Ma.1 STAT220 sigbjørn Fjelland

Sigbjørn Fjelland

2022-09-12

Problem 1.1

A: One numbers on dice

$\bar{A}$: Anything but A on a dice

$P(A) = 1/6 \qquad P(\bar{A}) = 5/6$

Possible outcome on three rolls:

$$\underbrace{A\,A\,\bar{A}}_{A \cap A \cap \bar{A} \,\ldots} \cup A\,\bar{A}\,A \cup \bar{A}\,A\,A$$

$$\Rightarrow P(AA\bar{A}) \cup P(A\bar{A}A) \cup P(\bar{A}AA)$$

$$= (1/6)^2 \cdot (5/6) + (1/6)^2 \cdot (5/6) + (1/6)^2 \cdot (5/6)$$

$$= 3(1/6)^2 \cdot (5/6) = \underline{5/72}$$

This result has not discounted that $\bar{A}$ can have 6 possible values such that we need to find:

$$6\left(P(AA\bar{A}) \cup P(A\bar{A}A) \cup P(\bar{A}AA) \right)$$

$$\Rightarrow 6 \cdot \frac{5}{72} = \frac{5}{12} \approx 0{,}416$$

$$\underline{\underline{\approx 4{,}2\,\%}}$$

Problem 1.2

$$P(A|B) > P(A) \quad | : P(A)$$

$$\Rightarrow \frac{P(A|B)}{P(A)} > 1$$

close to Bayes

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$\hookrightarrow \frac{P(A|B)}{P(A)} = \frac{P(B|A)}{P(B)}$$

the Same apply to $P(B|A)$

$$\frac{P(B|A)}{P(B)} > 1$$

Hence the occurrence of B make A more likely <u>and</u> vica versa

Problem 1.3

$$P(H) = P(T) = 1/2$$

a) $P(H\ H\ H\ H) = P(H) \cdot P(H) \cdot P(H) \cdot P(H)$

$$= (1/2)^4 = \underline{\underline{1/16}}$$

b) $P(T\ H\ H\ H) = \underbrace{P(T)}_{1/2} \cdot \underbrace{P(H) \cdot P(H) \cdot P(H)}_{(1/2)^3}$

$$= 1/2 \cdot (1/2)^3 = \underline{\underline{1/16}}$$

(Problem 1.3)

c) $A: HHHH \rightarrow P(A) = p^4$

$B: THHH \rightarrow P(A) = p^3(1-p)$

to obtain A before B

we have to throw 4

p

Heads in a row, such

that:

$P(B \text{ before } A) = 1 - P(A)$

$= 1 - p^4$

$= 1 - 1/16$

$= 15/16 \approx \underline{93,75\%}$

Problem 1.4

The sample space for throwing a coin till two heads apear has an infinite sample space with 2 throws as min.

$S = \{HTHTTHTHTTT\ldots HH\} \to \infty$

Within 4 throws of a fair coin?

$S = \{HTHH, TTHH\}$

We have that

$\#$ outcomes in total $= \left(\frac{1}{2}\right)^4$

· favorable $\#$ outcom $= 2$

$$\Rightarrow P(T = 4) = 2 \times \left(\frac{1}{2}\right)^4 = \frac{2}{16} = \underline{\underline{\frac{1}{8}}}$$

Problem 1.6

For $n=2$:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= P(E_1) + P(E_2) - P(E_1)P(E_2)$$

$$\leq P(E_1) + P(E_2)$$

Inductive step:

$$P\left(\bigcup_{i=1}^{n+1} E_i\right) = P(E_1) \cup P(E_2) \cup \ldots \cup P(E_n) \cup P(E_{n+1})$$

$$= P(E_1) + P(E_2) + \ldots + P(E_n) + P(E_{n+1})$$

$$- P(E_1) \cdot P(E_2) \cdots P(E_n) \cdot P(E_{n+1})$$

$$= \sum_{i=1}^{n+1} P(E_i) - \prod_{i=1}^{n+1} P(E_i)$$

$$\leq \sum_{i=1}^{n+1} P(E_i)$$

∎

Problem 1.8

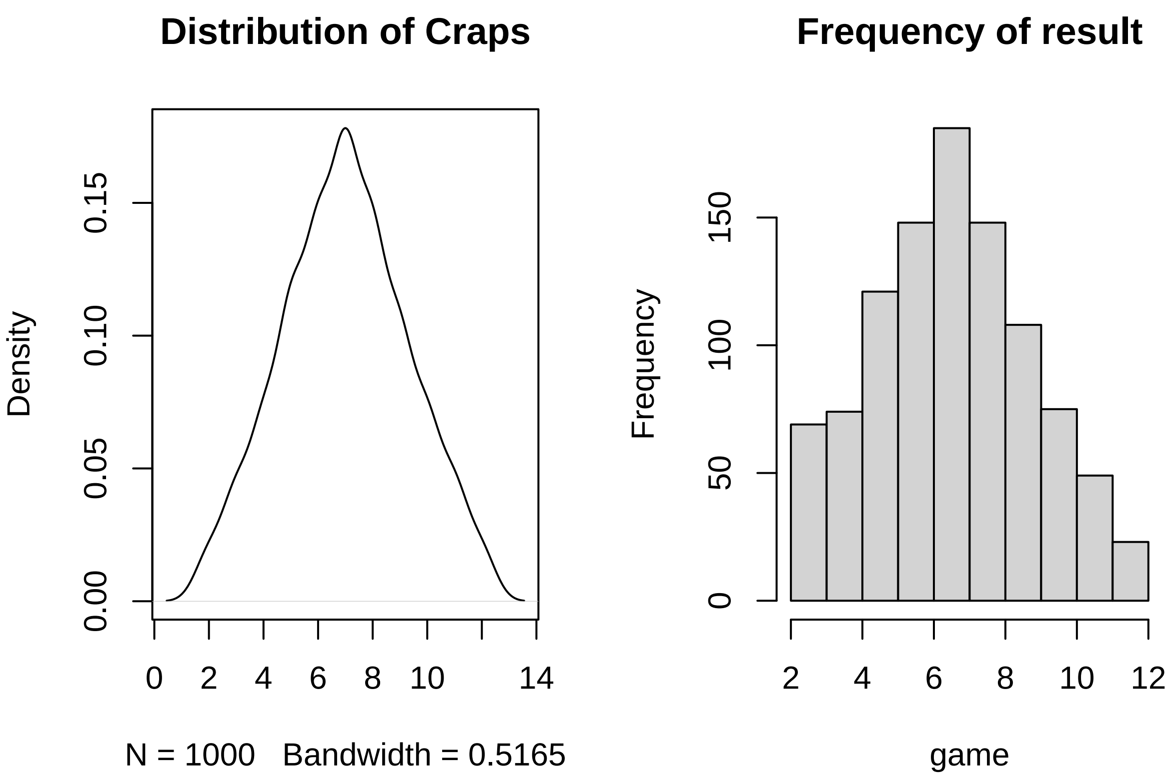
```r
craps <- function(n){

  result_vector <- c()

  for(i in 1:n){
    D<-sample(1:6, size = 2, replace = T, prob = rep(1/6,6) )
    result_vector <- c(result_vector, sum(D))
  }
 result_vector
}

game <- craps(1000)
game_infrence <- density(game)

#plots
par(mfrow=c(1,2))
plot(game_infrence, main = "Distribution of Craps")
hist(game, breaks = 12, main = "Frequency of result")
```



```r
print(game_infrence)
```

```
##
## Call:
##  density.default(x = game)
##
## Data: game (1000 obs.);  Bandwidth 'bw' = 0.5165
##
##        x                 y
##  Min.   : 0.4506   Min.   :0.0001918
##  1st Qu.: 3.7253   1st Qu.:0.0249313
##  Median : 7.0000   Median :0.0679298
##  Mean   : 7.0000   Mean   :0.0762640
##  3rd Qu.:10.2747   3rd Qu.:0.1260278
##  Max.   :13.5494   Max.   :0.1781401
```

Problem 1.9

X: Question

$P(X) = 1/3$ $\quad P(\overline{X}) = 2/3$

One wrong = {RRRW, RRWR, RWRR, WRRR}

No wrong = {RRRR}

$P(\text{one wrong}) = 4 \cdot (1/3)^3 \cdot (2/3) = \underline{8/81}$

$P(\text{No wrong}) = (1/3)^4 = \underline{1/81}$

$P(\text{at least 4 correct}) = 1/81 + 8/81 = 9/81$

$= 1/9 \approx \underline{\underline{11{,}1\,\%}}$

Problem 1.10

$$\{T_{(4)} = 7\} = \{X_6 = 3\}$$

$$NB(7, 4, p)$$

$$P(T_{(4)} = 7) = P(X = 3, n = 6)$$

$$= \binom{7-1}{4-1} p^{4-1} (1-p)^{7-4}$$

$$= \binom{6}{3} p^3 (1-p)^3$$

$$= \underline{\binom{6}{3} (p(1-p))^3}$$

$$\underset{p}{\arg\max}\,\{P(T_{(4)}=7)\} = \underset{p}{\arg\max}\,(p(1-p))$$

$$\frac{\partial}{\partial p}\, p(1-p) = 0$$

$$1-2p = 0$$

$$2p = 1$$

$$\underline{\underline{p = \frac{1}{2}}}$$

Problem 1.11

Assuming this is Negative Binomial

$i = 2$ and $i = 3$

$\{T_{(2)} = n\} = \{X_1 = n - r \ldots$