			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_2)$
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c_5			$(1 + \frac{1}{2}3 + \frac{1}{4}2)w(c_5)$
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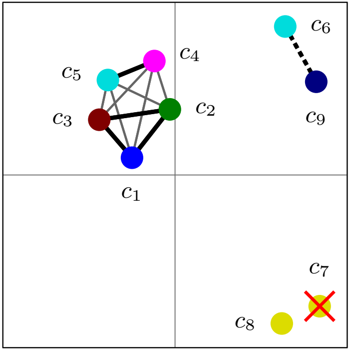
- level 0 weight 1

Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

Hough pyramid matching (HPM)

pyramid level 1

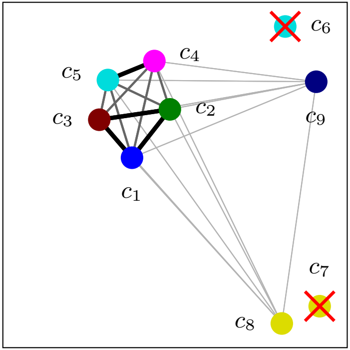


	p	q	similarity score
c_1			$(2+\frac{1}{2}2+\frac{1}{4}2)w(c_1)$
c_2			$(2+\frac{1}{2}2+\frac{1}{4}2)w(c_2)$
c_3			$(2+\frac{1}{2}2+\frac{1}{4}2)w(c_3)$
c_4			$(1+\frac{1}{2}3+\frac{1}{4}2)w(c_4)$
c_5			$(1+\frac{1}{2}3+\frac{1}{4}2)w(c_5)$
c_6			0
c_7			0
c_8			$(\frac{1}{4}6)w(c_8)$
c_9			$(\frac{1}{4}6)w(c_9)$

- correspondence c contributes by $w(c)$, based *e.g.* on visual word
- conflicting correspondences in the same bin b are erased
- in a bin b with n_b correspondences, each groups with $[n_b-1]_+$ others
- level 1 weight $\frac{1}{2}$

Hough pyramid matching (HPM)

pyramid level 2

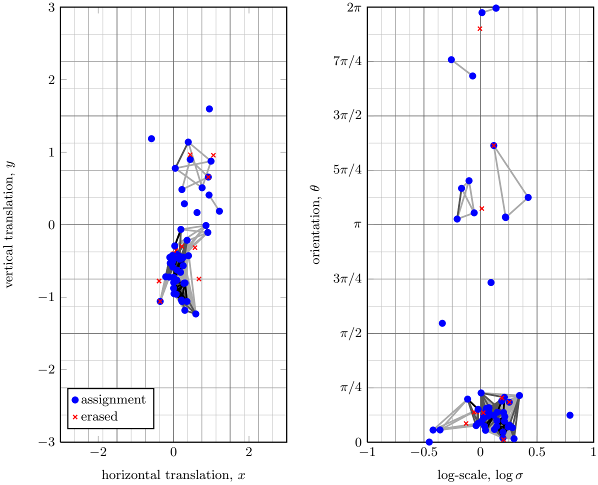


	p	q	similarity score
c_1			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_1)$
c_2			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_2)$
c_3			$(2 + \frac{1}{2}2 + \frac{1}{4}2)w(c_3)$
c_4			$(1 + \frac{1}{2}3 + \frac{1}{4}2)w(c_4)$
c_5			$(1 + \frac{1}{2}3 + \frac{1}{4}2)w(c_5)$
c_6			0
c_7			0
c_8			$(\frac{1}{4}6)w(c_8)$
c_9			$(\frac{1}{4}6)w(c_9)$

- correspondence c contributes by $w(c)$, based *e.g.* on visual word
- conflicting correspondences in the same bin b are erased
- in a bin b with n_b correspondences, each groups with $[n_b - 1]_+$ others
- level 2 weight $\frac{1}{4}$

Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

Hough pyramid matching (HPM)



- mode seeking: we are looking for regions where density is maximized in the transformation space

Hough pyramid matching (HPM)

- linear in the number of correspondences; no need to count inliers
- robust to deformations and multiple matching surfaces, invariant to transformations
- only applies to same instance matching

Tolias and Avrithis. ICCV 2011. Speeded-Up, Relaxed Spatial Matching.

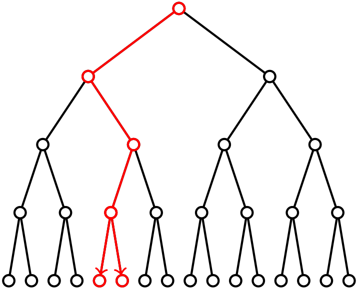
nearest neighbor search

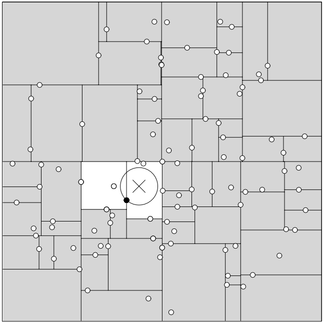
nearest neighbor search

- given query point y, find its nearest neighbor with respect to Euclidean distance within data set X in a d-dimensional space
- image retrieval: same problem; one or multiple queries depending on global or local representation
- image classification: nearest neighbor or naïve Bayes nearest neighbor classifier, again depending on representation

k-d tree

[Bentley 1975]



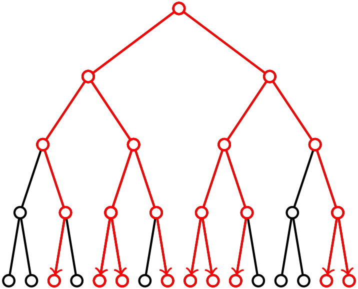


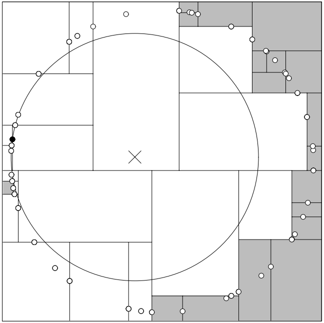
- index: recursively split at medoid on some dimension, make balanced binary tree
- search: descend recursively from root, choosing child according to splitting dimension and value
- backtracking becomes exhaustive at high dimensions

Bentley. CACM 1975. Multidimensional Binary Search Trees Used for Associative Searching.

k-d tree

[Bentley 1975]





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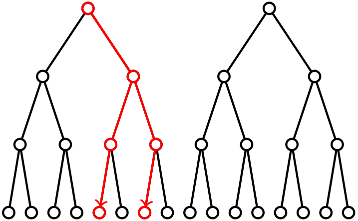
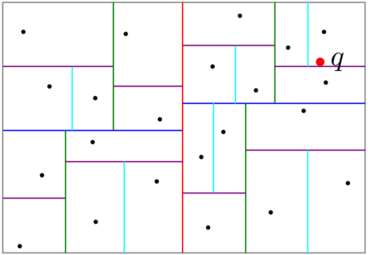
randomized k-d trees

[Silpa-Anan and Hartley 1975]



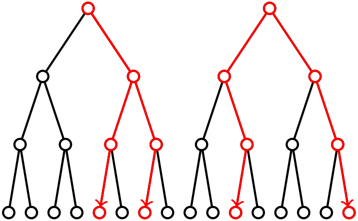


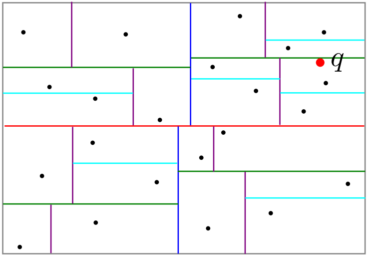
- index: same as before, but now multiple randomized trees
- search: descend trees in parallel according to shared priority queue
- still, points are stored, distances are exact

Silpa-Anan and Hartley CVPR 2008. Optimized KD-trees for fast image descriptor matching.

randomized k-d trees

[Silpa-Anan and Hartley 1975]





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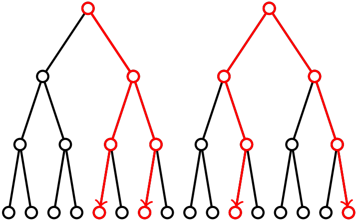

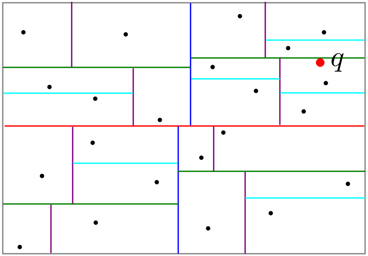

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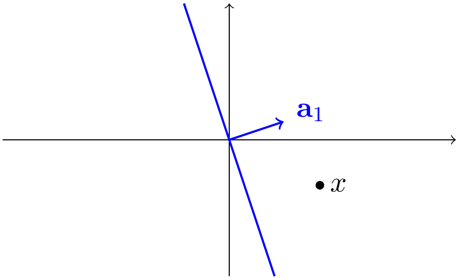
locality sensitive hashing (LSH)

[Charikar 2002]



- index: choose $\mathbf{a}_i \sim \mathcal{N}(0,1)$; encode each data point $x \in X$ by binary code $h(x) := (h_{\mathbf{a}_1}(x), \dots, h_{\mathbf{a}_k}(x)) \in \{-1,1\}^d$ with hash function

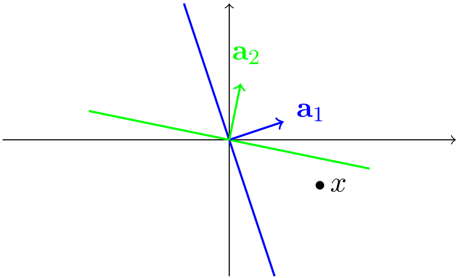
$$h_{\mathbf{a}}(x) = \operatorname{sgn}(\mathbf{a}^\top x)$$

- search: encode query y as $h(y)$ and search by Hamming distance
- not adapted to data distribution

Charikar. STOC 2002. Similarity Estimation Techniques From Rounding Algorithms.

locality sensitive hashing (LSH)

[Charikar 2002]



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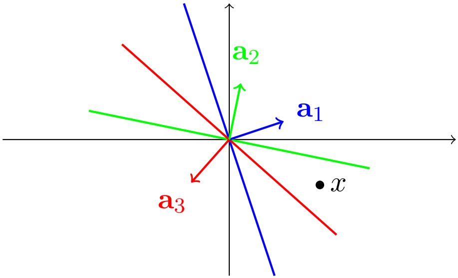

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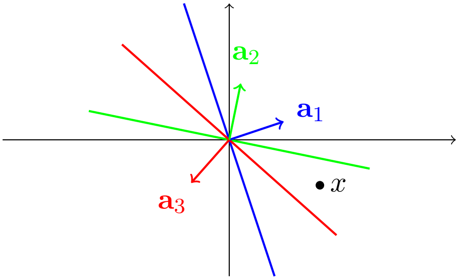

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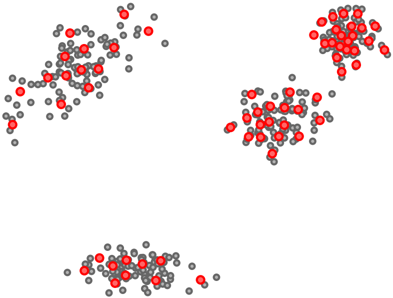
vector quantization (VQ)

[Gray 1984]

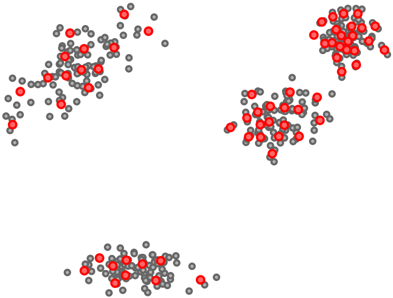


- index: cluster X into codebook $C = \{c_1, \ldots, c_k\}$; quantize each $x \in X$ to $q(x) = \min_{c \in C} \|x - c\|^2$ and encode it by $\log k$ bits
- search: pre-compute distances $\|y - c\|^2$ for $c \in C$ and approximate distances $\|y - x\|^2$ by $\|y - q(x)\|^2$ where $q(x) \in C$
- small distortion $\rightarrow$ large k, too large to store, too slow to search

Gray. SPM 1984. Vector quantization.

vector quantization (VQ)

[Gray 1984]



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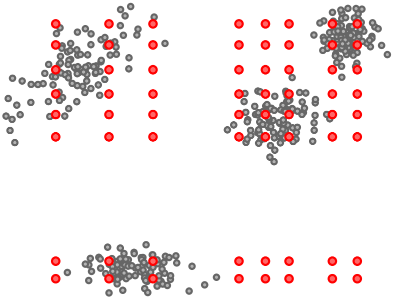
Gray. SPM 1984. Vector quantization.

product quantization (PQ)

[Jégou et al. 2011]

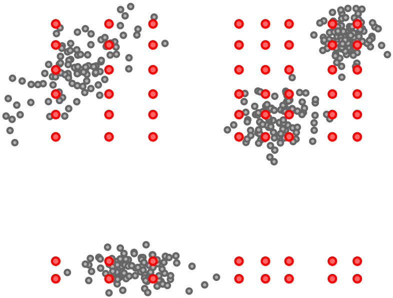


- index: decompose vectors as $x = (x^1, \dots, x^m)$, cluster X into codebook $C = C^1 \times \cdots \times C^m$ with k cells each and $|C| = k^m$
- search: pre-compute distances $\|y^j - c\|^2$ for $c \in C^j$ and approximate $\|y - x\|^2$ by $\|y - q(x)\|^2 = \sum_{j=1}^m \|y^j - q^j(x^j)\|^2$ where $q^j(x^j) \in C^j$
- a lot of centroids do not represent data and are unused

Jegou, Douze and Schmid. PAMI 2011. Product Quantization for Nearest Neighbor Search.

product quantization (PQ)

[Jégou et al. 2011]

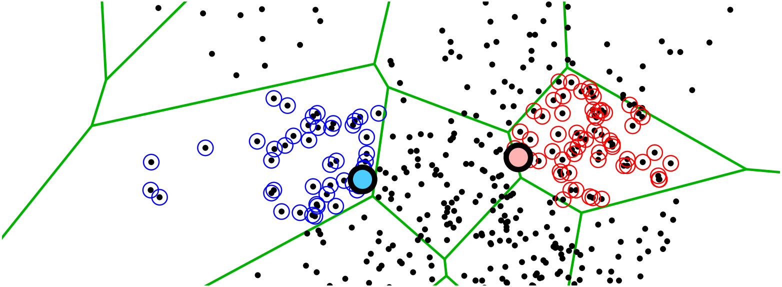


- index: decompose vectors as $x = (x^1, \ldots, x^m)$, cluster X into codebook $C = C^1 \times \cdots \times C^m$ with k cells each and $|C| = k^m$
- search: pre-compute distances $\|y^j - c\|^2$ for $c \in C^j$ and approximate $\|y - x\|^2$ by $\|y - q(x)\|^2 = \sum_{j=1}^m \|y^j - q^j(x^j)\|^2$ where $q^j(x^j) \in C^j$
- a lot of centroids do not represent data and are unused

Jegou, Douze and Schmid. PAMI 2011. Product Quantization for Nearest Neighbor Search.

inverted index

[Jégou et al. 2011]



- index: train a coarse quantizer Q of k cells; quantize each $x \in X$ to $Q(x)$, compute residual $r(x) = x - Q(x)$ and encode residuals by a product quantizer q
- search: quantize query y to a fixed number of nearest cells; exhaustively search by PQ only within those cells
- a lot of points in the coarse cells are too far away from query

Jegou, Douze and Schmid. PAMI 2011. Product Quantization for Nearest Neighbor Search.

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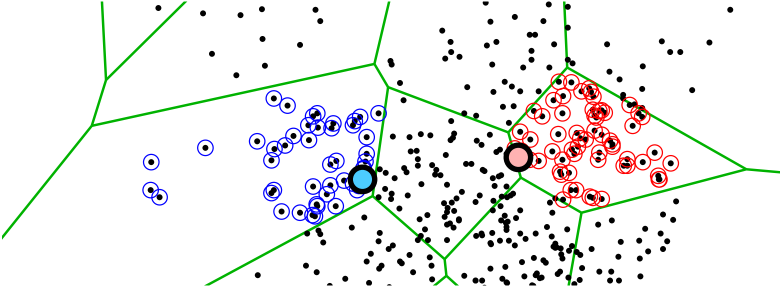

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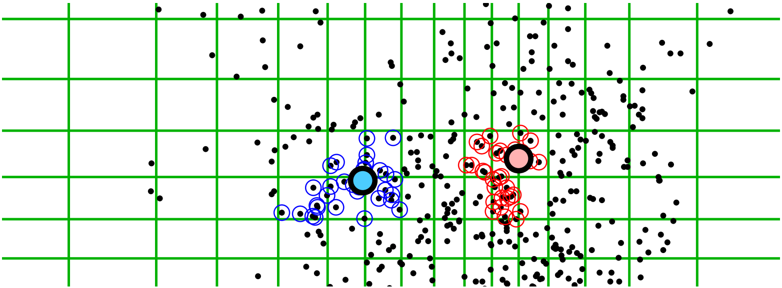
inverted multi-index

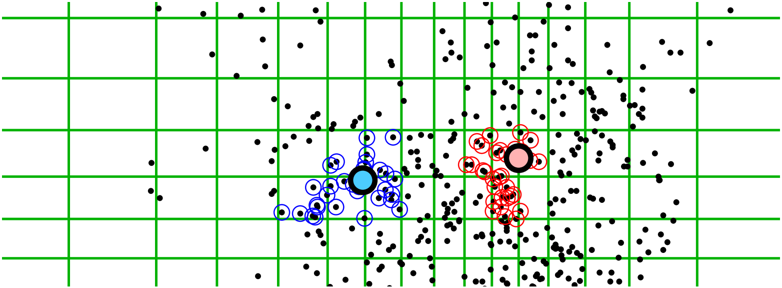
[Babenko and Lempitsky 2012]



- index: decompose vectors as $x = (x^1, x^2)$; train two coarse quantizers Q^1, Q^2 of k cells each, quantize each $x \in X$ to $Q^1(x^1), Q^2(x^2)$ and encode residuals by product quantizers q^1, q^2
- search: visit cells $(c^1, c^2) \in C^1 \times C^2$ in ascending order of distance to y by multi-sequence algorithm
- two coarse quantizers induce a finer partition than one

inverted multi-index

[Babenko and Lempitsky 2012]



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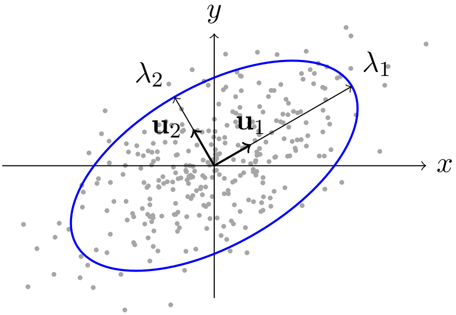
principal component analysis (PCA)



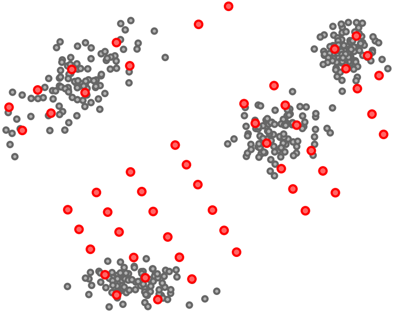
- given data $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$, compute empirical mean $\bar{\mathbf{x}} := \frac{1}{n} \sum_{i=1}^n \mathbf{x}_i$ and covariance matrix

$$S := \frac{1}{n} \sum_{i=1}^{n} (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^\top$$

- then diagonalize S by $S = U \Lambda U^\top$ where $U = (\mathbf{u}_1 \;\; \mathbf{u}_2)$ and $\Lambda = \operatorname{diag}(\lambda_1, \lambda_2)$

optimized product quantization (OPQ)

[Ge et al. 2013]

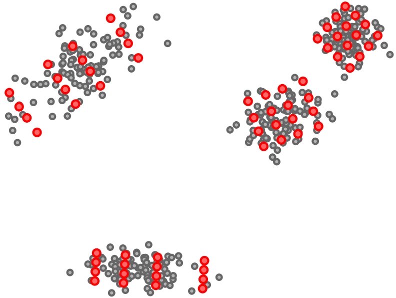


- no correlation: PCA-align by diagonalizing $\mathrm{cov}(X)$ as $U\Lambda U^\top$
- balanced variance: shuffle eigenvalues Λ by permutation π such that the product $\prod_i \lambda_i$ is constant in each subspace
- find codebook $\hat{C}$ by PQ on rotated data $\hat{X} := RX$ where $R := UP_\pi^\top$ and P_π is the permutation matrix of π

Ge, He, Ke and Sun. CVPR 2013. Optimized Product Quantization for Approximate Nearest Neighbor Search.

locally optimized product quantization (LOPQ)

[Kalantidis and Avrithis 2014]



- same as PQ with inverted index (or multi-index), but residuals are encoded by OPQ
- better on multimodal data: residual distributions closer to Gaussian assumption

Kalantidis and Avrithis. CVPR 2014. Locally Optimized Product Quantization for Approximate Nearest Neighbor Search.

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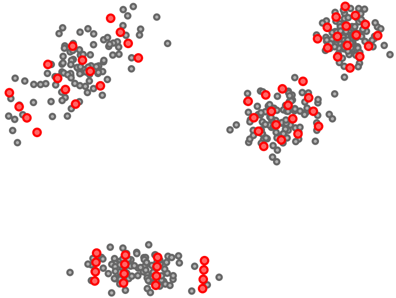

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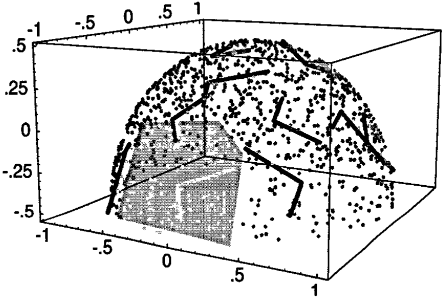
local principal component analysis

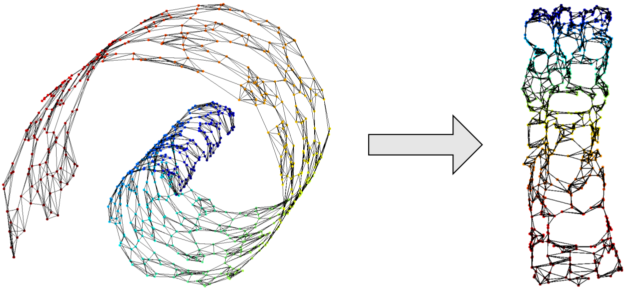
[Kambhatla & Leen 1997]



- cluster data, then apply PCA per cell
- captures the support of data distribution
  - multimodal (*e.g.* mixture) distributions
  - distributions on nonlinear manifolds

Kambhatla and Leen. NC 1997. Dimension Reduction By Local Principal Component Analysis.

manifold learning



- *e.g.* Isomap: apply PCA to the geodesic (graph) distance matrix
- *e.g.* kernel PCA: apply PCA to the Gram matrix of a nonlinear kernel
- other topology-preserving methods are only focusing on distances to nearest neighbors
- many classic methods use eigenvalue decomposition and most do not learn and explicit mapping from the input to the embedding space

summary

- bag of words: treating geometry separately from appearance, and quantizing descriptors
- BoW for instance and class recognition: what is common, what is different
- k-means, HKM, vocabulary tree, AKM, soft/multiple assignment, max pooling, burstiness
- beyond BoW—matching between sets of features/descriptors that cannot be expressed as dot product: HE, VLAD, ASMK
- design or learn embeddings: EMK, PMK, SPM, HPM?
- a sum of similarities is better than a sum of distances
- nearest neighbor search: inverted index, multi-index, trees, forests, hashing, compression
- PCA and beyond: we should learn the manifold

discussion

representation

- convolution is linear + translation invariant (or equivariant) and is the only function having these properties
- Gabor filters or histograms of gradient orientations are more or less the same thing and are just the first layer of extracting a representation
- they record responses at every possible position, scale and orientation, resulting in a 4-dimensional representation; rotation and change of scale in the image behave like translation in the representation space
- convolution means that for every pixel we are looking at some spatial neighborhood (in the image domain), but the image has only one channel (grayscale)
- histograms can be expressed as two stages of encoding + pooling; then we can generalize these operations for the next layers

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codebooks

- so, for the second layer we still have histograms of some kind but now they are over vectors (the filter responses of the first stage) rather than scalars (orientation and scale)
- to make a histogram we need a finite set of such vectors, and this we obtain through vector quantization (or sampling) of the layer one responses of a given dataset
- so, the concept that such representations are "hand-crafted" is incorrect; codebooks are learned from data in an unsupervised fashion
- codebook size, parameters in the encoding and pooling stages *etc.* are just hyperparameters that will we learn through cross-validation
- in contrast to layer one, there is no spatial neighborhood here (with the exception of HMAX) but there is depth, *i.e.* a number of channels corresponding to the dimensions of these vectors; we will combine both, resulting in 3-dimensional filter kernels

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local features

- depending on the task (*e.g.* stereopsis, motion estimation, instance recognition compared to class recognition), not all spatial regions are equally important
- classification works best with dense features, but still, through encoding, the responses to most "visual words" are zero; so there some sparsity in the representation, at least before pooling
- in order to make change of scale really behave like translation in the representation space, we also need scale normalization and a logarithm
- operators that detect local features can be expressed as convolution followed by some kind of competition, but they can require more than one layers with nonlinearities in between; we will follow this idea for more complex patterns
- when it comes a sparse set of local features, matching becomes easier to formulate compared to *e.g.* continuous distributions

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matching

- descriptors are really meant to be used for matching one image to another (*e.g.* for instance recognition) or one image to a pattern (for classification)
- we want to learn a descriptor such that dot product will be good enough for matching
- we can start by thinking about pairwise matching between two sets of descriptors and come up with (design or learn) a representation, maybe at a higher dimension, such that dot product will be approximating this pairwise matching process
- there should be some invariance to geometric transformations; whether this should be designed or learned is up to discussion

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