1. Solve each of the following sets of simultaneous congruenas

ⓐ $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$

<u>Sol^n</u>: Product of all moduli, $M = 3 \times 5 \times 7 = 105$

We can compute partial moduli, dividing M by each modulus:

$$M_1 = \frac{105}{3} = 35, \quad M_2 = \frac{105}{5} = 21, \quad M_3 = \frac{105}{7} = 15$$

Inverse of $M_i \bmod m_i$: where m_i are 3, 5, 7

1. $35 \bmod 3 = 2$ inverse of $35 \bmod 3 = 2$

2. $21 \bmod 5 = 1$ inverse of $21 \bmod 5 = 1$

3. $15 \bmod 7 = 1$ inverse of $15 \bmod 7 = 1$

total weighted sum $= 1 \cdot 35 \cdot 2 + 2 \cdot 21 \cdot 1 + 3 \cdot 15 \cdot 1$

$= 70 + 42 + 45$

$= 157$

$x \equiv 157 \bmod 105 \Rightarrow x = 52$ (or 1 reminder 52)

$\therefore x \equiv 52 \pmod{105}$

(b) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$

<u>Soln:</u> products of all moduli $M = 11 \cdot 29 \cdot 31$

$= 9889$

partial moduli:

$M_1 = \frac{9889}{11} = 899$, $M_2 = \frac{9889}{29} = 341$

$M_3 = \frac{9889}{31} = 319$

Modular inverse of M_i and m_i or y_i:

We know, $M_i y_i \equiv 1 \pmod{m_i}$

1. M_i ~~dod~~ mod m_i = $899 \bmod 11 = 8$

$8 \cdot y_1 = 1 \bmod (11)$ $8 \times 7 = 56 \equiv 1 \Rightarrow y_1 = 7$.

2. $341 \bmod 29 = 22$

$22 \cdot y_2 = 1 \bmod (29)$. $22 \times 4 = 88 \equiv 1 \Rightarrow y_2 = 4$

3. $319 \bmod 31 = 9$. $9y_3 = 1 \bmod (31) \Rightarrow 9 \times 7 = 63 \equiv 1 \Rightarrow y_3 = 7$

total ~~s~~um $= 5 \cdot 899 \cdot 7 + 14 \cdot 341 \cdot 4 + 15 \cdot 319 \cdot 7$

$= 84056$

$x \equiv 84056 \bmod 9889 \Rightarrow x = 4944$ or (8·rem 4944)

$x \equiv 4944 \pmod{9889}$

c) $x \equiv 5 \bmod 6$, $x \equiv 4 \bmod 11$, $x \equiv 3 \bmod 17$

Soln: Products of the moduli; $M = m_1 \times m_2 \times m_3$

$= 6 \cdot 11 \cdot 17 = 1122$

partial moduli : $M_1 = \frac{1122}{6} = 187$

$M_2 = 1122/11 = 102$

$M_3 = 1122/17 = 66$

Modular Inverse : $M_i y_i = 1 \bmod M_i$

1. $M_i \bmod m_i = 187 \bmod 6 = 1$

$1 \times 1 = 1 \Rightarrow y_1 = 1$

2. $102 \bmod 11 = 3$ $3 \times 4 = 12 \equiv 1 \Rightarrow y_2 = 4$

3. $66 \bmod 17 = 15$ $15 \times 8 = 120 \equiv 1 \Rightarrow y_3 = 8$

total sum $= 5 \cdot 187 \cdot 1 + 4 \cdot 102 \cdot 4 + 3 \cdot 66 \cdot 8$

$= 4151$

$x \equiv 4151 \bmod 1122 \Rightarrow x = 785$ or (3 reminder 785)

$x \equiv 785 \bmod 1122$