International Trade: Lecture 3

Technology Differences

Sifan Xue
NSD, Peking University

Fall 2024

Technology Differences as Source of Comparative Advantage

Differences in technological capabilities are another potential source of comparative advantage.

- We will study these first in a context where differences are exogenous.
- Endogenous growth literature makes them endogenous.

Note that technological differences are not the same as labor productivity differences.

- Labor productivity differences may arise due to different endowments of complementary factors.
- But technological differences and labor productivity differences may be difficult to distinguish in empirical research.

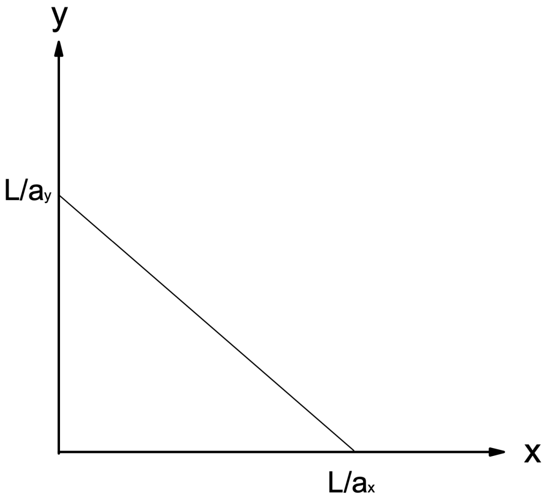
The Basic Ricardian Model

- Two goods. Two countries. CRS. Perfect competition.
- Input coefficients (a_x, a_y) and (a_x^*, a_y^*).
- Autarky: $wa_x \geq p, wa_y \geq 1$.
- If both goods are produced: $p = \frac{a_x}{a_y} \equiv a$;

$$p > a \Rightarrow y = 0,$$

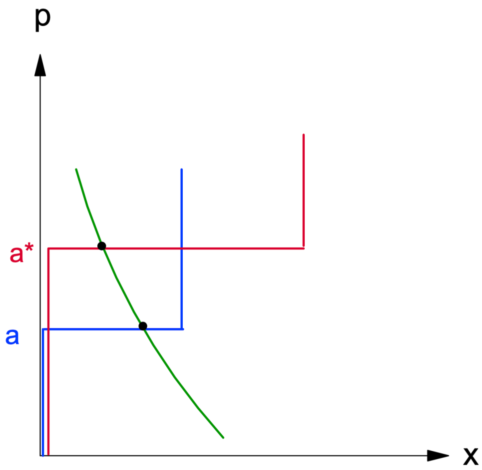
$$p < a \Rightarrow x = 0.$$

The Basic Ricardian Model: Production Possibility Frontier



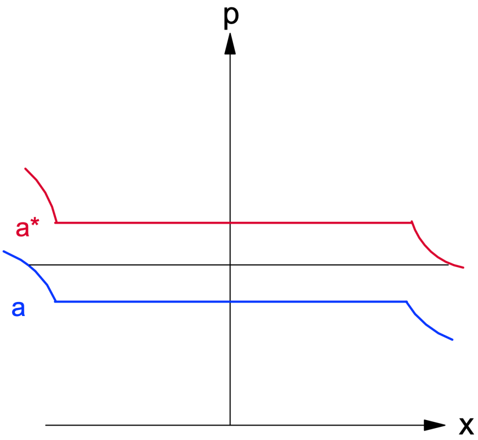
The Basic Ricardian Model: Autarky Equilibrium

Relative autarky prices and comparative advantage are governed by relative labor inputs.



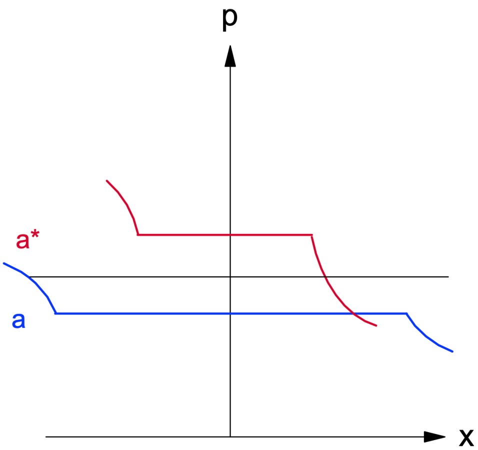
The Basic Ricardian Model: Trade Equilibrium

If countries of roughly similar size, both specialize, with free trade price strictly between the two autarky prices.



The Basic Ricardian Model: Trade Equilibrium (cont'd)

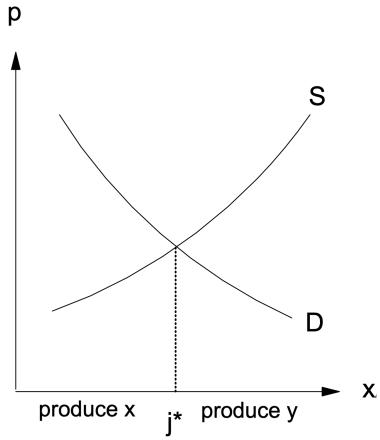
If one country is much larger, or demand pattern is sufficiently skewed, then incomplete specialization in one country.



Ricardian Model: Many Countries

Continuum of small countries. Rank them so that $a(j) = \frac{a_x(j)}{a_y(j)}$ is increasing.

Plot relative world supply and demand:



Ricardian Model: Many Goods: Dornbusch, Fischer, and Samuelson, AER 1977

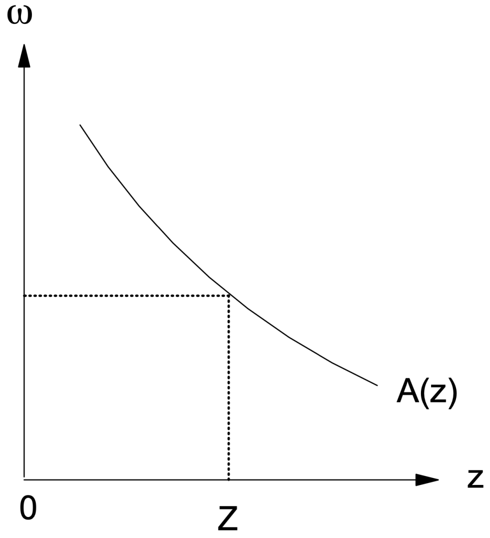
- Continuum of goods.
- Rank goods so that $\frac{a^*(z)}{a(z)}$ is falling for $z \in [0, 1]$.
  $\Rightarrow$ Home country has greatest comparative advantage in good $z = 0$.
- Define $A(z) = \frac{a^*(z)}{a(z)}$; $A'(z) < 0$.
- In equilibrium, home country produces z if and only if

$$wa(z) \leq w^* a^*(z),$$

i.e., if and on if

$$\omega \equiv \frac{w}{w^*} \leq A(z) \quad \Rightarrow \quad A(Z) = \omega.$$

Ricardian Model: Many Goods: Dornbusch, Fischer, and Samuelson, AER 1977 (cont'd)

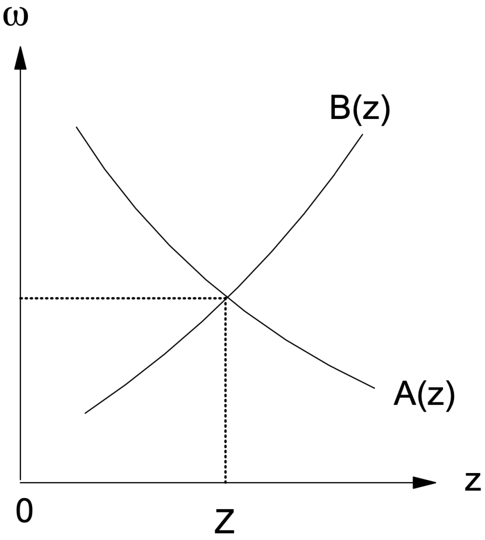


Ricardian Model: Many Goods: Dornbusch, Fischer, and Samuelson, AER 1977 (cont'd)

- Let demands be derived from generalized Cobb-Douglas preferences $u = \int_0^1 b(z) \ln z dz$ with spending shares $b(z)$ such that $\int_0^1 b(z)dz = 1$.
- Then $p(z)c(z) = b(z)wL$.
- Define $\phi(z) = \int_0^z b(s)ds$; spending share for goods in $[0, z]$.
- In equilibrium, $\phi(Z)(wL + w^*L^*)$ is spending on goods produced at home.
- This must match home factor income, wL.
- $\phi(Z)(wL + w^*L^*) = wL \quad \Rightarrow$

$$\omega = \frac{L}{L^*}\frac{\phi(Z)}{1-\phi(Z)} \equiv B(Z).$$

Ricardian Model: Many Goods: Dornbusch, Fischer, and Samuelson, AER 1977 (cont'd)



Eaton and Kortum, 2002 ECTA

This paper develops a general equilibrium model of international trade that explains the following stylized facts within a single unified framework:

- Trade diminishes dramatically with distance.
- Prices vary across locations with greater differences between places further apart.
- Factor rewards are far from equal across countries.
- Countries' relative productivities vary substantially across industries.

Eaton and Kortum, 2002 ECTA (cont'd)

- More specifically, the paper develops and quantifies a multi-country Ricardian model of trade (based on differences in technology) that incorporates a role for geography.
  Model captures competing forces:
  - Comparative advantage promotes trade.
  - Geographic barriers (transport costs, tariffs, quotas) diminish trade.
- Develops a probabilistic formulation of the Ricardian model with a continuum of goods.
- Allows the Dornbusch, Fischer, and Samuelson (1977) framework to be extended to many countries separated by geographic barriers.
- Derives structural relationships, whose parameters can be estimated using cross-country data on bilateral trade and prices, and which can be used to carry out counterfactuals.
- The theoretical framework can be used to analyze a variety of issues in international trade and has served as the basis for further work on firm heterogeneity and trade and in macroeconomics.

Endowments

- First, develop a partial equilibrium model of trade that takes countries' input costs as given.
- Second, close the model by endogenizing countries' input costs.
- The world consists of many countries $i \in \{1, \ldots, N\}$.
- There is a single primary factor of production, labor, which is mobile across industries but immobile across countries.
- Denote country i's input cost by c_i, which includes the cost of both labor and intermediate inputs, as specified below.
- The production technology exhibits constant returns to scale, and country i's efficiency in producing good $j \in [0, 1]$ is denoted by $z_i(j)$.
- Geographic barriers to trade are assumed to take the iceberg form, where a quantity $d_{ni} > 1$ must be shipped in order for one unit to arrive for $n \neq i$, and $d_{ii} = 1$.
    - Cross-country arbitrage ensures that geographic barriers to trade satisfy the triangle inequality: $d_{ni} \leq d_{nk} d_{ki}$.

Preferences

- Goods are homogeneous and the market structure is perfect competition.
- The cost to buyers in country n of purchasing good j from country i is:

$$p_{ni}(j) = \left(\frac{c_i}{z_i(j)}\right) d_{ni} \tag{1}$$

- Consumer expenditure minimization implies that the price actually paid by buyers in country n for good j is:

$$p_n(j) = \min\{p_{ni}(j); i = 1, \dots, N\} \tag{2}$$

- The representative consumer's preferences are a constant elasticity of substitution (CES) function defined over consumption of the goods $j \in [0, 1]$:

$$U = \left(\int_0^1 Q(j)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 0 \tag{3}$$

Technology

- Country i's efficiency in producing good j is the realization of a random variable Z_i (drawn independently for each good j and country i) from a probability distribution $F_i(z) = \Pr[Z_i \leq z]$.
- By the law of large numbers, $F_i(z)$ is also the fraction of goods for which country i's efficiency is below z.
- Country i's efficiency distribution is Fréchet (Type II extreme value distribution):

$$F_i(z) = e^{-T_i z^{-\theta}}, \quad T_i > 0, \quad \theta > 1 \tag{4}$$

- Where a higher value of T_i implies that a higher efficiency draw for any good j is more likely (so T_i captures absolute advantage).
- A higher value of θ implies less variability in efficiency across goods (so θ is an inverse measure of the degree of comparative advantage).

Prices

- From the pricing rule (1), country i presents country n with a distribution of prices $G_{ni}(p) = \Pr[P_{ni} \leq p] = 1 - F_i\left(\frac{c_i d_{ni}}{p}\right)$:

$$G_{ni}(p) = 1 - e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}} \tag{5}$$

- The distribution of the minimum prices (the prices actually paid by consumers) in country n:

$$G_n(p) = \Pr[P_n \leq p] = 1 - \prod_{i=1}^{N} [1 - G_{ni}(p)] \tag{6}$$

- Substituting (5) into (6), the distribution of the minimum prices takes the same form as the distribution of prices:

$$G_n(p) = 1 - e^{-\Phi_n p^{\theta}}, \quad \Phi_n = \sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta} \tag{7}$$

- Where Φ summarizes the determinants of the country n price distribution.

Three Properties of Prices

1. The probability that country i provides a good at the lowest price in country n is simply:

$$\pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}, \tag{8}$$

   which is country i's contribution to country n's price parameter Φ_n in (7) and, by the law of large numbers, is the fraction of goods that country n buys from country i.

2. The price of a good that country n *actually buys* from any country i also has the distribution $G_n(p)$.
   - Therefore, a source country with a higher state of technology, lower input cost, or lower geographic barriers exploits its advantage by selling a wider range of goods, exactly to the point at which the distribution of prices for what it sells in country n is the same as country n's overall distribution (expansion on the *extensive* margin).

Three Properties of Prices

3. With CES utility, and a Fréchet distribution for the prices actually paid by consumers, the price index takes the following form:

$$\mathsf{p_n} = \gamma \Phi_{\mathsf{n}}^{-\frac{1}{\theta}}, \quad \gamma = \left[\Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right) \right]^{\frac{1}{1-\sigma}}, \tag{9}$$

where Γ is the Gamma function and the parameter restriction $\sigma < 1 + \theta$ must be satisfied for the price index to be well-defined.

States of technology around the world, input costs around the world, and geographic barriers influence the price index through the parameter Φ_{n} for each country.

Trade Flows and Gravity

- An immediate corollary of Property 2 of the price distributions is that a country's average expenditure per good does not vary by source country.
  - All of the adjustment in trade flows is in the *extensive margin* of the number of goods and **not** in the intensive margin of average expenditure per good.
- Therefore, the fraction of goods that country n buys from country i, π_{ni}, is also the fraction of its expenditure on goods from country i:

$$\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_{k=1}^{N} T_k(c_k d_{nk})^{-\theta}} \tag{10}$$

- The exporter i's total sales are:

$$Q_i = \sum_{m=1}^{N} X_{mi} = T_i c_i^{-\theta} \sum_{m=1}^{N} d_{mi}^{-\theta} X_m \Phi_m^{-1} \tag{11}$$

Trade Flows and Gravity (cont'd)

- Using the expression for Q_i above to solve for $T_i c_i^{-\theta}$, and substituting it into (10), we obtain the following gravity equation for bilateral trade flows:

$$X_{ni} = \frac{\left(\frac{d_{ni}}{p_n}\right)^{-\theta} X_n}{\sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m} Q_i \tag{12}$$

- The gravity equation captures the effects of both "*bilateral*" and "*multilateral resistance*".
  - The term $\left(\frac{d_{ni}}{p_n}\right)^{-\theta} X_n$ captures the market size of destination n as perceived by exporter i.
  - The term $\sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m$ captures the total world market as perceived by exporter i.
- Since exporters with a higher state of technology, lower input cost, or lower geographic barriers expand along the extensive margin, the trade elasticity depends on θ (a technology parameter) and **not** σ (the elasticity of substitution).

Equilibrium Input Costs

- To close the model:
  - Decompose the cost of inputs into labor and intermediates.
  - Specify how wages and the price of intermediates are determined.
- The cost of an input bundle in country i is:

$$c_i = w_i^{\beta} p_i^{1-\beta} \tag{13}$$

where w_i is the wage and p_i is the price index capturing the cost of intermediates.

Welfare

- Combining the equation for input costs (13) with the expression for the price index (9) and the share of a country's trade with itself (10), we obtain an expression for real wage:

$$\frac{w_i}{p_i} = \gamma^{-1/\beta} \left(\frac{T_i}{\pi_{ii}} \right)^{1/\beta\theta} \tag{14}$$

- The real wage depends on:
  - Technology parameter T_i.
  - Share of purchases from home π_{ii}.
- Gains from trade follow immediately since $\pi_{ii} = 1$ under autarky and $\pi_{ii} < 1$ under trade.
- For given parameters, $\pi_{ii} < 1$ is a sufficient statistic for the welfare gains from trade (see Arkolakis et al. 2012).
- Country size influences the gains from trade through π_{ii}, and given the import share, trade gains are greater the smaller θ (more heterogeneity in efficiency) and β (larger share of intermediates).

Price Levels

- The price index in each country is a function of the price indices in all other countries and wages.
- The expression for the price index (9) can be re-written using the input cost equation (13) and the definition of Φ_n in (7) as:

$$p_n = \gamma \left[\sum_{i=1}^{N} T_i \left(w_i^{\beta} p_i^{1-\beta} d_{ni} \right)^{-\theta} \right]^{-1/\theta} \tag{15}$$

- The expression for the trade share (10) can also be re-written as a function of prices and wages using the equation for input cost (13), the price index (9) and the definition of Φ_n:

$$\frac{X_{ni}}{X_n} = \pi_{ni} = T_i \left(\frac{\gamma d_{ni} w_i^{\beta} p_i^{1-\beta}}{p_n} \right)^{-\theta} \tag{16}$$

Labor Market Equilibrium

- The theory is interpreted as a model of the manufacturing sector which is now embedded in the general equilibrium of the economy.
- Manufacturing labor income in country i is labor's share of country i's manufacturing sales around the world, including its sales at home:

$$w_i L_i = \beta \sum_{n=1}^{N} \pi_{ni} X_n, \tag{17}$$

  where L_i is manufacturing workers and X_n is total spending on manufactures.
- Total spending on manufactures includes both final consumption and intermediate expenditure:

$$X_n = \alpha Y_n + (1 - \beta) X_n, \tag{18}$$

  where Y_n denotes aggregate final expenditures and α is the fraction spent on manufactures.

Labor Market Equilibrium (cont'd)

- Since $w_n L_n = \beta X_n$, total spending on manufactures can be rewritten as:

$$X_n = \alpha Y_n + \left(\frac{1-\beta}{\beta}\right) w_n L_n \tag{19}$$

- Aggregate final expenditure Y_n is the sum of value-added in manufacturing and income generated in non-manufacturing:

$$Y_n = Y_n^M + Y_n^O \tag{20}$$

- Since labor is the sole primary factor of production, value-added in manufacturing equals payments to labor used in manufacturing, $Y_n^M = w_n L_n$:

$$Y_n = w_n L_n + Y_n^O \tag{21}$$

- **Assume:** incomplete specialization between manufacturing and non-manufacturing.
- **Assume:** non-manufacturing output can be traded costlessly and choose the non-manufacturing good as the numeraire.

Closing the Model

- Focus on Eaton-Kortum's *first approach* to closing the model:
  - Workers can move freely between manufacturing and non-manufacturing.
  - With incomplete specialization and costless trade in non-manufacturing, the wage is determined by productivity in non-manufacturing and aggregate expenditure is determined by endowments, $Y_n = w_n \bar{L}_n$.
- Combining the expressions for manufacturing labor income (17) and total spending on manufactures (18) yields a system of equations that determine manufacturing employment L_i in each country given $\{\pi_{ni}, w_n, Y_n\}$:

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} \left[(1-\beta) w_n L_n + \alpha \beta Y_n \right] \tag{22}$$

- **To solve for general equilibrium:** use the system of equations (15) and (16) to solve for prices and trade shares in each country given wages, then use equation (22) to determine manufacturing employment.

Quantitative Analysis: Gravity

- Divide gravity (10) by the analogous expression for the share of country i producers at home, and using the price index (9):

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \frac{\Phi_i}{\Phi_n} d_{ni}^{-\theta} = \left(\frac{p_i d_{ni}}{p_n} \right)^{-\theta}, \tag{23}$$

  where the LHS is country i's *normalized import share* in country n.

  - Country i's import share in country n relative to country i's import share at home.

- Now examine the partial correlation in the data between country i's *normalized import share* in country n and distance between countries i and n.

Trade and Geography

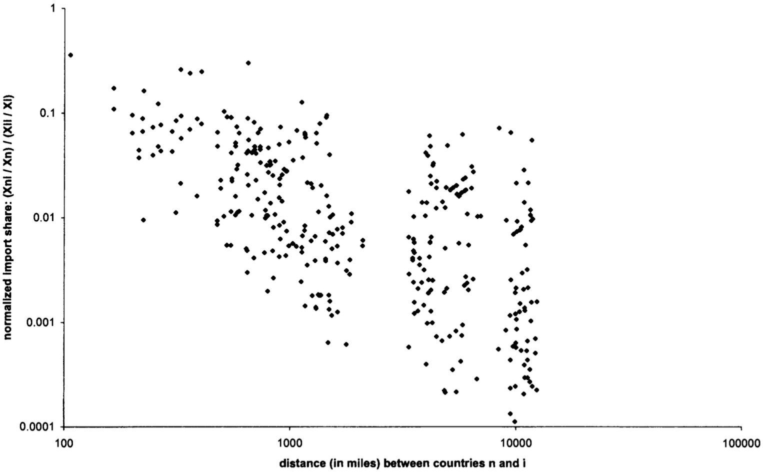


FIGURE 1.—Trade and geography.

Estimating Gravity

- To control for d_{ni} and $\frac{p_i}{p_n}$, use retail prices in 19 countries of 50 manufactured products j:

$$r_{ni}(j) = \ln p_n(j) - \ln p_i(j)$$

- Take as the measure of relative prices:

$$\ln\left(\frac{p_i}{p_n}\right) = -\frac{1}{50}\sum_{j=1}^{50} r_{ni}(j)$$

- In the model, $r_{ni}(j)$ is bounded above by $\ln d_{ni}$, with this bound attained for goods that n imports from i. Therefore measure $\ln d_{ni}$ by the second-highest value of $r_{ni}(j)$ across j:

$$\ln d_{ni} = \max 2_j\{r_{ni}(j)\}$$

- Combining these two components:

$$\left(\frac{p_i d_{ni}}{p_n}\right) = D_{ni} = \frac{\max_{2j}\{r_{ni}(j)\}}{\frac{1}{50}\sum_{j=1}^{50} r_{ni}(j)}$$

Trade and Prices

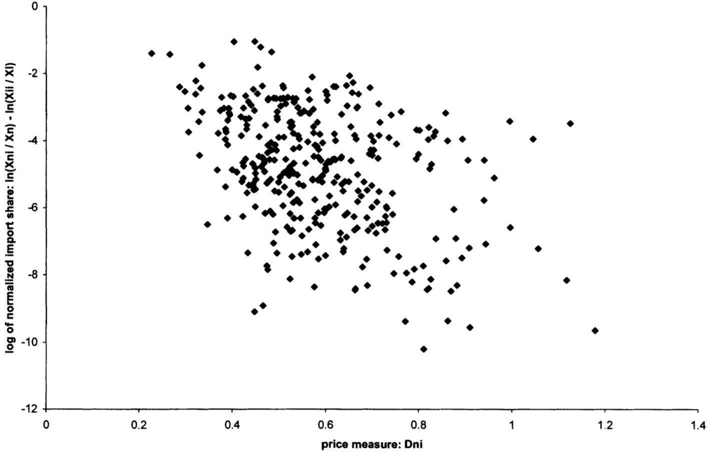


FIGURE 2.—Trade and prices.

Use the slope of this relationship to identify $\theta = 8.28$.

Estimating Gravity

- We have identified one of the two key parameters of the Fréchet distribution, $\theta = 8.28$.
- To identify the other key parameter of the Fréchet distribution, T_i, we use the solution for trade shares; Dividing through in (16) by the importer's home sales, we obtain:

$$\frac{X_{ni}}{X_{nn}} = \frac{T_i}{T_n} \left(\frac{w_i}{w_n} \right)^{-\theta\beta} \left(\frac{p_i}{p_n} \right)^{-\theta(1-\beta)} d_{ni}^{-\theta} \tag{24}$$

- Using the trade share (16) for home sales in *both* country i and country n yields:

$$\frac{p_i}{p_n} = \frac{w_i}{w_n} \left(\frac{T_i}{T_n} \right)^{-1/\theta\beta} \left(\frac{X_i/X_{ii}}{X_n/X_{nn}} \right)^{-1/\theta\beta} \tag{25}$$

- Using this expression for relative prices, we obtain the following expression for the normalized import share.

Estimating Gravity (cont'd)

- Normalized import share equation:

$$\ln \frac{X'_{ni}}{X'_{nn}} = -\theta \ln d_{ni} + \frac{1}{\beta} \ln \frac{T_i}{T_n} - \theta \ln \frac{w_i}{w_n}, \tag{26}$$

  where $\ln X'_{ni} \equiv \ln X_{ni} - \frac{1-\beta}{\beta} \ln \frac{X_i}{X_{ii}}$. Defining:

$$S_i \equiv \frac{1}{\beta} \ln T_i - \theta \ln w_i, \tag{27}$$

  we obtain:

$$\ln \frac{X'_{ni}}{X'_{nn}} = -\theta \ln d_{ni} + S_i - S_n, \tag{28}$$

  where S_i and S_n can be estimated from this gravity equation using exporter and importer fixed effects and controlling for distance.
- The Fréchet parameter T_i can be recovered by purging the exporter fixed effects S_i of wages w_i.

Parameterization

TABLE VIII
SUMMARY OF PARAMETERS

Parameter	Definition	Value	Source
θ	comparative advantage	8.28 (3.60, 12.86)	Section 3 (Section 5.2, Section 5.3)
α	manufacturing share	0.13	production and trade data
β	labor share in costs	0.21	wage costs in gross output
T_i	states of technology	Table VI	source effects stripped of wages
d_{ni}	geographic barriers	Table VII	geographic proxies adjusted for θ

Counterfactuals

- With mobile labor, treat total GDP and wages as fixed.
  - Set GDPs to their actual levels and wages to the baseline.
- With immobile labor, treat non-manufacturing GDP and manufacturing employment as fixed.
  - Set manufacturing employment to its actual level.
  - Set non-manufacturing GDP to actual GDP less the baseline value for labor income in manufacturing.
- Measure overall welfare as real GDP:

$$W_n = \frac{Y_n}{p_n^{\alpha}}$$

Raising Geographic Barriers

TABLE IX

THE GAINS FROM TRADE: RAISING GEOGRAPHIC BARRIERS

	Percentage Change from Baseline to Autarky					
	Mobile Labor			Immobile Labor		
Country	Welfare	Mfg. Prices	Mfg. Labor	Welfare	Mfg. Prices	Mfg. Wages
Australia	−1.5	11.1	48.7	−3.0	65.6	54.5
Austria	−3.2	24.1	3.9	−3.3	28.6	4.5
Belgium	−10.3	76.0	2.8	−10.3	79.2	3.2
Canada	−6.5	48.4	6.6	−6.6	55.9	7.6
Denmark	−5.5	40.5	16.3	−5.6	59.1	18.6
Finland	−2.4	18.1	8.5	−2.5	27.9	9.7
France	−2.5	18.2	8.6	−2.5	28.0	9.8
Germany	−1.7	12.8	−38.7	−3.1	−33.6	−46.3
Greece	−3.2	24.1	84.9	−7.3	117.5	93.4
Italy	−1.7	12.7	7.3	−1.7	21.1	8.4
Japan	−0.2	1.6	−8.6	−0.3	−8.4	−10.0
Netherlands	−8.7	64.2	18.4	−8.9	85.2	21.0
New Zealand	−2.9	21.2	36.8	−3.8	62.7	41.4
Norway	−4.3	32.1	41.1	−5.4	78.3	46.2
Portugal	−3.4	25.3	25.1	−3.9	53.8	28.4
Spain	−1.4	10.4	19.8	−1.7	32.9	22.5
Sweden	−3.2	23.6	−3.7	−3.2	19.3	−4.3
United Kingdom	−2.6	19.2	−6.0	−2.6	12.3	−6.9
United States	−0.8	6.3	8.1	−0.9	15.5	9.3

Notes: All percentage changes are calculated as $100\ln(x'/x)$ where x' is the outcome under autarky ($d_{ni} \to \infty$ for $n \neq i$) and x is the outcome in the baseline.

Reducing Geographic Barriers

TABLE X

THE GAINS FROM TRADE: LOWERING GEOGRAPHIC BARRIERS

	Percentage Changes in the Case of Mobile Labor					
	Baseline to Zero Gravity			Baseline to Doubled Trade		
Country	Welfare	Mfg. Prices	Mfg. Labor	Welfare	Mfg. Prices	Mfg. Labor
Australia	21.1	−156.7	153.2	2.3	−17.1	−16.8
Austria	21.6	−160.3	141.5	2.8	−20.9	41.1
Belgium	18.5	−137.2	69.6	2.5	−18.6	68.8
Canada	18.7	−139.0	11.4	1.9	−14.3	3.9
Denmark	20.7	−153.9	156.9	2.9	−21.5	72.6
Finland	21.7	−160.7	172.1	2.8	−20.9	44.3
France	18.7	−138.3	−7.0	2.3	−16.8	15.5
Germany	17.3	−128.7	−50.4	1.9	−14.3	12.9
Greece	24.1	−178.6	256.5	3.3	−24.8	29.6
Italy	18.9	−140.3	6.8	2.2	−16.1	5.7
Japan	16.6	−123.5	−59.8	0.9	−6.7	−24.4
Netherlands	18.5	−137.6	67.3	2.5	−18.5	65.6
New Zealand	22.2	−164.4	301.4	2.8	−20.5	50.2
Norway	21.7	−161.0	195.2	3.1	−22.9	69.3
Portugal	22.3	−165.3	237.4	3.1	−22.8	67.3
Spain	20.9	−155.0	77.5	2.4	−18.0	−4.4
Sweden	20.0	−148.3	118.8	2.7	−19.7	55.4
United Kingdom	18.2	−134.8	3.3	2.2	−16.4	28.5
United States	16.1	−119.1	−105.1	1.2	−9.0	−26.2

Notes: All percentage changes are calculated as $100\ln(x'/x)$ where x' is the outcome under lower geographic barriers and x is the outcome in the baseline.

Conclusion

- Comparative advantage generates potential welfare gains from trade.
- The extent to which these gains are realized is attenuated by geographic barriers.
- The paper develops a stochastic multi-country Ricardian model that captures these two forces parsimoniously.
- The model delivers equations relating bilateral trade between countries to parameters of technology and geography.
- Using data on bilateral trade, prices, and geography, the paper estimates the parameters of the model and undertakes counterfactual analysis of changes in model parameters.
- The paper provides a framework for the quantitative analysis of the determinants of trade, the pattern of trade, and welfare gains from trade.

Deckle, Eaton & Kortum (2007) Counterfactuals

- Eaton and Kortum (2002) with no outside sector.

Initial Equilibrium

- Wages:

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} w_n L_n$$

- Price Indices:

$$p_n = \gamma \left[\sum_{i=1}^{N} T_i \left(w_i^{\beta} p_i^{1-\beta} d_{ni} \right)^{-\theta} \right]^{-1/\theta}$$

- Trade Shares:

$$\pi_{ni} = \frac{X_{ni}^T}{X_n^T} = \frac{T_i w_i^{-\theta\beta} p_i^{-\theta(1-\beta)} d_{ni}^{-\theta}}{\sum_{k=1}^{N} T_k w_k^{-\theta\beta} p_k^{-\theta(1-\beta)} d_{nk}^{-\theta}}$$

Counterfactual Equilibrium

- Wages:

$$w_i' L_i = \sum_{n=1}^{N} \pi_{ni}' w_n' L_n$$

- Price Indices:

$$p_n' = \gamma \left[\sum_{i=1}^{N} T_i \left(w_i'^{\beta} p_i'^{1-\beta} d_{ni} \right)^{-\theta} \right]^{-1/\theta}$$

- Trade Shares:

$$\pi_{ni}' = \frac{T_i (w_i')^{-\theta\beta} (p_i')^{-\theta(1-\beta)} d_{ni}^{-\theta}}{\sum_{k=1}^{N} T_k (w_k')^{-\theta\beta} (p_k')^{-\theta(1-\beta)} d_{nk}^{-\theta}}$$

Comparative Statics

- Denote the relative value of a variable by $\hat{x} = \frac{x'}{x}$.
- For a given comparative static (e.g. change in trade costs), solve for the implied changes in wages and price indices using import shares and GDP in the initial equilibrium:

$$\hat{w}_i Y_i = \sum_{n=1}^{N} \frac{\pi_{ni} \left(\hat{w}_i\right)^{-\theta\beta} \left(\hat{p}_i\right)^{-\theta(1-\beta)} \left(\hat{d}_{ni}\right)^{-\theta}}{\sum_{k=1}^{N} \pi_{nk} \left(\hat{w}_k\right)^{-\theta\beta} \left(\hat{p}_k\right)^{-\theta(1-\beta)} \left(\hat{d}_{nk}\right)^{-\theta}} \hat{w}_n Y_n$$

$$\hat{p}_n = \left[\sum_{i=1}^{N} \pi_{ni} \left(\hat{w}_i\right)^{-\theta\beta} \left(\hat{p}_i\right)^{-\theta(1-\beta)} \left(\hat{d}_{ni}\right)^{-\theta}\right]^{-1/\theta}$$

$$\pi'_{ni} = \frac{\pi_{ni} \left(\hat{w}_i\right)^{-\theta\beta} \left(\hat{p}_i\right)^{-\theta(1-\beta)} \left(\hat{d}_{ni}\right)^{-\theta}}{\sum_{k=1}^{N} \pi_{nk} \left(\hat{w}_k\right)^{-\theta\beta} \left(\hat{p}_k\right)^{-\theta(1-\beta)} \left(\hat{d}_{nk}\right)^{-\theta}},$$

where $Y_i = w_i L_i$.

Examples of Subsequent Research

- Lucas, Robert E. and Fernando Alvarez (2007) "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade," *Journal of Monetary Economics*, 54(6), 1726-1768.
  - Closes the model without an outside sector to solve for endogenous factor prices.
- Rodriguez-Clare, Andres (2010) "Offshoring in a Ricardian World," *American Economic Journal: Macroeconomics*, 2(2).
  - Analyzes offshoring within the Eaton-Kortum framework.
- Ramondo, Natalia and Andres Rodriguez-Clare (2013) "Trade, Multinational Production and the Gains from Openness," *Journal of Political Economy*, 121(2), 273-322.
  - Analyzes both trade and multinational production within the Eaton-Kortum framework.

Examples of Subsequent Research (cont'd)

- David Donaldson (2018) "Railroads of the Raj: Estimating the Economic Impact of Transportation Infrastructure," *American Economic Review*, 108(4-5), 899-934.
    - Uses the Eaton-Kortum framework to evaluate the impact of the construction of the railway network in Colonial India.
- Arkolakis, Costas, Arnaud Costinot and Andres Rodriguez-Clare (2012) "New Theories, Same Old Gains?" *American Economic Review*, 102(1), 94-130.
    - Shows that in a class of international trade models, welfare can be expressed in terms of the trade share as in the Eaton-Kortum framework.