CE141-4-FY: Mathematics for Computing

Problem Sheet I.1

Work on the following questions, preferably in pairs, discussing what you are doing as you work.

1. Which of the following is a proposition? If you think that something is not a proposition, explain why it is not.

   (a) Essex has the highest mountains in England.

   (b) Is Colchester East of London?

   (c) Colchester was once a Roman town.

   (d) Colchester is a great place to live.

   (e) Eat more fruit and vegetables.

   (f) Wind farms are a profitable investment.

2. Which of the following strings, involving the propositions p, q and r, are valid logical expressions? If you think something is not a valid expression, explain why it is not.

   (a) $p \wedge (q)$

   (b) $p \vee \neg q$

   (c) $pq \vee r$

   (d) $\mathrm{T} \vee q$

   (e) $p(\vee q \wedge r)$

3. For the following "variable" propositions, identify the quantity (or quantities) on which each depends (in the sense that the bus example in the notes depends on the day of the week).

   (a) Olympic Games will be held next year.

   (b) The rocks 100 m below your feet contain oil.

   (c) The sea is visible from the window.

4. One way of thinking about a compound proposition is that it is a *function* of the (elementary) propositions on which it depends. For example, the “OR function” can be defined as follows:

$$f(p,q) := p \vee q$$

You can think of this function as a “machine” that takes the values (T or F) of its inputs (p and q), and outputs the corresponding value of $p \vee q$. (In digital circuit design an OR-gate is a physical implementation of such a machine.)

The “NAND function” is also frequently used in digital circuit design; it is defined as follows:

$$f(p,q) := \neg(p \wedge q)$$

(a) Construct a truth table for the NAND function.

(b) From your truth table, show that $f(p,p) = \neg p$

5. Construct truth tables for the following compound propositions:

(a) $f(p,q) := \neg p \vee q$

(b) $f(p,q) := \neg p \wedge q$

(c) $f(p,q) := \neg p \vee \neg q$

(d) $f(p,q) := \neg(p \vee q)$

(e) $f(p,q) := p \vee (\neg p \vee q)$

(f) $f(p,q) := (p \wedge \neg p) \vee q$

(g) $f(p,q) := p \wedge (q \vee p)$

6. Compare your answers to parts (c) and (d) of Question 5. What conclusions can you draw?

7. Compare your answers to Question 4 (a) and Question 5 (c). What conclusions can you draw? Look at the module notes to see if you can find a *name* for this result.

8. Two trains are travelling at the same speed on the railway line between Fort William and Mallaig (a single-track line in the Scottish Highlands). Consider the following propositions:

p := "Train 1 is travelling towards Mallaig"

q := "Train 2 is travelling towards Mallaig"

r := "It is possible for the trains to collide"

The proposition r can be expressed as a compound proposition in p and q: $r = f(p,q)$.

(a) Construct the truth table for this compound proposition.

(b) Construct the truth table for the function

$$g(p,q) := (p \wedge \neg q) \vee (\neg p \wedge q).$$

Hence show that $g \equiv f$. (I.e. f and g are identical functions.)

9. Mallaig is West of Fort William. Consider, in the context of Question 8, the proposition

s := "A collision between the trains will occur"

This cannot be exressed as a compound proposition in p and q. Why not? Can you think of another elementary proposition, t, such that $s = f(p,q,t)$ for an appropriate "three input" function f.

10. Construct the truth tables for the following compound propositions:

(a) $f(p,q,r) := (p \vee q) \vee r$

(b) $f(p,q,r) := (p \wedge q) \wedge r$

(c) $f(p,q,r) := p \vee (q \wedge r)$

(d) $f(p,q,r) := (p \vee q) \wedge r$

(e) $f(p,q,r) := ((p \wedge \neg q) \wedge r) \vee ((\neg p \wedge q) \wedge r)$

(f) $f(p,q,r) := ((p \wedge \neg q) \wedge \neg r) \vee ((\neg p \wedge \neg q) \wedge r) \vee ((\neg p \wedge q) \wedge \neg r) \vee ((p \wedge q) \wedge r)$

(g) $f(p,q,r) := (\neg p \vee \neg q) \vee \neg r$

(h) $f(p,q,r) := \neg((p \wedge q) \wedge r)$

11. A compound proposition depends on three elementary propositions: p, q and r. The compound proposition can be expressed in words as

"An odd number of the propositions p, q and r are true"

Construct the truth table for this compound proposition.

12. Which of the compound propositions in Question 10 (if any) is identical to that in Question 11?

13. Compare the truth tables of parts (g) and (h) in Question 10. What conclusions can you draw?

14. By constructing truth tables for the compound propositions

$$p \vee (q \vee r), \quad q \vee (p \vee r),$$

and comparing your answers with that for part (a) of Question 10, show that

$$(p \vee q) \vee r \equiv p \vee (q \vee r) \equiv q \vee (p \vee r)$$

15. Question 14 goes part way to justifying the syntax $p \vee q \vee r$ (containing no brackets). How many re-arrangements of the elementary propositions p, q and r, and the brackets (), would be needed to **fully** justify this?

16. How would you justify the syntax $\neg p \vee \neg q \vee \neg r$?

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