Oblig 5

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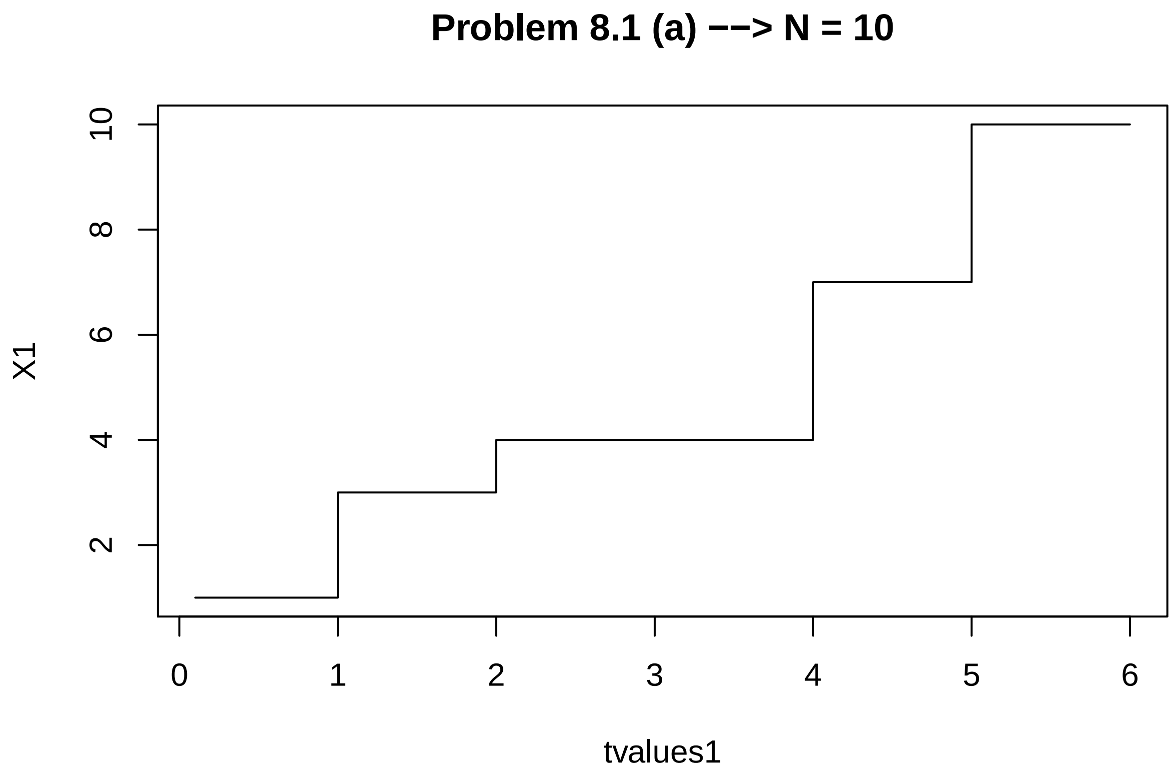
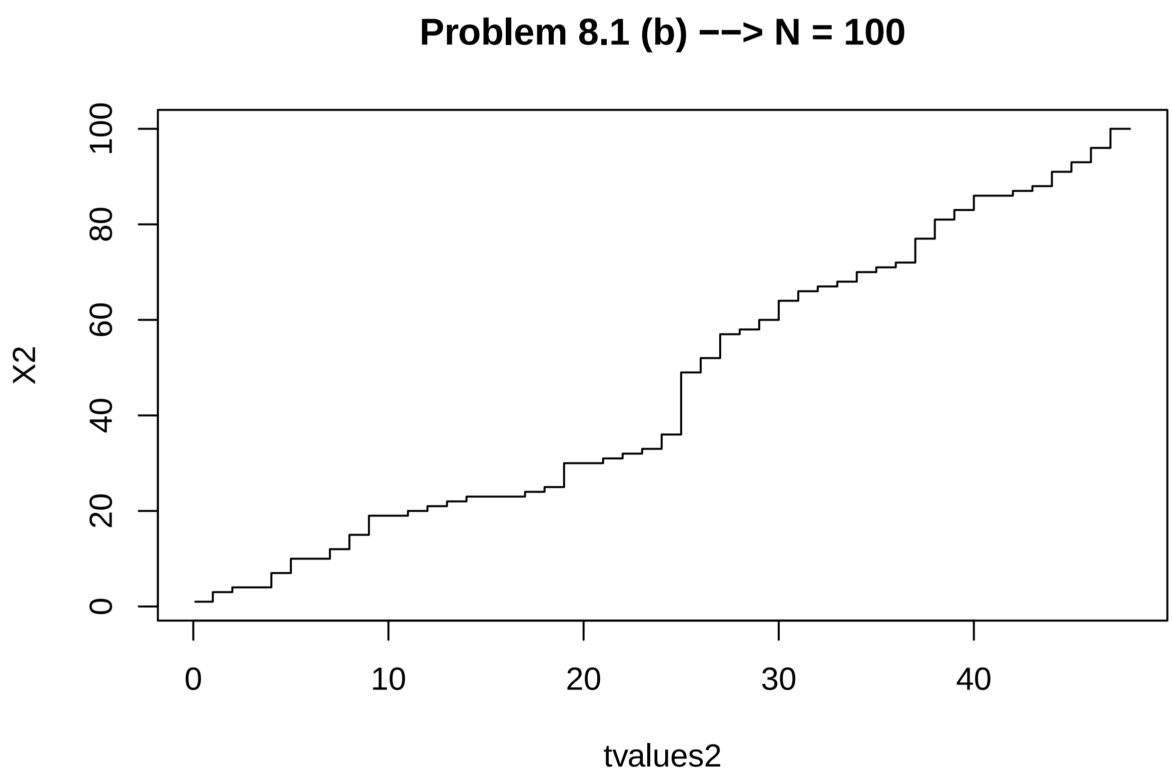
Problem 8.1

```
set.seed(123)
lb<-0.5
delta<-0.1
N<-c(10, 100, 1000)

T1<-rpois(N[1],lb)
S1<-rep(0,N[1])
S1[1]<-T1[1]

for(k in 2:N[1]) S1[k]<- S1[k-1] + T1[k]

tmax1<- S1[N[1]]+ 0.5/lb
tvalues1<-seq(delta, tmax1,delta)
ntvalues1<- length(tvalues1)
X1<-rep(0, ntvalues1)
for(t in 1: ntvalues1) X1[t]<- length( which( S1<=tvalues1[t] ))
plot(tvalues1, X1, type="s", main = 'Problem 8.1 (a) --> N = 10')
```

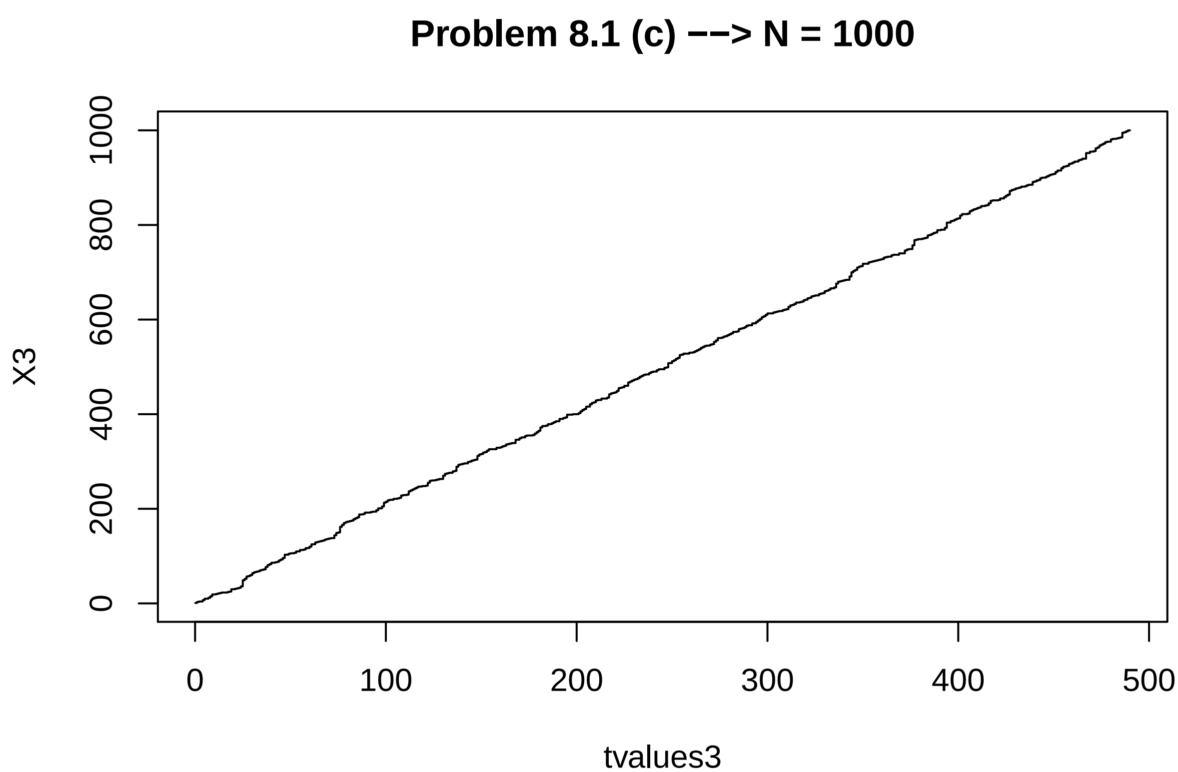


Problem 8.1 (a) −−> N = 10
X1
2
4
6
8
10
0
1
2
3
4
5
6
tvalues1

```r
set.seed(123)
T2<-rpois(N[2],lb)
S2<-rep(0,N[2])
S2[1]<-T2[1]

for(k in 2:N[2]) S2[k]<- S2[k-1] + T2[k]

tmax2 <- S2[N[2]]+ 0.5/lb
tvalues2<-seq(delta, tmax2, delta)
ntvalues2<- length(tvalues2)
X2<-rep(0, ntvalues2)
for(t in 1: ntvalues2) X2[t]<- length( which( S2<=tvalues2[t] ))

plot(tvalues2, X2, type="s", main = 'Problem 8.1 (b) --> N = 100')
```

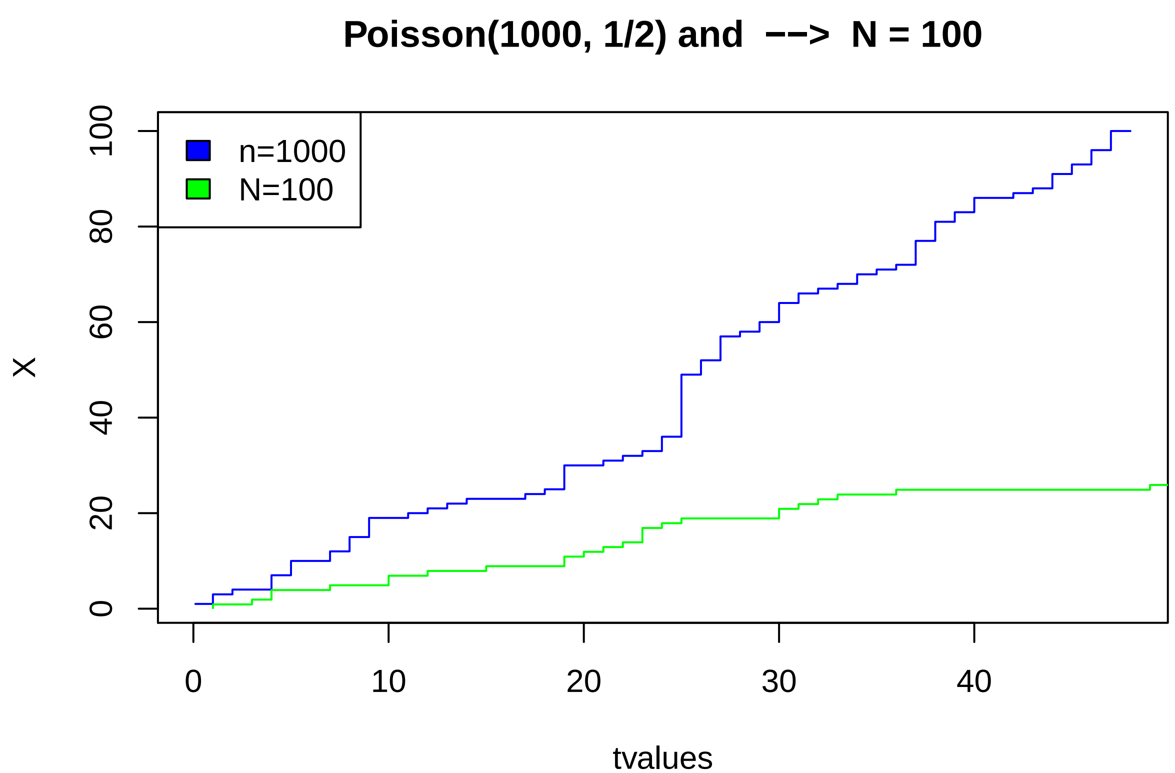


```r
set.seed(123)
T3<-rpois(N[3],lb)
S3<-rep(0,N[3])
S3[1]<-T3[1]

for(k in 2:N[3]) S3[k]<- S3[k-1] + T3[k]

tmax3 <- S3[N[3]]+ 0.5/lb
tvalues3<-seq(delta, tmax3,delta)
ntvalues3<- length(tvalues3)
X3 <-rep(0, ntvalues3)
for(t in 1: ntvalues3) X3[t]<- length( which( S3<=tvalues3[t] ))
plot(tvalues3, X3, type="s", main = 'Problem 8.1 (c) --> N = 1000')
```



d)

```
set.seed(123)
POI<-rpois(n=1000 ,lambda=lb)
S <-rep(0,N[2])
S[1]<-POI[1]
for(k in 2:N[2]) S[k]<- S[k-1] + POI[k]

tmax<- S[N[2]]+ 0.5/lb
tvalues<-seq(delta, tmax,delta)
ntvalues<- length(tvalues)
X<-rep(0, ntvalues)
for(t in 1: ntvalues) X[t]<- length( which( S<=tvalues[t] ))
plot(tvalues, X, type="s",col= 'blue', main = 'Poisson(1000, 1/2) and  -->  N = 100')
lines(y=tvalues2, X2, type="s", col= 'green')

legend("topleft",
c("n=1000","N=100"),
fill=c("blue","green")
)
```



Problem 8.2

a) The probability for a poisson distributed random variables is given by:

$P(X=k) = \frac{\lambda^k}{k!} \cdot exp(-\lambda)$

It is known that the mean rate is 3 messages pr hour such that
$E(X) = \lambda = 3$ and the probability that X is 0 afer 1 hour is:

$P(X=0) = \frac{3^0}{0!} \cdot exp(-3) = 0.0497$

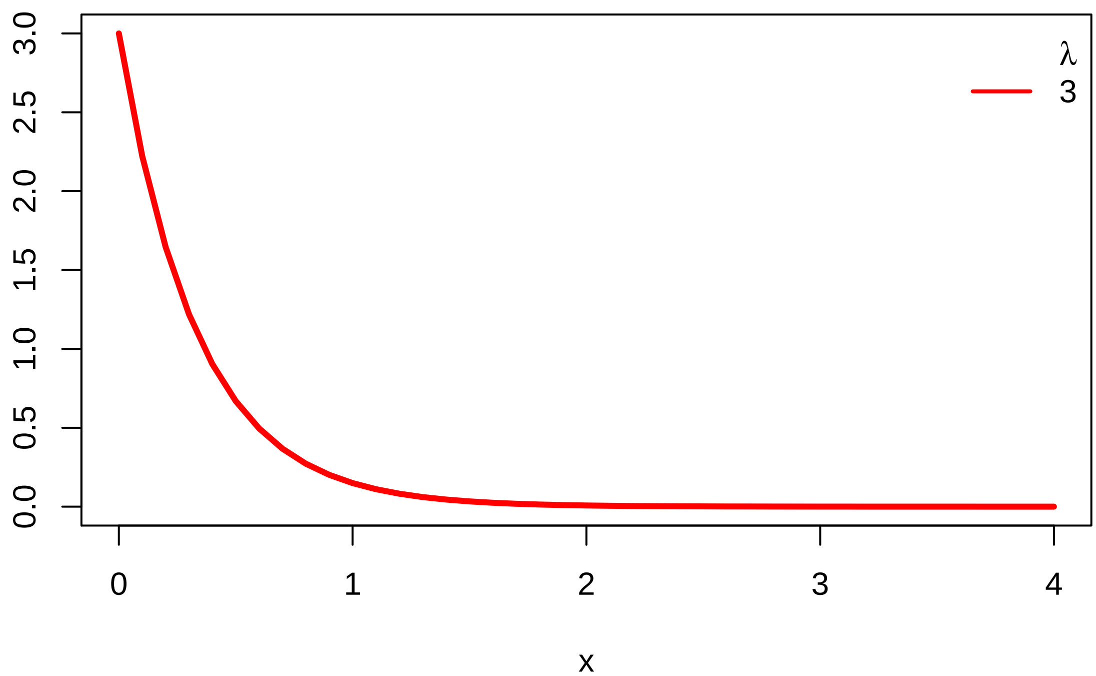
From $08:00 \rightarrow 12:00$ it is 4 hours and since poisson is indipendent there is 4 random variables $\{X_1, X_2, X_3, X_4\}$ wich gives us probability that X is 0 from $08:00 \rightarrow 12:00$:

$P((X_1=0) \cap (X_2=0) \cap (X_3=0) \cap (X_4=0))$
$= (\frac{3^0}{0!} \cdot e^{-3})^4 = (e^{-3})^4$
$= e^{-12} = \underline{\underline{6.14 \cdot 10^{-6}}}$

b) Since events is poisson distributed with time intervals $\{X_1, X_2, ..., X_n\}$ hrs, and rate $E(X_n) = \lambda$ of 3 with non overlaping events, the distribution is exponential with expnential parameter λ:

$f(x) = \lambda \cdot e^{-\lambda X} = 3 \cdot e^{-3 \cdot X}$

```r
x <- seq(0, 4, 0.1)
plot(x, dexp(x, 3), type = "l",
     ylab = "", lwd = 3, col = "red")
legend("topright", c(expression(paste(, lambda)), "3"),
       lty = c(0, 1), col = c("red"), box.lty = 0, lwd = 2)
```



Problem 8.3

$P(N(S)=i|N(t)=N)=\frac{P(N(S)=i\cap N(t)-N(s)=n-i)}{P(N(t)=n}=\frac{P(N(s)=i)\cdot P(N(t-s)=n-i)}{P(N(t)=n)}$

$=\frac{n!}{i!(n-1!)}\cdot\frac{s^i\cdot(t-s)^{n-i}}{t^n}$