Hands on #6

① shared in github repository.

② shared in github repository.

③ Mathematically derive the average runtime complexity of the non-random pivot version of quick sort.

let,
the size of the array $= n$.

Partition step takes $\Theta(n)$ time to rearrange elements around the pivot.

If the pivot's rank is k, we get two subarrays of sizes k and $n-1-k$.

Now, the recurrence,

$$T(n) = \Theta(n) + \frac{1}{n} \sum_{k=0}^{n-1} \left(T(k) + T(n-1-k) \right)$$

$$T(n) = \theta(n) + \frac{1}{n}\sum_{k=0}^{n-1}\left(T(k) + T(n-1-k)\right)$$

$$T(n) = cn + \frac{1}{n}\sum_{k=0}^{n-1}\left[T(k) + T(n-1-k)\right]$$

Combining the terms,

$$T(n) = cn + \frac{2}{n}\sum_{k=0}^{n-1} T(k)$$

lets define, $S(n) = \sum_{j=0}^{n} T(j)$

then it becomes,

$$T(n) = S(n) - S(n-1)$$

$$S(n) - S(n-1) = cn + \frac{2}{n} S(n-1)$$

$$S(n) = S(n-1)\left(1 + \frac{2}{n}\right) + cn$$