

# lecture 2: visual representation

## deep learning for vision

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# logistics

- **planning**: to be updated gradually
- material marked as xxxx\* is optional

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# outline

introduction

receptive fields

visual descriptors

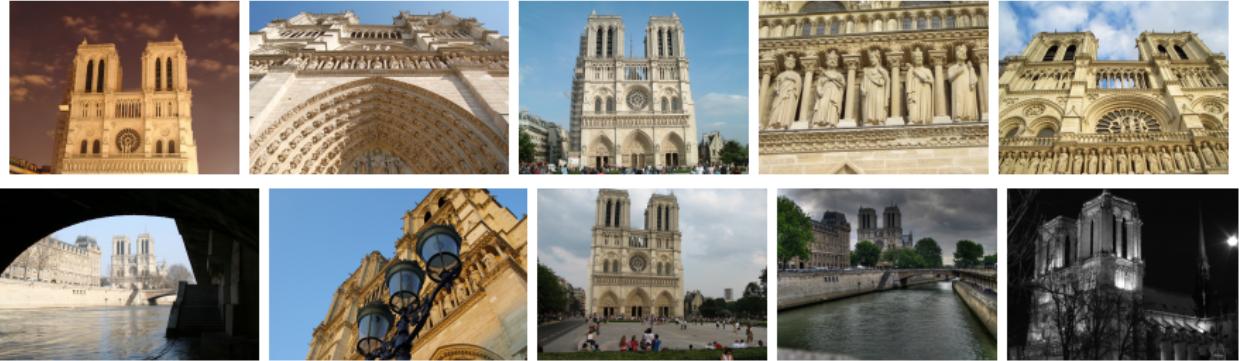
feature hierarchy

# introduction

# image retrieval challenges

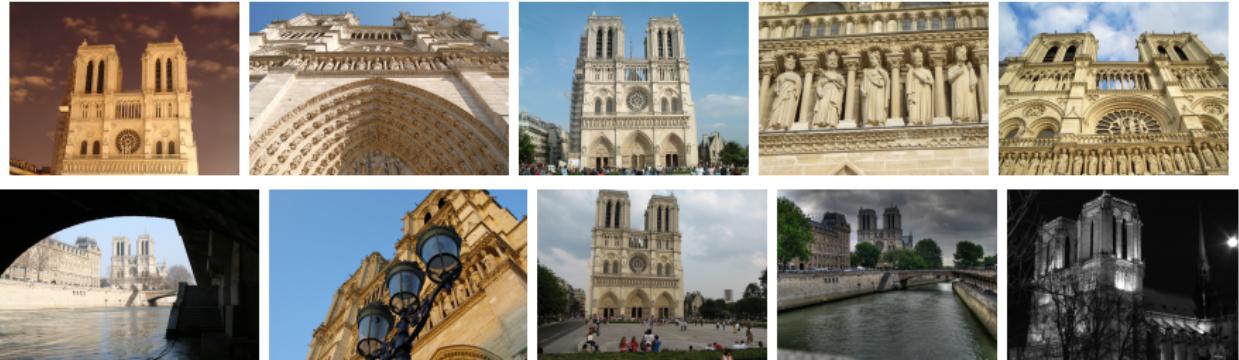


# image retrieval challenges



- scale
- viewpoint
- occlusion
- background clutter
- lighting

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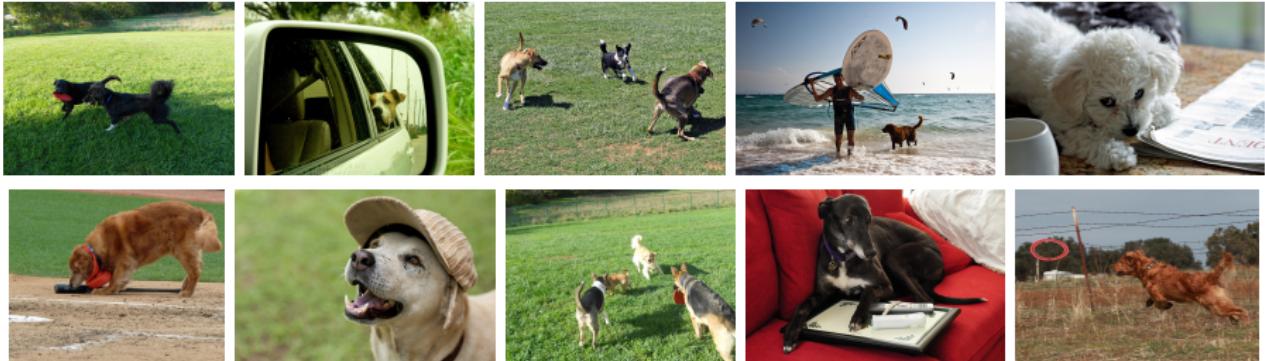


- scale
- viewpoint
- occlusion
- background clutter
- lighting
- discriminative power
- distractors

# image classification challenges

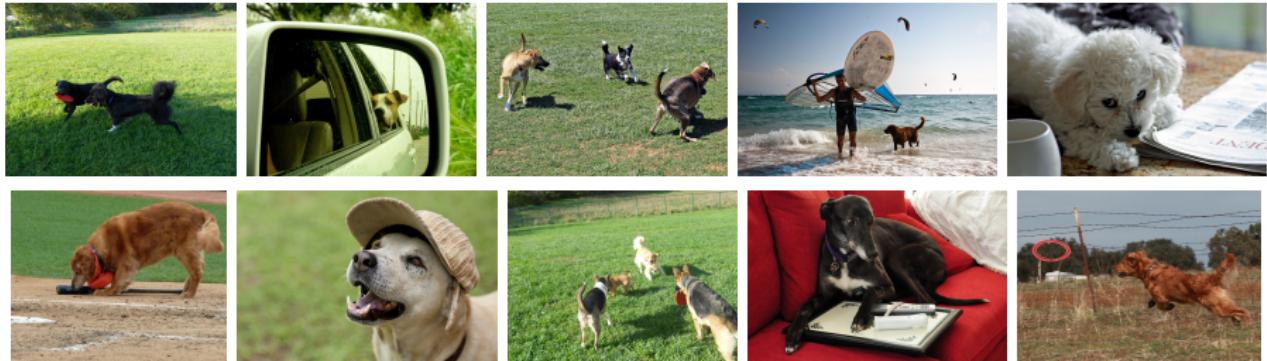


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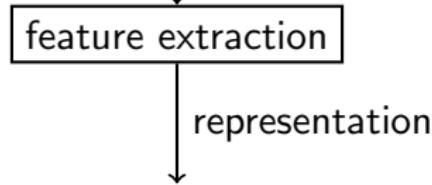


- scale
- viewpoint
- occlusion
- background clutter
- lighting
- number of instances
- texture/color
- pose
- deformability
- intra-class variability

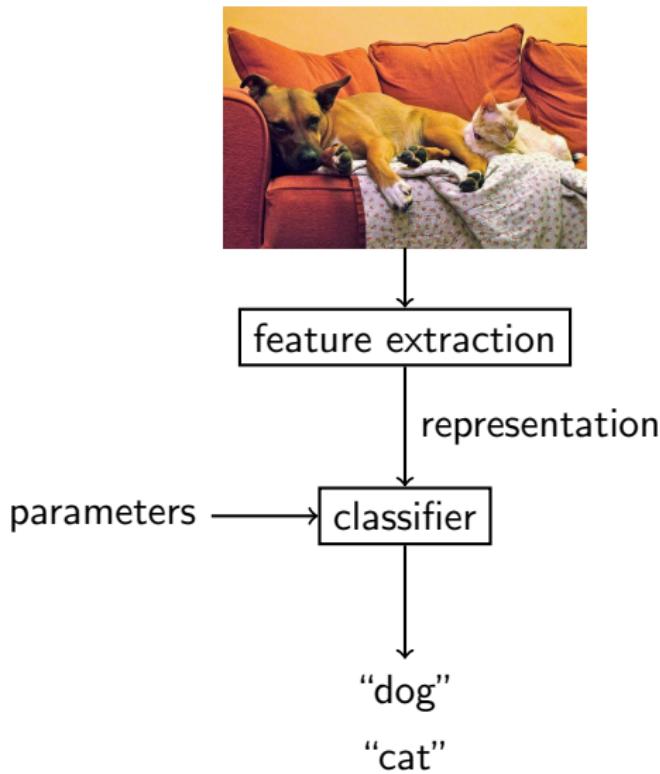
# data-driven approach



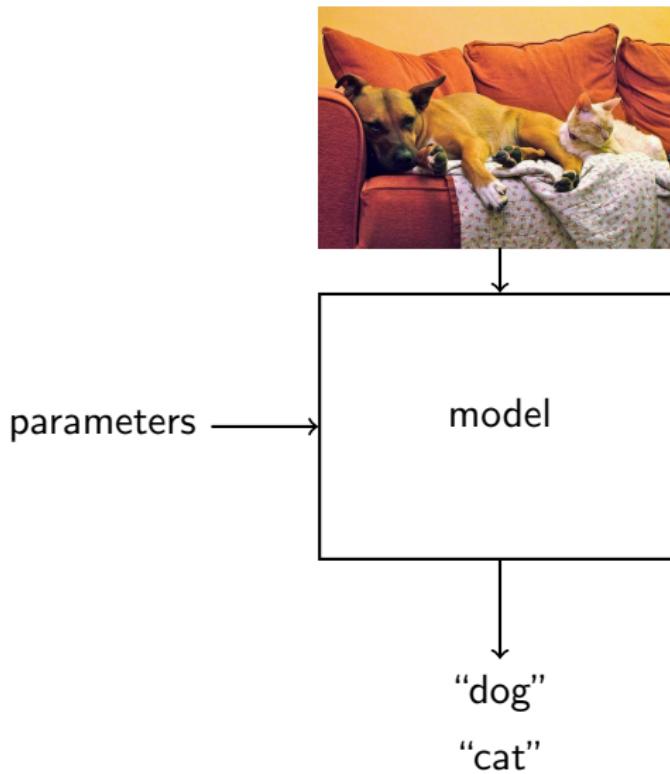
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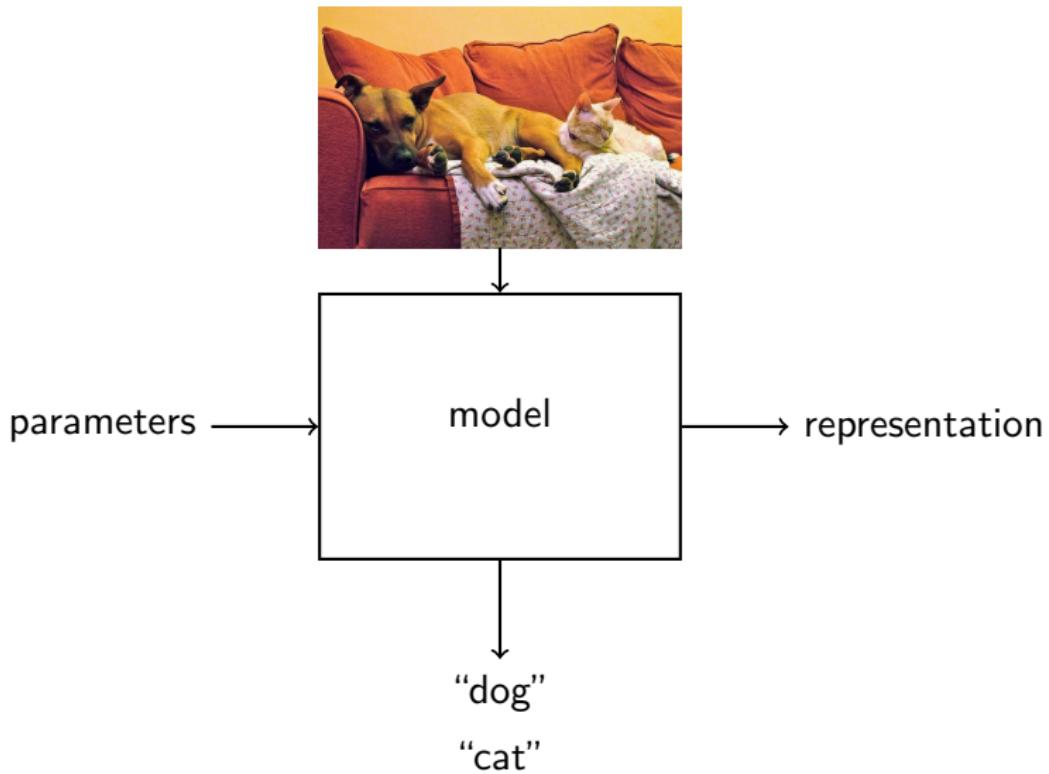
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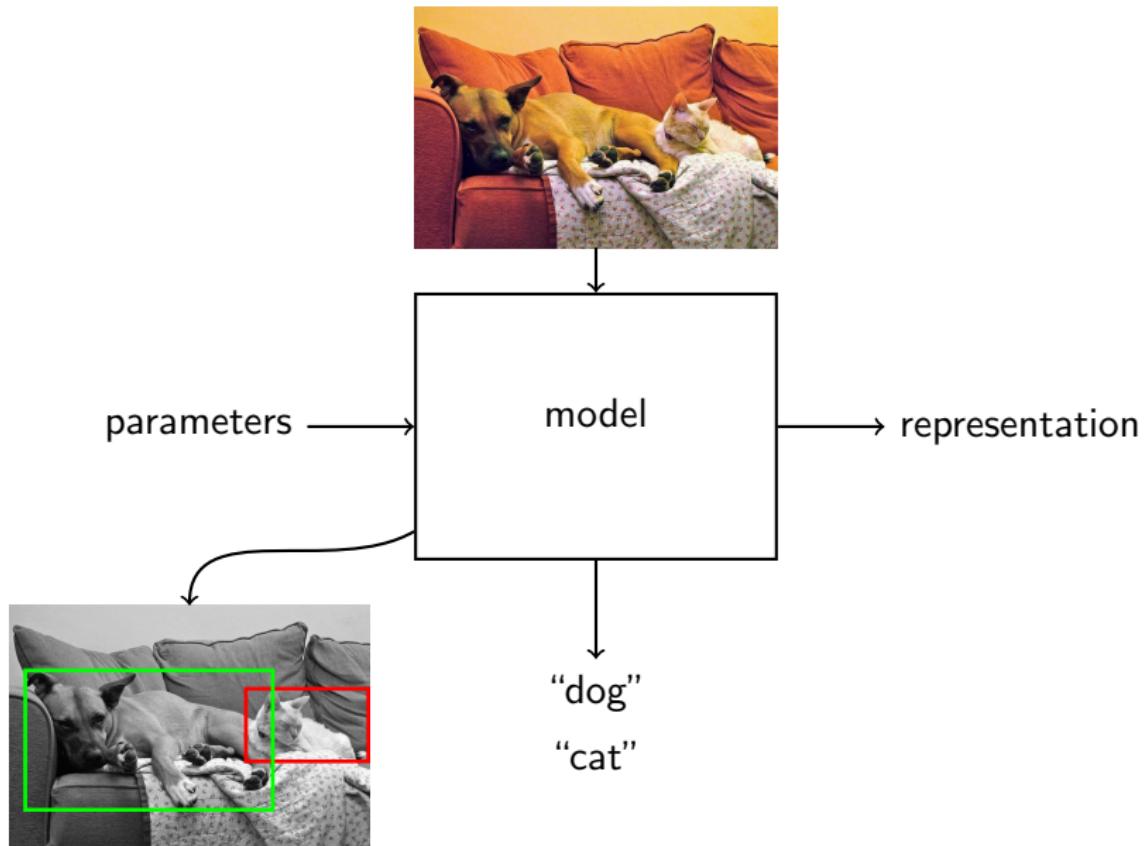
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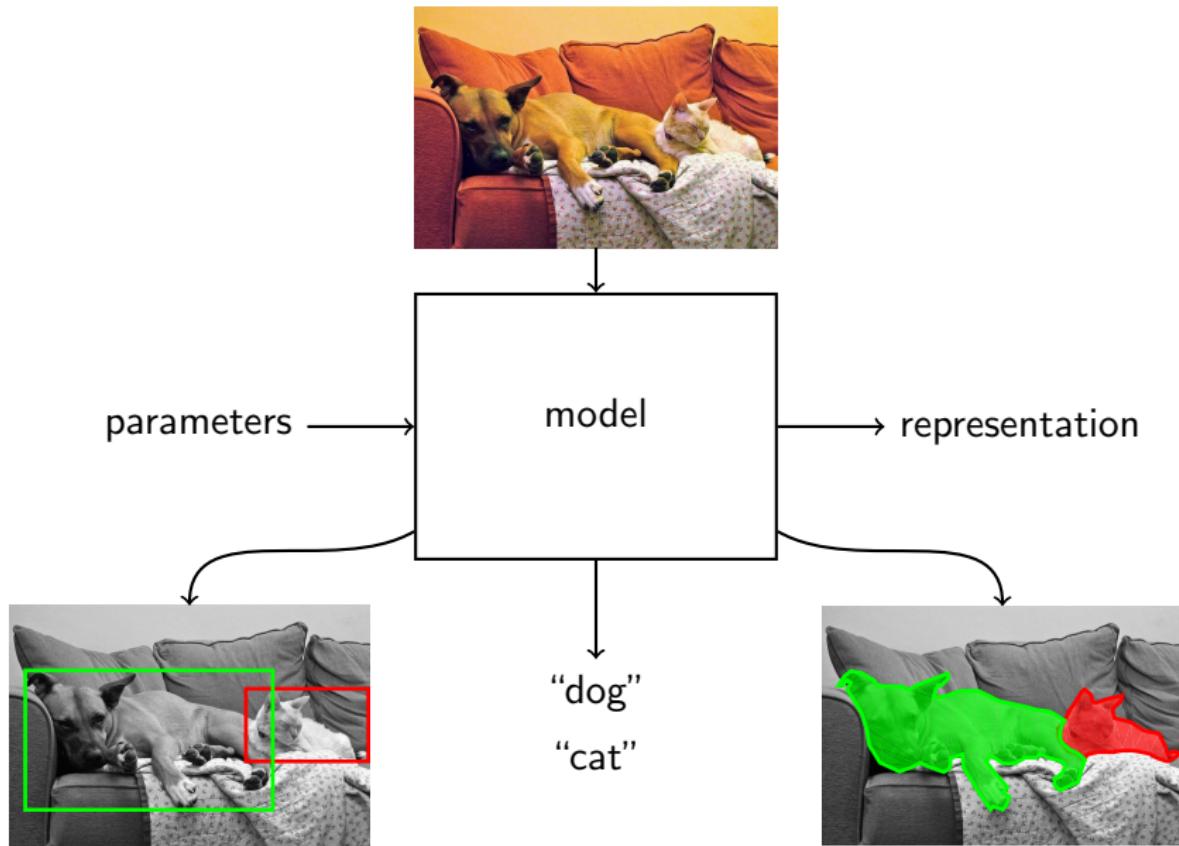
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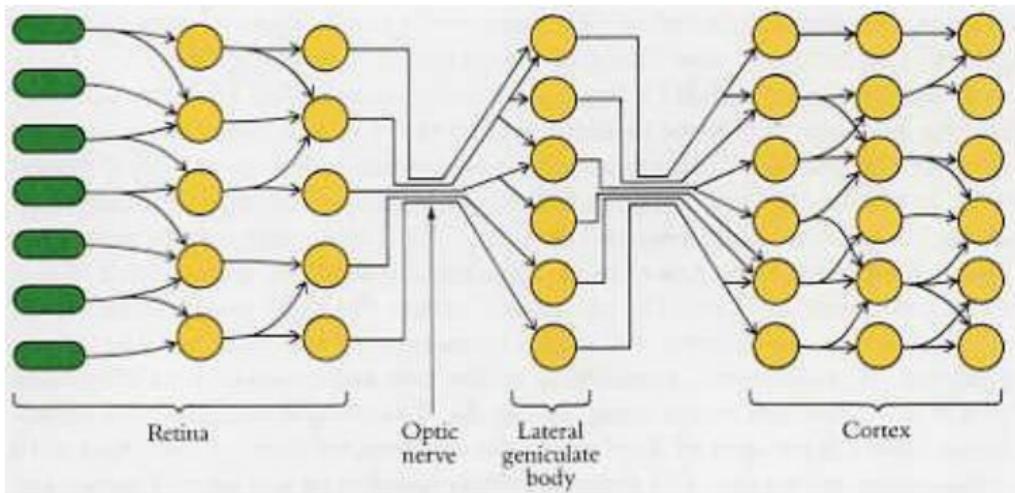


# data-driven approach



# receptive fields

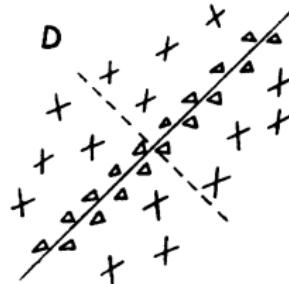
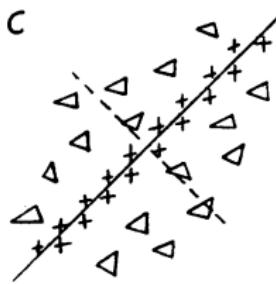
# topographic mapping: translation equivariance



- as you move along the retina, the corresponding points in the cortex trace a continuous path
- each column represents a two-dimensional array of cells
- a translation in the input causes a translation in the representation

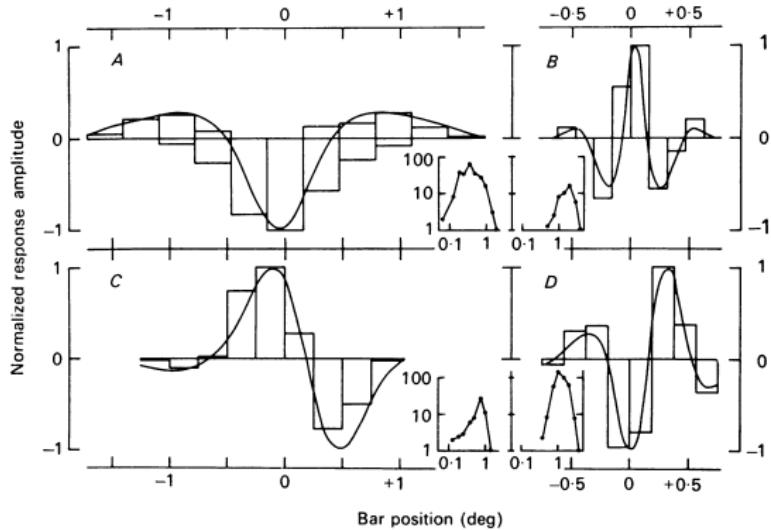
# receptive fields

[Hubel and Wiesel 1962]



- A: 'on'-center LGN; B: 'off'-center LGN; C, D: simple cortical
- ×: excitatory ('on'), △: inhibitory ('off') responses
- localized responses, orientation selectivity

# linearity



- simple cells perform linear spatial summation over their receptive fields
- spatial response (by oriented bars of varying position)
- frequency response (by oriented gratings of varying frequency)

Movshon, Thompson and Tolhurst. JP 1978. Spatial Summation in the Receptive Fields of Simple Cells in the Cat's Striate Cortex.

# linear time-invariant (LTI) systems

- discrete-time signal:  $x[n], n \in \mathbb{Z}$
- translation (or shift, or delay):  $s_k(x)[n] = x[n - k], k \in \mathbb{Z}$
- linear system (or filter): system commutes with linear combination

$$f\left(\sum_i a_i x_i\right) = \sum_i a_i f(x_i)$$

- time-invariant (or translation equivariant): system commutes with translation

$$f(s_k(x)) = s_k(f(x))$$

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# convolution

- unit impulse  $\delta[n] = \mathbb{1}[n = 0]$
- every signal  $x$  expressed as

$$x[n] = \sum_k x[k]\delta[n - k] = \sum_k x[k]s_k(\delta)[n]$$

- if  $f$  is LTI with impulse response  $h = f(\delta)$ , then  $f(x) = x * h$ :

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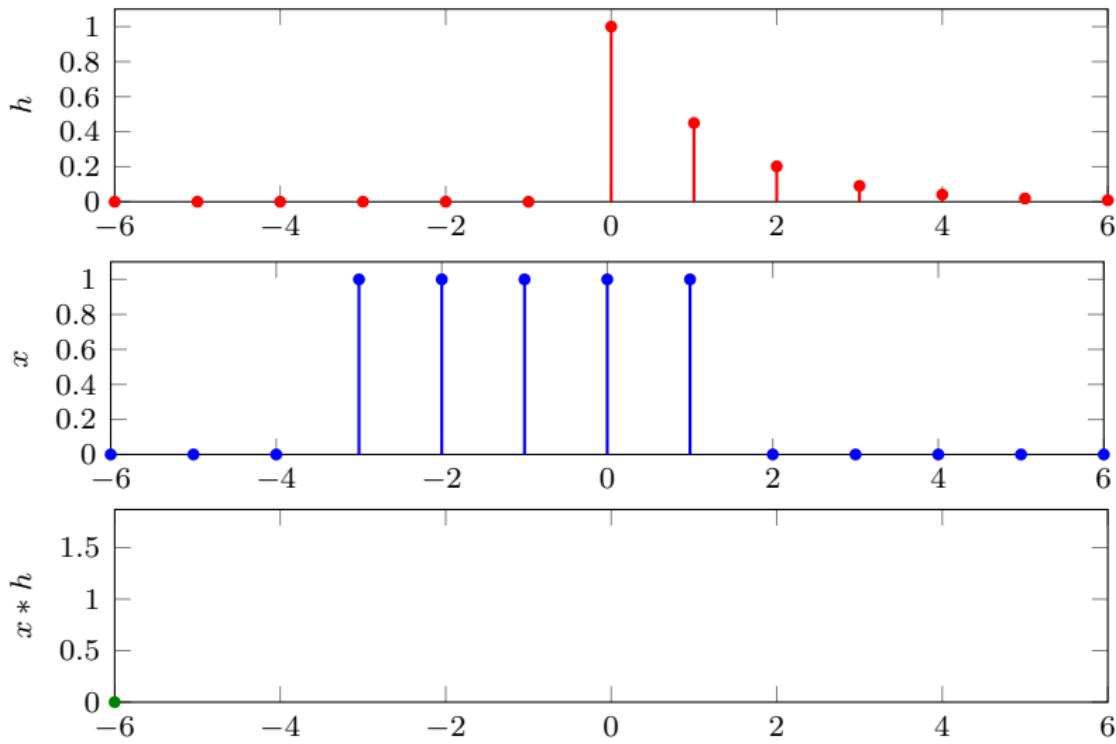
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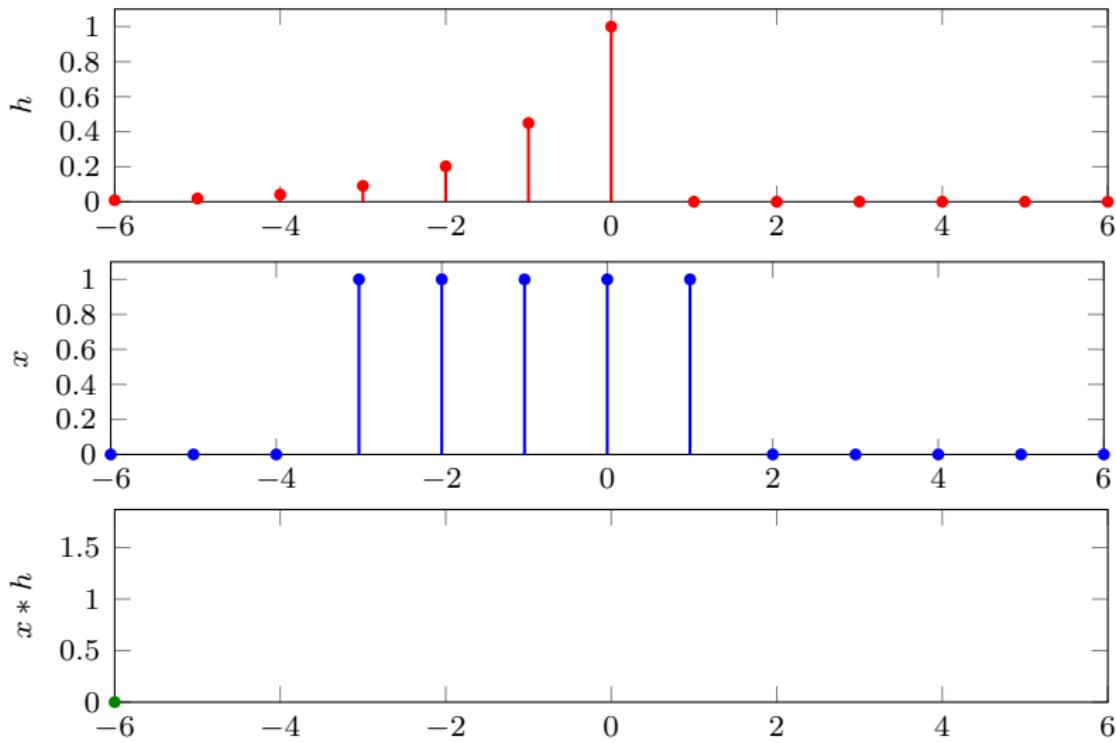
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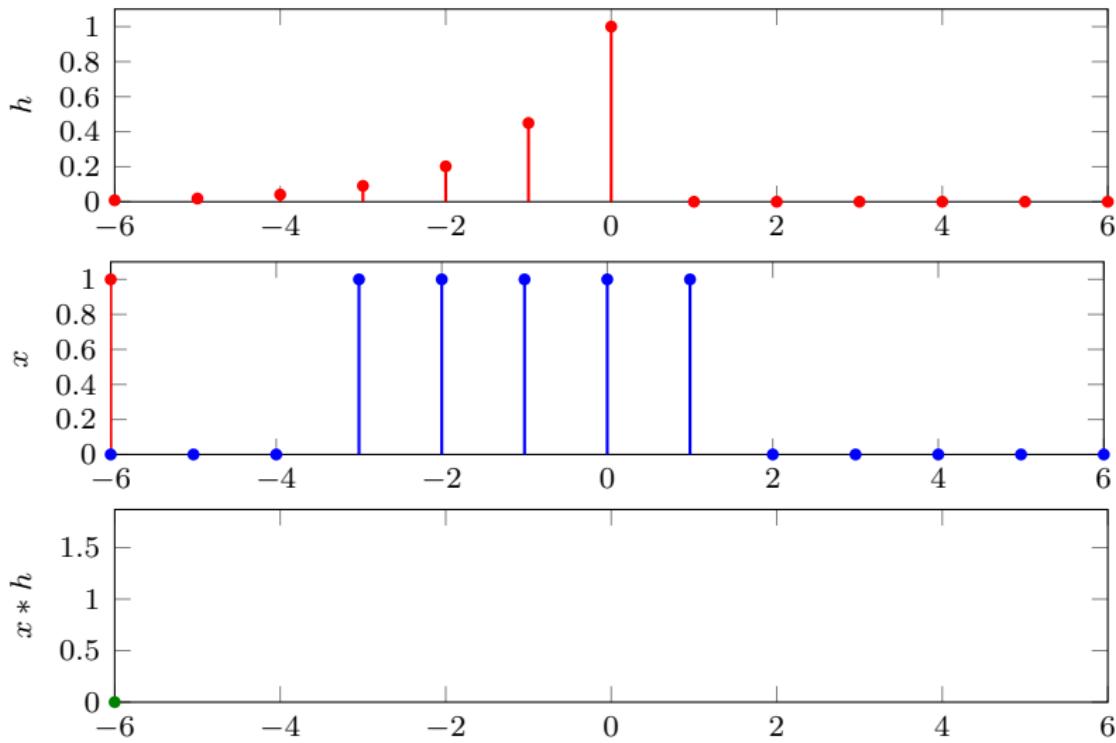
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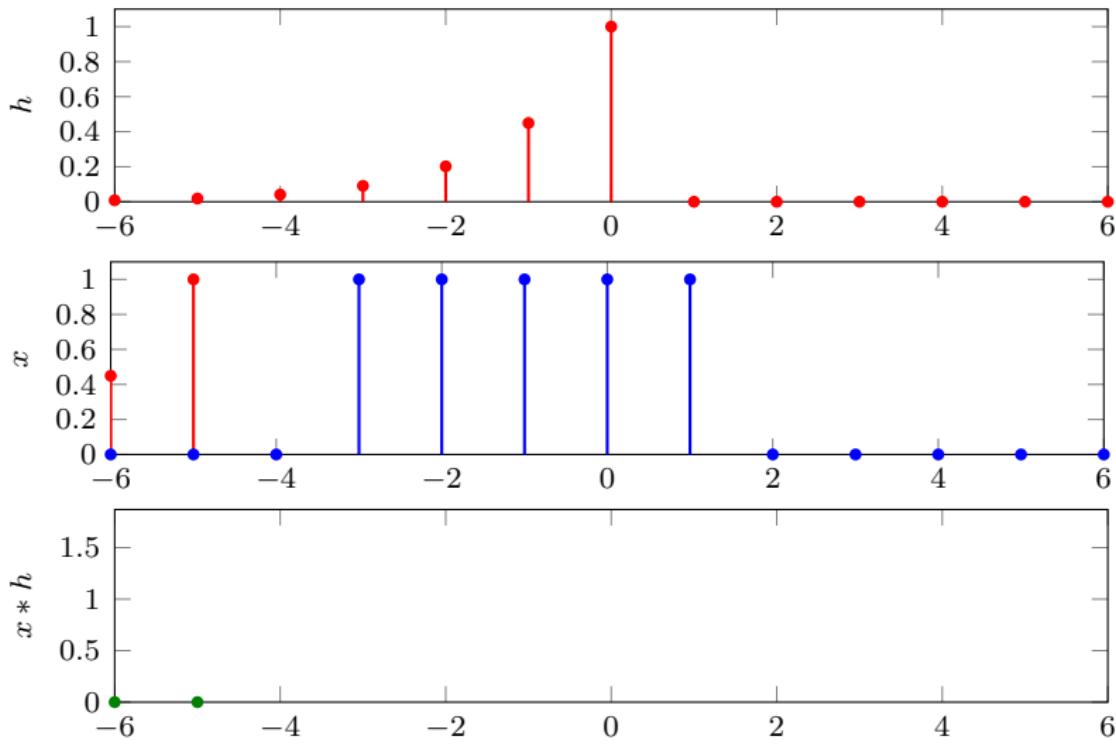
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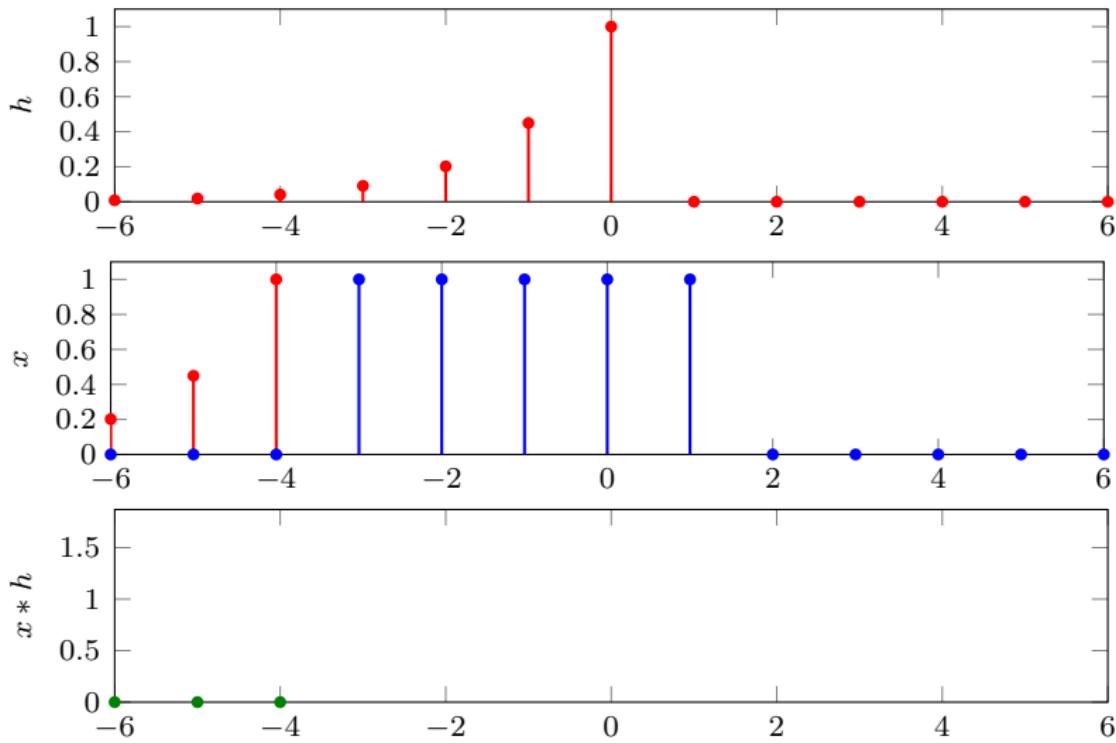
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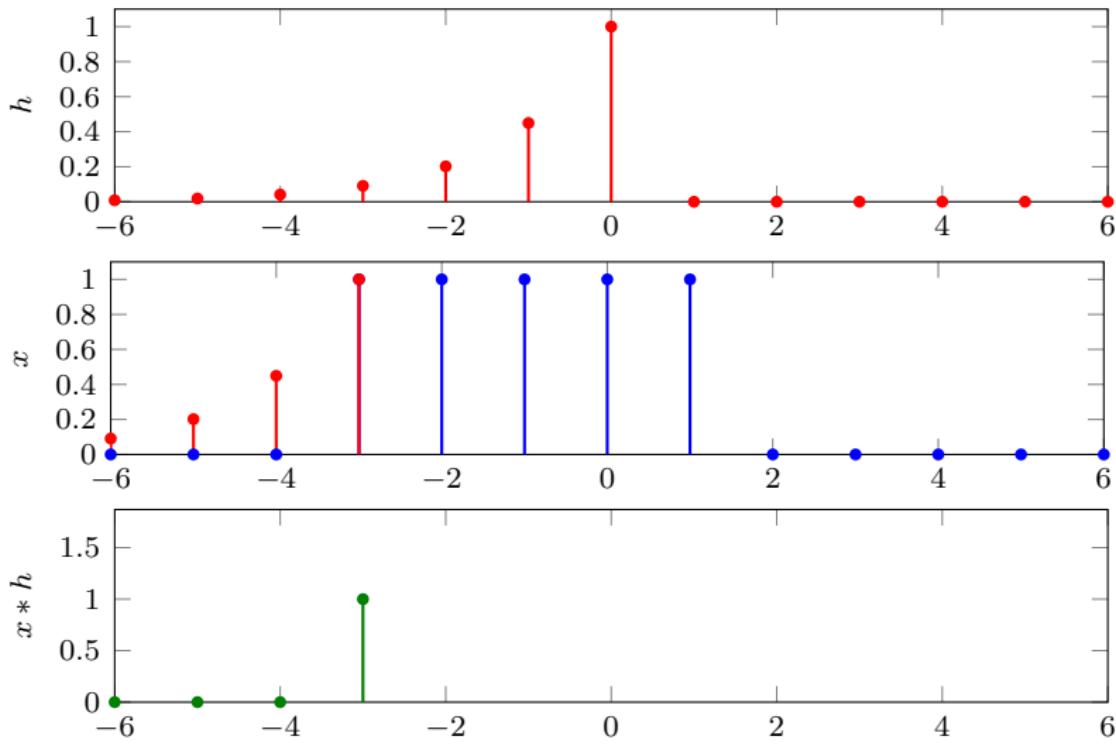
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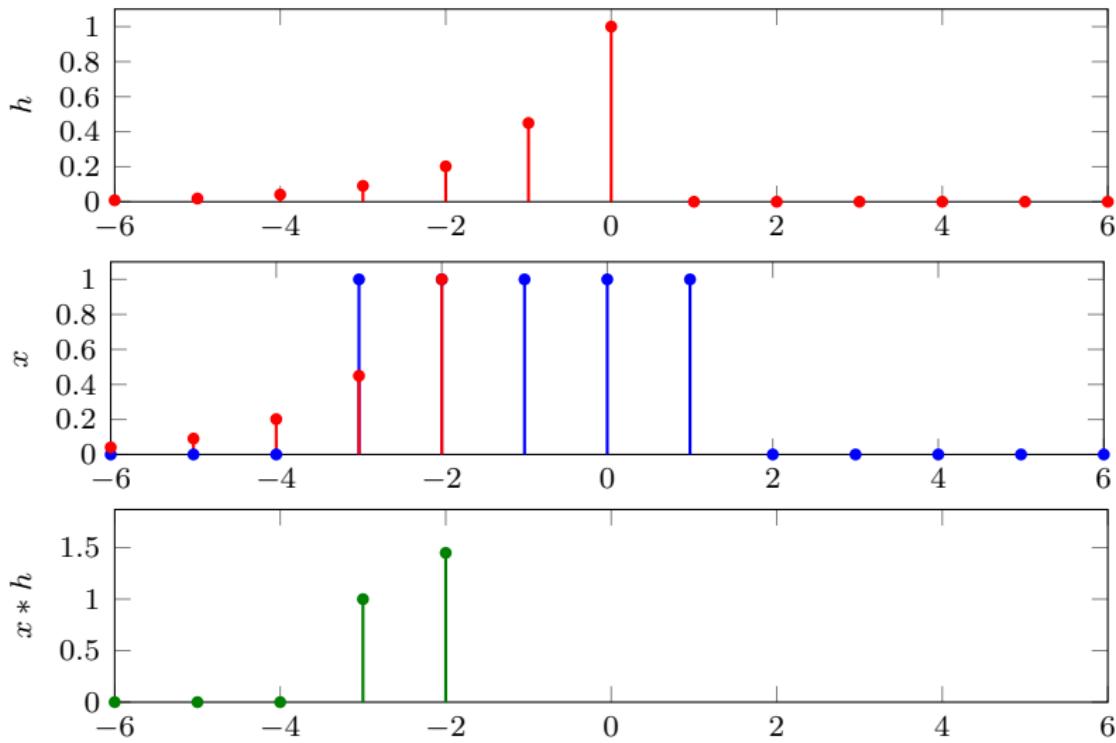
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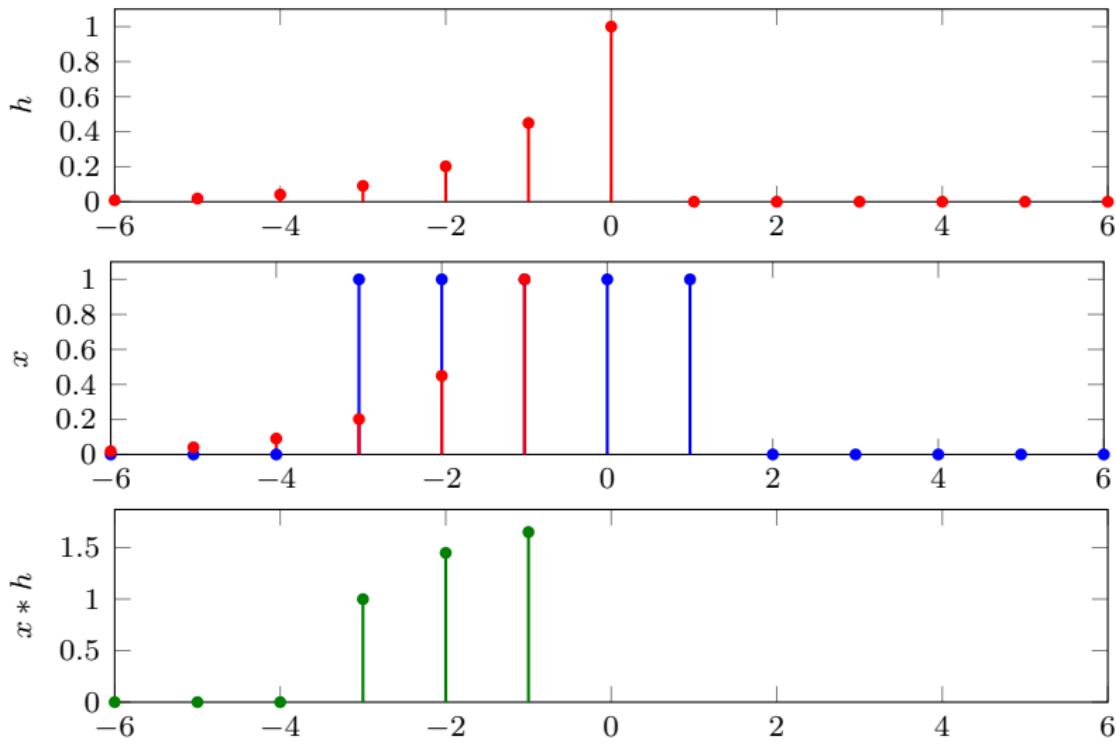
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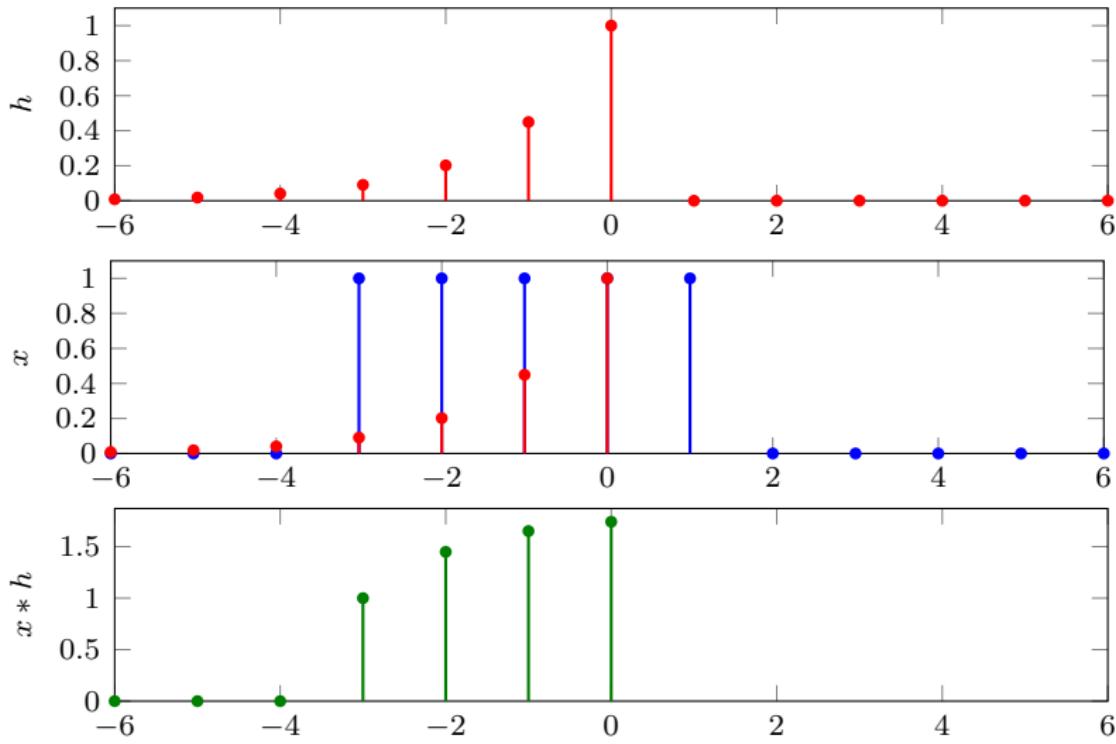
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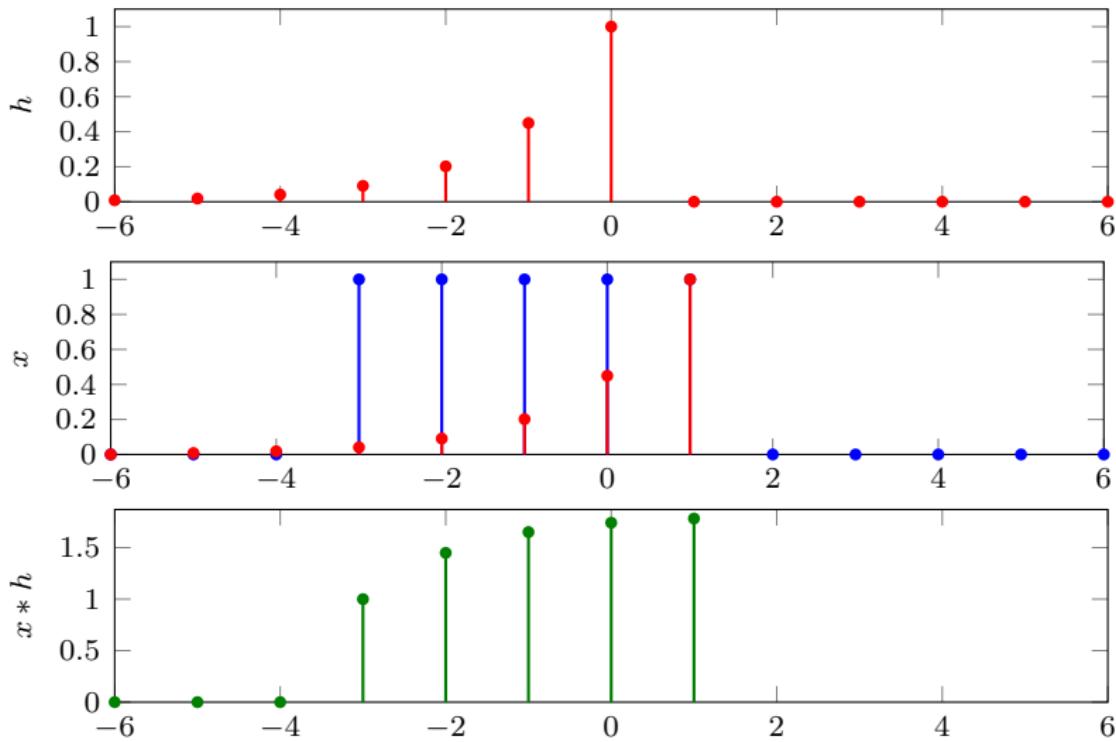
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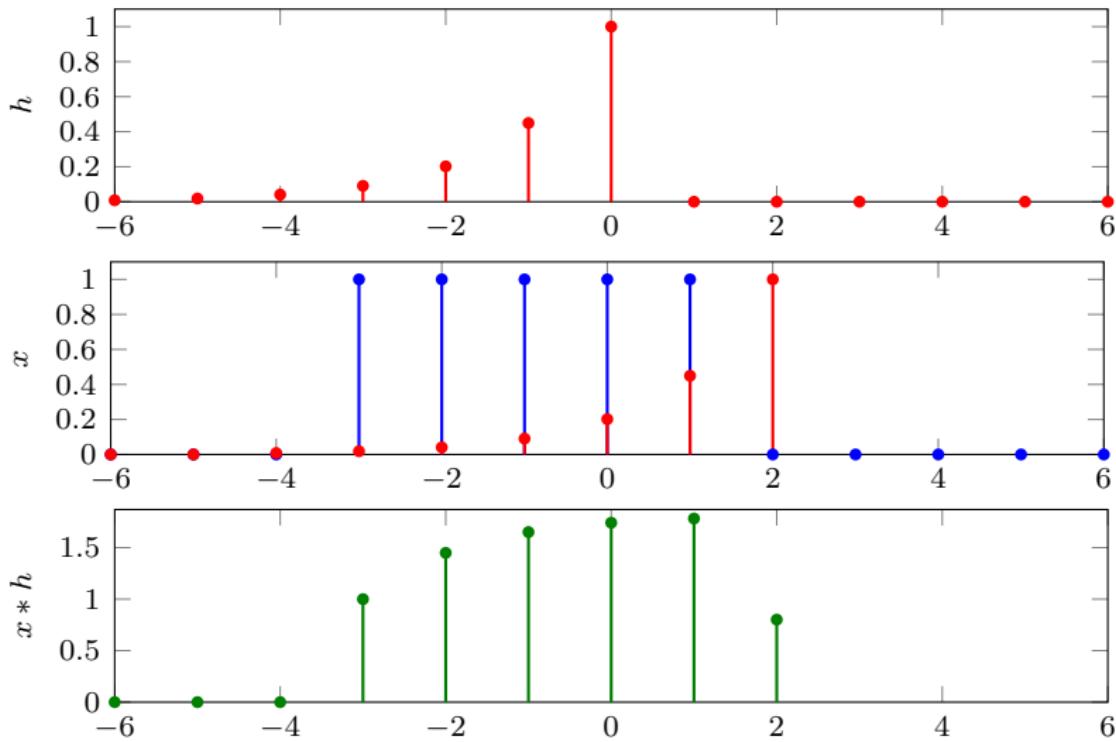
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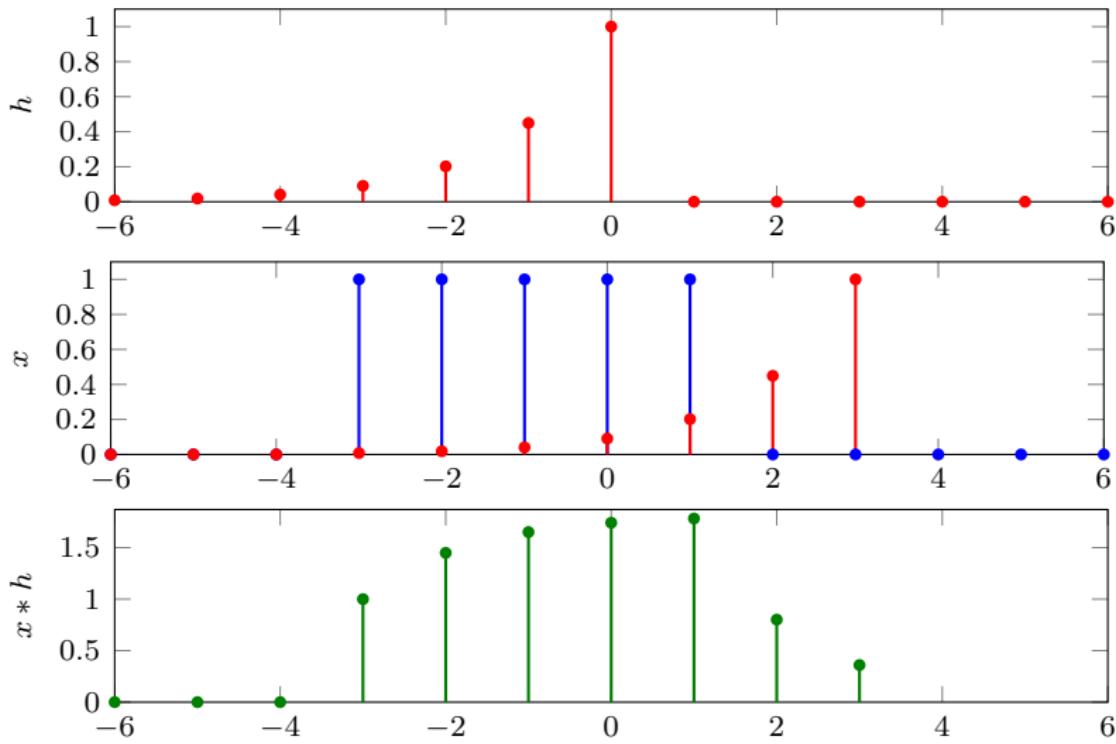
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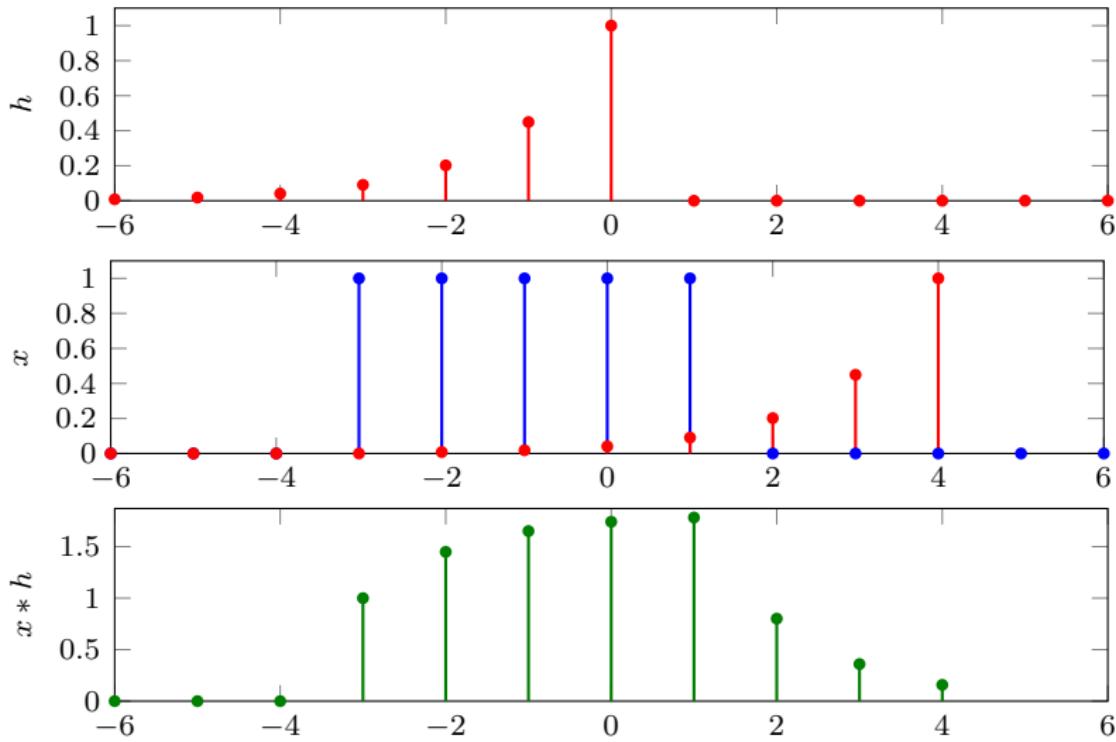
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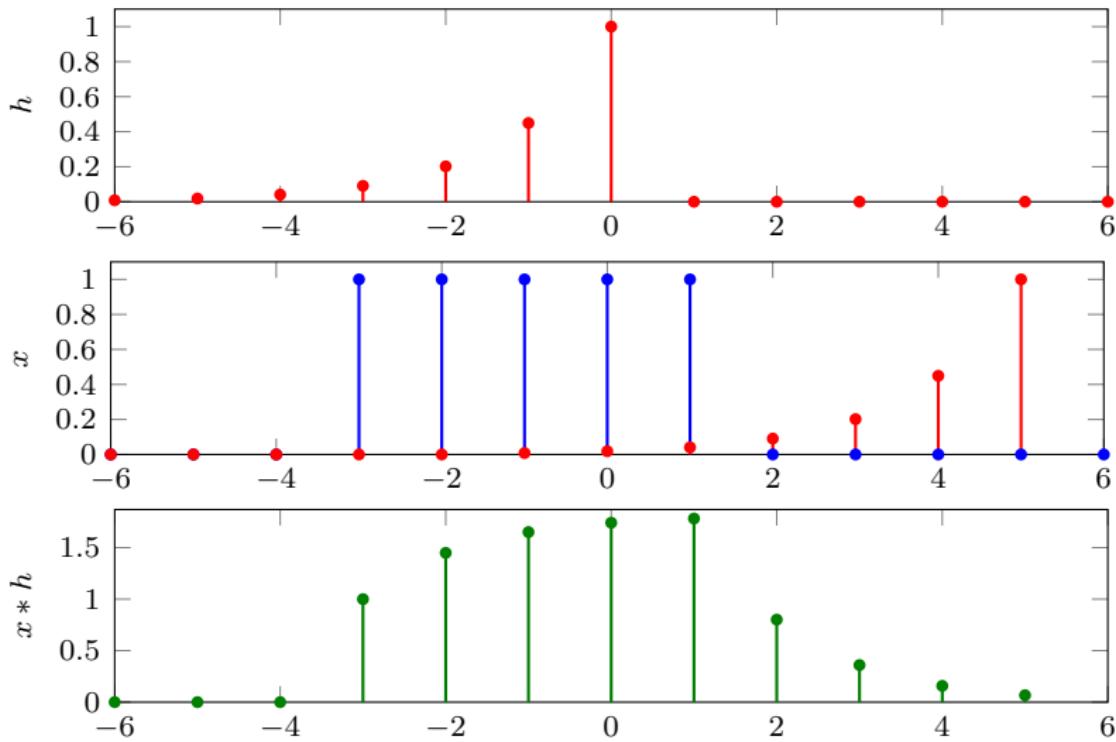
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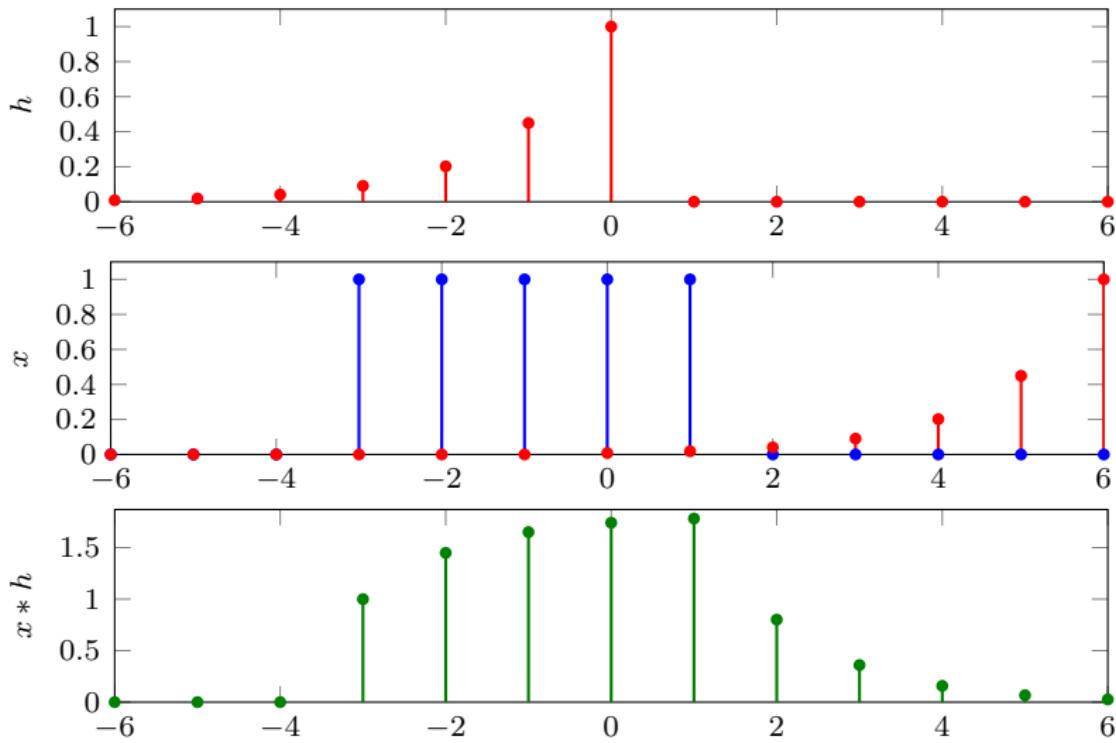
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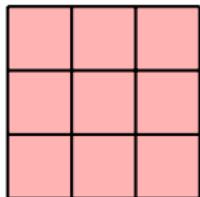
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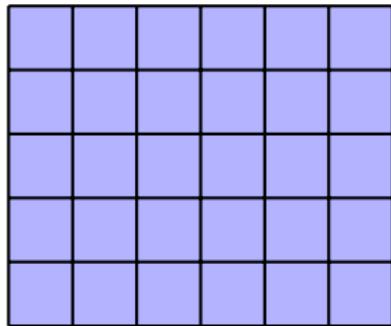
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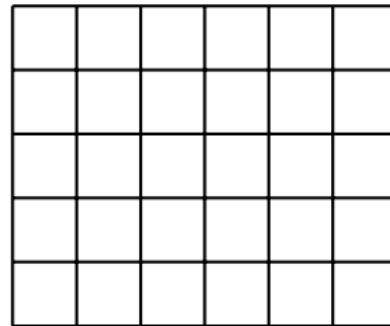
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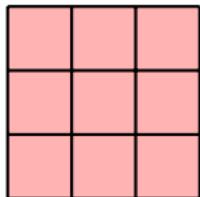


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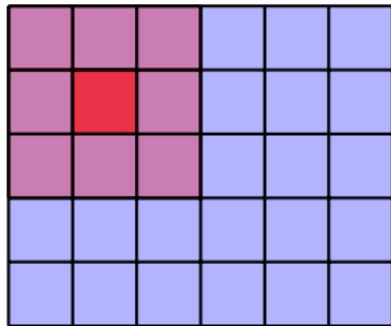
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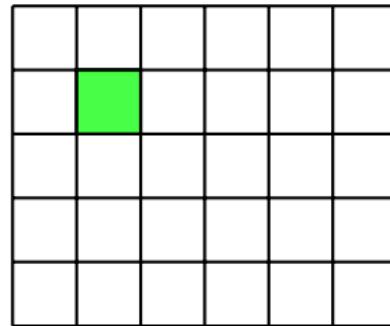
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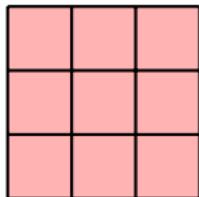


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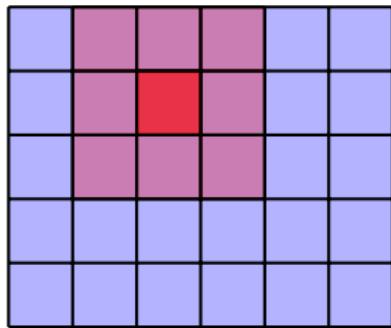
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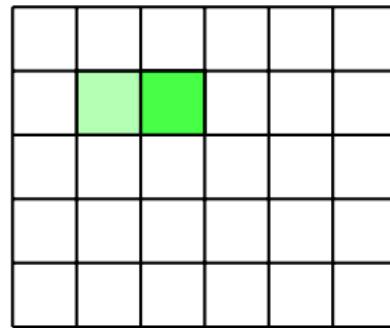
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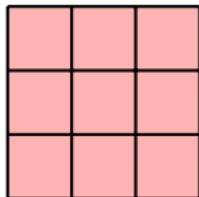


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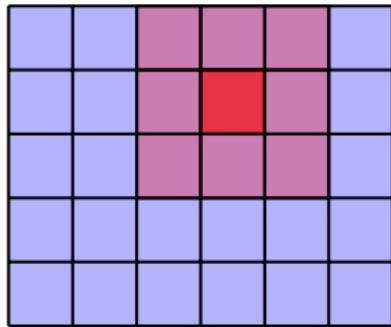
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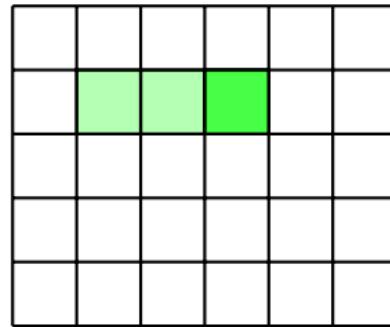
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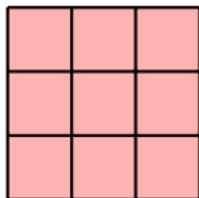


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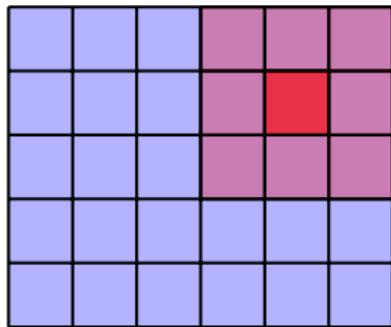
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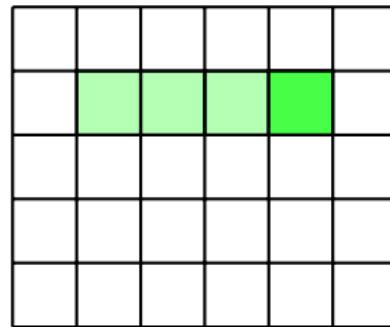
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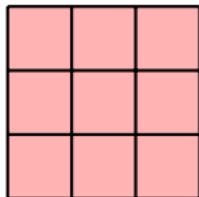


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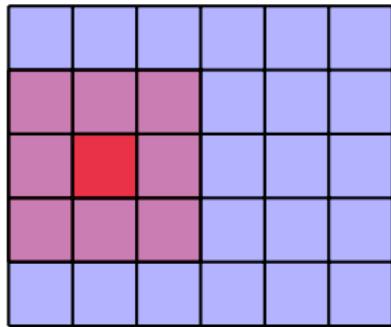
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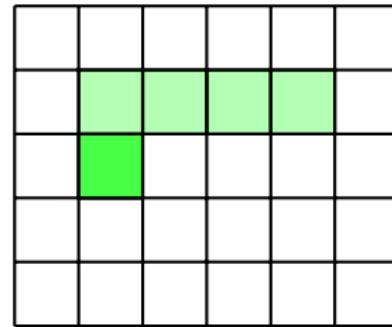
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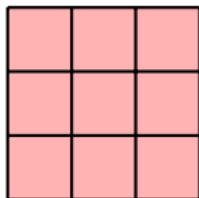


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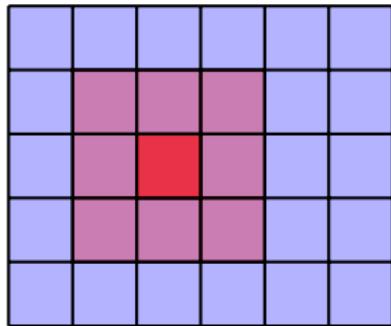
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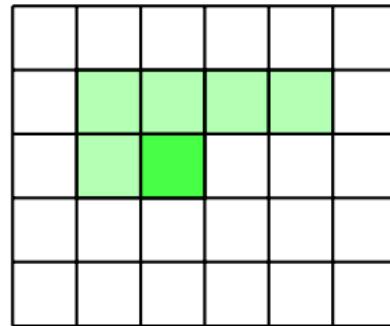
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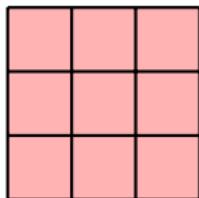


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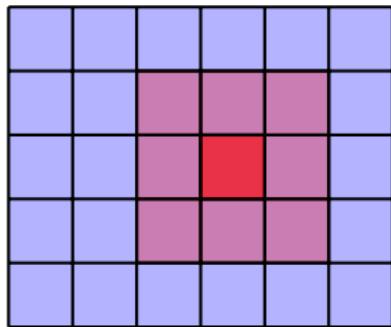
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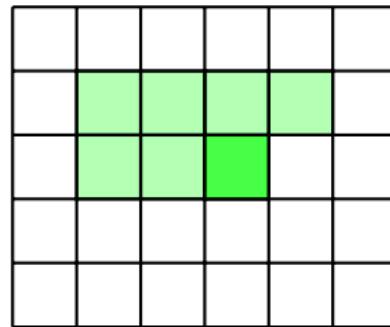
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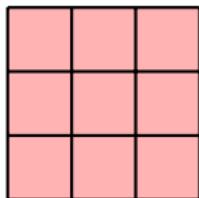


$x$



$x * h$

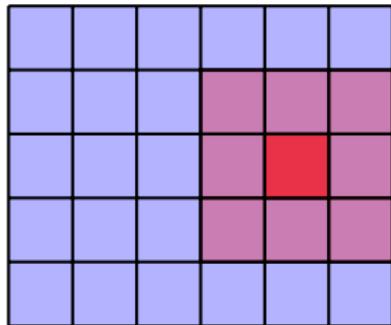
## 2d convolution



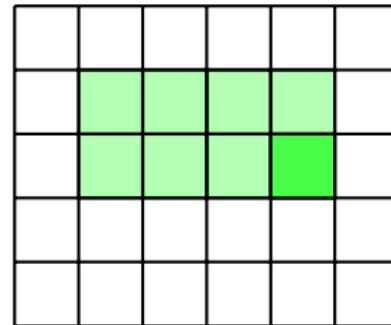
$h$

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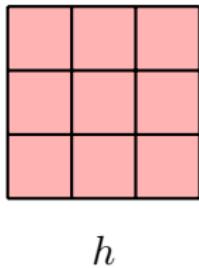
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$x$

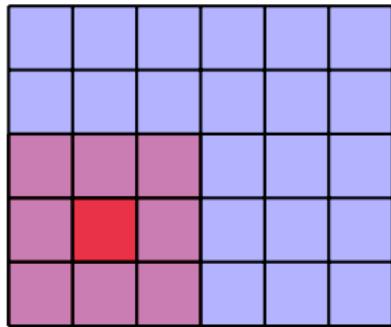


# 2d convolution

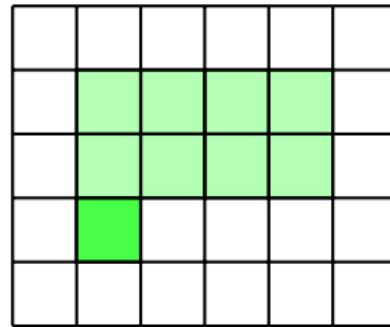


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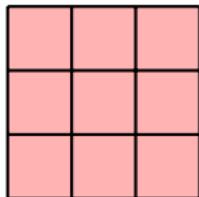


$x$



$x * h$

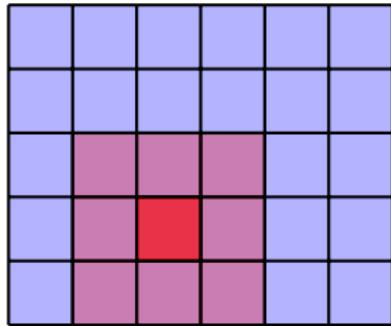
## 2d convolution



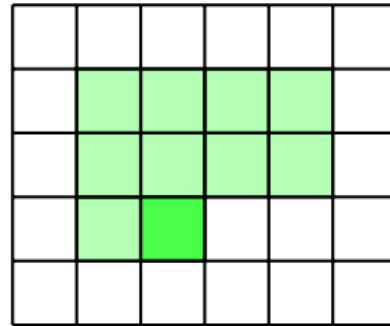
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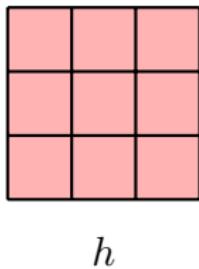


$x$



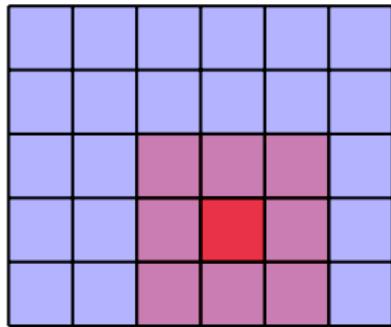
$x * h$

## 2d convolution

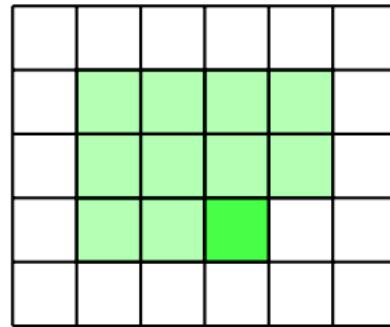


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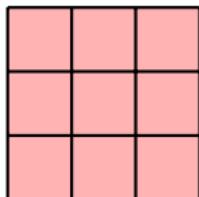


$x$



$x * h$

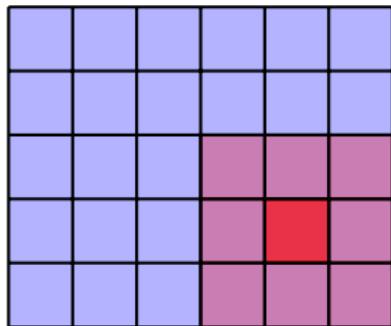
## 2d convolution



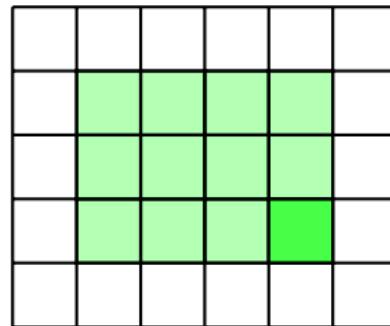
$h$

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$x$



$x * h$

# continuous time

- continuous-time signal:  $x(t)$ ,  $t \in \mathbb{R}$
- translation (or shift, or delay):  $s_\tau(x)(t) = x(t - \tau)$ ,  $\tau \in \mathbb{R}$
- LTI system definition: same
- Dirac delta “function”  $\delta$ : every signal  $x$  expressed as

$$x(t) = \int x(\tau)\delta(t - \tau)d\tau$$

- convolution:  $f$  LTI, impulse response  $h = f(\delta)$  implies

$$f(x)(t) = (x * h)(t) := \int x(\tau)h(t - \tau)d\tau$$

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# Fourier transform

- time (or space) → frequency

$$X(f) = \int x(t)e^{-j2\pi ft} dt$$

- frequency → time (or space)

$$x(t) = \int X(f)e^{j2\pi ft} df$$

- measurements



bar (+)



bar (-)



grating

# Fourier transform

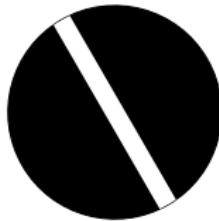
- time (or space) → frequency

$$X(f) = \int x(t)e^{-j2\pi ft} dt$$

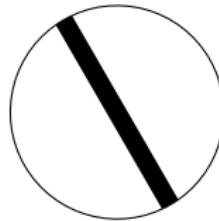
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grating

# Fourier transform

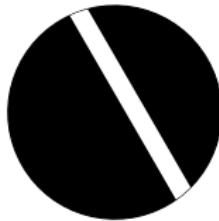
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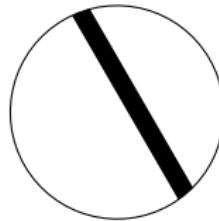
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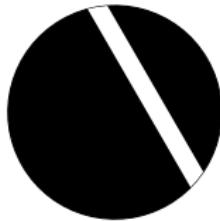
- time (or space) → frequency

$$X(f) = \int x(t)e^{-j2\pi ft} dt$$

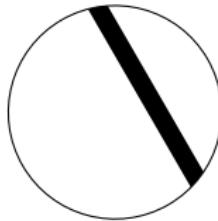
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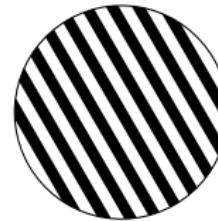
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bar (+)



bar (-)



grating

# Fourier transform

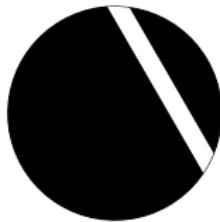
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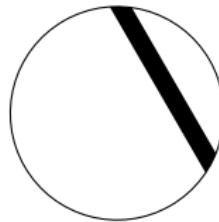
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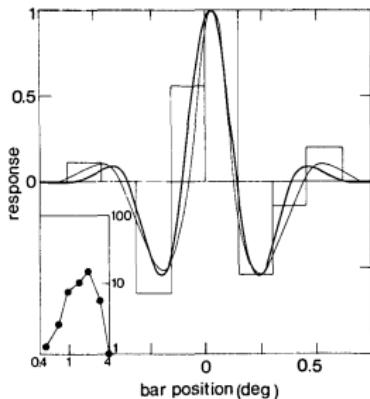


bar (-)



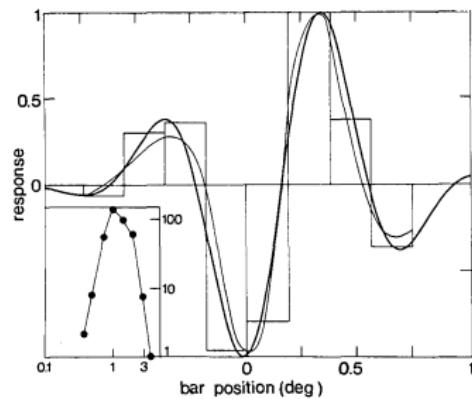
grating

# mathematical model



symmetric

$$e^{-a^2(x-x_0)^2} \cos(2\pi f_0(x - x_0))$$

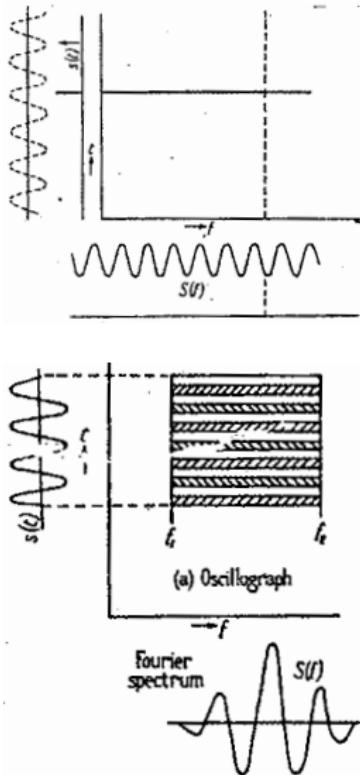


antisymmetric

$$e^{-a^2(x-x_0)^2} \sin(2\pi f_0(x - x_0))$$

- (thin) experimental: inverse Fourier of grating stimuli responses
- (thick) least-squares fit of Gabor elementary signal

# Gabor elementary signals



- “effective duration”

$$\Delta t = [2\pi(t - \bar{t})^2]^{1/2}$$

- “effective bandwidth”

$$\Delta f = [2\pi(f - \bar{f})^2]^{1/2}$$

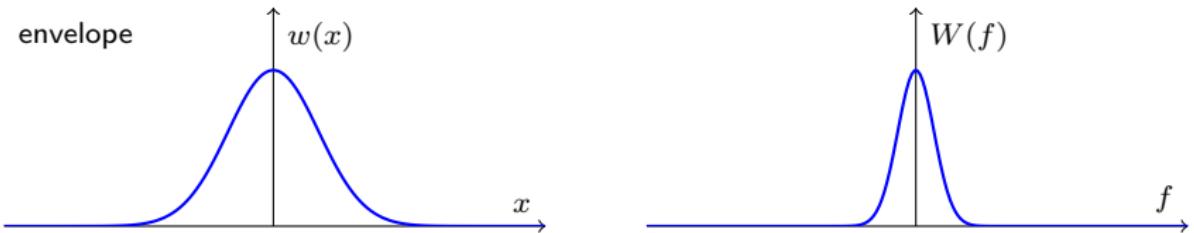
- uncertainty principle

$$\Delta t \Delta f \geq \frac{1}{2}$$

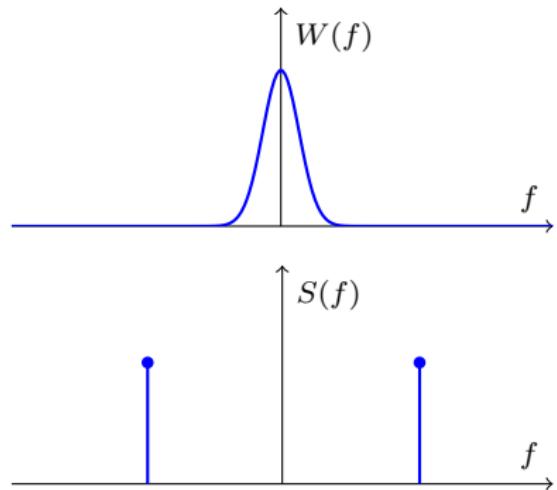
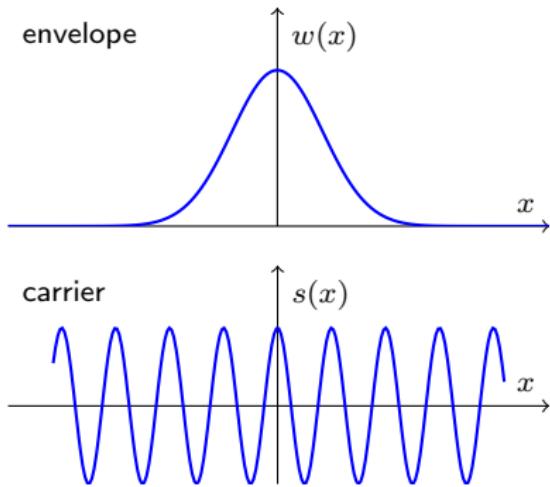
- minimal solution

$$\psi(t) = e^{-a^2(t-t_0)^2} e^{j2\pi f_0(t-t_0)}$$

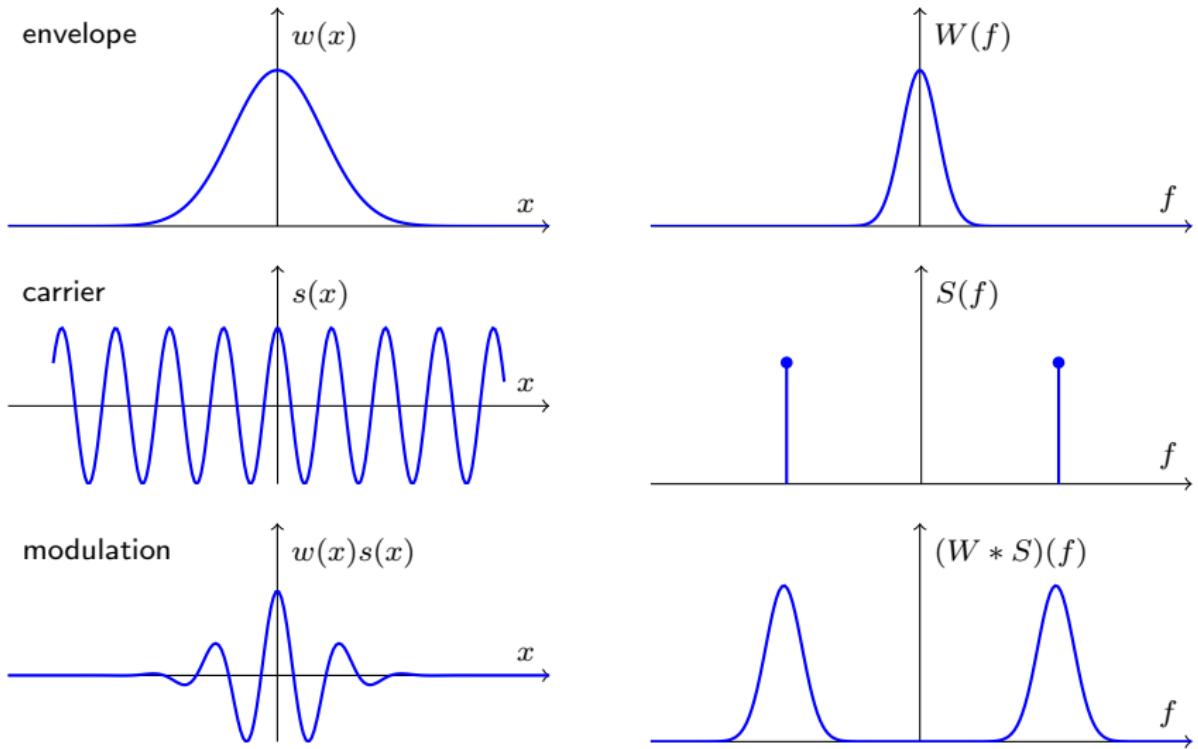
# convolution theorem & modulation



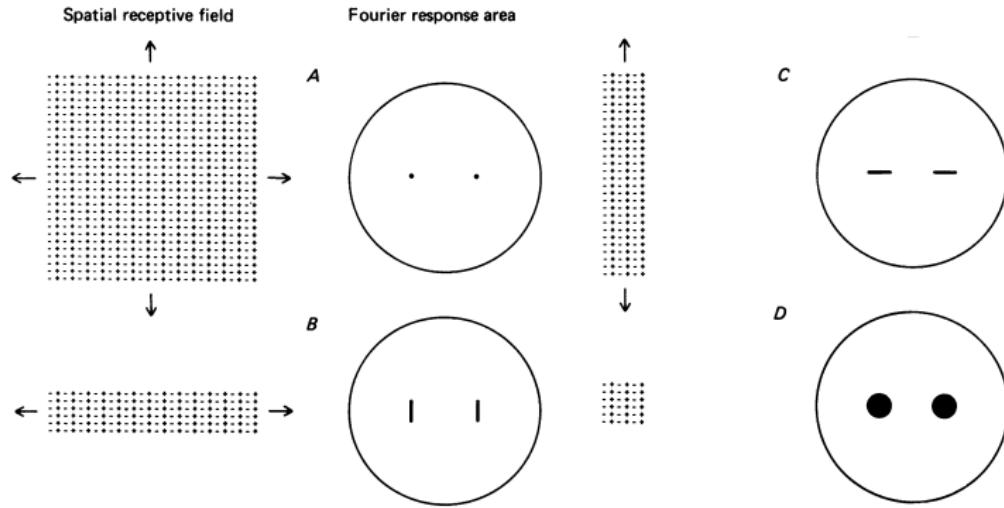
# convolution theorem & modulation



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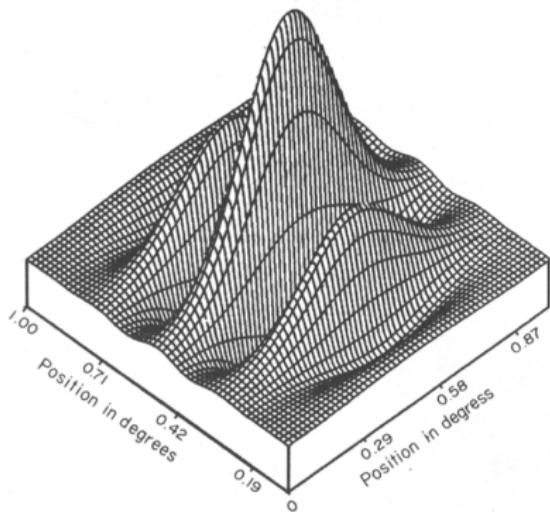
## 2d space/frequency considerations



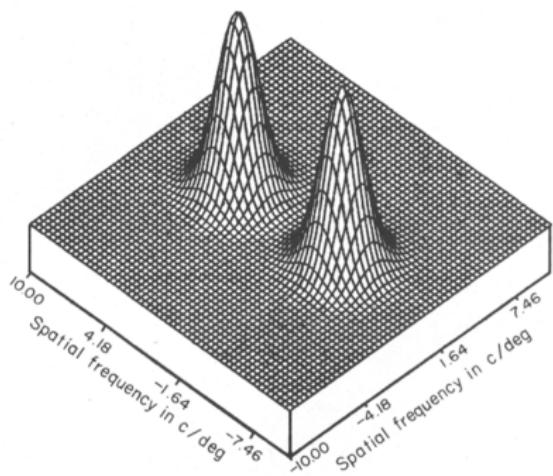
- responses to gratings at different frequencies and orientations
  - localized in space and frequency, in both dimensions

## 2d space/frequency considerations

(a) Excitability profile

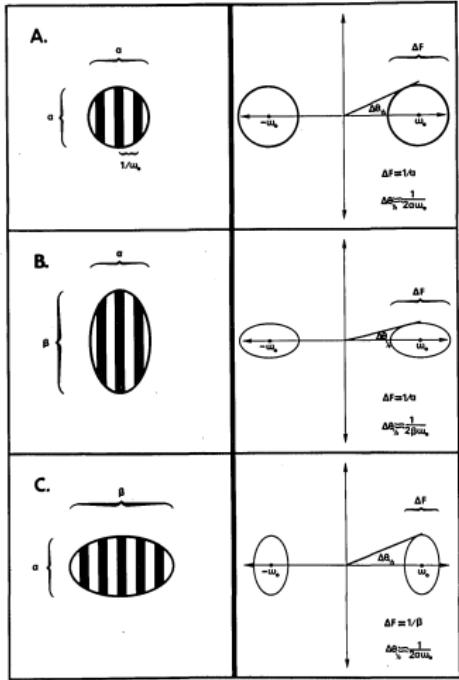


(b) 2-D Fourier transform of profile



- spatial frequency and orientation are separable
- by inverse Fourier, hypothesize a 2d spatial 'receptive field profile'

# 2d Gabor filters



- 2d uncertainty principle

$$\Delta x \Delta u \geq \frac{1}{4}$$

- minimal solution

$$f(\mathbf{x}) = e^{-\pi w_{\mathbf{x}_0, A}(\mathbf{x})} e^{j2\pi c_{\mathbf{x}_0, \mathbf{u}_0}(\mathbf{x})}$$

$$F(\mathbf{u}) = e^{-\pi w_{\mathbf{u}_0, A^{-1}}(\mathbf{u})} e^{j2\pi c_{\mathbf{u}_0, \mathbf{x}_0}(\mathbf{u})}$$

- envelope & carrier signals

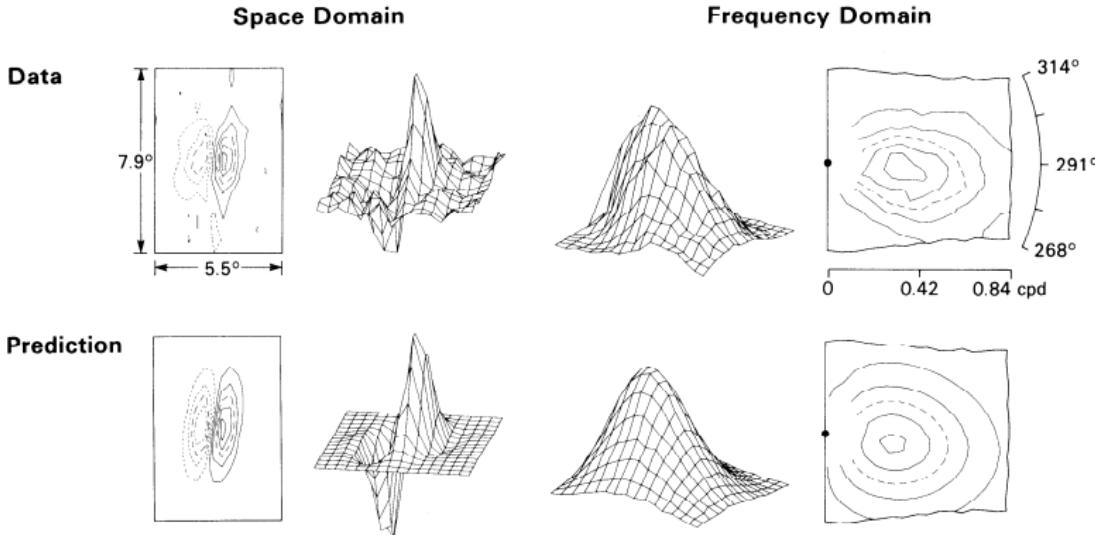
$$w_{\mathbf{x}_0, A}(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_0)^\top A^2 (\mathbf{x} - \mathbf{x}_0)$$

$$c_{\mathbf{x}_0, \mathbf{u}_0}(\mathbf{x}) = \mathbf{u}_0^\top (\mathbf{x} - \mathbf{x}_0)$$

$$A = \text{diag}(a, b)$$

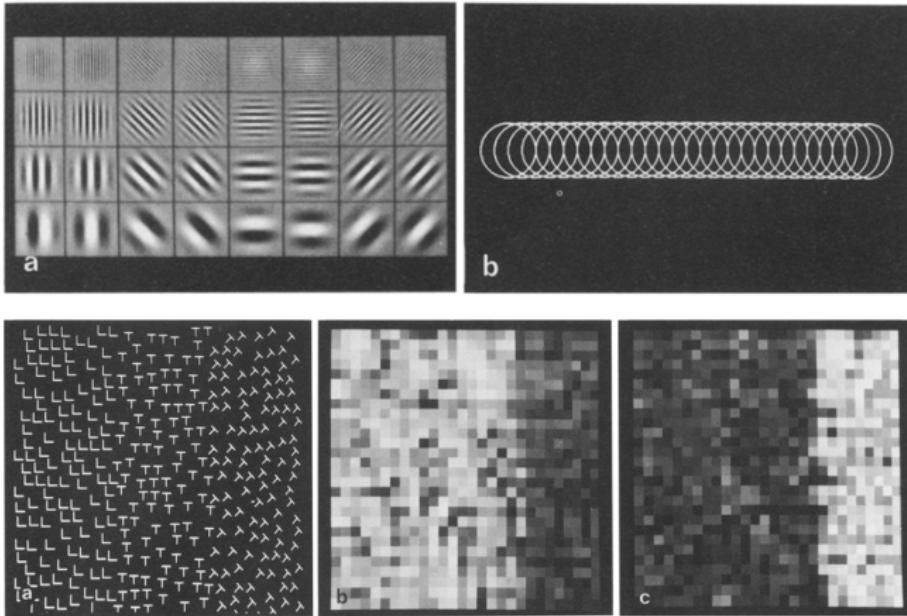
Daugman. JOSA 1985. Uncertainty Relation for Resolution in Space, Spatial Frequency, and Orientation Optimized By Two-Dimensional Visual Cortical Filters.

# Gabor hypothesis verified



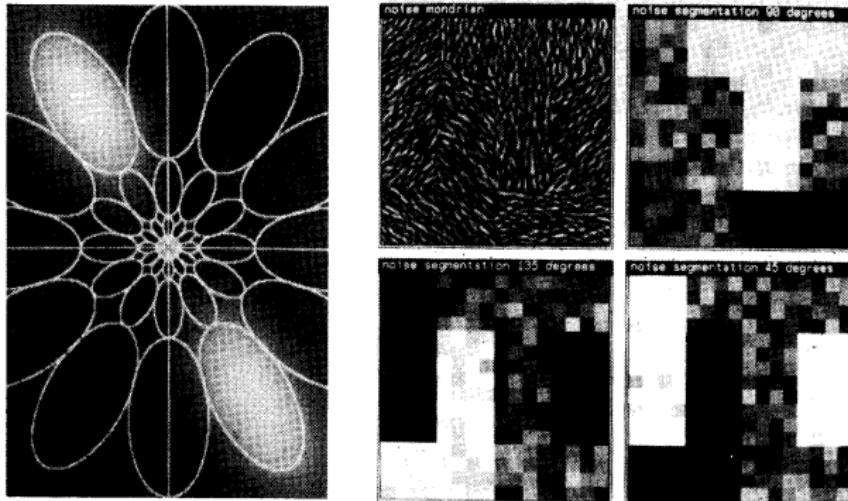
- compare spatial data to Gabor fitted to inverse Fourier of frequency data, and vice versa
- error unstructured and indistinguishable from random

# texture segmentation



- sample image on spatial uniform cartesian grid
- filter each spatial cell at different frequencies and orientations

# “textons”

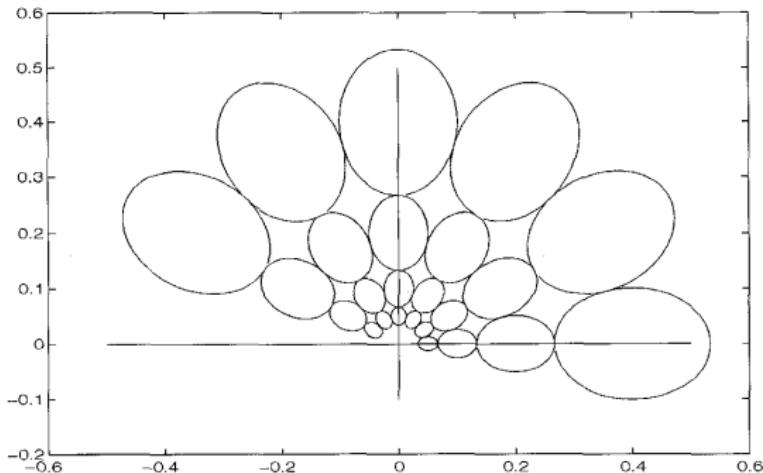


- see filter bank as frequency sampling on log-polar grid
- cluster filter (vector) responses into “textons”
- apply to iris recognition

# visual descriptors

# texture descriptors

[Manjunath and Ma 1996]



- same frequency sampling scheme
- filtering and global pooling in space domain
- popularized as part of MPEG-7 standard

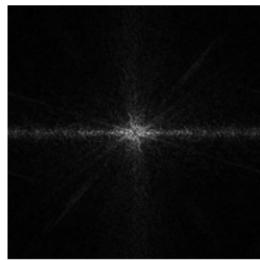
# global descriptors



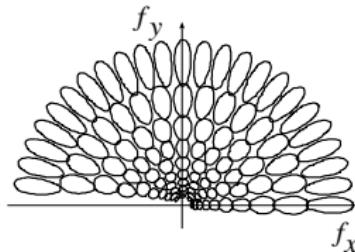
image



pre-processing



power spectrum



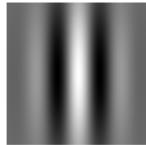
filter bank

- sampling scheme adapted to power spectrum statistics
- filtering and global pooling in frequency domain

## sampling the frequency plane



frequency



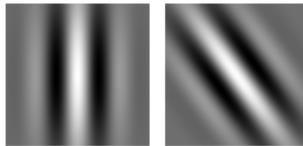
space

- space ( $\mathbf{x}$ ) and frequency ( $\mathbf{u}$ ) rotate together by  $\theta$
- scaling envelope ( $A$ ) and carrier ( $\mathbf{u}_0$ ) together
- 4d representation: position, scale, orientation

## sampling the frequency plane



frequency



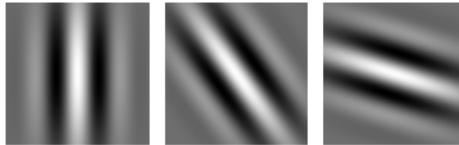
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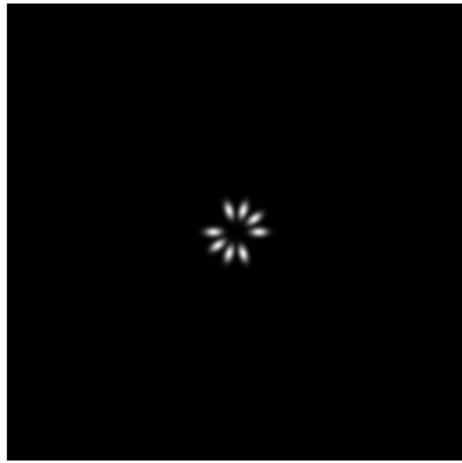
frequency



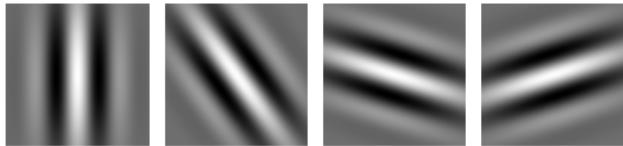
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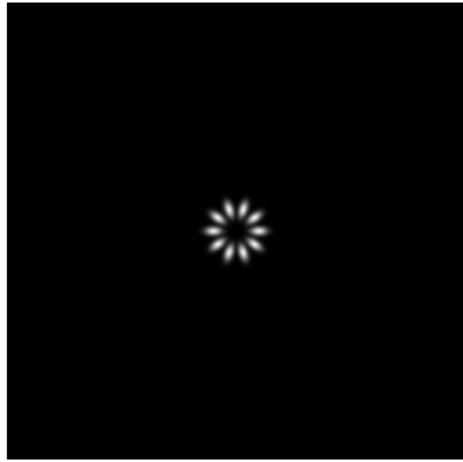
frequency



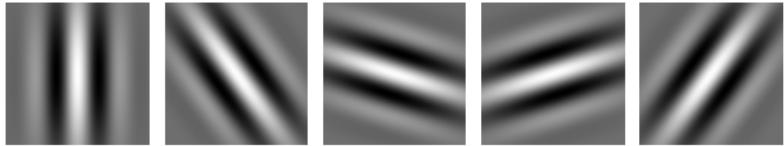
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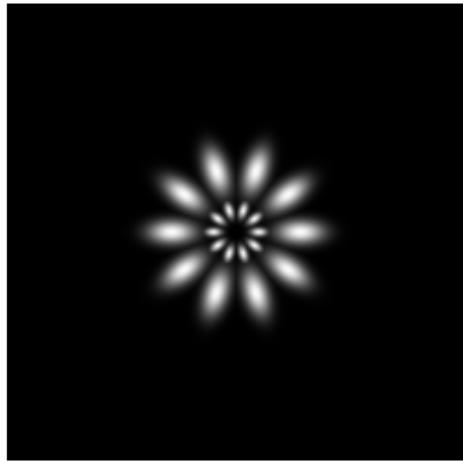
frequency



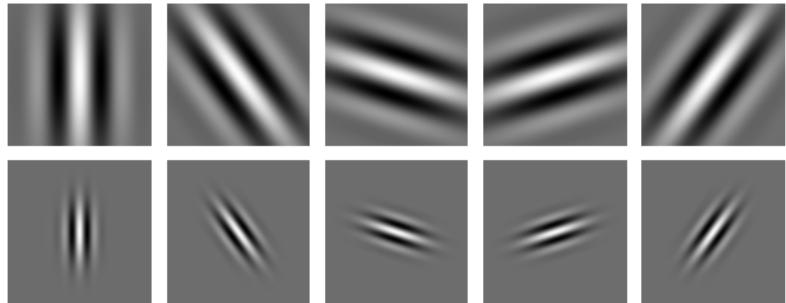
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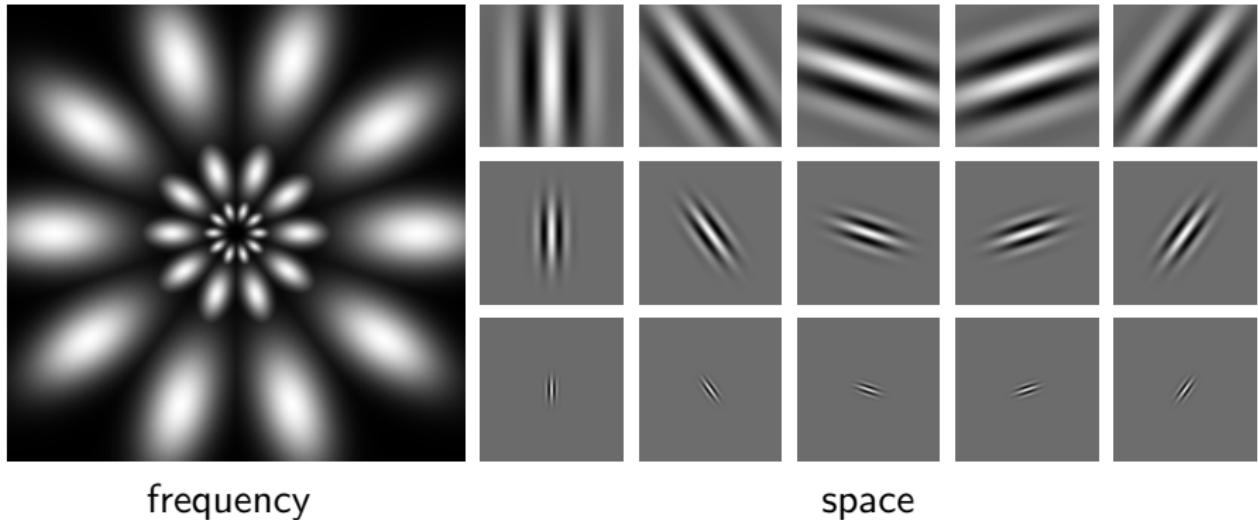
frequency



space

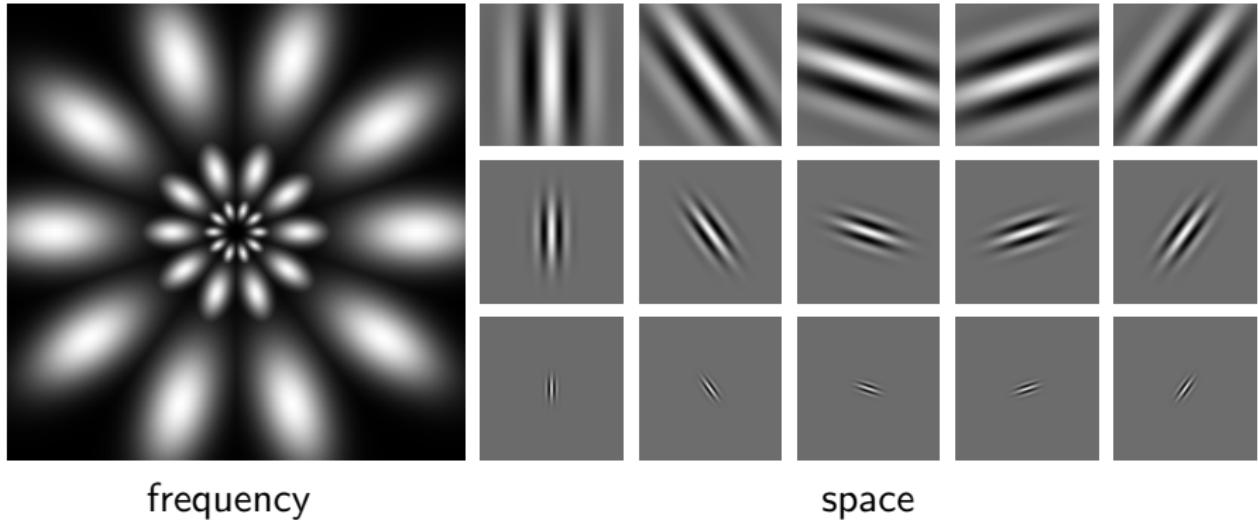
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## from images to vectors

- suppose an image  $f(\mathbf{x})$  is represented in frequency by  $|F(\mathbf{u})|^2$
- suppose a template  $h(\mathbf{x})$  (another image or an attribute) is also represented in frequency by

$$|H(\mathbf{u})|^2 = \sum_{n=1}^N h_n |G_n(\mathbf{u})|^2$$

where  $\{G_n\}$  is a Gabor filter bank; let  $\mathbf{h} = [h_1, \dots, h_N]$

- now define the vector  $\mathbf{f} = [f_1, \dots, f_N]$  with

$$f_n = \int |F(\mathbf{u})|^2 |G_n(\mathbf{u})|^2 d\mathbf{u}$$

- and measure the similarity of  $f, h$  by the inner product

$$\int |F(\mathbf{u})|^2 |H(\mathbf{u})|^2 d\mathbf{u} = \sum_{n=1}^N f_n h_n = \langle \mathbf{f}, \mathbf{h} \rangle$$

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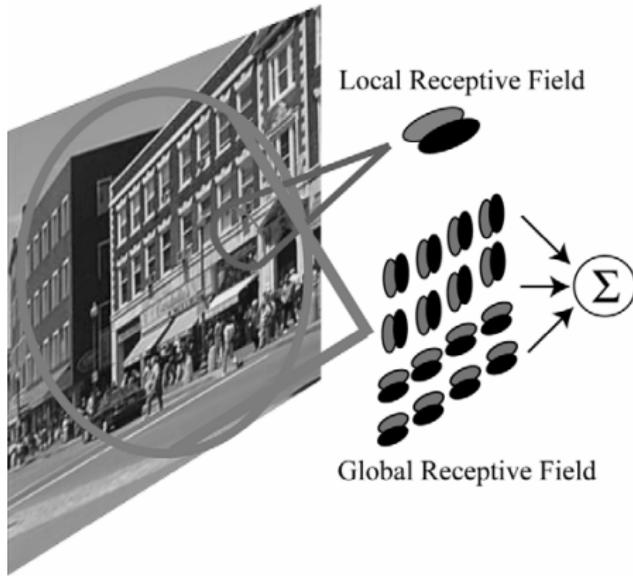
- now define the vector  $\mathbf{f} = [f_1, \dots, f_N]$  with

$$f_n = \int |F(\mathbf{u})|^2 |G_n(\mathbf{u})|^2 d\mathbf{u}$$

- and measure the similarity of  $f, h$  by the inner product

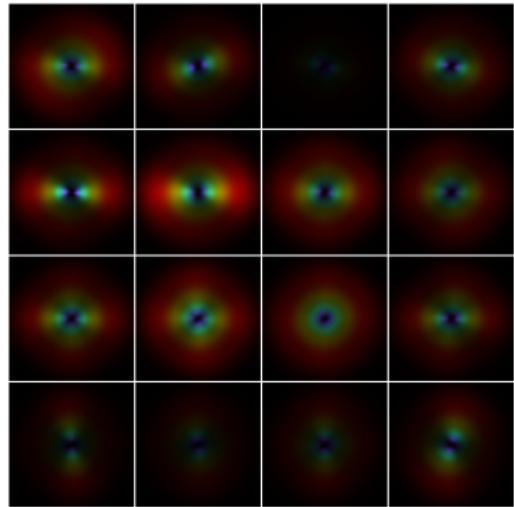
$$\int |F(\mathbf{u})|^2 |H(\mathbf{u})|^2 d\mathbf{u} = \sum_{n=1}^N f_n h_n = \langle \mathbf{f}, \mathbf{h} \rangle$$

## global vs. local receptive fields



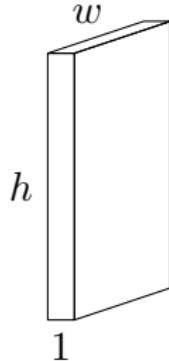
- pool filter responses only locally
  - next level in hierarchy can apply different spatial weights

# the gist descriptor



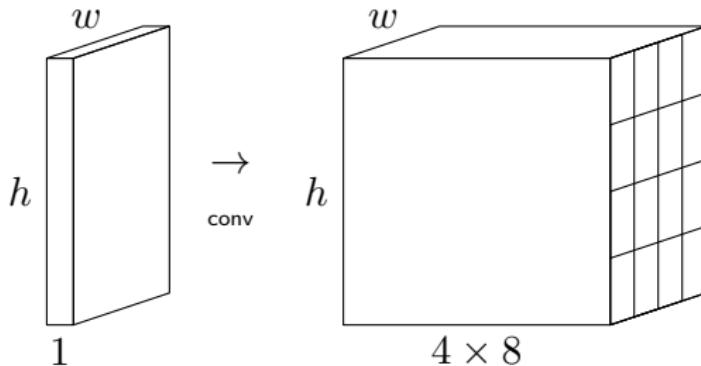
- apply filter bank to entire image in frequency domain
- partition image in  $4 \times 4$  cells
- average pooling of filter responses per cell

# gist pipeline



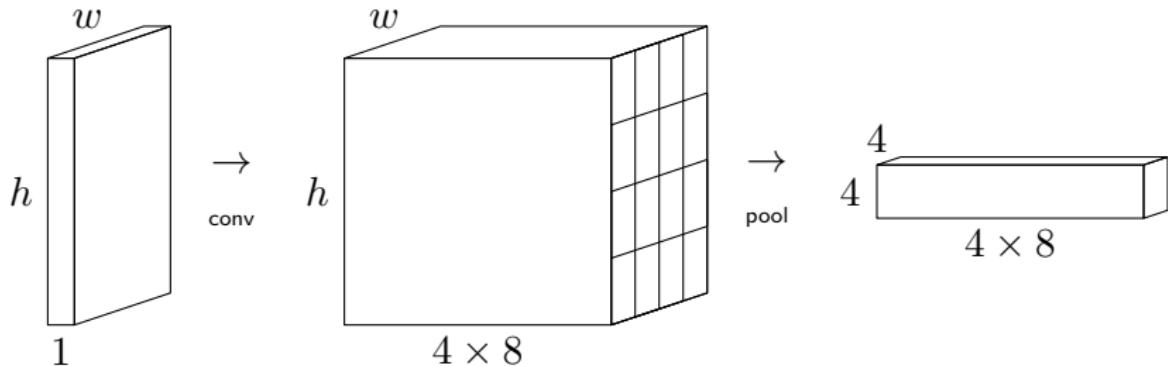
- 3-channel RGB input → 1-channel gray-scale
- apply filters at  $4 \text{ scales} \times 8 \text{ orientations}$
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# gist pipeline



- 3-channel RGB input → 1-channel gray-scale
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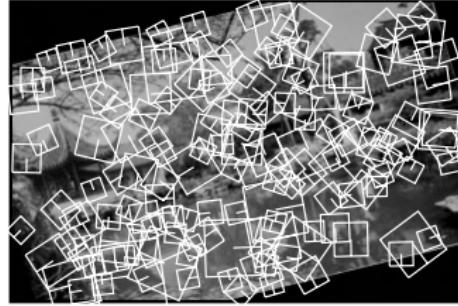
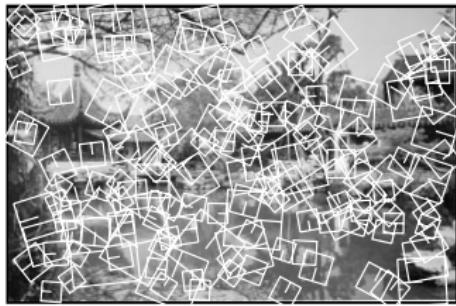
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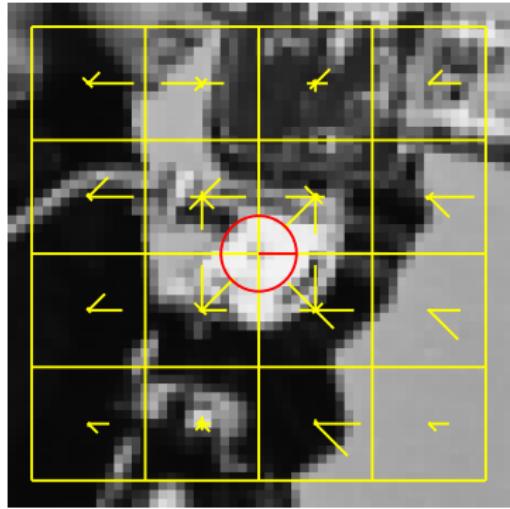
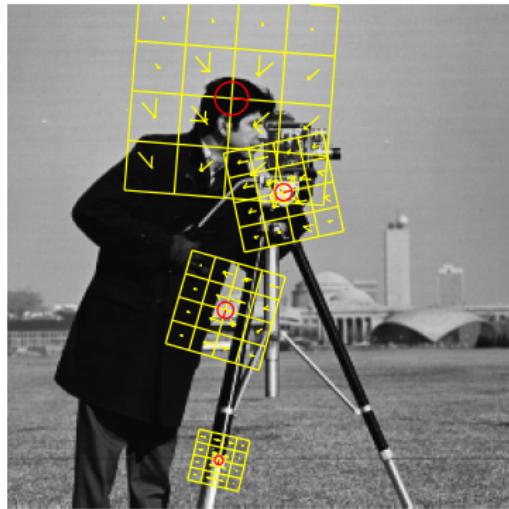
# scale-invariant feature transform

[Lowe 1999]



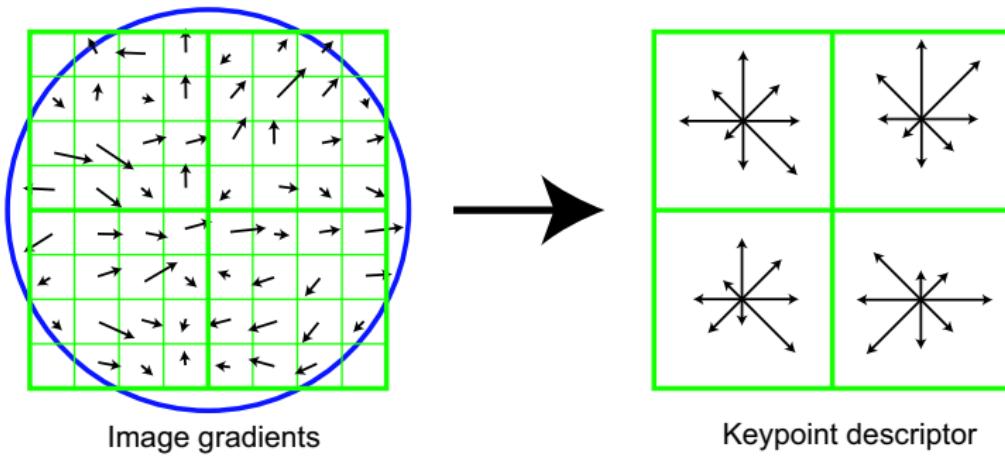
- detect a sparse set of “stable” features (rectangular patches), **equivariant** to translation, scale and rotation

# scale-invariant feature transform



- for each patch
  - normalize with respect to scale and orientation
  - construct a histogram of gradient orientations

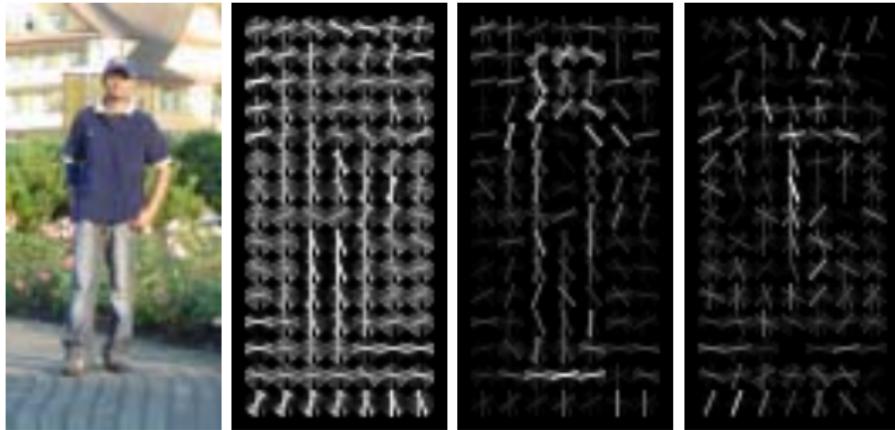
# the SIFT descriptor



- votes in 8-bin orientation histograms weighted by magnitude and by Gaussian window on patch
  - histograms pooled over  $4 \times 4$  cells, trilinear interpolation
  - 128-dimensional descriptor, normalized, clipped at 0.2, normalized

# histogram of oriented gradients

[Dalal and Triggs 2005]



- applied to person detection by sliding window and SVM
- classifier learns positive and negative weights on positions and orientations
- shifts focus back to dense features for classification

# the HOG descriptor



- applied densely to adjacent cells of  $8 \times 8$  pixels
- no scale or orientation normalization; just single-scale
- normalized by overlapping blocks of  $3 \times 3$  cells—redundant

## so what is a histogram?

- consider a histogram  $h$  over integers  $C = \{0, 1, 2, 3, 4\}$ , computed from the following samples:

$$\begin{array}{rcl} C & = & \{ 0 \ 1 \ 2 \ 3 \ 4 \ } \\ \hline 3 & \rightarrow & ( 0 \ 0 \ 0 \ 1 \ 0 ) \\ 2 & \rightarrow & ( 0 \ 0 \ 1 \ 0 \ 0 ) \\ 0 & \rightarrow & ( 1 \ 0 \ 0 \ 0 \ 0 ) \\ 3 & \rightarrow & ( 0 \ 0 \ 0 \ 1 \ 0 ) \\ 2 & \rightarrow & ( 0 \ 0 \ 1 \ 0 \ 0 ) \\ 2 & \rightarrow & ( 0 \ 0 \ 1 \ 0 \ 0 ) \\ \hline h & = & ( 1 \ 0 \ 3 \ 2 \ 0 ) \ / \ 6 \end{array}$$

- each sample is **encoded** (*hard-assigned*) into a vector in  $\mathbb{R}^5$ ; all such vectors are **pooled** (*averaged*) into one vector  $h \in \mathbb{R}^5$
- encoding is always **nonlinear** and pooling is **orderless**
- $C$  is a **codebook** or **vocabulary**

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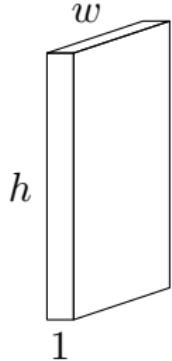
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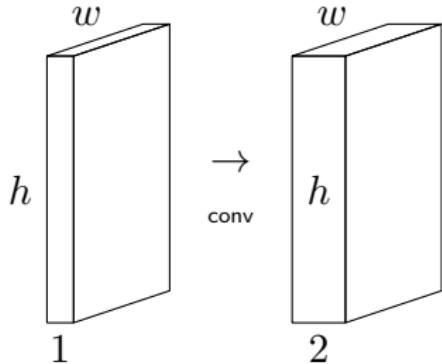
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# SIFT (HOG) pipeline



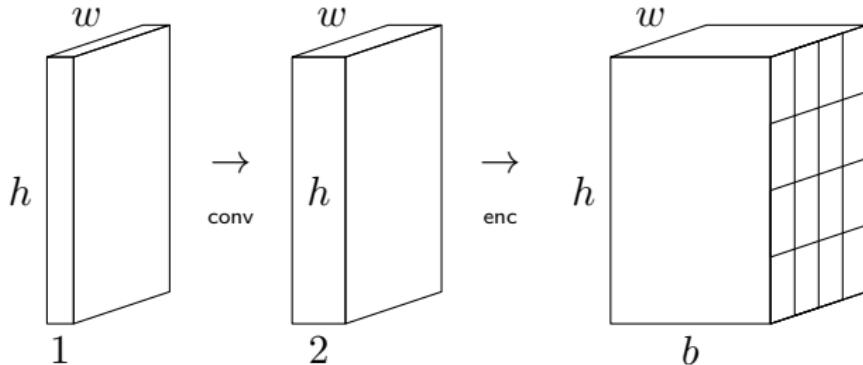
- 3-channel patch (**image**) RGB input → 1-channel gray-scale
  - compute gradient magnitude & orientation
  - encode into  $b = 8$  (**9**) orientation bins
  - average pooling on  $c = 4 \times 4$  ( $\lfloor w/8 \rfloor \times \lfloor h/8 \rfloor$ ) cells
  - descriptor of length  $c \times b = 128$  (**block-normalize** →  $c \times (3 \times 3) \times b$ )

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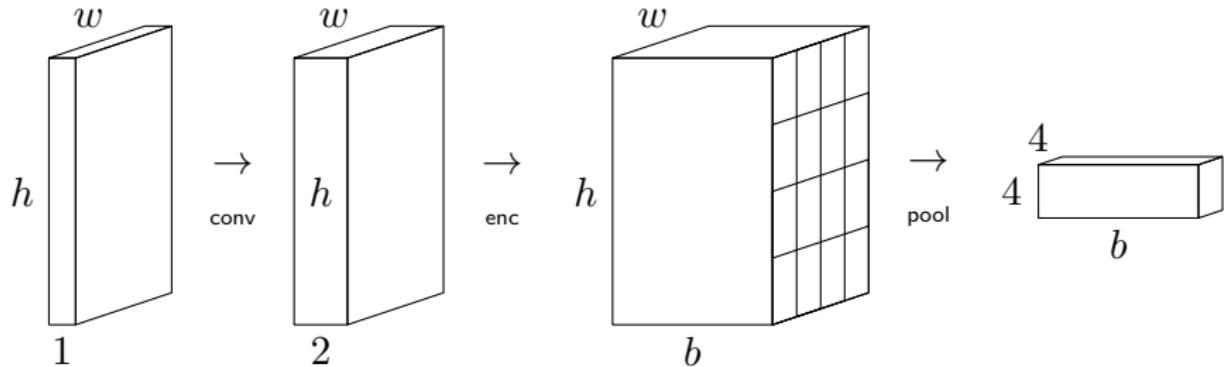
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# feature hierarchy

## back to Gabor

- let us use the following edge pattern



- rotate it by all  $\theta \in [0, 2\pi]$
- for each  $\theta$ , filter (take dot product) with a bank of antisymmetric Gabor filters at 5 orientations, single scale
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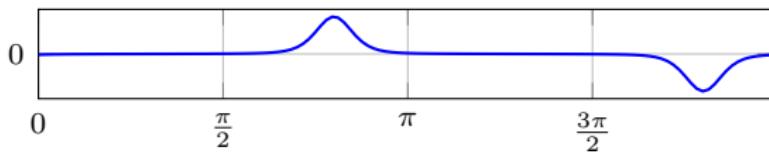
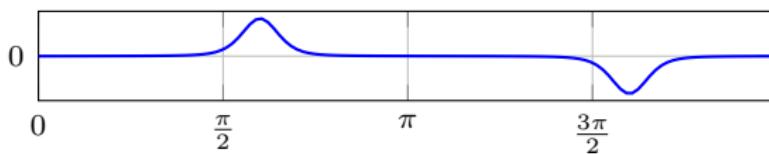
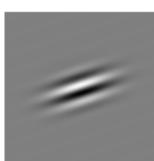
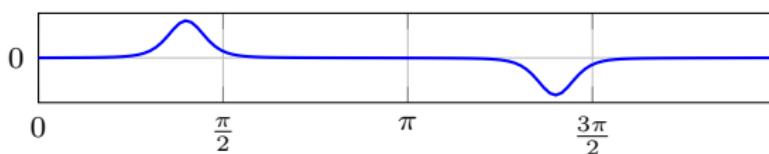
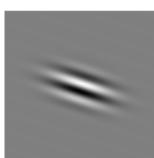
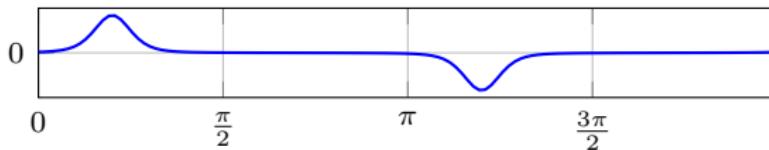
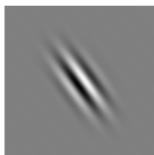
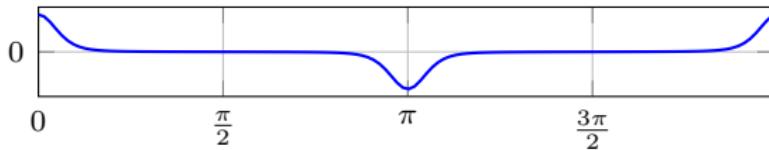
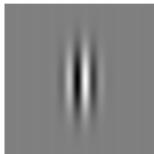
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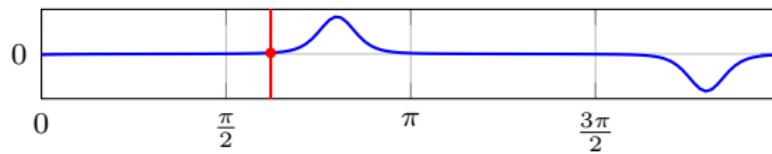
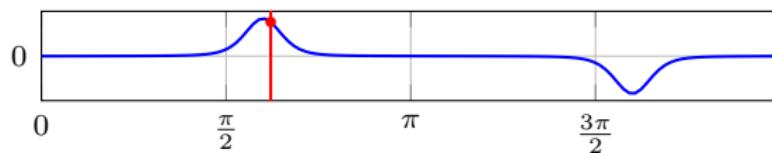
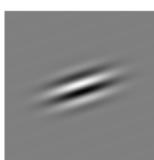
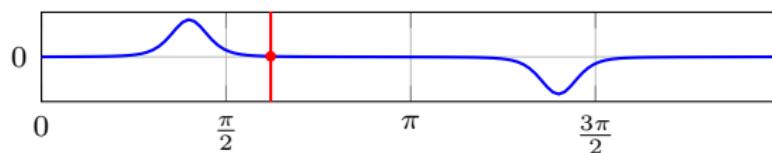
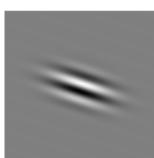
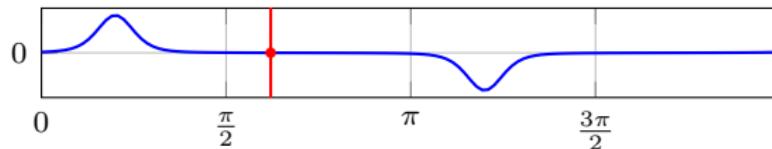
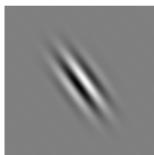
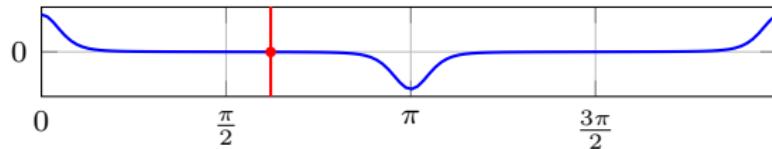
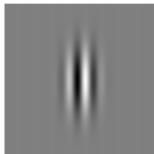


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# nonlinear mappings

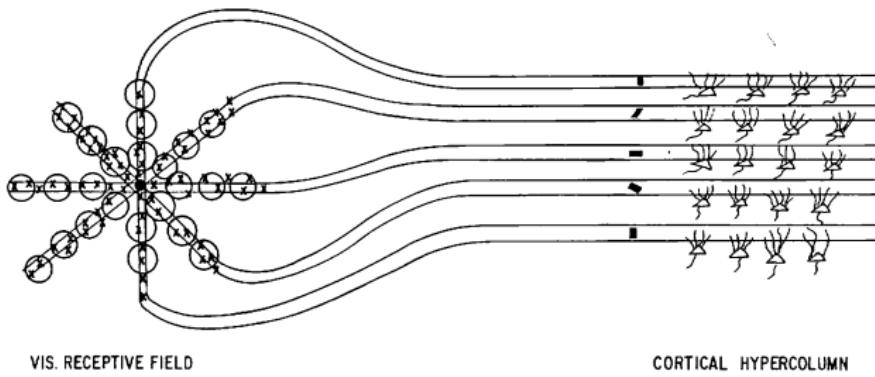
- Q: we said convolution is linear; now, once we have a gradient orientation measurement, why do we need a nonlinear function?
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## on manifolds

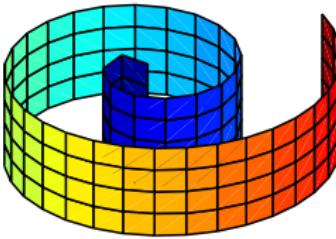
- an image of resolution  $320 \times 200$  is a vector in  $\mathcal{I} = \mathbb{R}^{64,000}$ ; are all such vectors equally likely?
- an object seen at different scales and orientations only spans a 2-dimensional smooth manifold in  $\mathcal{I}$

and we would like to express scale and orientation as two natural coordinates

- how would we go about expressing perspective transformation? attributes of handwritten characters? poses of a human body? occluded surfaces? species of dogs?

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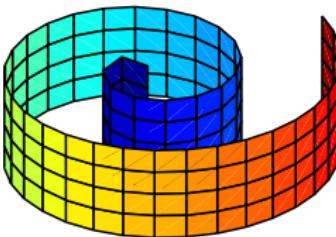


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# feature hierarchy

- at each level, nonlinearly encode each local (e.g. pixel) representation according to a codebook, followed by pooling
- scale and orientation are just two dimensions; a codebook is just a dense grid
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# feature hierarchy

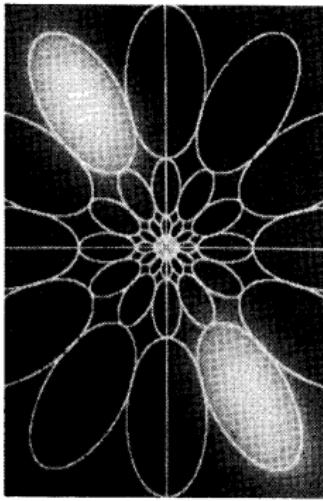
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# back to textons

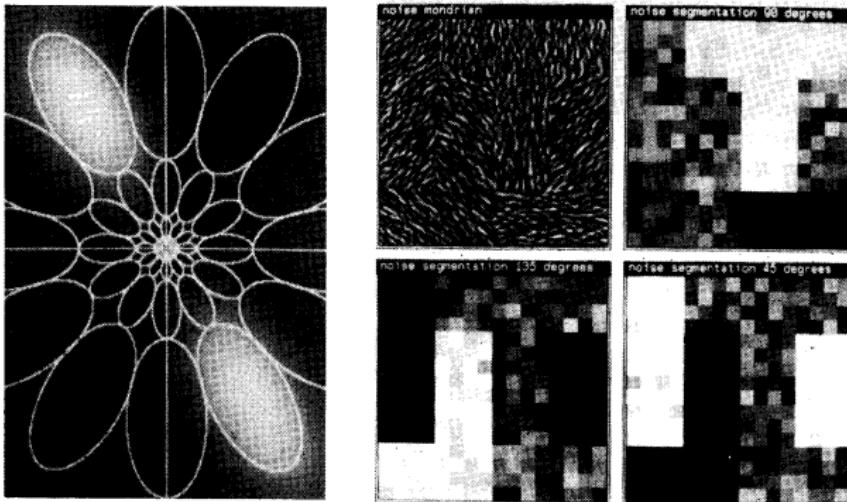
[Daugman 1988]



- see filter bank as frequency sampling on log-polar grid
- cluster  $3 \times 6$  filter (vector) responses into “textons”
- apply to iris recognition

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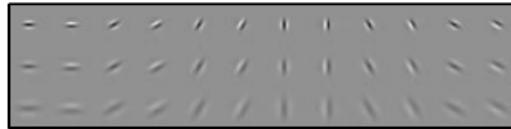
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# textons

[Malik et al. 1999]



oriented filter bank



image



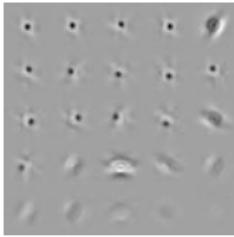
texture segmentation

- textons (re-)defined as clusters of filter responses
- regions described by texton histograms

# textons



image



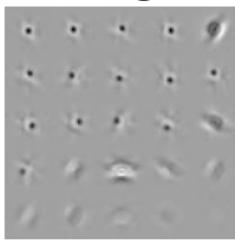
textons

- each pixel mapped to a filter response vector of length  $3 \times 12$
- vectors clustered by  $k$ -means into  $k = 25$  “texton” centroids
- each pixel assigned to a texton
- each texton has a “channel” of pixel assignments

# textons



image



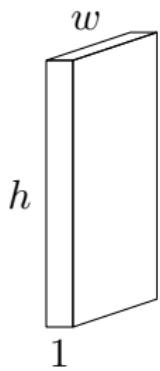
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channels

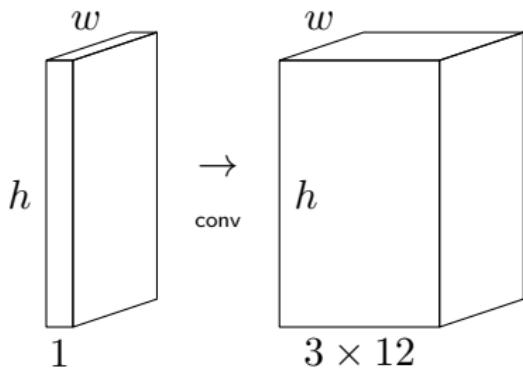
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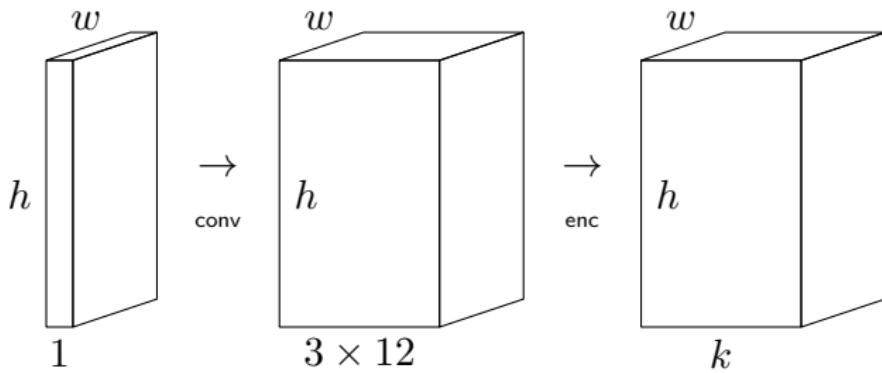
- 3-channel RGB input → 1-channel gray-scale
  - apply filters at 3 scales × 12 orientations
  - point-wise encoding (hard assignment) on  $k = 25$  textons
  - stride-1 average pooling on overlapping neighborhoods

# texton pipeline



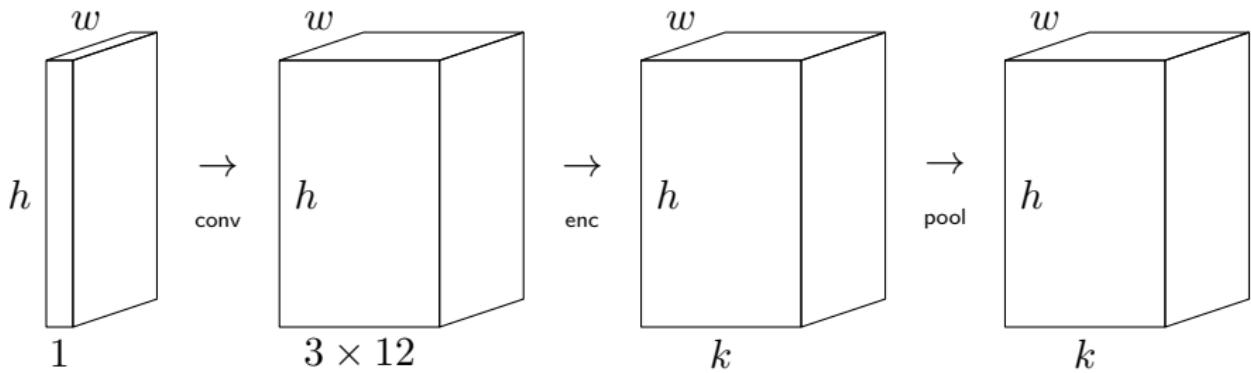
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# bag of words (BoW)

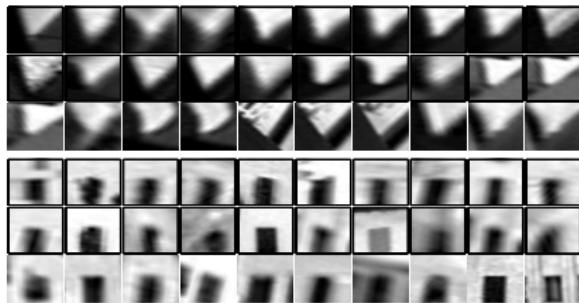
[Sivic and Zisserman 2003]



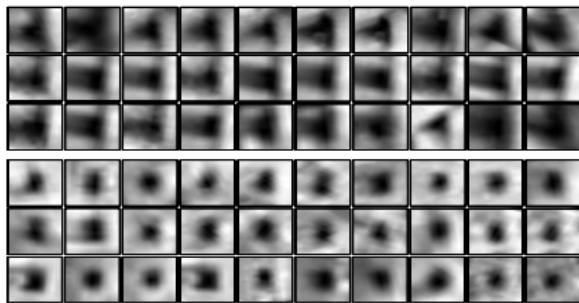
- two types of sparse features detected
- SIFT descriptors extracted from a dataset of video frames

# bag of words: retrieval

[Sivic and Zisserman 2003]



Harris affine  
6k words



maximally stable  
10k words

- “visual words” defined as clusters of SIFT descriptors learned from the dataset
- images described by visual word histograms
- matching is reduced to sparse dot product → fast retrieval

# bag of words: classification

[Csurka et al. 2004]



features

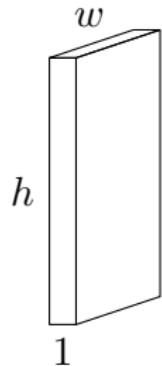


visual words

		
phones, books, cars	bikes, buildings, cars	buildings, cars, faces

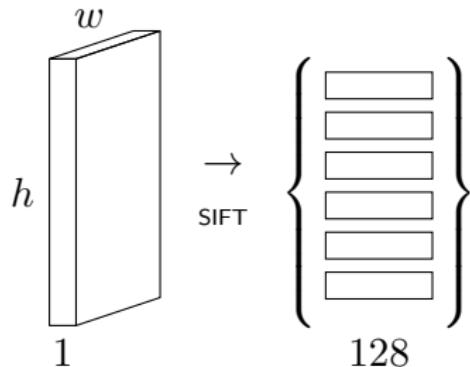
- same representation,  $k = 1000$  words, naive Bayes or SVM classifier
- features soon to be replaced dense multiscale HOG or SIFT

# bag of words pipeline



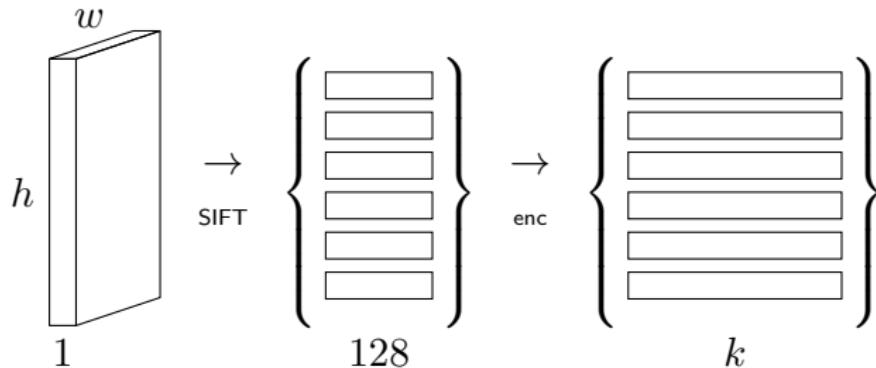
- 3-channel RGB input  $\rightarrow$  1-channel gray-scale
- set of  $\sim 1000$  features  $\times$  128-dim SIFT descriptors
- element-wise encoding (hard assignment) on  $k \sim 10^4$  visual words
- global sum pooling,  $\ell^2$  normalization

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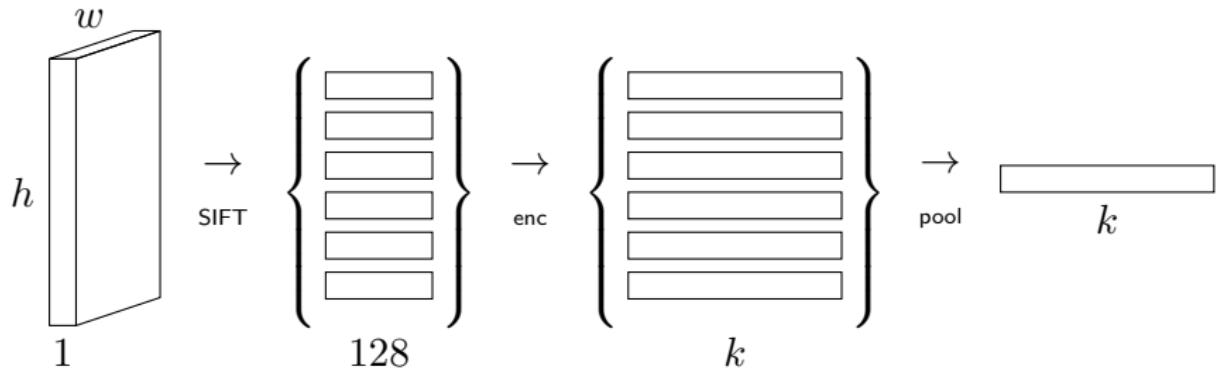
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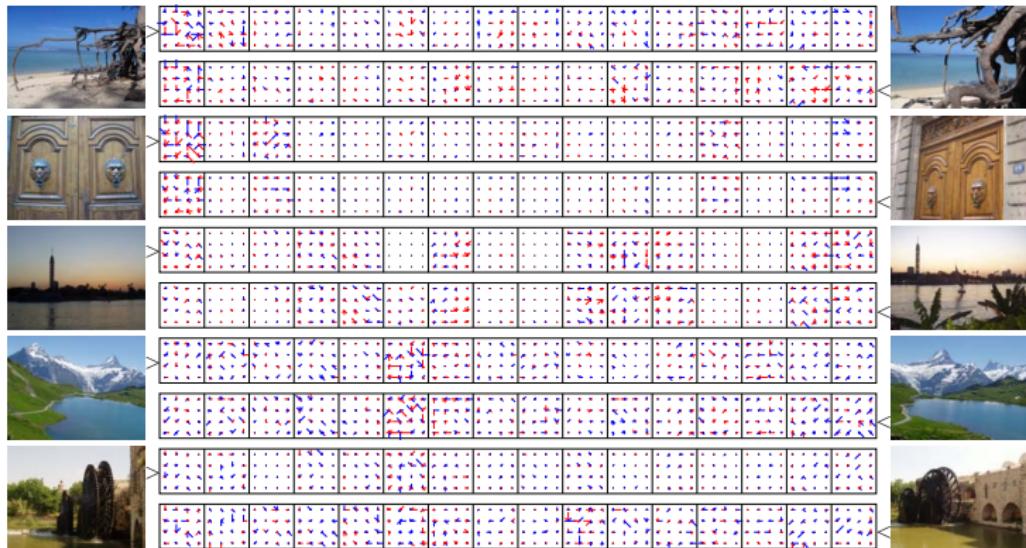
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## vector of locally aggregated descriptors (VLAD)\*

[Jégou et al. 2010]



- encoding yields a vector per visual word, rather than a scalar frequency
  - this vector is 128-dimensional like SIFT descriptors

# VLAD definition\*

- input vectors:  $X = \{x_1, \dots, x_n\} \subset \mathbb{R}^d$
- vector quantizer:  $q : \mathbb{R}^d \rightarrow C \subset \mathbb{R}^d$ ,  $C = \{c_1, \dots, c_k\}$

$$q(x) = \arg \min_{c \in C} \|x - c\|^2$$

- residual vector

$$r(x) = x - q(x)$$

- residual pooling per cell

$$V_c(X) = \sum_{\substack{x \in X \\ q(x)=c}} r(x) = \sum_{\substack{x \in X \\ q(x)=c}} x - q(x)$$

- VLAD vector (up to normalization)

$$\mathcal{V}(X) = (V_{c_1}(X), \dots, V_{c_k}(X))$$

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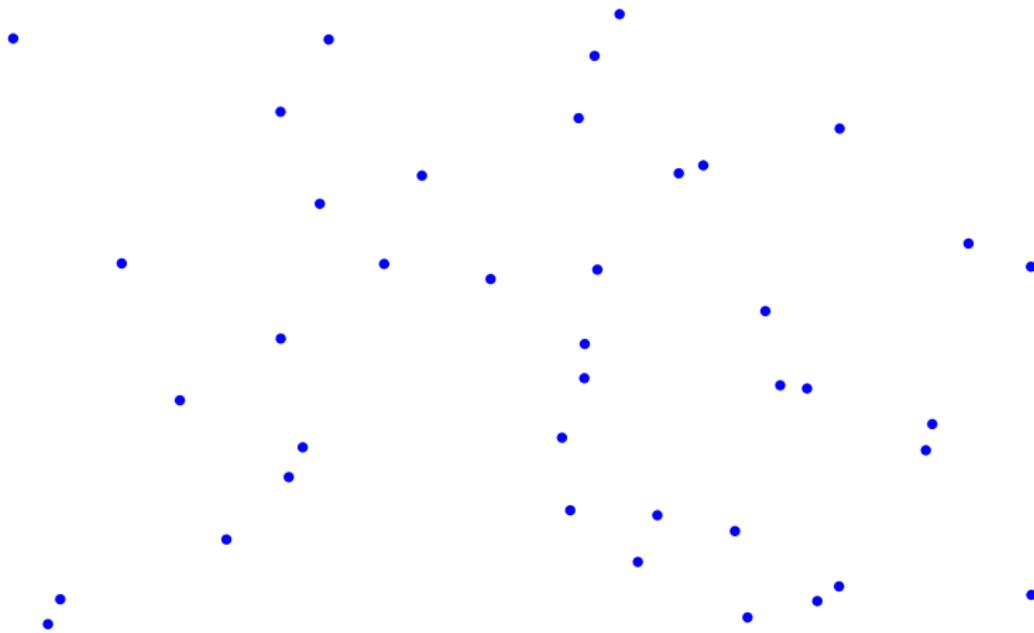
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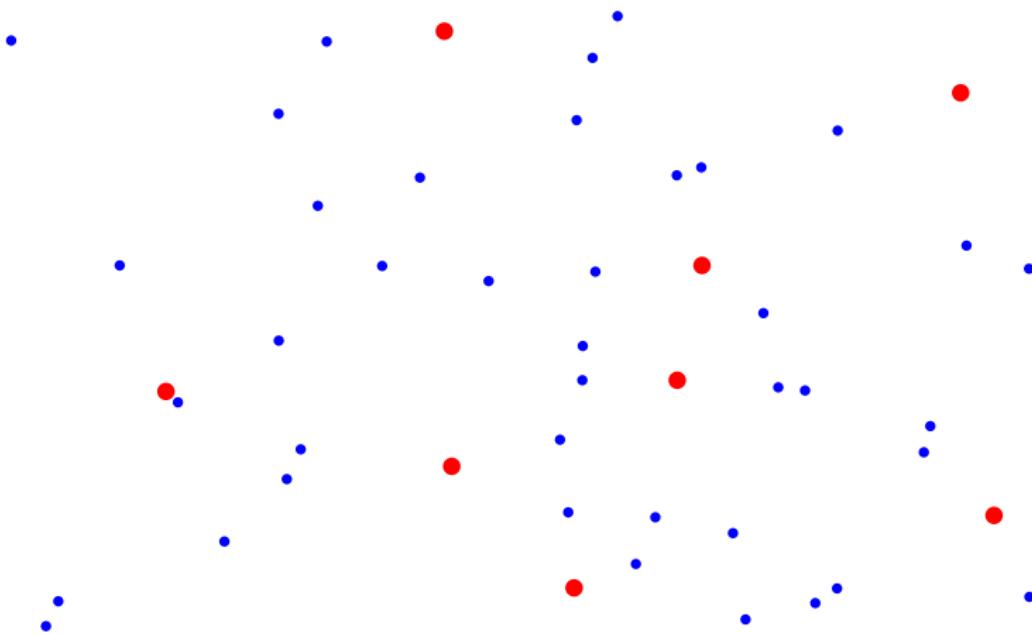
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# VLAD geometry\*



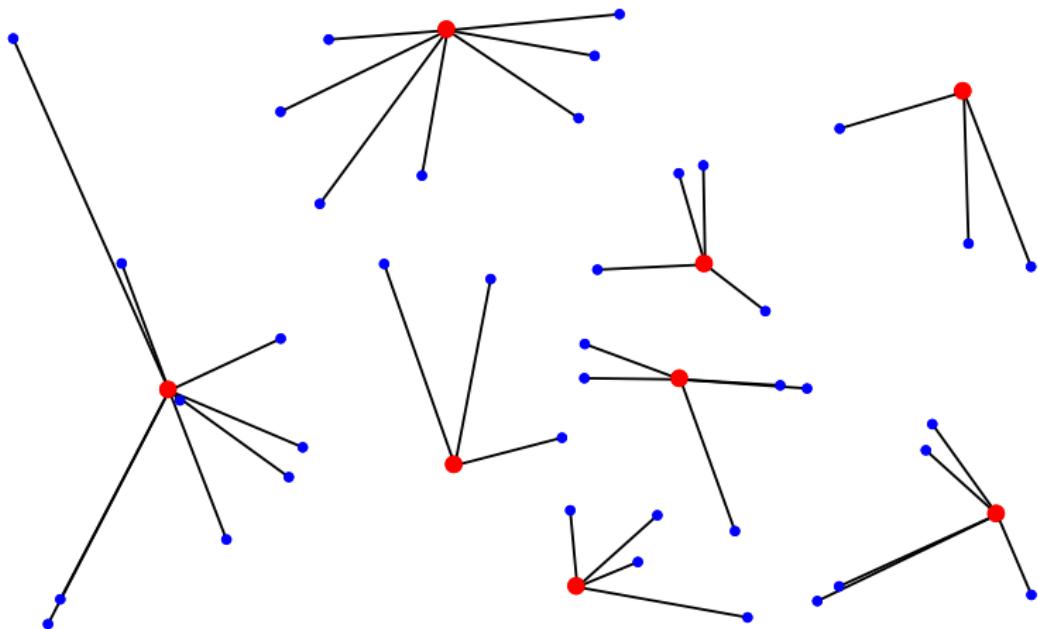
- **input vectors** – codebook – residuals – pooling

# VLAD geometry\*



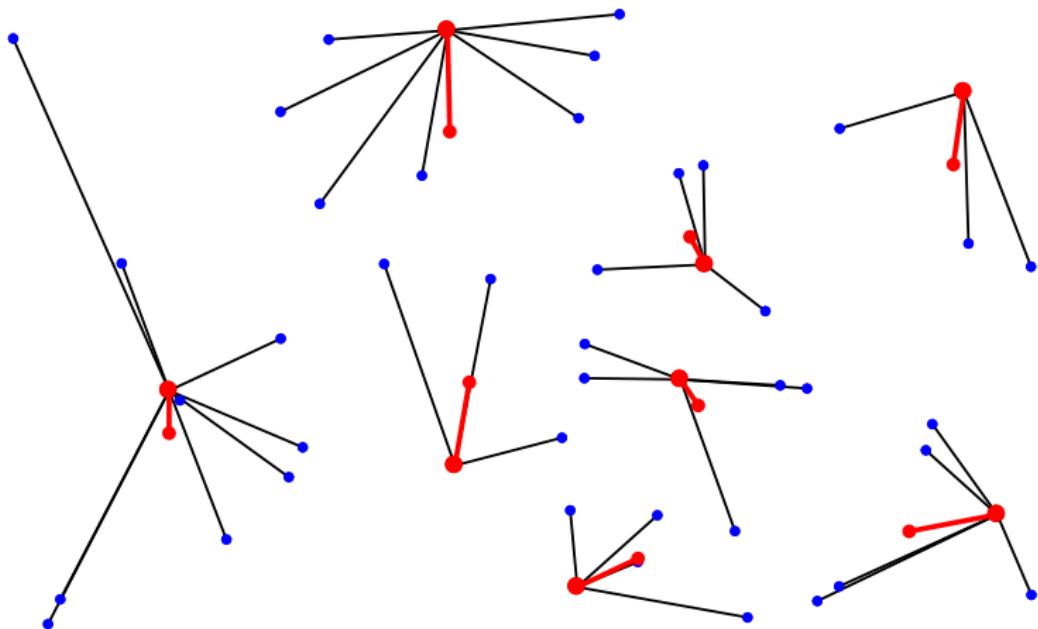
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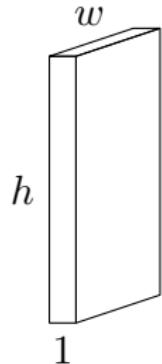
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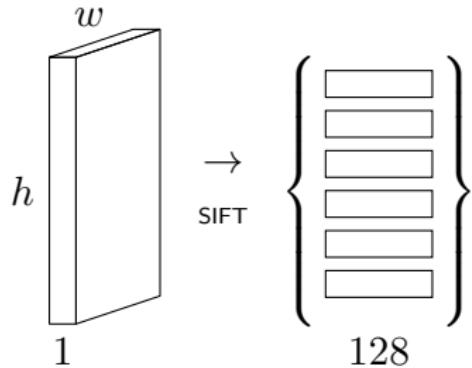
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# VLAD pipeline\*



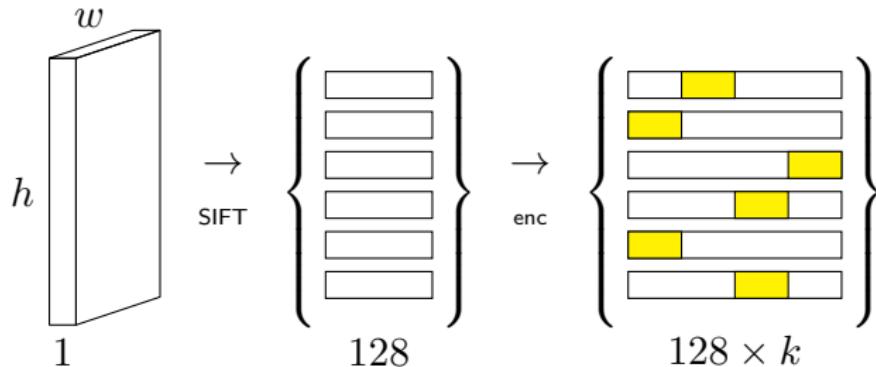
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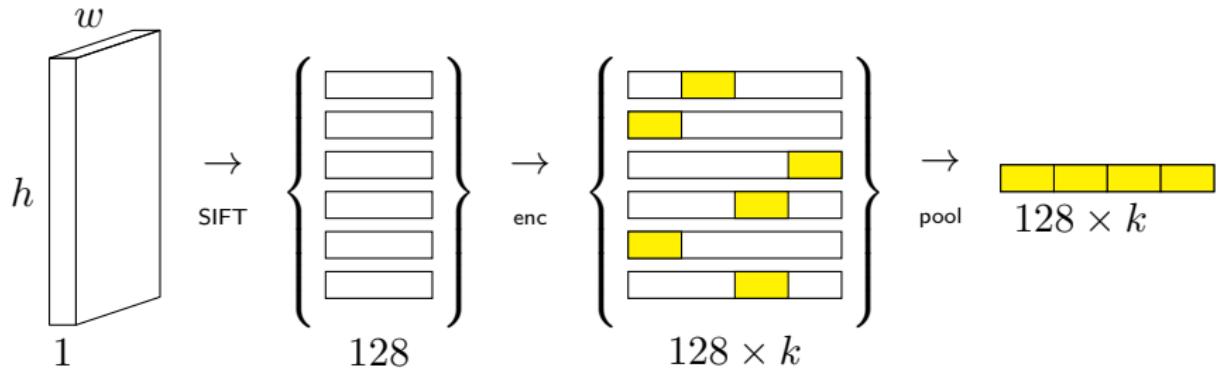
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## probabilistic interpretation\*

- if  $p(X|C)$  is the likelihood of i.i.d observations  $X$  under a uniform isotropic Gaussian mixture model with component means  $C$

$$p(X|C) \propto \prod_{x \in X} e^{-\frac{1}{2}\|x - q(x)\|^2}$$

- then the VLAD vector is proportional the gradient of  $\ln p(X|C)$  with respect to the model parameters  $C$

$$\mathcal{V}(X) \propto \nabla_C \ln p(X|C) = [\nabla_{c_1} \ln p(X|C), \dots, \nabla_{c_k} \ln p(X|C)]$$

- if we were to optimize  $C$  to fit the data  $X$ , then  $\hat{\mathcal{V}}(X)$  would be the direction in which to modify  $C$

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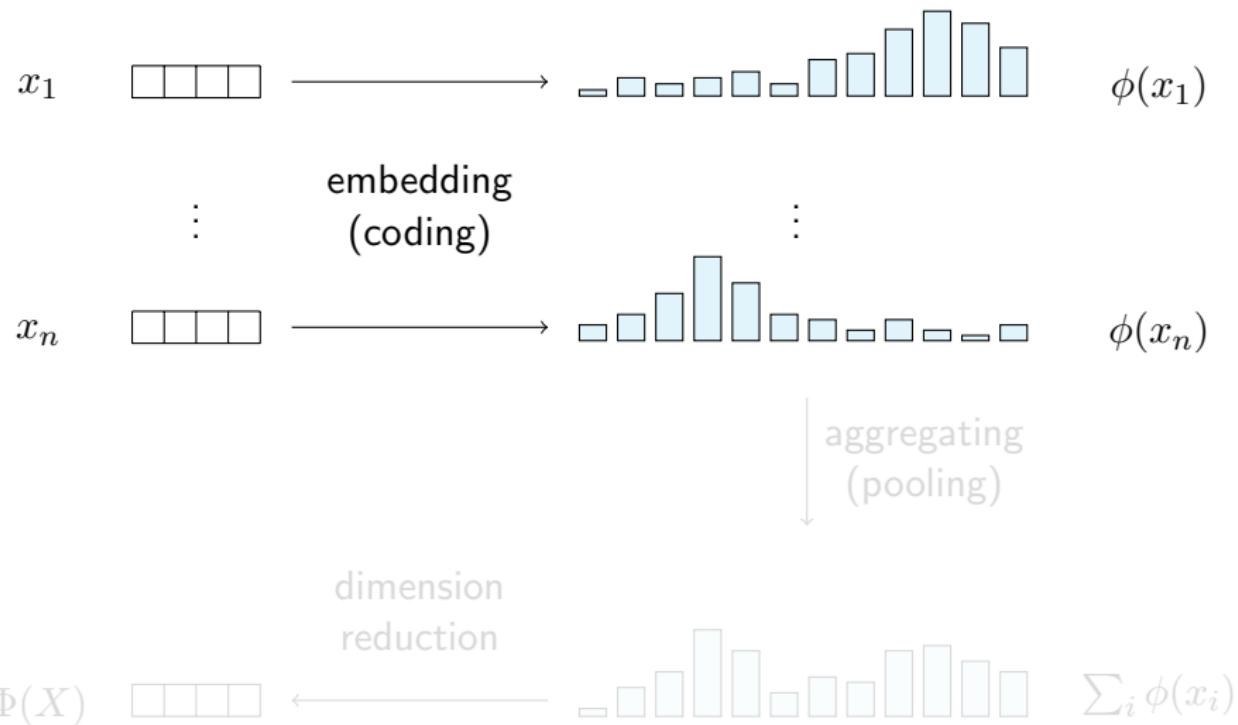
# Fisher kernel\*

- the Fisher kernel generalizes to a non-uniform diagonal Gaussian mixture model

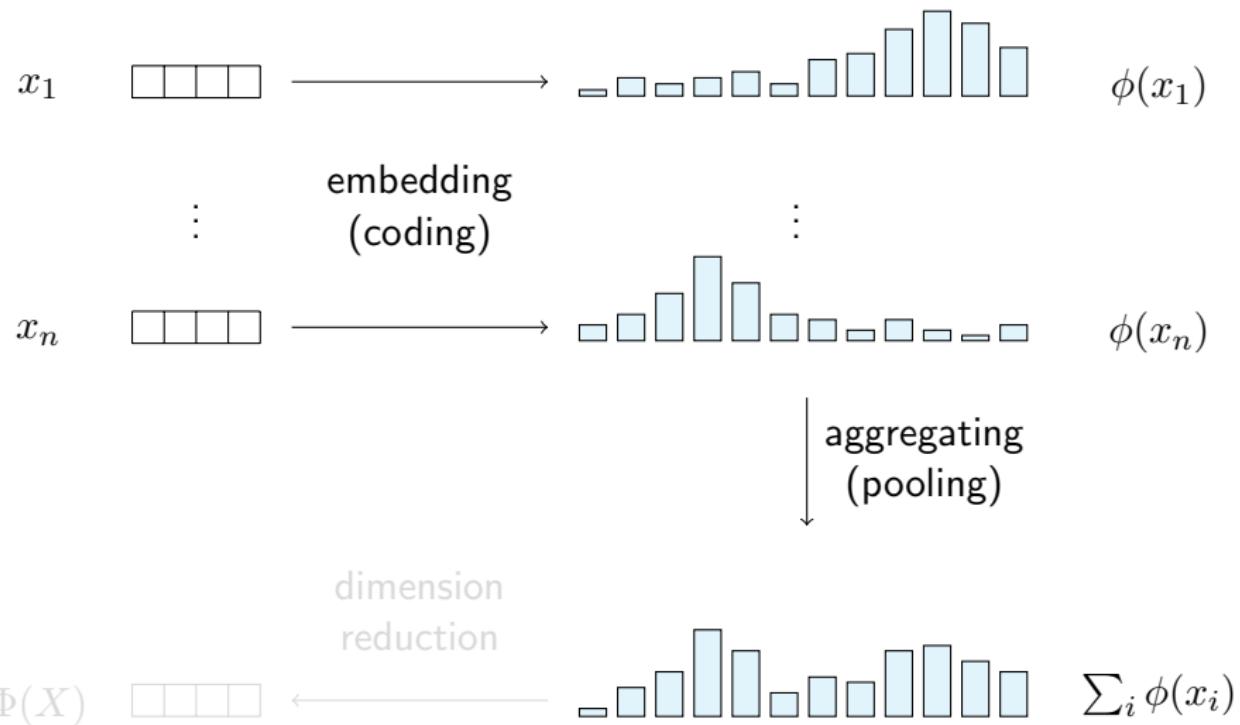
order statistics	parameter	model
0	mixing coefficient $\pi$	BoW
1	means $\mu$	VLAD
2	standard deviations $\sigma$	Fisher

# embeddings in general\*

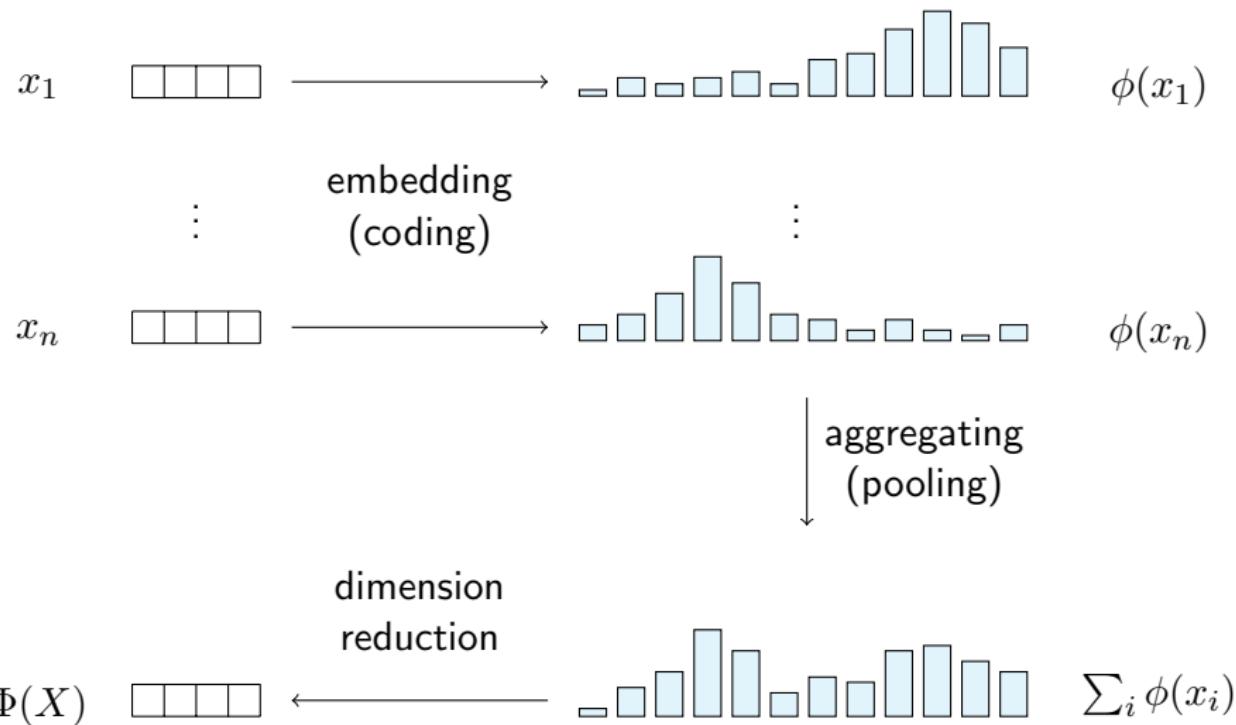
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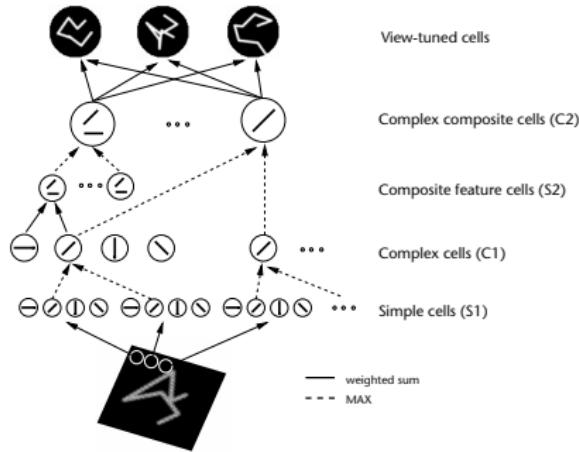


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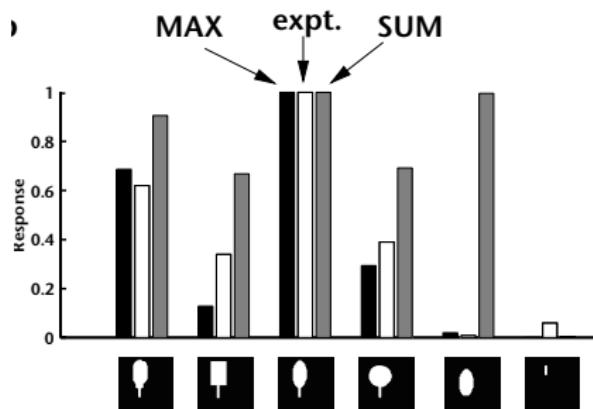


# HMAX

[Riesenhuber and Poggio 1999]



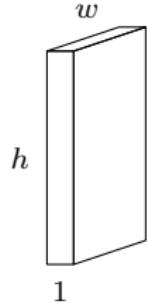
hierarchical model



sum vs. max pooling

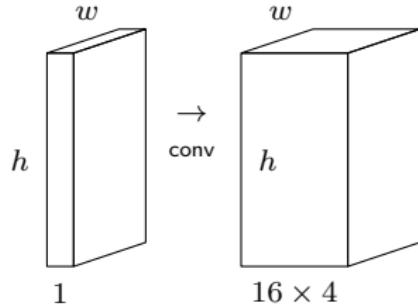
- computational model consistent with psychophysical data
- advocates non-linear max pooling

# (simplified) HMAX pipeline



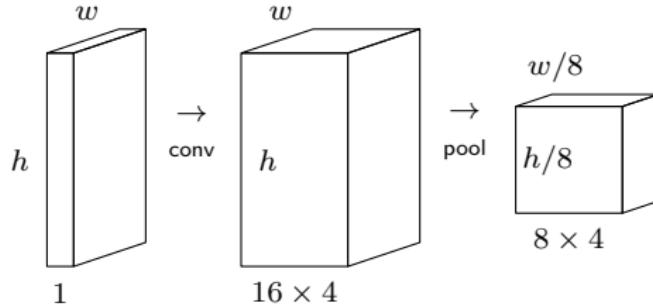
- 3-channel RGB input → 1-channel gray-scale
- **S1** apply filters at  $16 \text{ scales} \times 4 \text{ orientations}$
- **C1** max-pooling over  $8 \times 8$  spatial cells and over 2 scales
- **S2** convolutional RBF matching of input patches  $X$  to  $k = 4072$  prototypes  $P_i$  ( $n_i \times n_i$  patches at 4 orientations) extracted **at random** during learning: activations  $Y_i = \exp(-\gamma \|X - P_i\|^2)$
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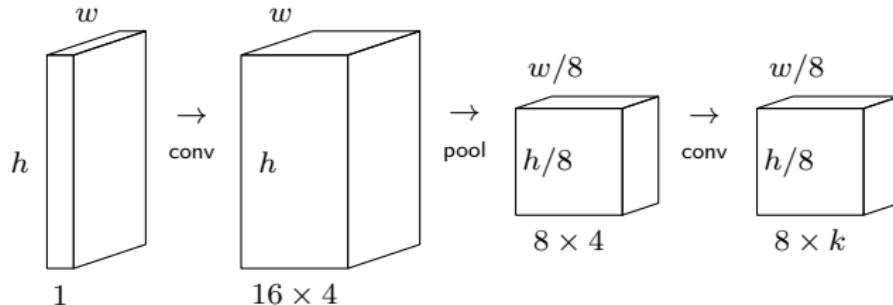
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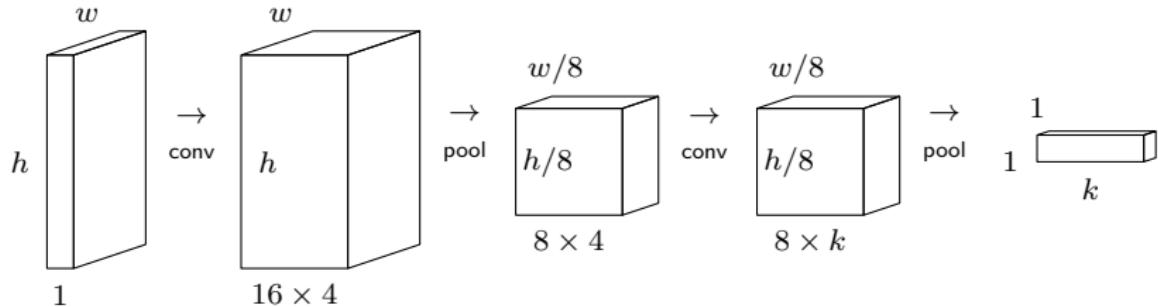
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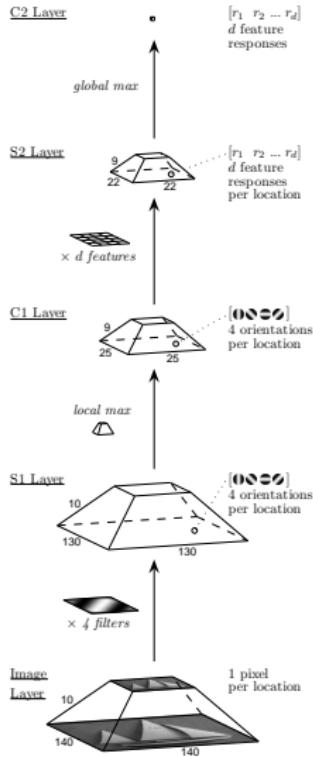
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# HMAX improvements\*

[Mutch and Lowe 2006]



- image pyramid
- S1 inhibition: non-maxima suppression over orientations
- strided C1 max pooling (50% overlap)
- C1 sparsification: dominant orientations kept

## summary

- neuroscience background, convolution, Gabor filters
- texture analysis, frequency sampling, visual descriptors
- dense *vs.* sparse features
- gist, SIFT, HOG
- pooling Gabor filter responses as orientation histograms
- feature hierarchy, codebooks, encoding, pooling
- textons, BoW, VLAD\*, Fisher kernel\*, HMAX
- hard *vs.* soft encoding, max *vs.* sum pooling