

# STAT220 Oblig 4-5

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Library in use:

```
library(tinytex)
library(matrixcalc)
library(shape)
library(diagram)
library(igraph)
```

```
##
## Attaching package: 'igraph'

## The following object is masked from 'package:matrixcalc':
##
##      %s%

## The following objects are masked from 'package:stats':
##
##      decompose, spectrum

## The following object is masked from 'package:base':
##
##      union
```

## Problem 6.1

Vi har gitt transition matrix P:

```
library(matrixcalc)
P=matrix(c(0.7, 0.2, 0.1, 0.0, 0.6, 0.4, 0.5, 0.0, 0.5), nrow=3, ncol=3, byrow = TRUE)
dimnames(P)=list(c('0','1','2'), c('0','1','2'))
print(P)
```

```
##      0  1  2
## 0 0.7 0.2 0.1
## 1 0.0 0.6 0.4
## 2 0.5 0.0 0.5
```

a) Since  $P_{i,j}^m = P(x_{m+n} = j | x_n = i)$  and  $P^{(m)} = P^m$ :

$\Rightarrow P^{(3)} = P^3$  og vi får følgende matrise

```
P_3 = matrix.power(P,3)
print(P_3)
```

```
##      0      1      2
## 0 0.478 0.264 0.258
## 1 0.360 0.256 0.384
## 2 0.570 0.180 0.250
```

```
print(P_3[1,2])
```

```
## [1] 0.264
```

slik at:  $P(x_3 = 1|x_0 = 0) = \underline{0.264}$

samme egenskap gjelder også for  $P^{(3)} = P^3$

```
P_4 = matrix.power(P,4)
print(P_4)
```

```
##          0          1          2
## 0 0.4636 0.2540 0.2824
## 1 0.4440 0.2256 0.3304
## 2 0.5240 0.2220 0.2540
```

```
print(P_4[1,2])
```

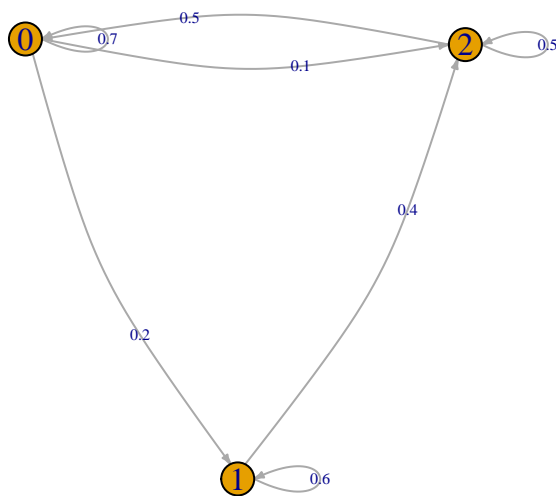
```
## [1] 0.254
```

og vi får da:

$P(x_3 = 1|x_0 = 0) = \underline{0.254}$

b) As we se from the graphic presentation below, it is possible to go from any state to another trough one or more steps:

```
h <- graph_from_adjacency_matrix(P, weighted = "prob")
E(h)$prob <- ifelse(is.nan(E(h)$prob), NA, E(h)$prob)
plot(h, edge.label = round(E(h)$prob, 2), edge.arrow.size = .25, edge.curved=-0.2, edge.label.cex = .5)
```



from state 0:  $0 \rightarrow 2 \rightarrow 1$

from state 1:  $1 \rightarrow 0 \rightarrow 2$

from state 2:  $2 \rightarrow 1 \rightarrow 0$  and  $1 \rightarrow 0$

It is therfore irreducible.

Periodicity: Since the period of a state is the largest  $d$  that satisfy following properties: -  $p_{ii}^{(n)} = 0$  whenever  $n$  is not devicible by  $d$ . - The period  $i$  is shown by  $d(i)$ . - If  $P_{ii}^{(n)} = 0$ , for all  $n > 0 \rightarrow i = \infty$

and...

$i$  is periodic if  $d(i) > 1$  and aperiodic if  $d(i) = 1$

It is aperiodic since there are several sequences of steps to go from a state and back again ( $i \rightarrow i$ ), including the fact that all the states are self periodic the markov chain is aperiodic.

Transient or recurrent?  $\rightarrow$  It is recurrent! It might stop and loop at a position or between state 2 and 0 which are communicating, but eventually it will occur in any of the states by a certainty of 100% as in the formal definition:  $f_{ii} = P(X_n = i, \text{ for some } n \geq 1 | w_0 = i)$  for any state  $i$  is Recurrent if  $f_{ii} = 1$  and transient if Recurrent if  $f_{ii} < 1$ .

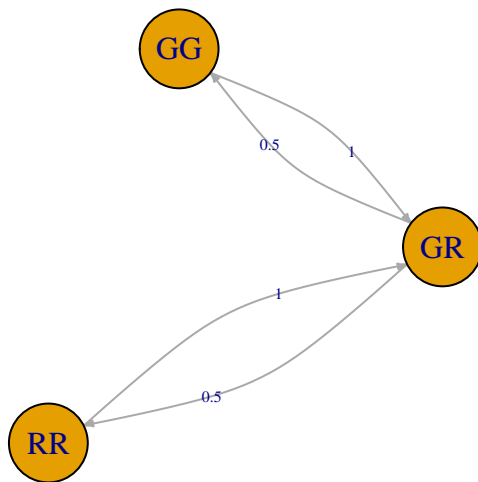
### Problem 6.2

a)  $G : \text{Green } R : \text{Red}$

We start initially at  $GR$  and pick a random ball with  $p = 1/2$ . We are then sent to a state of  $RR$  or  $GG$  where the probability of being returned to State  $GR$  is 100%

```
library(matrixcalc)
A=matrix(c(0, 1, 0, 0.5, 0, 0.5, 0, 1, 0), nrow=3, ncol=3, byrow = TRUE)
dimnames(A)=list(c('GG','GR','RR'), c('GG','GR','RR'))

f <- graph_from_adjacency_matrix(A, weighted = "prob")
E(f)$prob <- ifelse(is.nan(E(f)$prob), NA, E(f)$prob)
plot(f, edge.label = round(E(f)$prob, 2), vertex.size=40, edge.arrow.size = .25, edge.curved=0.3, edge.l
```



Which gives following matrix

```
print(A)

##      GG GR  RR
## GG 0.0  1 0.0
## GR 0.5  0 0.5
## RR 0.0  1 0.0
```

and following for picking red ball in the future entering the state of  $GR \rightarrow P(X_n = j | X_0 = 1) : \#$  her må jeg få sendt ett spørsmål

```
print(A[2,])
```

```
##  GG  GR  RR
## 0.5 0.0 0.5
```

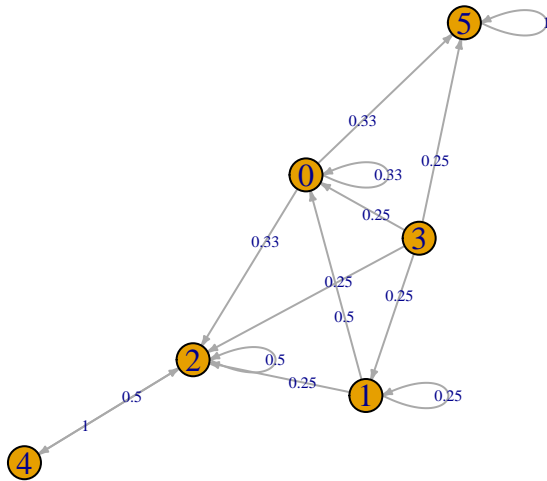
b)  $d(GR) = 2$  og  $d(GG) = d(RR) = 4$  som gir største felles nevner 2 og dermed er den periodisk med periode 2.

### Problem 6.3

a)

```
B=matrix(c((1/3), 0.0, (1/3), 0.0, 0.0, (1/3),  
          (1/2), (1/4), (1/4), 0.0, 0.0, 0.0,  
          0.0, 0.0, (1/2), 0.0, (1/2), 0.0,  
          (1/4), (1/4), (1/4), 0.0, 0.0, (1/4),  
          0.0, 0.0, 1.0, 0.0, 0.0, 0.0,  
          0.0, 0.0, 0.0, 0.0, 0.0, 1.0), nrow=6, ncol=6, byrow = TRUE)  
dimnames(B)=list(c('0','1','2','3','4','5'), c('0','1','2','3','4','5'))
```

```
g <- graph_from_adjacency_matrix(B, weighted = "prob")  
E(g)$prob <- ifelse(is.nan(E(g)$prob), NA, E(g)$prob)  
plot(g, edge.label = round(E(g)$prob, 2), edge.arrow.size = .25, edge.label.cex = .5)
```



b) As we se from the graph above:

**Transient States:**

State 1, 2, 3 and 4 will at some pint possibly loop, but ultimatly it will end up in state 5 where it will loop for a infinate time.

**Recurrent States:**

State 5 is self-recurrent

c) Communicating States:

$(2 \leftrightarrow 4)$ ,  $(1 \leftrightarrow 1)$ ,  $(2 \leftrightarrow 2)$  and  $(5 \leftrightarrow 5)$

d)