International Trade

Lecture 2: Introduction to Trade Models

Sifan XUE

NSD

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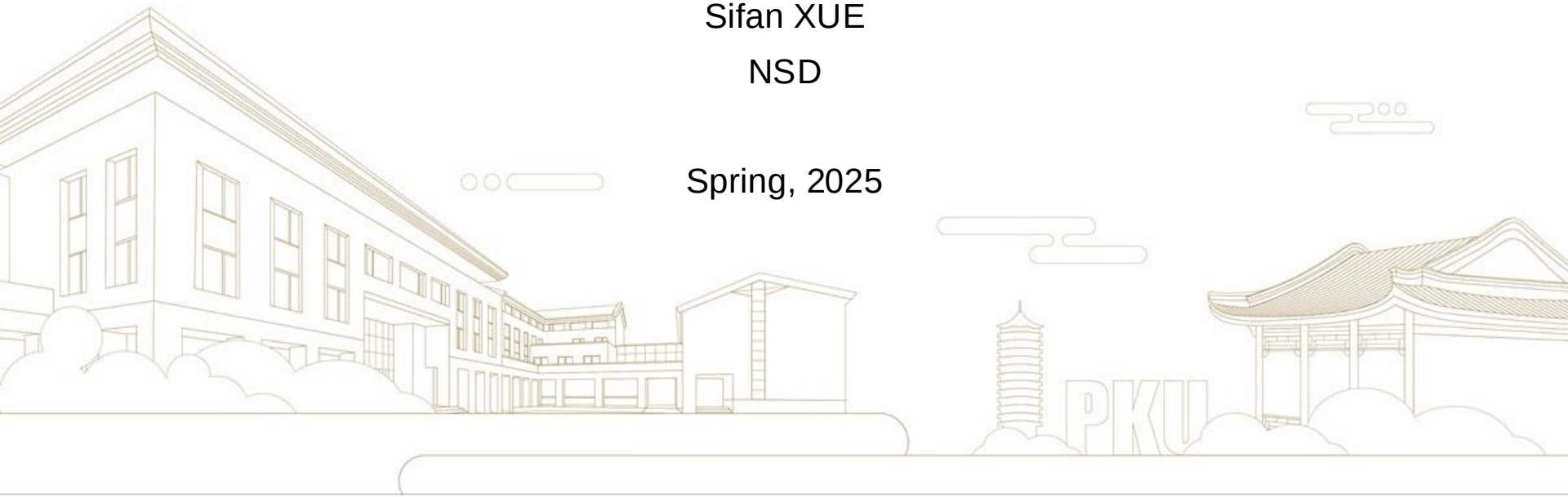


Outline for Today

- Endowment model
  - Illustration of trade model structure under simplest structure (no production)
  - Review of microeconomics used in trade models
  - Application in the trade of natural resources

Introduction to Trade Models

- What questions do we answer in trade models?
  - Why trade happens?
    - Differential relative prices – we elaborate from next page
  - What determines trade patterns – which country export which good? (What determines differences in relative prices?)
    - Abundance of endowment/factor; production technology; preferences; …
  - What are the impacts of trade?
    - Prices of goods, production patterns, factor prices, …
  - What are the welfare effects of trade (compared to autarky)?
    - Expansion of budget constraint; utility changes; real income changes, …

Equalization of Relative Prices

- Why arbitrage when relative prices are not equal?
  - Assume two goods, A and B
  - Two countries, H (home) and F (foreign)
  - $P_A^H = 2$, $P_B^H = 1$, $P_A^F = 3$, $P_B^F = 2$
- What are the relative prices of A (relative to B) in the two countries?
  - $P_A^H / P_B^H =$ _____
  - $P_A^F / P_B^F =$ _____

Equalization of Relative Prices

- What are the meanings of relative prices?
  - $P_A^H / P_B^H = 2$; $P_A^F / P_B^F = 3/2$
  - In country H, A is _____ times as expensive as B
  - If you sell 1 unit of A in country H, you can buy _____ unit(s) of B
  - If you sell 1 unit of B in country H, you can buy _____ unit(s) of A
  - If you sell 1 unit of A in country F, you can buy _____ unit(s) of B
  - If you sell 1 unit of B in country F, you can buy _____ unit(s) of A
  - Relative price describes the rate of exchange on the market

Equalization of Relative Prices

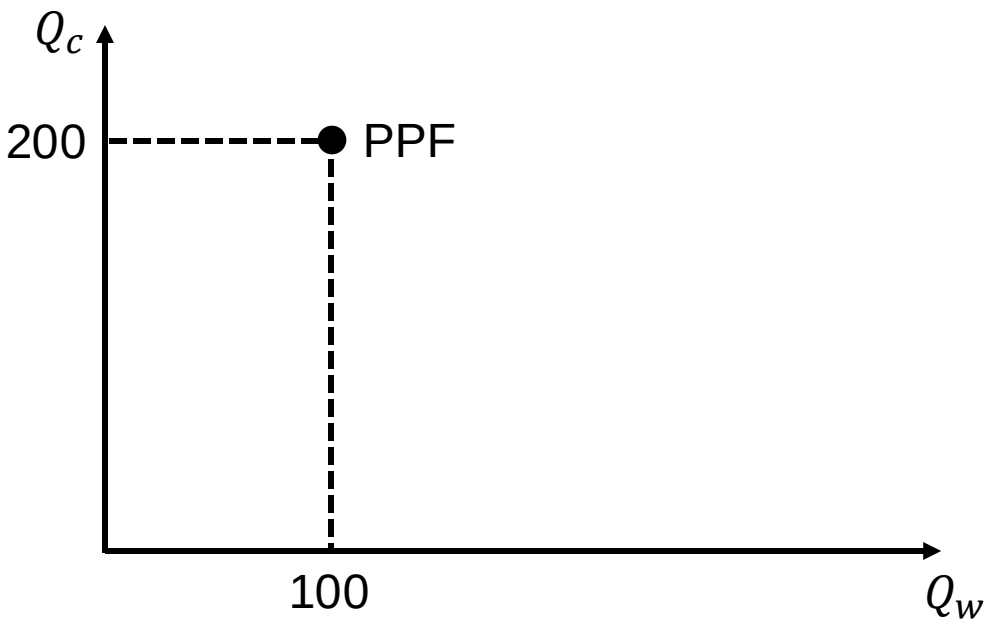
- When the two countries opens to trade with each other with unequal relative prices:
  - $P_A^H / P_B^H = 2$; $P_A^F / P_B^F = 3/2$
  - An arbitrager in country H sells 1 unit of A in exchange for _____ units of B, Following the relative price in H
  - He then transports the _____ units of B to country F
  - And sells the _____ units of B in exchange for _____ units of A in country F
  - When he transport the _____ units of A back to H
  - He earns _____ units of A as profit

Equalization of Relative Prices

- When the relative prices are not equal, and two countries open to trade with each other
  - Begin with $P_A^H / P_B^H = 2$; $P_A^F / P_B^F = 3/2$
  - An arbitrager can always make profit by selling at a higher relative price in one country and buying at a lower relative price in the other country
  - When he sells A and buys B in H, relative price of A in H ↓
  - When he sells B and buys A in F, relative price of A in F ↑
  - The arbitrage will continue to happen until relative prices of A equalize in the two countries

Endowment Model: An Example

- A representative agent (Or think of it as a unit continuum of symmetric agents, who are both consumers and sellers); Perfectly competitive market
- Two goods: wheat (W) and cloth (C)
- Endowment: $Q_w = 100$ of wheat and $Q_C = 200$ of cloth
- What is the shape of PPF?

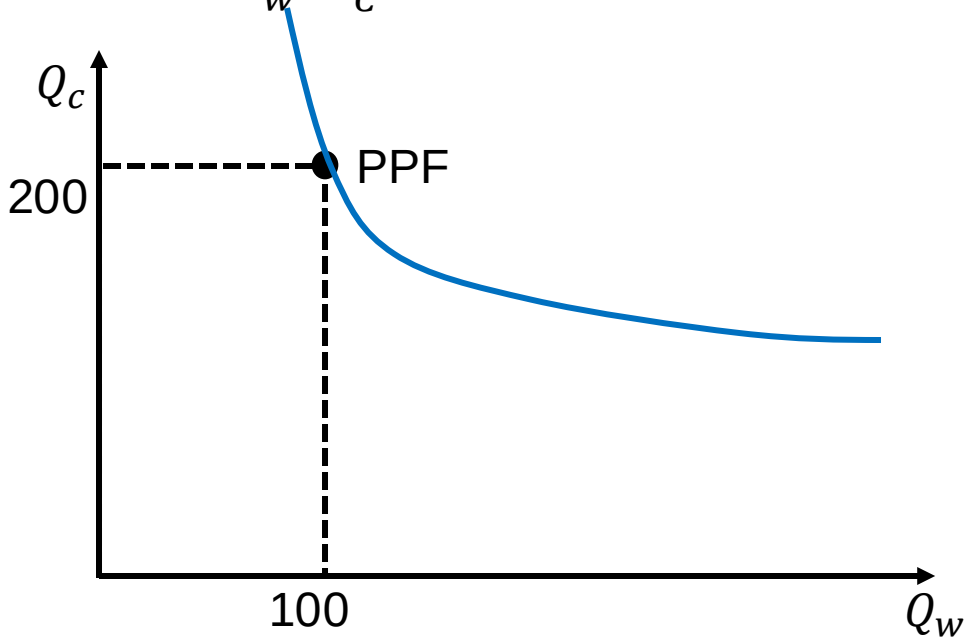


Endowment Model: PPF and IC

- Consumption C_w and C_C, prices P_w and P_C, for wheat and cloth
- Under autarky, $C_w = Q_w$ and $C_c = Q_c$
- Cobb-Douglas preferences: $U = C_w^{0.2} C_C^{0.8}$
- Indifference curves:

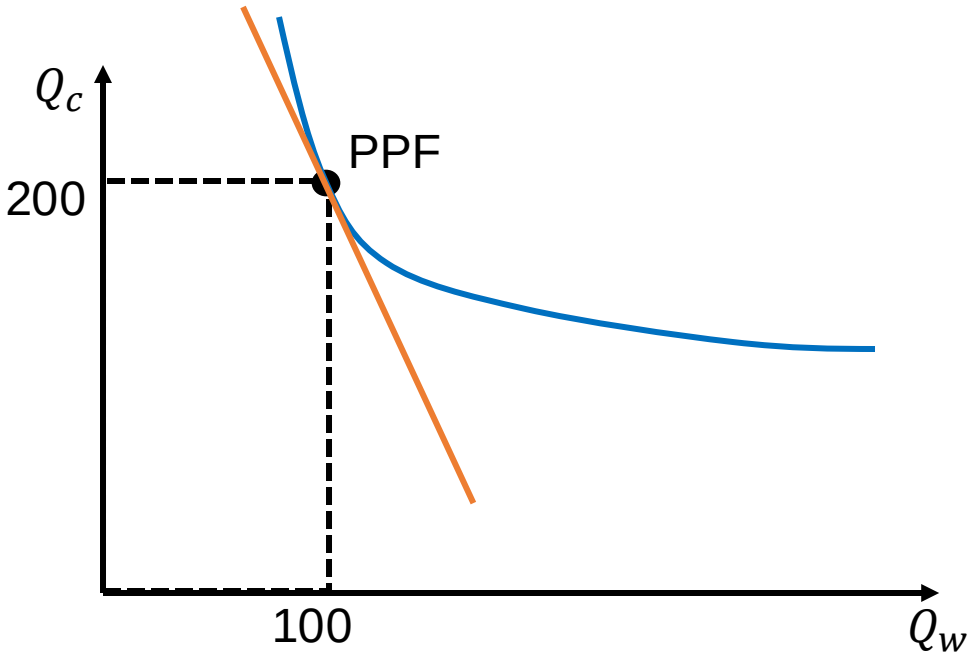
All points on an indifference curve have the same level of utility;

Combinations of two goods that you can consume and be equally satisfied.



Endowment Model: Income

- Denote an agent's income by I
- Budget constraint: $P_w C_w + P_c C_c \leq I$
- What's the slope of the budget line? Why?



Solution of the Relative Price

- Set up the maximization problem (Lagrange)
- First order condition:

$$\frac{MU_c}{P_c} = \frac{MU_w}{P_w}$$

Equilibrium budget line: pass the PPF, tangent to indifference curve

- Together with the budget constraint, one can solve for the relative price

Solution of Autarky Prices

- We solve that $P_w = 0.2I/C_w$ and $P_c = 0.8I/C_c$
- And remember the budget constraint is $I = P_w C_w + P_c C_c$
- Under autarky equilibrium, $C_w = Q_w = 100$ and $C_c = Q_c = 200$
- Three equations to solve three unknowns, but the system is not independent
- Normalize one of the three variables

Solution of Autarky Prices

- We can multiply all prices and income by any positive number and we still have an equilibrium – remember the Walras theorem in general equilibrium analysis (intermediate microeconomics)?
- We have only 1 effective price, and take the other price as numeraire (e.g., set $P_c = 1$)

Solution of Autarky Prices

- If we choose cloth as the numeraire and set $P_c = 1$
- The equations are

$$P_w = 0.2I/100$$

$$P_c = 0.8I/200$$

$$I = 100P_w + 200P_c$$

- Now we have the solution of prices:

$$P_w = \qquad P_c = \qquad I =$$

- Only relative price matters in trade

Trade of 2 Countries

- Now we assume the home country trades with a foreign country
- Both countries share the same preferences $U^j = \left(C_w^j\right)^{0.2}\left(C_C^j\right)^{0.8}$,

for $j = H, F$

- Home endowment: $Q_w^H = 100$ of wheat and $Q_C^H = 200$ of cloth
- Foreign endowment: $Q_w^F = 200$ of wheat and $Q_C^F = 200$ of cloth

Patterns of Trade

- $P_w^j = \frac{0.2I^j}{Q_w^j}$ and $P_c^j = \frac{0.8I^j}{Q_c^j} = 1$ (cloth as numeraire)
- Home endowment: $Q_w^H = 100$ of wheat and $Q_C^H = 200$ of cloth
- Autarky price in H: $P_w^H = 0.5$ and $P_c^H = 1$
- Foreign endowment: $Q_w^F = 200$ of wheat and $Q_C^F = 200$ of cloth
- Autarky price in F: $P_w^F =$ _____ and $P_c^F = 1$
- Wheat is relatively more expensive in _____ than _____ under autarky – so _____ exports wheat to _____
- Vice versa, _____ exports cloth to _____
- **Cause of trade:** differences in endowments of two goods
- **The country (relatively) more abundant in its endowment of one good export the good to the country relatively more scarce in the endowment of that good.**

World Trade Equilibrium

- The world is taken as a larger economy
- Representative of world has the same preference $U = C_w^{0.2} C_C^{0.8}$
- Where $C_w = C_w^H + C_w^F$ and $C_c = C_c^H + C_c^F$ are total consumption
- The total endowment of the two goods:
- $Q_w = Q_w^H + Q_w^F = 300$; $Q_C = Q_c^H + Q_c^F = 400$
- What are the prices in the world trade equilibrium?

World Prices

- World preference: $U = C_w^{0.2} C_C^{0.8}$
- World endowment: $Q_w = 300$; $Q_C = 400$
- Prices of wheat and cloth should equalize in two countries, so that there is no arbitrage

$$P_w = P_w^H = P_w^F \text{ and } P_c = P_c^H = P_c^F$$

- We still need an international numeraire whose price equal to 1 in all countries – we continue to set $P_c = 1$

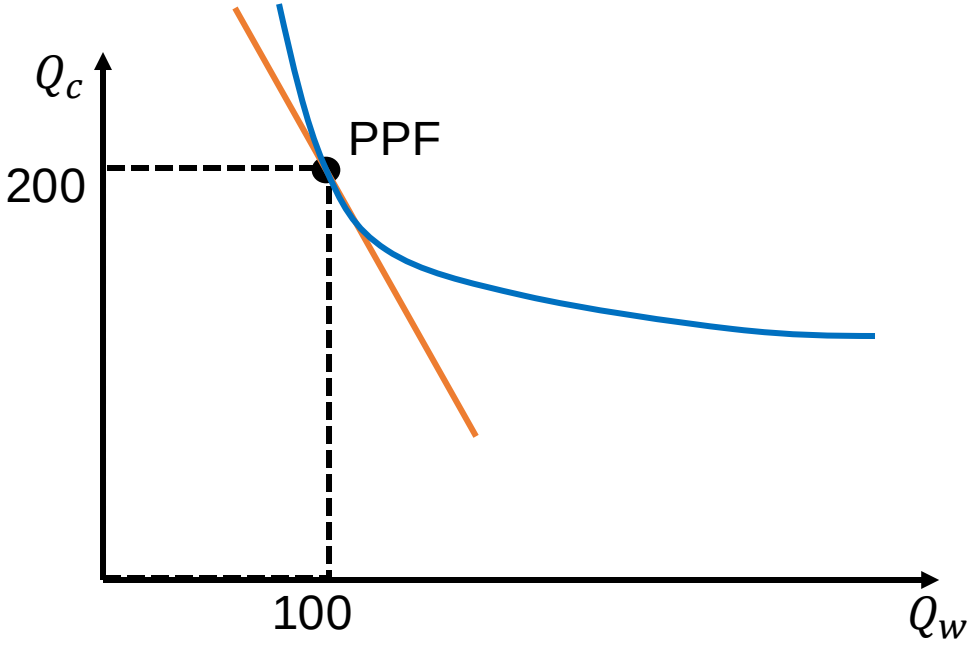
World Prices

- World preference: $U = C_w^{0.2} C_C^{0.8}$
- World endowment: $Q_w = 300$; $Q_C = 400$
- $P_w = 0.2I/Q_w$ and $P_c = 0.8I/\mathrm{Q}_c = 1$ (cloth as numeraire)
- The world price: $P_w = 1/3$ and $P_c = 1$
- Relative price of wheat is greater than the autarky relative price in F (1/4) and smaller than the autarky relative price in H (1/2), i.e., between the autarky relative prices

Home Country: Autarky vs. Trade

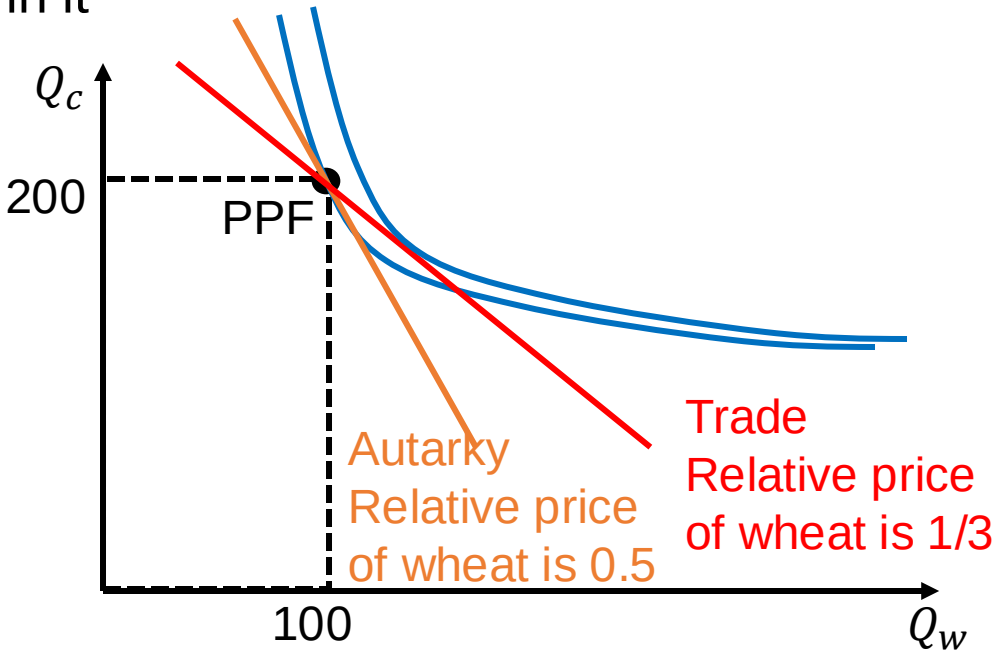
- Home under autarky: consumption equal to endowment (PPF)
- New budget line under trade equilibrium ($P_w = 1/3$ and $P_c = 1$):

$$P_w C_w^H + P_c C_c^H = P_w Q_w^H + P_c Q_c^H$$



Home Country: Autarky vs. Trade

- Wheat becomes relatively cheaper with trade
- **"Gain from Trade":** by revealed preference; Expansion of PPF
- Scarcity in wheat compensated by trading with a partner more abundant in it



Solve Consumption and Trade Flows

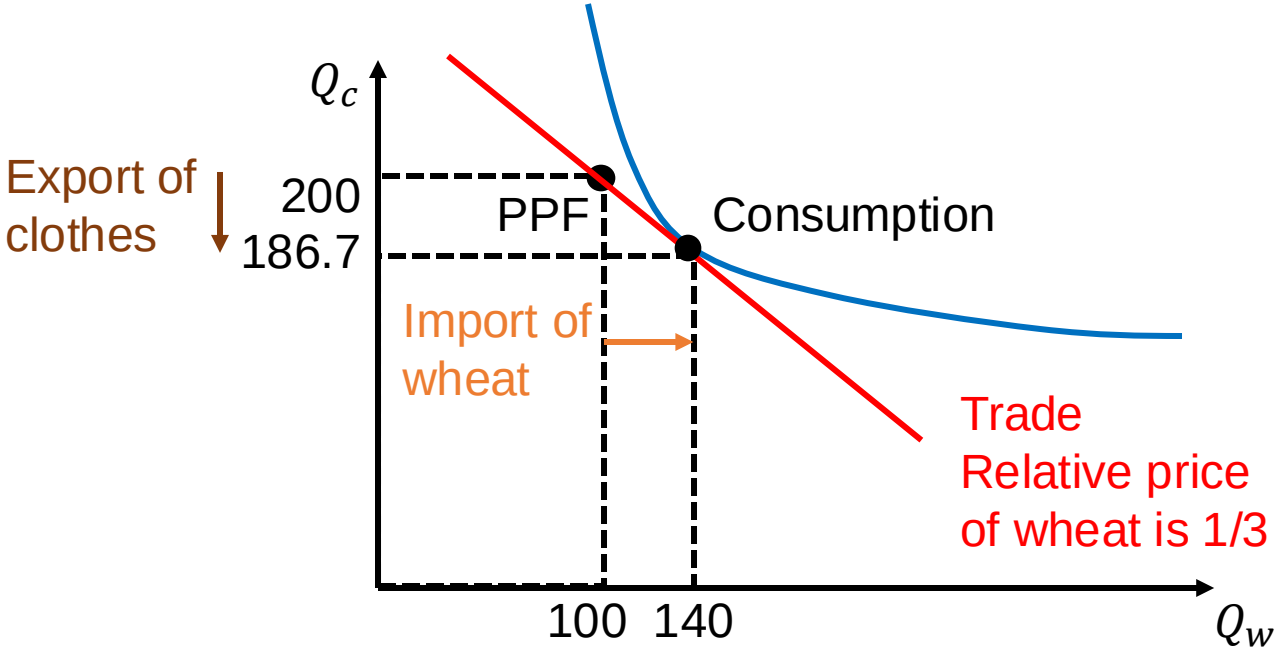
- FOC for Home consumption choice at equilibrium:
- $P_w = \frac{0.2I^H}{C_w^H}$ and $P_c = \frac{0.8I^H}{C_c^H}$, $I^H = P_w Q_w^H + P_c Q_c^H$
- Where $P_w = 1/3$ and $P_c = 1$, $Q_w^H = 100$, $Q_C^H = 200$
- Solve consumption: $I^H = \frac{1}{3} * 100 + 1 * 200 = \frac{700}{3}$

$$P_w = \frac{1}{3} = 0.2 * \frac{I^H}{C_w^H} \Rightarrow C_w^H = 140, P_c = 1 = 0.8 * \frac{I^H}{C_c^H} \Rightarrow C_c^H = \frac{560}{3}$$

- Export of clothes $EX_c^H = Q_c^H - C_c^H = 200 - \frac{560}{3} = \frac{40}{3}$
- Import of wheat $IM_w^H = C_w^H - Q_w^H = 140 - 100 = 40$

Home Country: Autarky vs. Trade

- Consumption in trade equilibrium: C_c^H =186.7, $C_w^H = 140$
- Export of clothes $EX_c^H = Q_c^H - C_c^H = 13.3$
- Import of wheat $IM_w^H = C_w^H - Q_w^H = 40$



Trade Balance

- Value of export of clothes $EV_c^H = P_c(Q_c^H - C_c^H) = 13.3$
- Value of import of wheat $IV_w^H = P_w(C_w^H - Q_w^H) = 13.3$
- The two values cancels out with each other: trade is balanced

Summary of Numeric Example

- Under autarky, two countries have different relative prices because of differences in endowment abundance
- When two countries trade with each other, countries export the good more abundant in endowment and import the good more scarce in endowment
- In equilibrium, equalized relative price lies between the autarky prices of the two countries
- Both countries gain in welfare from trade as if PPF expands
- Value of import and export are equal in both countries

The Endowment Model: Generalized

- Generalize to multiple goods and countries
- Endowment economy
- G goods, labeled $g = 1, \ldots, G$
- Each economy has an endowment of each good

$$Q_g^i \ (g = 1, \ldots, G; \ i = H, F_1, F_2, \ldots)$$

- Representative agent w/ common preferences across countries

The Endowment Model: Generalized

- Preferences are Cobb-Douglas

$$U^i = \prod_{g=1}^{G} \left(C_g^i\right)^{\beta_g}, \text{ with } \sum_{g=1}^{G} \beta_g = 1$$

- Assume an income I^i of the representative agent of country i:

$$P_g^i C_g^i = \beta_g I^i$$

- The expenditure share on good g is β_g

Generalized Endowment Model: Autarky

- With no trade, equilibrium requires that consumption must equal the endowment (demand = supply),

$$C_g^i = Q_g^i$$

- Then, in equilibrium we have

$$P_g^i C_g^i = P_g^i Q_g^i$$

- Since $P_g^i C_g^i = \beta_g I^i$, we must have

$$P_g^i = \beta_g I^i / Q_g^i$$

- Relative price of two goods are determined by β_g and Q_g^i

Generalized Endowment Model: World

- Now think of the “world economy” – all countries open up to trade
- Still assume: no trade costs
- Thus, same (relative) prices everywhere in the world
- As the Cobb-Douglas preference derives (and that the preferences are the same in all countries)

$$P_g^w = \beta_g I / Q_g^w$$

- P_g^w are the prices in the world economy, and I is the world income
- Q_g^w are the total endowments (sum of all country-specific endowments)

The “Home” Economy

- Now consider a country we call “Home” under the trade equilibrium
- The economy has endowments $Q_g^H < Q_g^w$
- Prices in home are the world prices, $P_g^H = P_g^w = \beta_g I / Q_g^w$
- Total income I^H in Home: $I^H = \sum_{g=1}^{G} P_g^w Q_g^i$
- Consumption: $C_g^H = \frac{\beta_g I^H}{P_g^w}$

Trade in the “Home” Economy

- How do we know if Home export or import good g?
- Export if home consumption is smaller than home endowment
- Import if home consumption is higher than home endowment
- Value of Home export in good g ($EV_g^H < 0$ means import):

$$EV_g^H = P_g Q_g^H - P_g C_g^H$$

Trade in the “Home” Economy

- Can we predict the pattern of trade good by good using a comparison of autarky prices? $P_g^H > P_g^w \Leftrightarrow IM_g^H > 0$?
- Consider Nepal, a mountainous, mostly vegetarian country.
  - Nepal produces three goods: beef, leather, and guided hiking trips.
  - Mountainous ⇒ few cattle ⇒ Autarky leather price is high.
  - Vegetarian ⇒ low demand for beef ⇒ Autarky beef price is low.
  - Now let Nepal trade. With access to cheap cattle from abroad, it shifts resources from cattle raising to tourism. Suppose, for example, that it specializes in tourism. Then Nepal will import beef despite autarky beef price being lower than the world price.
- We would still have “on average, countries import goods that are relatively expensive in autarky and export goods that are relatively cheap in autarky.”

Trade Balance

- Now that we have $EV_g^H = P_g Q_g^H - \beta_g I^H$
- We can show that the "Home" economy has trade balance:

$$\sum_{g=1}^{G} EV_g^H = 0$$

Exports

- Now that we have $EV_g^H = P_g Q_g^H - \beta_g I^H$
- Plug in the price conditions: $P_g = \beta_g I / Q_g^w$
- Plus an algebra trick: $I^H = I \cdot \frac{I^H}{I}$
- Finally we have

$$EV_g^H = \beta_g I \left(\frac{Q_g^H}{Q_g^w} - \frac{I^H}{I} \right)$$

- That says a country exports a good if (and only if) its share of the world's endowment of that good is higher than the country's share of world income:

$$\frac{Q_g^H}{Q_g^w} > \frac{I^H}{I}$$

Application to Oil Trade

- Key expression

$$EV_g^H = \beta_g I \left(\frac{Q_g^H}{Q_g^w} - \frac{I^H}{I} \right)$$

- If a country owns **10%** of the world's oil, but has **1%** of the world's income, then it exports oil in value equal to 9% of world's oil consumption:

$$\frac{EV_g^H}{\beta_g I} = \frac{Q_g^H}{Q_g^w} - \frac{I^H}{I} = 10\% - 1\% = 9\%$$

Application to Oil Trade

- Key expression

$$E_g^H = \beta_g I \left(\frac{Q_g^H}{Q_g^w} - \frac{I^H}{I} \right)$$

- If a country owns **10%** of the world's oil, but has **20%** of the world's income, then it import oil in value equal to 10% of world's oil consumption:

$$\frac{EV_g^H}{\beta_g I} = \frac{Q_g^H}{Q_g^w} - \frac{I^H}{I} = 10\% - 20\% = -10\%$$

Application to Oil Trade

- Real data: relative oil production, GDP in 2014, trade estimation and real import/export:

Economy	Oil production relative to world production (%)	GDP relative to world GDP (%)	Estimation of net export (1000 bbl/d)	Real net export in 2012 (1000 bbl/d)
Costa Rica	0.0003%	0.0633%	-54.3	-46.3
France	0.0656%	3.6142%	-3,060.0	-1,943.2
Saudi Arabia	12.4966%	0.9533%	9,953.8	10,546.0
Spain	0.0430%	1.7940%	-1,509.9	-1,557.7

Source: EIA of the United States, The World Bank, IEA

- The "estimation of net export" column equals

$$(\text{Column 2} - \text{Column 3}) \times 86.23 \text{ million bbl per day (2012 world production)}$$