International Trade: Lecture 2

Factor Endowments and Comparative Advantage

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Fall, 2024

Factor Endowments as Basis for Trade

Comparative advantage requires differences in autarky prices.

One source of these differences can be differences in *factor endowments*.

Idea: Goods can be traded more cheaply than factors.

- When factor endowments differ, factors are expensive where they are relatively scarce.
- Creates incentive for factor trade.
- If factors are costly to move (or there are legal restrictions), it can give rise to systematic differences in autarky prices: Goods whose production "requires" scarce factors will be expensive.
- Goods trade can at least partly alleviate factor scarcity. Countries can import factor services embodied in goods.

Factor content of trade

The Heckscher-Ohlin Model

Two factors: “capital” and “labor” (or “skilled” and “unskilled”).

- Inelastic factor supplies.
- Countries differ in factor endowments: $\frac{K}{L} \neq \frac{K^*}{L^*}$.

Two industries.

- Factors are mobile across sectors: $w_x = w_y = w$, $r_x = r_y = r$.
- CRS technologies, no joint production, perfect competition.
- Sectors differ in factor intensity.

$$\frac{a_{K_x}}{a_{L_x}}\left(\frac{w}{r}\right) \neq \frac{a_{K_y}}{a_{L_y}}\left(\frac{w}{r}\right)$$

- Identical, homothetic preferences for the two goods

The Heckscher-Ohlin Model: Production Equilibrium in One Country

Look at supply-side equilibrium, taking prices as given.

- Profits non-positive:

$$\phi(w, r) \geq p; \quad (x \geq 0) \tag{1}$$

$$wa_{L_x}(w/r) + ra_{K_x}(w/r) \geq p \tag{1'}$$

- Profits non-positive in numeraire sector:

$$\gamma(w, r) \geq 1; \quad (y \geq 0) \tag{2}$$

$$wa_{L_y}(w/r) + ra_{K_y}(w/r) \geq 1 \tag{2'}$$

The Heckscher-Ohlin Model: Production Equilibrium in One Country (cont'd)

- Labor-market clearing:

$$a_{L_x}(w/r)x + a_{L_y}(w/r)y \leq L; \quad (w \geq 0) \tag{3}$$

- Capital-market clearing:

$$a_{K_x}(w/r)x + a_{K_y}(w/r)y \leq K; \quad (r \geq 0) \tag{4}$$

- Goods-market clearing (in autarky):

$$\frac{D_x}{D_y}(p) = \frac{x}{y}$$

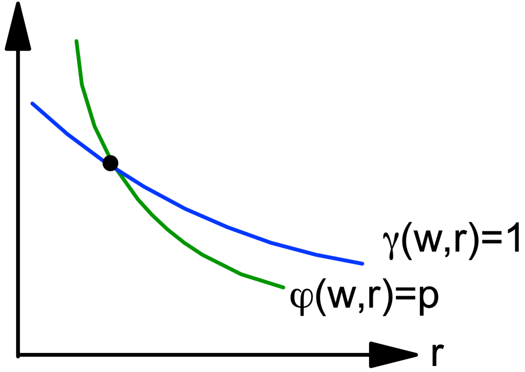
- We will generally assume that the labor and capital market clearing conditions hold as equalities, but we must check that both goods are produced in equilibrium. In any event, equations (1) - (4) determine $\{x, y, w, r\}$ as a function of $\{p, K, L\}$.

The Heckscher-Ohlin Model: Trade Equilibrium

Suppose that two countries trade freely. Free trade equalizes the relative price, p. The foreign country has a system of equilibrium equations analogous to the domestic country.

(i) Let us hypothesize that both countries are incompletely specialized: $x, y, x^*, y^* > 0$**.**

With incomplete specialization, we can graph the equilibrium conditions as equalities.



Slope?

- $\phi_w dw + \phi_r dr = 0$
- $\frac{dw}{dr} = -\frac{\phi_r}{\phi_w} = -\frac{a_{Kx}}{a_{Lx}}$

Single crossing?

- No factor intensity reversal (x is capital-intensive industry).
- (1) and (2) uniquely determine w and r.

The Heckscher-Ohlin Model: Trade Equilibrium (cont'd)

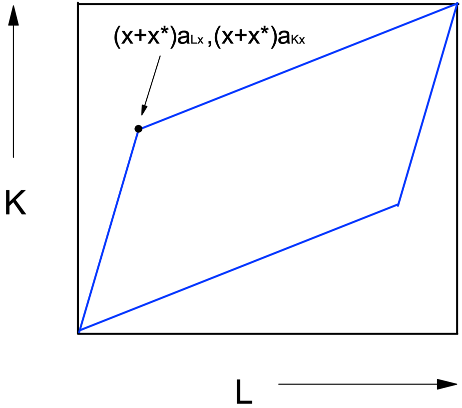
(i) Trade may reproduce the "integrated equilibrium"

- It follows that if both countries happen to produce both goods in a trade equilibrium (incomplete specialization), factor prices will be *equalized* by trade.
- If this is the case, trade reproduces the integrated equilibrium
  - i.e., the immobility of factors does not affect the aggregate amounts produced or consumed in the world.
  - Here, the embodiment of factor services in goods allows goods trade to substitute fully for the non-tradability of factors.

The Heckscher-Ohlin Model: Trade Equilibrium (cont'd)

(ii) When can trade reproduce the integrated equilibrium?

This is the same question as (1) when are factor prices equalized by trade? (2) when are both countries incompletely specialized in the trade equilibrium?



The Heckscher-Ohlin Model: Trade Equilibrium (cont'd)

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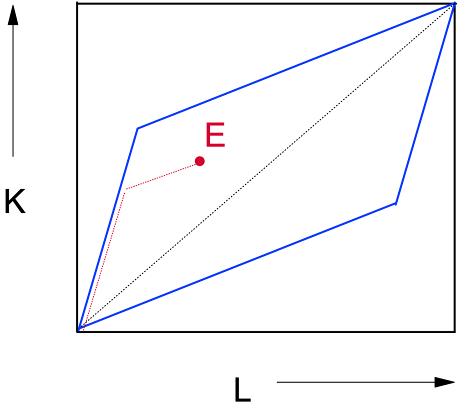

The Heckscher-Ohlin Model: Trade Equilibrium (cont'd)

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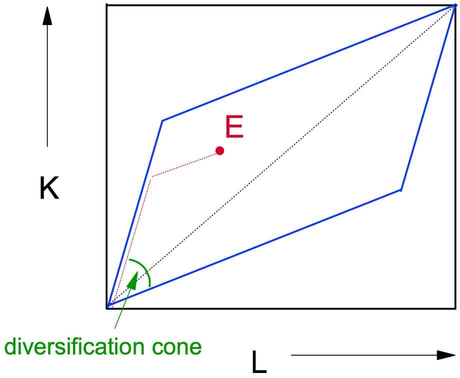


FPE obtains if endowment ratios are not "too" different. (Size of parallelogram depends on factor intensity differences across industries.)

The Heckscher-Ohlin Model: Trade Equilibrium (cont'd)

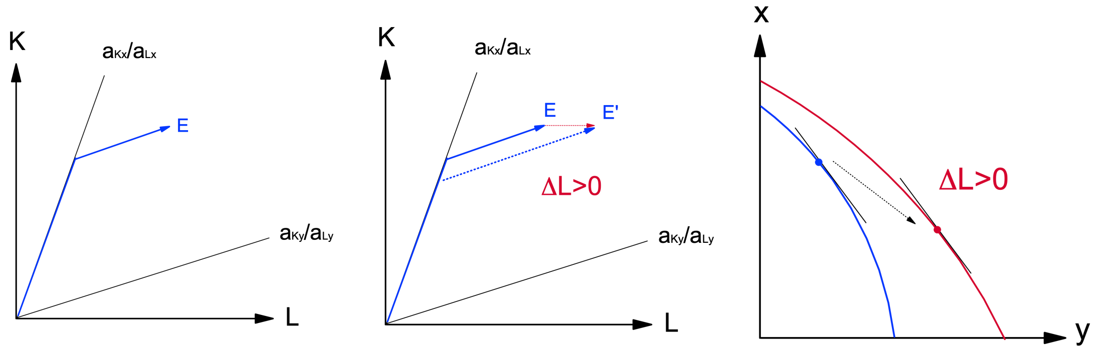
(iii) Implications of FPE for trade pattern

- Incomplete specialization $\Rightarrow$ FPE $\Rightarrow$ $a_{ij} = a^*_{ij}$
- Factor markets must clear in both countries with the same factor intensities, despite different relative factor endowments.
- This implies that relative outputs cannot be the same!
- Differences in relative outputs reflect differences in relative factor endowments.

The Heckscher-Ohlin Model: The Rybczynski Theorem

Theorem (Rybczynski)

When the endowment of one factor is increased at constant prices, this increases the output of the good that uses the factor **more than proportionally**, and decreases the output of the other good.



The Heckscher-Ohlin Model: The Rybczynski Theorem (Algebra)

$$xa_{K_x} + ya_{K_y} = K$$

$$a_{K_x}dx + a_{K_y}dy = dK$$

$$\frac{xa_{K_x}}{K}\frac{dx}{x} + \frac{ya_{K_y}}{K}\frac{dy}{y} = \frac{dK}{K}$$

$$\boxed{\lambda_{K_x}\hat{x} + \lambda_{K_y}\hat{y} = \hat{K}}$$

Similarly $\lambda_{L_x}\hat{x} + \lambda_{L_y}\hat{y} = \hat{L}$

The Heckscher-Ohlin Model: The Rybczynski Theorem (Algebra)

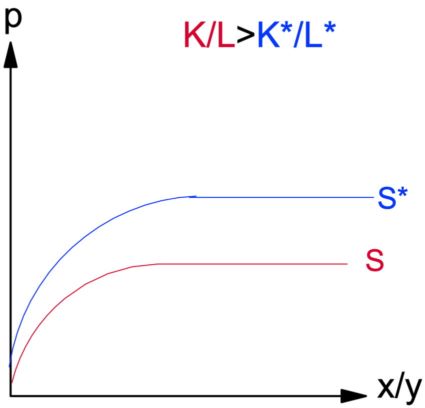
$$
\begin{aligned}
x a_{K_x} + y a_{K_y} &= K \\
a_{K_x} dx + a_{K_y} dy &= dK \\
\frac{x a_{K_x}}{K}\frac{dx}{x} + \frac{y a_{K_y}}{K}\frac{dy}{y} &= \frac{dK}{K} \\
\boxed{\lambda_{K_x}\hat{x} + \lambda_{K_y}\hat{y} = \hat{K}} \\
\text{Similarly } \lambda_{L_x}\hat{x} + \lambda_{L_y}\hat{y} &= \hat{L}
\end{aligned}
$$

$$
\begin{aligned}
\hat{x} &= \frac{\lambda_{L_y}\hat{K} - \lambda_{K_y}\hat{L}}{\Delta}, \hat{y} = \frac{\lambda_{K_y}\hat{L} - \lambda_{L_y}\hat{K}}{\Delta} \\
\Delta &= \lambda_{K_x}\lambda_{L_y} - \lambda_{L_x}\lambda_{K_y} \\
&= \lambda_{K_x}\left(1 - \lambda_{L_x}\right) - \lambda_{L_x}\left(1 - \lambda_{K_x}\right) \\
&= \lambda_{K_x} - \lambda_{L_x} > 0 \\
\Rightarrow \quad & \boxed{\hat{x} - \hat{y} = \frac{\hat{K} - \hat{L}}{\Delta}}
\end{aligned}
$$

Magnification Effect: If $\hat{K} > \hat{L}$, then: $\hat{x} > \hat{K} > \hat{L} > \hat{y}$

The Heckscher-Ohlin Model: Trade Pattern

(iv) Trade Pattern: Factor endowment ratios predict autarky price differences, which in turn predict the pattern of trade.



The Heckscher-Ohlin Model: Trade Pattern

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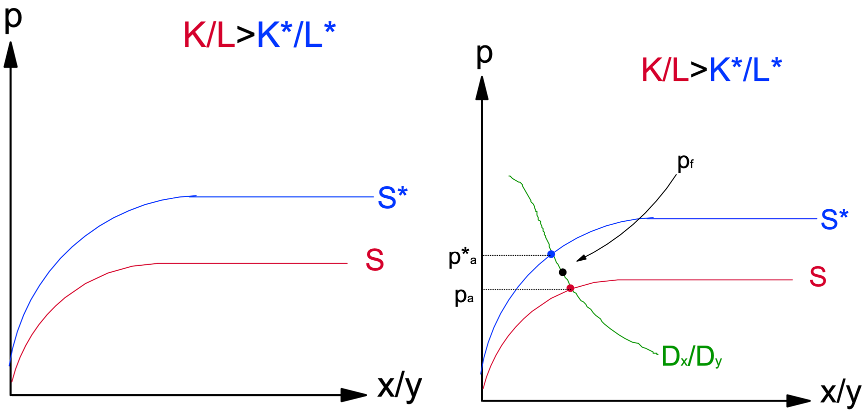
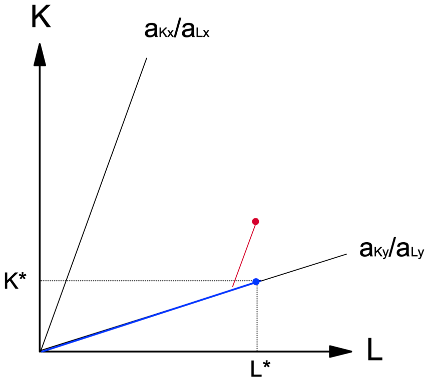


The Heckscher-Ohlin Model: Trade Pattern (cont'd)

Or specialization in at least One Country: Must have either $x^* = 0$ or $y = 0$ or both.



Consider p s.t. $x^* = 0$ but FPE; note $x > 0$

The Heckscher-Ohlin Model

Theorem (Heckscher-Ohlin)

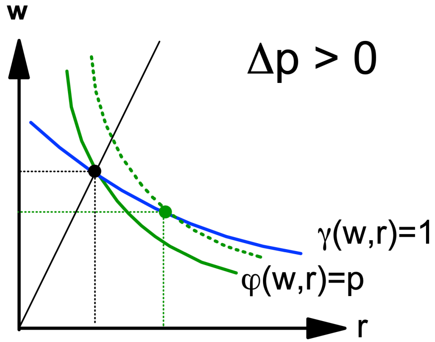
Each country exports the good that makes relatively intensive use of its relatively abundant factor.

Corollary

In each country, trade causes an increase in the relative price of the good that is exported.

The Heckscher-Ohlin Model: Distributional Implications

Factor prices are determined by commodity prices. Relative to autarky, trade increases the relative price of the good that uses the abundant factor intensively.



$\hat{r} > \hat{p} > 0 > \hat{w}$

The Heckscher-Ohlin Model: The Stolper-Samuelson Theorem

Theorem (Stolper-Samuelson)

In the Heckscher-Ohlin model, an increase in the relative price of a good increases the real return to the factor used intensively in producing that good, and decreases the real return to the other factor.

The Heckscher-Ohlin Model: The Stolper-Samuelson Theorem (Algebra)

$$wa_{L_x} + ra_{K_x} = p$$

$$wda_{L_x} + a_{L_x}dw + rda_{K_x} + a_{K_x}dr = dp \text{ (Total differentiation)}$$

$$a_{L_x}dw + a_{K_x}dr = dp \text{ (Application of envelope theorem)}$$

$$\frac{wa_{L_x}}{p}\frac{dw}{w} + \frac{ra_{K_x}}{p}\frac{dr}{r} = \frac{dp}{p}$$

$$\theta_{Lx}\hat{w} + \theta_{Kx}\hat{r} = \hat{p}$$

$$\text{Similarly } \theta_{Ly}\hat{w} + \theta_{Ky}\hat{r} = 0$$

$$\Rightarrow \hat{w} = \frac{\theta_{Ky}\hat{p}}{\theta_{Ky} - \theta_{Kx}} \quad , \quad \hat{r} = \frac{\theta_{Ly}\hat{p}}{\theta_{Ly} - \theta_{Lx}}$$

$$\Rightarrow \begin{cases} \theta_{Ky} - \theta_{Kx} < 0 & \Rightarrow \quad \hat{w} < 0 \\ \theta_{Ly} > \theta_{Ly} - \theta_{Lx} > 0 & \Rightarrow \quad \hat{r} > \hat{p} > 0 \end{cases}$$

The Specific-Factors Model

Introduce another simple model with trade based on factor endowments.

- good for applied work;
- highlights distributional implications of trade.

Three-factor, two-good model:

- One "mobile" factor; Two factors specific to a sector.
- sometimes seen as a "short-run" model with sector specificity

$$w a_{L_x}(w/r_x) + r_x a_{K_x}(w/r_x) = p$$

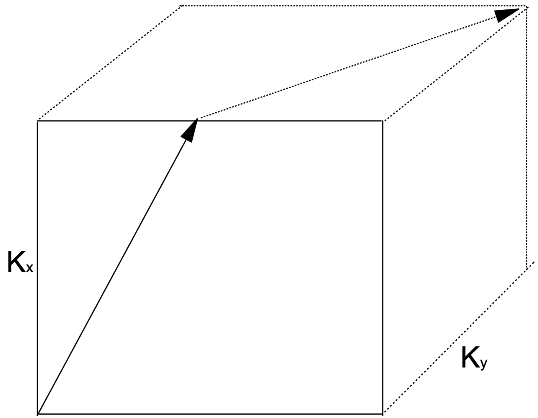
$$w a_{L_y}(w/r_y) + r_y a_{K_y}(w/r_y) = 1$$

$$x a_{L_x}(w/r_x) + y a_{L_y}(w/r_y) = L$$

$$x a_{K_x}(w/r_x) = K_x, \quad y a_{K_y}(w/r_y) = K_y$$

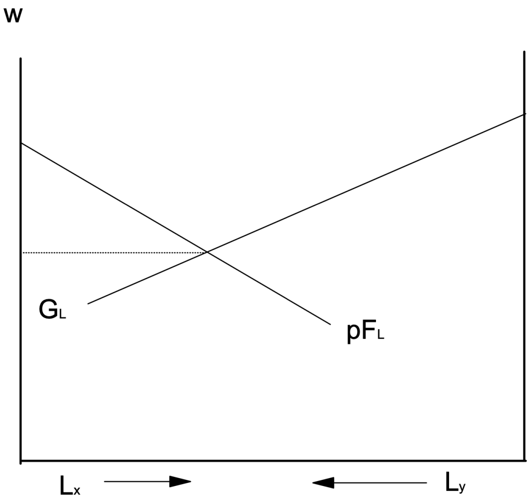
First two equations have three factor prices, so factor prices are not fully determined by commodity prices, even with incomplete specialization.

Specific-Factors Model: Factor Price Equalization Set

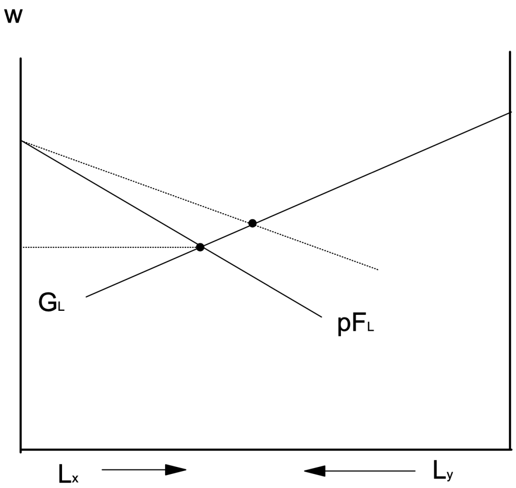


The factor price equalization (FPE) set is two-dimensional (a plane) in three-dimensional space.

Specific-Factors Model: Beaker Diagram

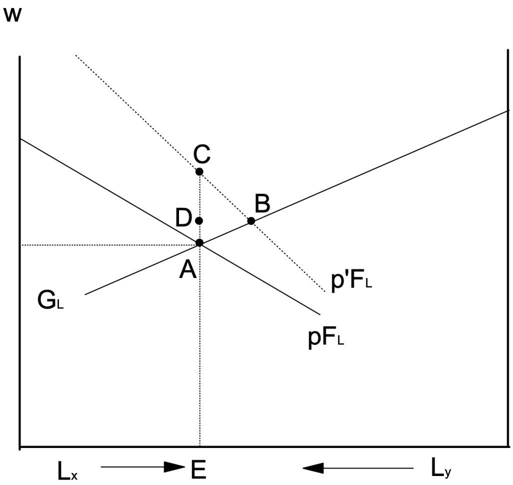


Specific-Factors Model: Pattern of Trade



$$\hat{K}_x > 0 \Rightarrow \hat{K}_x > \hat{L}_x > 0 > \hat{L}_y \Rightarrow \hat{x} > 0 > \hat{y}$$

Specific-Factors Model: Effect of Trade on Income Distribution

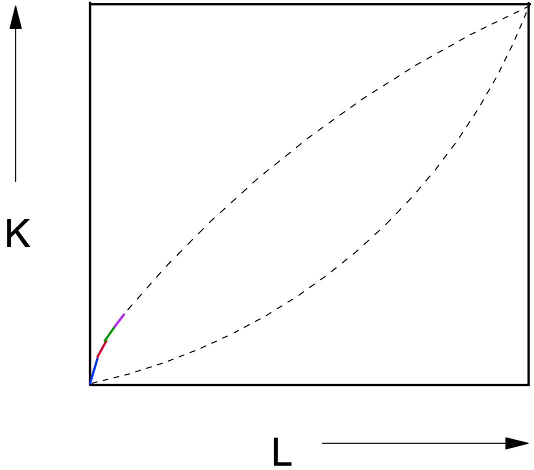


$$\hat{p} = AC/AE, \hat{w} = AD/AE$$

$$\underbrace{\hat{r}_x > \hat{p}}_{\frac{L_x}{K_x}\uparrow \Rightarrow MPK_x \uparrow \Rightarrow \frac{r_x}{p}\uparrow} > \hat{w} > 0 > \hat{r}_y$$

Two Factors, Many Goods: Dornbusch, Fischer, and Samuelson, QJE 1980

- Define $a(i) = \frac{a_K(i)}{a_L(i)}$. Assume $a(i)$ is continuous, differentiable, and $a'(i) < 0$.
- Factor price equalization?



Two Factors, Many Goods: Pattern of Trade with FPE

- With factor price equalization, either country can produce any good from a "competitiveness" standpoint.
- Factor market clearing requires:

$$\int a_{L_x}(i)x(i)di = L$$

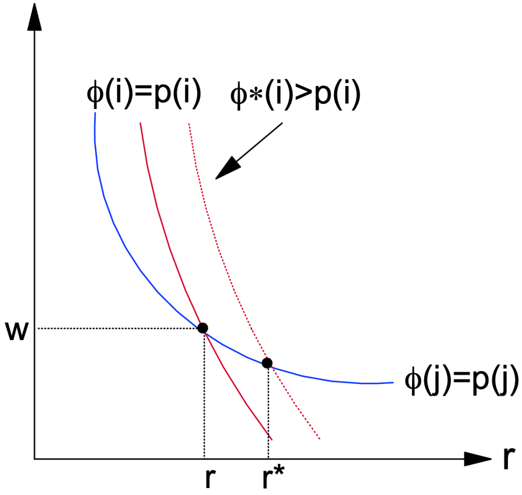
$$\int a_{K_x}(i)x(i)di = K$$

$$x(i) + x^*(i) = \bar{x}(i) \quad \forall i$$

- This can be achieved in many ways. So the pattern of production and trade is not unique.
  - On average, the capital rich country has a more capital intensive production bundle.
  - Capital rich country exports "capital services".

Two Factors, Many Goods: Pattern of Trade without FPE

- Can rank goods by factor intensity.
- If $\frac{r}{w} < \frac{r^*}{w^*}$ and $\phi(j) = \phi^*(j)$, then $\phi(i) < \phi^*(i)$ for $i < j$.
- The capital-rich country produces all goods up to some dividing point i^*, the labor-rich country produces the others. The marginal good i^* is determined, together with factor prices, to ensure that factor markets clear.



Factor Endowments: General Theory

- What can we say about trade patterns when there are m nontradable primary factors and n tradable goods?
- Assume constant returns to scale (CRS) and identical technologies.
- Factor Price Equalization?
  - When will trade equalize factor prices?
  - This is the same as asking, when will trade reproduce the integrated equilibrium?
  - We can represent the **integrated equilibrium** as the competitive outcome of a closed economy with factor endowment vector $\bar{V} = V + V^*$.
  - Let $A(\bar{w})$ be the matrix of input output coefficients, with typical element $a_{ij} = \partial c^j(\mathbf{w})/\partial w_i$

Factor Endowments: General Theory: Factor Price Equalization

- Profit maximization condition:

$$\bar{w}^T A(\bar{w}) \geq \bar{p}^T$$

- Factor markets clearing:

$$A(\bar{w})\bar{x} = \bar{V}$$

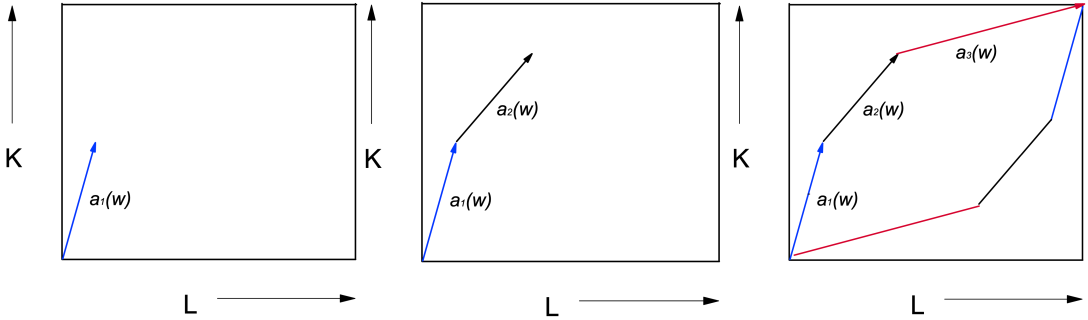
- Product markets clearing:

$$\bar{x} = \left(\bar{w}^T \bar{V}\right) D(p)$$

- $n-1$ relative prices, n outputs, m factor prices.

Factor Endowments: General Theory: Factor Price Equalization (cont'd)

As before, we can depict the equilibrium in factor space



Factor Endowments: General Theory: Factor Price Equalization (cont'd)

- In a *trade equilibrium*, the factor markets must clear separately in each country. If trade can reproduce the integrated equilibrium, there are no consequences of the immobility of factors.
- We can hypothesize that trade reproduces integrated equilibrium and check for contradictions.
- Suppose
  - $\mathbf{w} = \mathbf{w}^* = \mathbf{w}$
  - $A(\mathbf{w}) = A(\mathbf{w}^*) = A(\mathbf{w})$
- Must find $\mathbf{x}, \mathbf{x}^* \geq 0$ such that
  - $\mathbf{x} + \mathbf{x}^* = \mathbf{x}$
  - $A(\mathbf{w})\mathbf{x} = \mathbf{V}$
  - $A(\mathbf{w})\mathbf{x}^* = \mathbf{V}^*$

Factor Endowments: General Theory: Factor Price Equalization (cont'd)

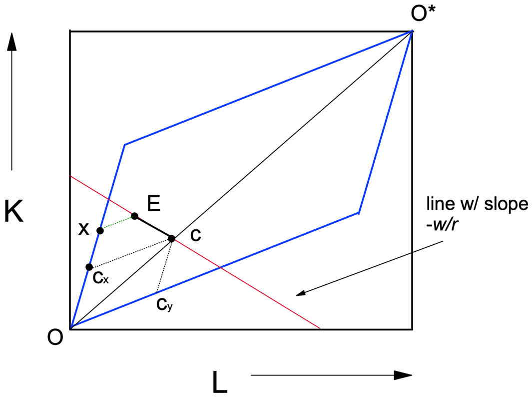
This is possible any time the endowment point lies inside the "parallelogram"; i.e., if endowments are not too different.

- The n goods form a set of at most n dimensions in the m dimensional factor space. (Less than n dimensions if factor use in one industry is a linear combination of some others.) So FPE is "unlikely" if $m > n$.
- When $m < n$, there is more than one allocation that reproduces the integrated equilibrium. All of these allocations satisfy the conditions for equilibrium; i.e., factor markets and goods markets clear. So production patterns in each country are indeterminate.

The Factor Content of Trade

- Intuitively, if factors could move freely so that they were available in the same proportions everywhere, then there would be no need for commodity trade.
- We can think of goods trade as a partial (or complete) substitute for factor movements.
    - Trade allows countries to acquire services of factors that are in scarce supply at home and sell services of factors of factors that are in abundant supply.
    - This motivates discussion of the *factor content of trade*.
- We can begin by looking at the 2×2 Heckscher-Ohlin case in this light.

The Factor Content of Trade (cont'd)



- GDP at point c is the same as GDP at point E. So GDP/GDP* = Oc/cO*.
- The home GDP share is:

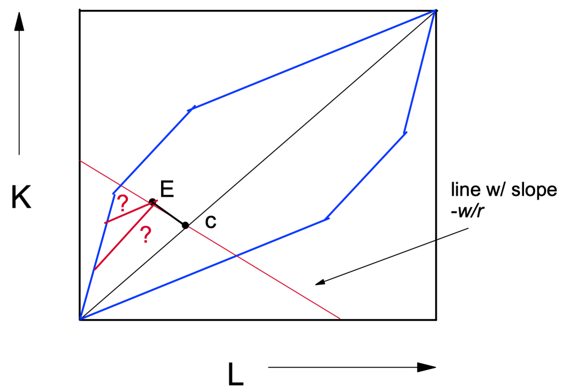
$$s = \frac{\text{GDP}}{\text{GDP} + \text{GDP}^*} = \frac{\text{Oc}}{\text{OO}^*}$$

- With identical, homothetic preferences, s is the home share of consumption of every good. Also, s is home share of consumption of services of every factor.
    - c_x is home consumption of resources used in producing good x.
    - c_y is home consumption of resources used in producing good y.
    - Ec is vector of trade in factor services.

The Factor Content of Trade (cont'd)

Now look at 2x3 case with FPE.

Pattern of trade is indeterminate, but the factor content of trade is E_c.



The Factor Content of Trade: General Case with Universal FPE

Let $s = (w \cdot V)/(w \cdot \bar{V})$; s is home income share.

Let V^t be the vector of factor content of net exports.

$$V^t = -A(w)M = A(w)(x - c) = A(w)(x - s\bar{x}) = V - s\bar{V}.$$

Order the factors so that $\frac{V_1}{\bar{V}_1} > \frac{V_2}{\bar{V}_2} > \ldots > \frac{V_m}{\bar{V}_m}$

Theorem (Travis-Vanek)

A country exports the services of all factors $1, 2, \ldots, J$ and imports the services of factors $J+1, J+2, \ldots, M$, for some J.

Proof of Travis Vanek: $V_i^t = V_i - s\bar{V}_i \Rightarrow \frac{V_i^t}{\bar{V}_i} = \frac{V_i}{\bar{V}_i} - s$. Therefore, $\frac{V_i^t}{\bar{V}_i} > \frac{V_{i+1}^t}{\bar{V}_{i+1}}$.

The Factor Content of Trade: Case without FPE

- Without FPE:
- We immediately confront problem of which input coefficients to use in calculating factor content: Best to use coefficients of country that produces the good (but not always how it has been done in empirical work).
- Define
  - E^{kj}: the vector of *gross exports* from country k to country j
  - $V^{kj} = A(w^k)E^{kj}$, the factor content of k's gross exports to j
  - $\tilde{V}^{kj} = V^{kj} - V^{jk}$, the net factor content of k's trade with j

The Factor Content of Trade: Case without FPE (cont'd)

Theorem (Helpman, EJ 1984)

$\left(w^j - w^k\right)\tilde{V}^{kj} \geq 0$; i.e., on average, bilateral factor content of trade confirms relative factor scarcities, as measured by equilibrium factor prices.

The Factor Content of Trade: Case without FPE (cont'd)

Lemma

In a free trade equilibrium, $p \cdot \left(x^k + E^{jk}\right) \leq R\left(p, V^k + V^{jk}\right)$, where $R\left(p, V\right) = \max_x p \cdot x$ such that (x, v) feasible.

Proof of Lemma: Country k has access to the same technology as j. Therefore, it could produce goods with value $p \cdot E^{jk}$ with the factors V^{jk} by using the same techniques.

The Factor Content of Trade: Case without FPE (cont'd)

Proof of Theorem:

$$p \cdot \left(x^k + E^{jk}\right) \leq R\left(p, V^k + V^{jk}\right) \overbrace{\leq}^{\text{(Why?)}} R\left(p, V^k\right) + w^k \cdot V^{jk} = p \cdot x^k + w^k \cdot V^{jk}$$

$$\Rightarrow \quad p \cdot E^{jk} \leq w^k \cdot V^{jk}$$

But $p \cdot E^{jk} = w^j \cdot V^{jk}$ (Why?)

Therefore $\left(w^k - w^j\right) V^{jk} \geq 0$

Similarly, $\left(w^j - w^k\right) V^{kj} \geq 0$

Now add.

The Pattern of Trade: Case with FPE

- Cannot make predictions about individual goods, because production pattern is indeterminate.
- Have seen that

$$V^t = -A(w)M = V - s\bar{V}$$

- So,

$$\left(V - s\bar{V}\right)^T A(w)M \leq 0,$$

i.e., a country tends to import goods that make intensive use of its scarce factors.

The Pattern of Trade: Case without FPE

- Ethier proves similar result using mean value theorem. For thus, must evaluate $A(\cdot)$ at some unobservable intermediate value $\tilde{w}$ (a linear combination of w and w^*).
- Dixit-Norman consider case where home and foreign endowments differ by dV. Then

$$\left(dV\right)^T A(w)M \leq 0.$$