

Question: Is set of odd numbers with binary operations (+), i.e.  $\langle O, + \rangle$  an abelian group? If not explain the reasons with necessary notations.

Solution:

$$O = \{ \dots -3, -1, 1, 3, 5, 7, 9, 11, 13, 15, \dots \}$$

1. Closure: if  $a, b \in O$  then  $a+b \in O$

$$\text{Let, } a = 3$$

$$b = 5$$

$$\text{then, } 3+5=8 \text{ not } \in O$$

So, condition fails this requirement.

2. Identify element: 0 is even number, so in  $e+a \in G$  is work out.

3. Associative: If  $\forall a, b, c \in O$  then  $a+(b+c) = (a+b)+c \in O$

$$\text{Let, } a = 3$$

$$b = 5$$

$$c = -5$$

$$\text{then, } 3+(5-5) = (3+5)-5 = 3 \in O$$

So, this condition is satisfying.

4. Inverse Element: If  $a, a' \in O$  and  $(a+a') = e \in O$

$$\text{Let } a = 3$$

$$a' = -3$$

$$\text{then, } 3 - 3 = 0 \in O$$

So, the set is not satisfying the inverse property.

5. Commutative: If  $\forall a, b \in O$ , then  $(a+b) = b+a \in O$

$$\text{Let } a = 5$$

$$b = 7$$

then,

$$5 + 7 \neq 7 + 5 = 12 \text{ not } \in O$$

So, this condition is satisfying.

Here, given odd numbers set is not an abelian because it is not satisfying all condition of abelian.