Name: Tanvir Ahmmed Sifat

ID: IT24646

① Let G be a group of order pq, where p and q are distinct primes. Prove that G is abelian.

Answer: False.

Reason: Counter example S_3, $|S_3| = 6 = 2 \cdot 3$ but S_3 is non-abelian.

② Prove that if G is a group of order p^2 where p is prime, then G is abelian if and only if it has subgroups of order p.

Answer: False.

Reason: Every group of order p^2 is abelian.

There are exactly ~~the~~ two types: C_{p^2} and $C_p \times C_p$.

~~There~~ correct equivalence is $G \cong C_p \times C_p \Leftrightarrow G$ has $p+1$ subgroups of order p". Both groups are abelian.

(3) Let G be a finite group and H be a proper subgroup of G, Prove that the union of all conjugates of H cannot be equal to G.

Answer: False.

Reason: For finite G the union of conjugates of a proper subgroup can not be cover G. A finite group cannot be a union of finitely many proper subgroups.

(5) Prove that in any group G, the set of elements of finite order forms a subgroup of G.

Answer: False.

Reason: In abelian group yes, but not always, Example: infinite dihedral group two reflections (order 2) multiply to a rotation of infinite order, so ~~clous~~ closure fails.

⑥ Let G be a finite group and p be the smallest prime dividing $|G|$. Prove that any subgroup of index p in G is normal.

Answer: True.

Reason: Action on cosets gives homomorphism into S_p; using smallest prime property the image must force the subgroup to be normal.

⑨ Let G be a finite group and p be a prime number. If G has exactly one subgroup of order p^k for each $k \leq n$, where p^n divides $|G|$, Prove that G has a normal Sylow p subgroup.

Answer: True

Reason: Unique subgroup of order p^n is the unique Sylow p-subgroup, and uniqueness implies normality.