Ma2

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Various packages in use:

```
set.seed(98765432)
library(igraph)
```

```
##
## Attaching package: 'igraph'

## The following objects are masked from 'package:stats':
##
##     decompose, spectrum

## The following object is masked from 'package:base':
##
##     union
```

```
library(expm)
```

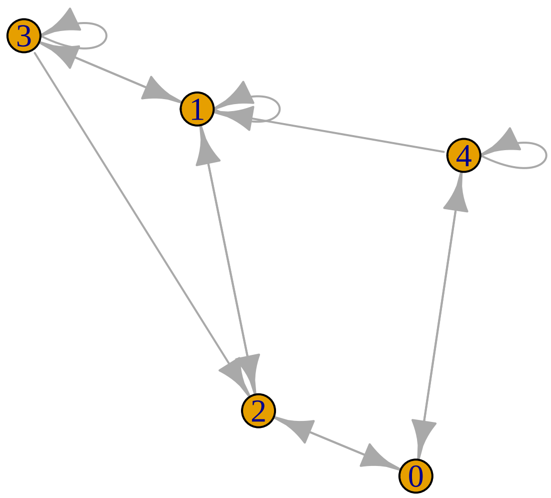
```
## Loading required package: Matrix

##
## Attaching package: 'expm'

## The following object is masked from 'package:Matrix':
##
##     expm
```

w -> wide t -> tilde h -> hat m -> matrix

problem 2.1

a)

```
P <- matrix(nrow = 5, ncol = 5, dimnames = list(c("0","1","2","3","4"),
                                                c("0","1","2","3","4")))
P[1,] <- c(0.000, 0.000, 0.498, 0.000, 0.502)
P[2,] <- c(0.000, 0.527, 0.366, 0.107, 0.000)
P[3,] <- c(0.451, 0.549, 0.000, 0.000, 0.000)
P[4,] <- c(0.000, 0.474, 0.114, 0.412, 0.000)
P[5,] <- c(0.328, 0.249, 0.000, 0.000, 0.423)

graph_P <- graph_from_adjacency_matrix(P, weighted=TRUE,diag=TRUE)
plot(graph_P)
```



problem 2.1

b)

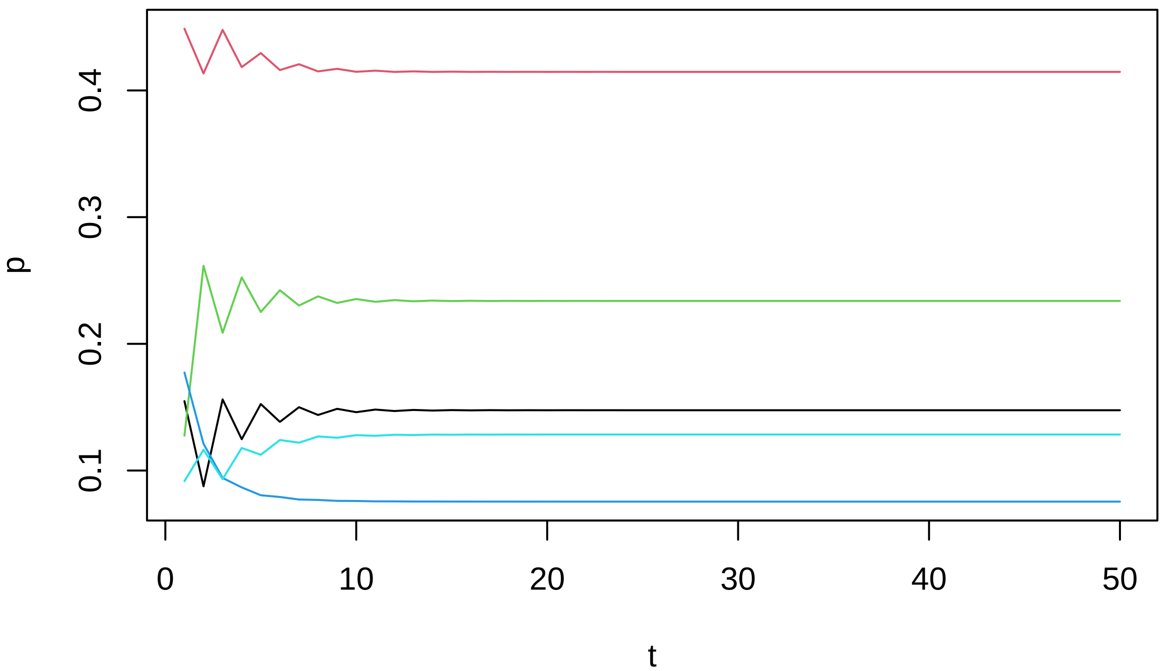
```r
irreducibility <- function(P, M){
            P.new <- P
            for(i in 2:M){
                Pk <- P %^% i
                P.new <- P.new + Pk
            }

            P.bar <- (1/M)*P.new

            if(all(P.bar > 0)){
                print("Transition Matrix is Irreducible")

            }
            else{
                print("YTransition Matrix is Reducible")
            }

}

irreducibility(P,5)
```

```
## [1] "Transition Matrix is Irreducible"
```

problem 2.1

c)

```
K = 50
M = 5
p0 <- as.vector(c(0.013, 0.214, 0.197, 0.375, 0.201))

K <- 50
pk <- matrix(nrow = K, ncol = M)
for (i in 1:K) {
    pk[i, ] <- p0 %*% (P%^%i)
}


plot(x = 1:K, y = pk[, 1], type = "l", col = 1, xlab = "t", ylab = "p",
ylim = c(min(pk), max(pk)))

lines(x = 1:K, y = pk[, 2], type = "l", col = 2)
lines(x = 1:K, y = pk[, 3], type = "l", col = 3)
lines(x = 1:K, y = pk[, 4], type = "l", col = 4)
lines(x = 1:K, y = pk[, 5], type = "l", col = 5)
```



problem 2.1

d) Stationary distribution by computing eigenvalues of P

```
pi <- eigen(t(P))$vectors[,1]/sum(eigen(t(P))$vectors[,1])

pi_confirm <- pi %*% P

print(pi_confirm - pi)
```

```
##                 0             1            2            3             4
## [1,] 5.551115e-17 -1.665335e-16 1.110223e-16 4.163336e-17 -5.551115e-17
```

e) Stationary distribution by iterting 50 times: $\pi = \pi \cdot P^{50}$

```
pi_converged <- p0 %*% (P%^%50)
```

As we see the difference is small

```
print(pi-pi_converged)
```

```
##                0            1             2            3             4
## [1,] 4.76654e-11 2.764483e-11 -5.232206e-11 -4.16657e-12 -1.882194e-11
```

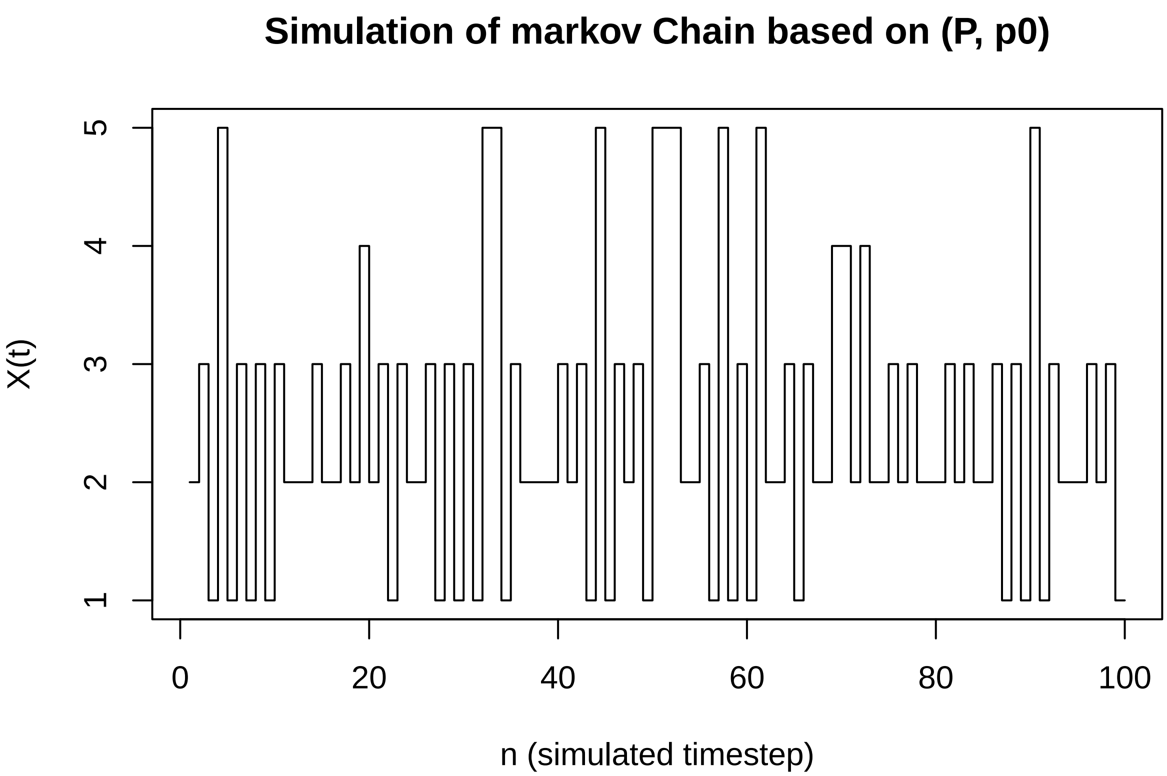
problem 2.2
a) function for simulating large samples

```
simulMarkov<-function(p0,mP,n){
M<-dim(mP)[1]
X<-rep(0,n)
X[1]<-sample(M,1,prob=p0)
for(t in 2:n) X[t]<-sample(M,1,prob=mP[X[t-1],])
return(X)
}
```

```
n<-100
X<-rep(0,n)
X[1]<-sample(M,1,prob=p0)
for(t in 2:n) X[t]<-sample(M,1,prob=P[X[t-1],])
```

problem 2.2
b)

```
plot(x = 1:n, y = X, main = "Simulation of markov Chain based on (P, p0)",
     xlab = "n (simulated timestep)", ylab = "X(t)", type = "S")
```



##problem 2.2
c)

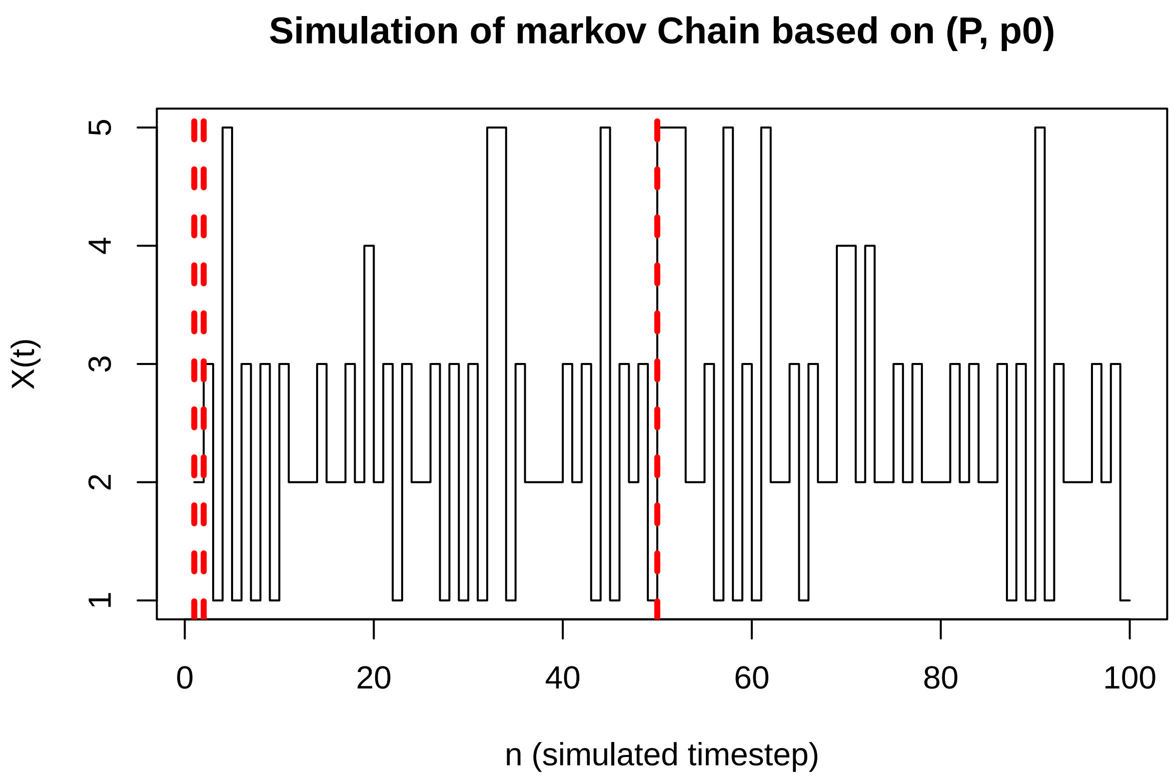
```
iz<-4
S_all<-which(X==iz)
Tn<-length(S_all)

Tn
```

```
## [1] 4
```

problem 2.2

d)

```
tau<-S_all[2:Tn]-S_all[1:(Tn-1)]

plot(x = 1:n, y = X, main = "Simulation of markov Chain based on (P, p0)",
     xlab = "n (simulated timestep)", ylab = "X(t)", type = "S")

abline(v = tau, col="red", lwd=3, lty=2)
```



problem 2.3

a)

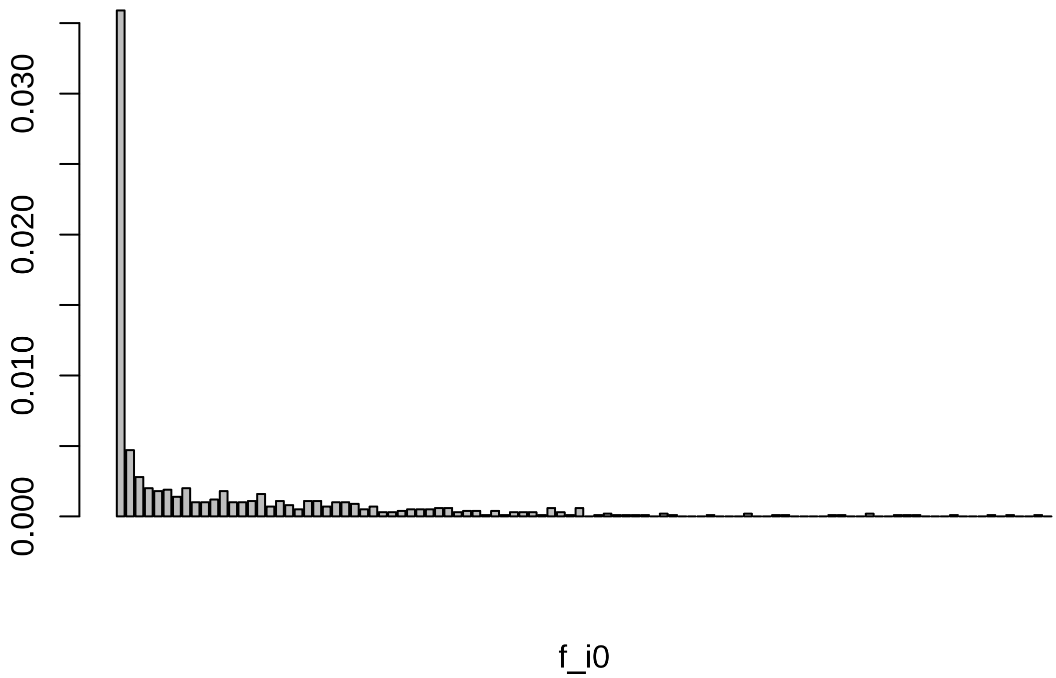
```
#set.seed(1234)
X.10000 <- simulMarkov(p0=p0, mP=P,n=10000)

iz.10000 = 4 #X.10000[1]
S_all<-which(X.10000 == iz.10000)

Tn.10000<-length(S_all)

Tn.10000
```

```
## [1] 834
```

b)

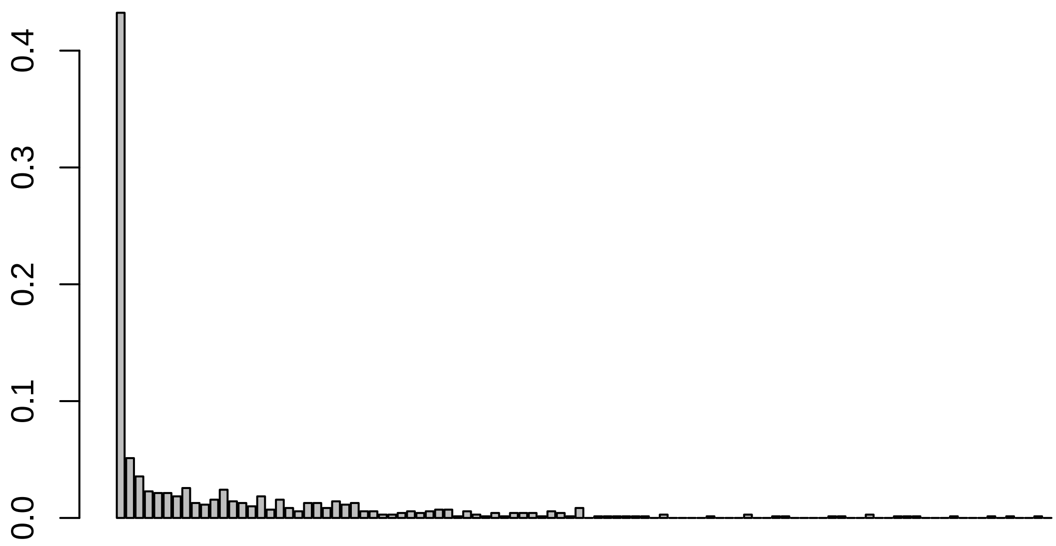
```
tau.10000<-S_all[2:Tn.10000]-S_all[1:(Tn.10000-1)]

f_i0<-c(1:100)
for(j in 1:100){
f_i0[j]<-(1/10000)*length(which(tau.10000==j))
```

```
}


barplot(f_i0, xlab = "f_i0")
```



c)

```
set.seed(123)
i0 <- 4
Tn.1 <- length(which(simulMarkov(p0 = p0, mP=P, n=10000) == i0))
tau.1 <- S_all[2:Tn.1] - S_all[1:(Tn.1 - 1)]
K <- 100

f_hat <- numeric(K)

for (s in 1:K) {

    f_hat[s] <- length(which(tau.1 == s))
  }

  f_hat <- f_hat / (Tn.1 - 1)


barplot(f_hat, xlab = "f_hat")
```



f_hat

i)

```
mu_hat_i0 <- (1/(Tn.1-1))*sum(tau.1[-1]) #AKA tou_bar
mu_hat_i0
```

```
## [1] 12.21479
```

ii)

```
mu_tilde_i0 <- sum((1:K) * f_hat)
mu_tilde_i0
```

```
## [1] 11.5633
```

iii)

```
pi_i0 <- pi_converged[i0]
pi_i0
```

```
## [1] 0.07545784
```

```
rbind("1 / pi_i0" = 1 / pi_i0,
"mu_hat_i0" = mu_hat_i0,
"mu_tilde_i0" = mu_tilde_i0)
```

```
##                  [,1]
## 1 / pi_i0   13.25243
## mu_hat_i0   12.21479
## mu_tilde_i0 11.56330
```

d)

```
n = 10000
m = n -1
X = simulMarkov(p0 = p0, mP=P, n=n)
```

```r
wtpi <- whpi <- numeric(5)
    for(i in 1:5){
          wtpi[i]<-m^(-1)*length((which(X==i)))
    }

wtpi
```

```
## [1] 0.13981398 0.42854285 0.23202320 0.07760776 0.12211221
```

```r
sum(wtpi)
```

```
## [1] 1.0001
```

e)

Emprircal Transition matrix:

```r
P_hat <- matrix(nrow = 5, ncol = 5, 0)
wtpi <-c()
whpi <-c()
for (t in 1:(length(X) - 1)) P_hat[X[t],
              X[t + 1]] <- P_hat[X[t], X[t + 1]] + 1

  for (i in 1:5) P_hat[i, ] <- P_hat[i, ] / sum(P_hat[i, ])

P_hat
```

```
##           [,1]      [,2]      [,3]       [,4]      [,5]
## [1,] 0.0000000 0.0000000 0.4907010 0.00000000 0.5092990
## [2,] 0.0000000 0.5376896 0.3633606 0.09894982 0.0000000
## [3,] 0.4312204 0.5687796 0.0000000 0.00000000 0.0000000
## [4,] 0.0000000 0.4484536 0.0992268 0.45231959 0.0000000
## [5,] 0.3259623 0.2571663 0.0000000 0.00000000 0.4168714
```

problem 2.4

we apply the amse matrix as in 2.3 (e)

a)

```r
P_hat
```

```
##           [,1]      [,2]      [,3]       [,4]      [,5]
## [1,] 0.0000000 0.0000000 0.4907010 0.00000000 0.5092990
## [2,] 0.0000000 0.5376896 0.3633606 0.09894982 0.0000000
## [3,] 0.4312204 0.5687796 0.0000000 0.00000000 0.0000000
## [4,] 0.0000000 0.4484536 0.0992268 0.45231959 0.0000000
## [5,] 0.3259623 0.2571663 0.0000000 0.00000000 0.4168714
```

comparing by subtracting P from $\hat{P}$ b)

```r
P-P_hat
```

```
##             0            1           2            3            4
## 0 0.000000000  0.000000000 0.007298999  0.000000000 -0.007298999
## 1 0.000000000 -0.010689615 0.002639440  0.008050175  0.000000000
## 2 0.019779646 -0.019779646 0.000000000  0.000000000  0.000000000
## 3 0.000000000  0.025546392 0.014773196 -0.040319588  0.000000000
## 4 0.002037674 -0.008166257 0.000000000  0.000000000  0.006128583
```

as we see the difference is about two to thre decimals