Gotta Collect 'em All

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Abstract
When you look at a classical probability problem, It's so scary at the start. But that's the hardest part. And trust me, it get's so much worse. I finally know where I belong. This is the way the weak grow to be so annoying to everyone around them. I've got a feeling, With you, I'm becoming Meme. I've worked hard, Come so far. It's my story, I'm the star. And the road I'm on Could go until forever. Every day there's something new. Every day I share these horrible insights with you. I've got a feeling, With you, I'm becoming Meme. We're looking for new horizons. Let's go explore new horizons. We're looking for new ways to take this problem and complicate it. Let's go explore all the ways that we've done that. I've got a feeling, With you, I'm becoming Meme.[5]

Keywords
Coupon Collector's Problem — Probability — Computer Simulation — Pokémon — A flimsy excuse of a paper to justify the purchase of more Pokémon cards

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Contents

1. Coupon Collecting and *YOU*

The Coupon Collector's Problem is one of the most well-known problems in probability theory among people who study probability theory (read: nerds). For those unfamiliar with terminology, a coupon is a piece of paper you had to bring in while shopping in a physical store to save money on certain items you bought[10]. These are no longer commonly seen, because CVS accidentally bought every single one that existed, and has to now put them on their receipts while the coupon factories work through this backlog.

The Coupon Collector's Problem, however, is a weird corruption of this. Instead of using the coupons to procure necessary items to ensure your survival (like 35 cents off a combination laser-pointer/foot massager), one is instead trying to get one of every single coupon being distributed at a time, presumably to flaunt ones wealth and/or collection of combination laser-pointer/foot massagers. Now, the easiest way to do so would be to ask the coupon distributors for the missing coupons, but unfortunately, in this telling of the story, they are not so kind as to listen to mere mortal's trifling concerns, and as such, are given a random coupon from the collection each time. The question then begs how many coupons would one have to collect in order to have one of each kind.

In the end, there are some simple math(s) one can go through to calculate the expected value[1], but the solution is for n coupons, one would expect to get nH_n coupons on average before getting one of every coupon, where H_n is the nth Harmonic number[1]. Because we don't like doing math(s) in this math(s)-inspired paper, we will instead be replacing the math(s) with other math(s) and approximating the Harmonic numbers by the natural log function[2], because it takes way too much effort to get Excel to do the Harmonic calculations, but the natural log is a default function, and laziness is very important.

Part of the underlying assumptions for the Coupon Collector's Problem is that all the coupons are equally likely to be obtained, and that what coupon is received at one time has no impact on the other coupons[4]. Variations on this problem have been studied numerous times to try and convert it to something more akin to real life, whether by allowing duplicate coupons to be traded between collections[4] assuming the coupons are

[1]The Harmonic numbers are the sums of $1+\frac{1}{2}+...\frac{1}{n}$.
[2]Specifically, $H(n) \approx \ln n + \gamma^3 + \frac{1}{2n}$.
[3]Euler's constant, $\gamma \approx .577215665$. No, Euler's constant isn't e, but that's still used because of the natural logs, and γ shows up a lot within contexts of natural logs. Also, this 3 was a footnote, and not γ being raised to the third power.
[4]or in actual math(s) parlance, the coupons follow IID: independent and identically distributed.

not equally likely[8], the coupons come in unique batches without repeats[9], or there are multiple collections going on at the same time and wanting to know how long until all of them are finished[6]. One great example of an adaptation that deals with multiple of these is looking at collecting trading cards.

2. What trading cards mess up

Opening up packs of trading cards is one way that the Coupon Collector problem can be translated to a more modern scenario. People frequently get tons of packs, and then open them all to see what they get. Usually, there are three main reasons people open up lots of packs: They are opening them up, trying to pull rare and valuable cards to resell and turn a profit; they are attempting to pull a few specific cards that they like or need for a deck, usually the rarer ones[5]; or, most pertinently, they are trying to collect every last card in the set oh hey wait that's what this paper is about.

The manner of getting the cards in a TCG is slightly different than that in the normal coupon collector's problem. First, the cards have differing rarities, and they have different distributions for each. Classically, these can be sorted into "common"(a plurality of the cards in a set that usually make up a large portion of each pack), "uncommon"(usually a smaller group that has a couple slots in a pack), "rare"(high end cards that are some of the heaviest hitters, but only appearing once or rarely twice in a pack), and "ultra rare"(the top end cards that aren't even guaranteed to show up in a single pack) cards. Because the cards are grouped in packs, the cards gotten in a pack usually aren't allowed to be duplicates, one does end up with a slightly wider spread that expected, which helps some, but this also means that they aren't independent of each other. And lastly, the different rarities can be construed as separate Coupon Collector problems, and completing the set is just the finishing time for the slowest of these. All told, looking at these together makes this an interesting problem, and one worthy of devoting

For this paper, I will be looking at the Pokémon TCG in particular for a couple of reasons: Firstly, as one of the largest media franchises in the world, there are a lot of players, and thus a lot of data to pull from[3]. Secondly, the author[6] has been a longstanding fan of the series, and so it has some sentimental value. Thirdly, with the newest set *Temporal Forces* releasing on the extended submission deadline and rotating into tournament legality on the publication date[7], this would allow Sigbovik to be on the cuttingest of cutting edges, an honor which I am sure they strive to follow.

[5]but sometimes it's not like literally how have I never pulled one of the future cards from Paradox Rift *despite* four of them just being normal uncommons like please i need those future booster energy capsules for my future box lite deck cmooooooon

[6]and her Zorua plushie she is holding on to, who deserves full coauthorship

[7]Please note, that although *Temporal Forces* has not been released, these packs were legitimately obtained at a pre-release event that the author finished tied for third despite not having actively played in fifteen years.

Figure 1. A sampling of the packs looked at for this paper, and totally not an excuse for me to have bought a ton of cards during Pokemon day[7].

Since the beginning of the Scarlet and Violet era of the TCG, the packs have followed this specific structure: Four common cards, chosen without duplicates; three uncommon cards, chosen without duplicates; two reverse holofoil cards, chosen from the pool of all common, uncommon and rare cards, that are not duplicates of each other but *may* be duplicates of other cards in the pack, one holofoil rare card, one basic energy card that is identical to those in any other set, and one code card for the online TCG. Additionally, the ultra rare cards can replace any of the three holofoil slots, depending on the type of ultra rare card and the set that it is in.

3. Rifting the Para-da problems in a... uh...

[8]

3.1 Individual Rarities

To help with this calculation, the author created a highly accurate simulation of opening packs of Pokémon cards from a perfectly specified set. To this, the data from the most recent standard set, *Paradox Rift*, was pulled, and a quick program was written up in Python. Simulated packs were opened up 10000 times, with how quickly each set was finished recorded for each, and the total time to complete the entire set.

Now, one might naïvely expect that the odds of finishing each of the subsets to be equal to the expected value of getting all of them if they were guaranteed divided by the odds of getting them in the first place. And after the ten thousand simulations... uh... yeah that's pretty close to the case. For n cards with a p chance of getting one of them, the expected number of packs is

$$E(n,p) \approx \frac{n\ln n + \gamma n + \frac{1}{2}}{p}$$

[8]look this is supposed to be a pun on paradox rift cmon gimme a break

Table 1. *Paradox Rift* card counts and predicted odds[3]

Rarity	Count	Odds
Common	81	
Uncommon	54	
Rare	27	
Double Rare/ex	20	15.57%
Ultra Rare/Full Art	28	6.64%
Illustration Rare	34	7.70%
Special Illustration Rare/Alt Art	15	2.11%
Hyper Rare/Gold Card	7	1.22%

For the standard commons and uncommons, which appear in more than one per booster pack, the "probablility "is just the number of times they appear in a booster pack, accounting for the reverse holo slots[9].

Table 2. *Paradox Rift* calculated and simulated averages

Rarity	Calculated	Sim. Mean	Sim. Median
Common	82.47	81.08	78
Uncommon	69.49	68.40	65
Rare	105.07	101.33	96
DR	458.93	484.25	455
UR	1648.46	1743.70	1647
IR	1811.87	1893.34	1789
SIR	2335.34	2441.37	2266
HR	1447.57	1572.38	1420

On looking at the higher rarities, though, the expected value seems a tad low. The calculated values were over 100 packs shy compared to the simulation for Ultra, Special Illustration, and Hyper Rares. The problem likely comes from trying to set up a Poisson-esque distribution with a Geometric distribution feeding in, and so a very unlucky set of pulls can end up skewing the results[11]. Given all this, instead of trying to come up with the exact way to match better, I opted for the easy route of just looking at the median, which tended to match much better, because moving the goalposts is a completely valid strategy for Sigbovik.

The expected chance of finishing before a certain point is also able to be calculated from bastardising pre-existing formulae[2]. In the end, it, too, is just the calculated value divided by the probability of getting one in the first place, but there is a small correction term added to account for the unluckiness in getting the rarer cards in the first place. For n cards with a p chance of getting one of them, to have less than an α chance of not having a complete set, the expected

[9] *Paradox Rift* makes this nice and easy, as the commons, uncommons, and rares make up exactly[10] 1/2, 1/3, and 1/6 of the total cards available for reverse holos.

[10] Although you do have to subtract about the 1/9 chance that the second reverse holo is replaced by one of the ultra rares, and for the rares, you also need to worry about its replacement.

[11] One unlucky run took 8975 packs to open up all the special illustration rares, more than 3.5x the average, and nearly running up against my hard-coded 10,000 pack limit

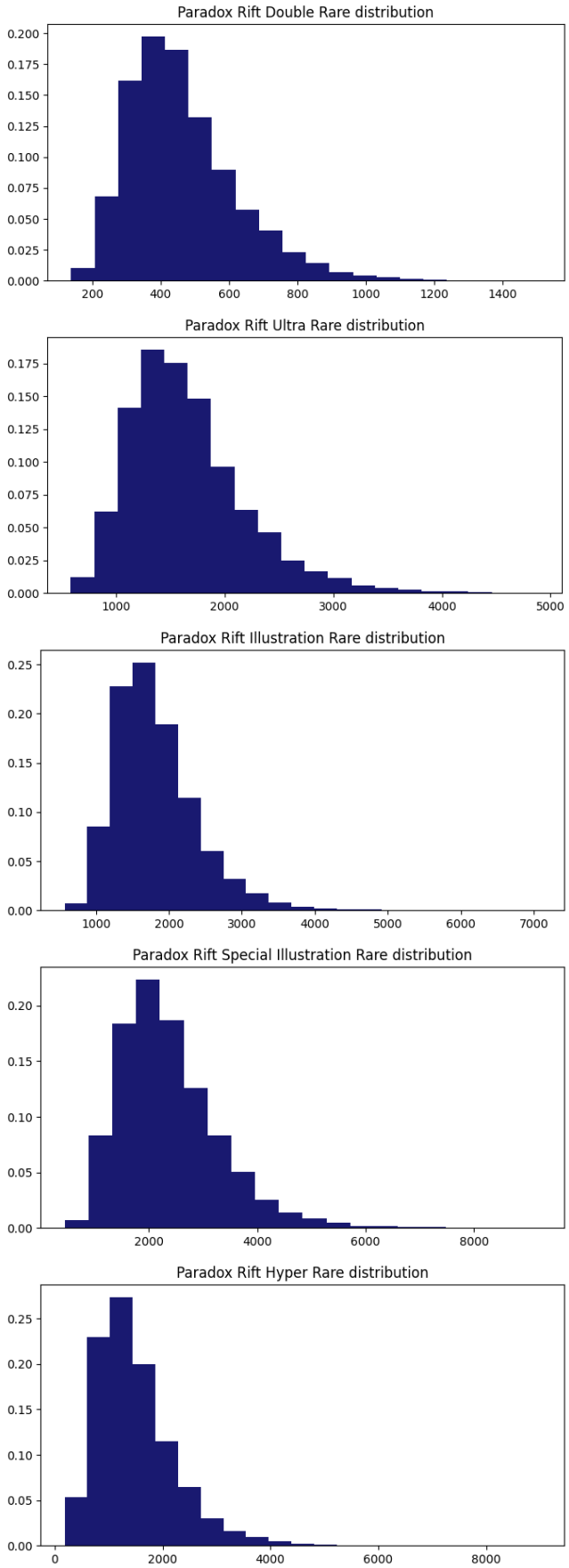

Figure 2. Simulation distributions for 10,000 simulations of *Paradox Rift*

number of packs is[12]:

$$E(n,p,\alpha) \approx \frac{(n\ln n)(1-\log_n \alpha) - \ln p}{p}$$

3.2 For the whole set

Calculating out the needed number of packs to complete the whole set, however, is a bit harder. For one, the two simplest approaches are patently wrong:

1. One could expect that the expected time to finish up all the sets is the sum of all the times for each rarity, but that would be simulating having to wait to start a set until the previous ones are finished. An extreme case of this would be if one set had... let's say 120 different cards in one of the rarities[13] with an expected time to finish of around 2500 packs, while the rest of the rarities (discounting standard commons, uncommons, and rares) had no more than a dozen cards and would be thought to finish in less than 1000 packs. In this case, most often the last rarity to finish would be that single large one, and the expected number of packs should be close to it.
2. One could also expect that the expected time to finish would be the largest value, as all the other one would be finished. However, just because they are likely to be finished first doesn't mean that they will be first. Again, a case for this might be having three rarities being extremely close to finishing at the same time with the probabilities accounting for the different sizes[14]. In this case, although the three sets would *average* finishing around the same time, it would not be a surprise to have one of them finishing early and another finishing later to even out the differences. And since the set is only complete when all are done, the expected time should be later.

So now we are somewhere between the largest value of any of them and the sum of all the values. Such a great range. Thankfully, math(s) come(s) to our rescue, and the expected value is[6]

$$E(s_1,...s_k) = nH(S) - \frac{(H(S)-1)\sum_{i<j} s_i s_j}{S(S-1)} + \mathcal{O}\left(\frac{1}{n}\right), S = \sum_{i=1}^{k} s_i$$

$$P(t<x) = \prod_i 1 - n_i^{1-\frac{p_i x + 1 + \ln(1-P)}{n_i \ln n_i}}$$

Uh... the... verification of these will be left as an exercise for the reader, because this is very sensible and easily translated into our new format accounting for only having a chance at getting a coupon each time. Rather than bog ourselves down with the math(s) here, instead, we will look at a way to get an estimate having already calculated the prior results.

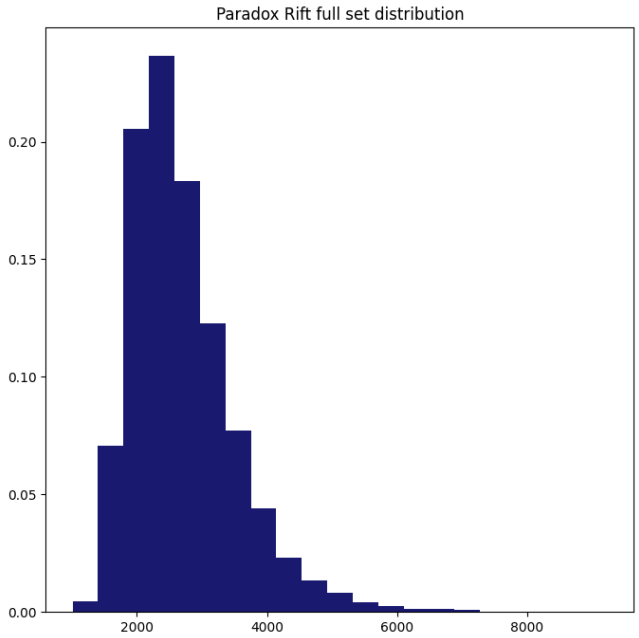

Figure 3. Simulation distributions for 10,000 simulations of *Paradox Rift*

The way this was done was just simulating all of the Scarlet and Violet era sets[15], and then using everyone's favorite linear regression on sensible candidate equations to see which came the closest for its simplicity. Candidate equations had to follow these sensible guidelines:

1. $F(E) = E$
2. $F(E_0, E_1, ...E_k) \geq E_0, F(E_1, ...E_k)$
3. $F(E_1, ...E_k, 0) = F(E_1, ...E_k)$

These guidelines state roughly that the expected value of only one prior expected value must equal itself, that adding another expected value cannot decrease the total values, and that adding nothing cannot change the current expected value. Upon searching with this, one simple equation that fulfilled all these conditions and matched very well was raising each number to a derived power, adding them together, and then taking that derived root:

$$F(E_1, E_2, ...E_k) = \left(\sum_{i=1}^{k} E_i^{\lambda}\right)^{\frac{1}{\lambda}}$$

[12] Since $p < 1$, $\ln p < 0$, and so subtracting the log adds to the term

[13] Spoiler alert for a few sections from now: *Paldean Fates* shiny rares.

[14] Spoiler alert again: *Scarlet & Violet Base Set* has Ultra, Illustration, and Special Illustration rares at around 1200 packs.

[15] Initially I was also going to include late Sword and Shield era sets, but given that Silver Tempest has rarities of Pokémon $\mathcal{V}$, Pokémon $\mathcal{V}$MAX/$\mathcal{V}$STAR, Full art Pokémon $\mathcal{V}$, Full art trainers, Alternate art Pokémon $\mathcal{V}$, Trainer Gallery Pokémon, Trainer Gallery $\mathcal{V}$/$\mathcal{V}$MAX/Trainers, Trainer Gallery Gold $\mathcal{V}$MAX, Radiant Rares, Rainbow Rares, *and* Gold Rares... yeah no

And with a simple regression gives a value of $\lambda \approx 3.88902$, which is close enough to 4 that I'm willing to just call it that to make it even simpler. Evaluating this for the prior values gives an expectation of 2826 packs, which is very close to the simulated 2813. For expected values for a certain α, it turns out that the solutions for λ varies linearly[16]. The calculated value is $\lambda = \frac{100}{27} + \sqrt{2}\Phi(\alpha)$, where Φ is the normal inverse[17]. Sadly this is a lot more complicated than the formula for just the means, but if you want to know when you will be 69.420% likely to have completed a set of cards, well, you deserve it.

4. All the sets Here Together

4.1 Second through sixth verse, same as the first

Of course, now that these methods have been thoroughly tested through exactly one series, it's time to expand these into the other five sets from the Scarlet & Violet era. One thing that needs to be brought up is that some of the other sets have special rarities only found in them, namely *Pokémon 151*'s Holofoil energies, and *Paldean Fate*'s Shiny Pokémon[18]. Thankfully, with how the expected values were found before, these only need to be slotted in to the slots where they are found, and simple as that, the entire results can be ran with nary a problem[19].

Table 3. Scarlet & Violet era expected pulls. These data were all run at the same time, hence the difference between the *Paradox Rift* values from before and in this table, as they were different simulations.

Set	DR	UR	IR	SIR	HR	Full Set
SVI	271	1093	1183	1188	802	1557
PAL	425	1516	1953	1564	1457	2309
OBF	556	559	490	471	289	765
MEW	280	848	646	589	287	974†
PAR	462	1654	1823	2375	1503	2713
PAF	185	175	77	1299	947	2585‡

†: *Pokémon 151*'s Holofoil energy has an expected value of 89 packs.
‡: *Paldean Fates* has Shiny Rares with an expected value of 2535 packs, and Shiny Ultra Rares with 487 packs.

From these, we can see that there is quite a variation in the sets. The quickest set is by far *Obsidian Flames*, which has only 54 cards above the normal rarity. This small size, combined with fairly generous pull rates[20], leads to a set that you are likely to complete well before most any other sets' Illustration Rares. *Paradox Rift* is actually the slowest set to complete, because although *Paldean Fates* has a worse set in its 120 Shiny Rares, that's the only bad one, while all of the non-Double Rares in *Paradox Rift* take a while.

Figure 4. All the different high rarities released in Scarlet & Violet. The top row has only been found in a single set: Holofoil Energy from *Pokémon 151*, Shiny Rare and Shiny Ultra Rare from *Paldean Fates*, ACE SPEC from *Temporal Forces*. The bottom two rows have been in all sets: Double Rare (from *Paradox Rift*), Ultra Rare (from *Obsidian Flames*), Hyper Rare (from *Paldean Fates*), Illustration Rare (from *Scarlet & Violet Base*), and Special Illustration Rare (from *Temporal Forces*).

The largest factor does tend to be the size of the special pulls. Although how they end up distributed and the exact odds do have some weight (hence why *Paradox Rift* is slower than *Paldean Fates*, despite having 60 fewer cards), a set with a large number of cards that need to be pulled will just necessarily take longer than a shorter one.

Now with all these calculations done, we just have to ask...

4.2 Okay but are packs worth it?

NO

[16] namely because two values were looked at

[17] Single-sided, so for $\alpha = .95$, it is *not* 2 (1.96) but instead 1.645

[18] Which thankfully are more common to get than the 1/4096 chance of shinies in the video games

[19] Okay maybe a little of a problem, considering that *Pokémon 151* has "Demigod"packs with the (special) illustration rares for one of the starter lines instead of any reverse holofoil cards. This took like an hour to encode, but I did it just for accuracy. It made effectively no difference.

[20] The first three sets all had fairly similar rates for the rarities. For *Paradox Rift*, although Double Rares went from around 13.5% to 15.5%, this was balanced out by Special Illustration Rares dropping from around 3.2% down to 2.2%, and Hyper Rares dropping from 1.8% to 1.4%. Since those tend to take longer to complete than Double Rares, this drop in odds has lengthened the completion time of the latter sets.

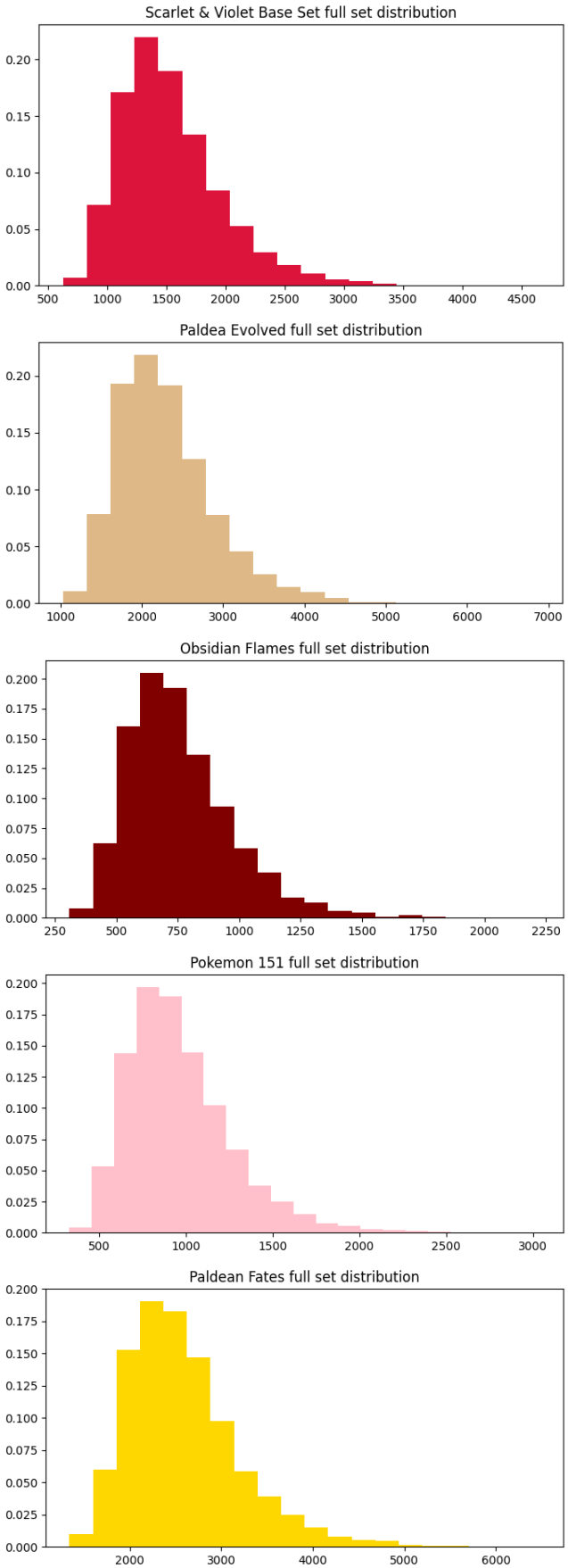

Figure 5. Simulation distributions for 10,000 simulations of the released Scarlet & Violet TCG sets

This should not be much of a surprise. The expected value of packs is based on the expectations of what could be contained in them, while usually only a handful of cards contain the value of the set[21]. On average, the entire set tends to sell for around one ninth the cost of the packs needed to pull all the cards. This difference is so severe, the only rarities that are likely to be pulled in the number of packs it takes to spend as much as the full set are the *Pokémon 151* Holo Energies (89 vs 132), and the Double, Ultra, and Illustration rares from *Paldean Fates* (185, 175, and 77 vs 224). And none of those cards are the ones providing the value in the set[22].

Table 4. Scarlet & Violet era expected costs from both pulling from packs and the price of all the cards in the set. All data was pulled from TCGPlayer and was accurate at the time of writing. Pack price is the most efficient purchase (usually a booster box of 36 packs), not the price for a single pack.

Set	Packs	Pack	Total	Set	Ratio
SVI	1549	$2.36	$3656	$313	11.68
PAL	2345	$2.64	$6191	$706	8.77
OBF	749	$2.31	$1730	$201	8.61
MEW	943	$6.00	$5660	$792	7.14
PAR	2691	$2.38	$6403	$702	9.12
PAF	2582	$4.08	$10,537	$915	11.52

Thus from all of this, the best shot of managing to get a complete set is if you just go out and buy all of the cards in it. Actually, you should probably first get some booster packs, because trying to find people willing to sell random new commons and uncommons is hard, so you'll probably have to go online, where for such worthless cards, you'll have to pay shipping and that will be so much more than the cost of the cards[23]. Failing that, if you want to go with pulling the cards, the next best thing is to get some friends. Although trying to complete two sets takes longer (and thus costs more money), it doesn't take twice as long, and you'll save yourself a little money in the end. In fact, the more people that you get in your trade network, the less each will have to pay on average. And if some people are only concerned with getting a few cool cards that they like, that helps even more.

5. Temporally Forcing through new data

With all that said on the current sets, let's turn our eyes to the new one. *Temporal Forces* will release on the 22nd of March, 2024[24]. One of the biggest factors for this set is a brand new rarity of cards for people to chase... or well... brand new for

[21] To wit, the author just opened up a *Paldea Evolved* booster pack as she wrote this sentence, and although she did get a hit in the double rare Slowking ex, that card goes for less than what the pack would have. It's still a very cute card, and she'll absolutely take it.

[22] Most expensive card, and only one over a dollar: PAF054 Charizard ex - $2.41

[23] Source: I got those darn Future Booster Energy Capsules I needed in my deck for three cents each, and spent $1.22 on shipping.

[24] The Extended Submission Deadline

Scarlet and Violet. Bringing back a second old mechanic[25], ACE SPEC cards return for the first time since *Plasma Blast* in Black and White.

Now, as the set has not been officially released[28], the odds of pulling any specific card is not yet known. However, with the full set list already shown, as well as the expectation that the pull rates are *probably* going to be similar to the other sets, we can at least make an estimate going through what we know of the past sets. Thus, for each of the sets ($i \in [\text{SVI, PAL, OBF, MEW, PAR, PAF}]$),

$$\hat{\mu}_{\text{TEF}} \approx \frac{1}{6}\sum \bar{x}_i$$

$$\hat{\sigma}^2_{\text{TEF}} \approx \frac{1}{6}\sum s_i^2 + \frac{1}{5}\sum (\bar{x}_i - \hat{\mu}_{\text{TEF}})^2$$

Now, I could just sit on my laurels, run the simulation with these values, and go on with my day like nothing had happened. But that's not what a scientist does. They do data collection. And they **WRITE STUFF DOWN**. Also, uh, since ACE SPEC cards are new there's no prior data to make an estimate from. I mean, there's *guesses* from the Black and White era on their frequency, but not of the "open up thousands of packs of cards and meticulously record all the pulls"that the other data were from. So there's nothing else to do but go to a couple of pre-release events and grab the data from there.

Table 5. Calculations for *Temporal Forces* pull rates, and expected values from them.

Rarity	Average	+Sample	Calc	Sim
DR	14.31%	14.59%	342	353
UR	6.59%	6.61%	952	960
IR	7.73%	7.64%	1063	1052
SIR	2.73%	2.17%	1353	1294
HR	1.72%	1.51%	974	963
ACE		7.28%	250	279
Full			1589	1560

After around 300 packs worth of data[29], it looks like *Temporal Forces* will have similar pull rates to the last few sets, with the slightly higher Double Rares in exchange for lower Special Illustration and Hyper Rares. ACE SPEC cards do

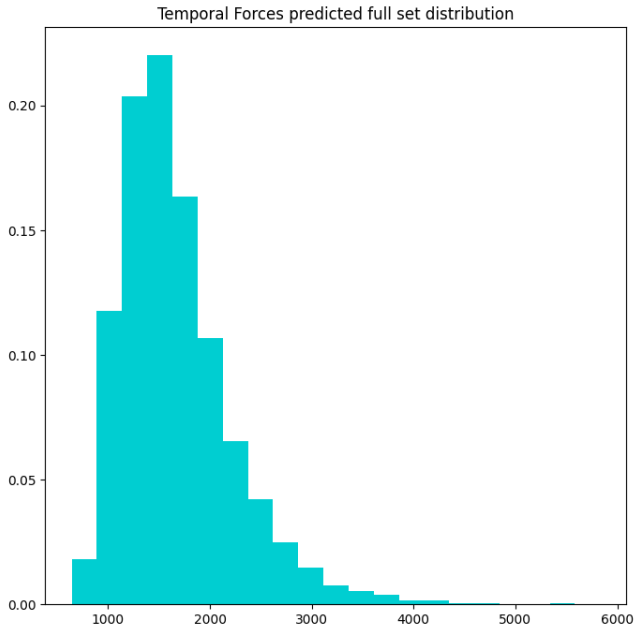

Figure 6. Simulation distributions for 10,000 simulations of *Temporal Forces* with estimated odds.

appear to be fairly common, and with them being the sole card to appear in the first reverse holo slot in this set[30], this just makes them bonus cards to pull, with around 36% of packs expected to contain at least some form of special card, higher than any of the other non-premium sets[31]. And with the overall size of the hits being on the same order as *Scarlet & Violet Base*, this looks like it will have the best pulls this side of *Obsidian Flames*, and with lots more cool cards to boot.

Will it be worth it, though? Again, probably not. The most cost-effective way to purchase packs for preorder (a case of booster boxes) is running $610 for 216 packs, good for an average price of $2.82 per pack. This means that to purchase all those packs, the cost right now is $4480. With the current value of the cards at $1203, this means it is only 3.72x more expensive to pull the cards rather than just buying them. However, given that the cards aren't actually out yet, it'll probably take about a month before the cards actually stabilise in price, and will likely correct to a much lower value than this. It may still turn out to be one of the best relative valued sets, but even now in this extremely favorable environment, it's so much cheaper to just buy the cards rather than trying to pull them.

[25] ex cards[26] were revealed to be part of the set at its reveal, having last been seen in the Ruby and Sapphire era.

[26] Not to be confused with EX cards, last seen during XY. Yes, they're different, and the capitalisation matters. So technically in Expanded, you could run four copies of AOR042 Tyranitar EX and four copies of OBF066 Tyranitar ex. Heck, throw in four copies of AOR043 M Tyranitar EX[27], four copies of BTS097 Tyranitar V, and four copies of any normal Tyranitar. They're all different.

[27] oh god i just realised with legends z-a showing megas are coming back, we might get the return of M EX pokémon and will have to deal with both ex and EX cards.

[28] But there have been Prereleases, as noted

[29] Specifically, in 302 packs, there were 51 Double Rares, 21 Ultra Rares, 20 Illustration Rares, 2 Special Illustration Rares, 1 Hyper Rare, and 22 ACE SPEC.

[30] Oh I guess I never mentioned where they go. Double and Ultra Rares take the place of the normal rare, while Illustration, Special Illustration, and Hyper Rares replace the second reverse holo. In *Paldean Fates*, all the Shiny cards also replaced the first reverse holo, while *Pokémon 151*'s Holofoil Energies replaced the energy.

[31] oh yeah you probably noticed that *Pokémon 151* and *Paldean Fates* had much higher booster prices than the other sets. Premium just means the booster packs can't be bought normally and only come in special collections.

Acknowledgments

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Appendix: so *Temporal Forces* released...

With the extendeded deadline, *Temporal Forces* has released, and as such, I have the actual data for the set. And... uh... the odds were very much messed with. Double Rares are now even more common than they were in *Paradox Rift* and *Paldean Fates* at around 17% of packs rather than 15.5%, and ACE SPEC cards were a fair ways less likely, at 5% rather than 7.3%, although the large gap was more likely given the lack of data for the value.

The biggest shock, however, is that both Special Illustration Rares and Hyper Rares have had their rates dropped to around half, from an estimate of 2.2% to a real of 1.2% for SIR and from 1.5% to 0.7% for HR. This means they take SO much longer, and *Temporal Forces* goes from being probably the easiest set to collect since *Obsidian Flames* to the WORST set by far.

And as for buying packs vs singles, the cheapest option is still the booster box case, down to $570 for 216 packs at $2.64 per pack. At this rate, the expected cards would set you back $7,300, and, as of the time of writing, the current set has a value of $837[32], for an expected overcost of 8.73x the value of the set. As the set continues to spread out, the price is likely to keep dropping, so this will be even more overpriced as time goes on while the set is in print, potentially getting to levels above any of the prior sets.

Table 6. *Temporal Forces* predicted and actual averages

	Simulated		Calculated	
Rarity	Pred.	Act.	Pred.	Act.
DR	353	311	348	296
UR	960	994	955	944
IR	1052	1081	1051	1052
SIR	1094	2578	1073	2504
HR	863	2189	858	2044
ACE	279	386	250	364
Set	1460	3033	1408	2769

[32]Yes price data is still from TCGPlayer

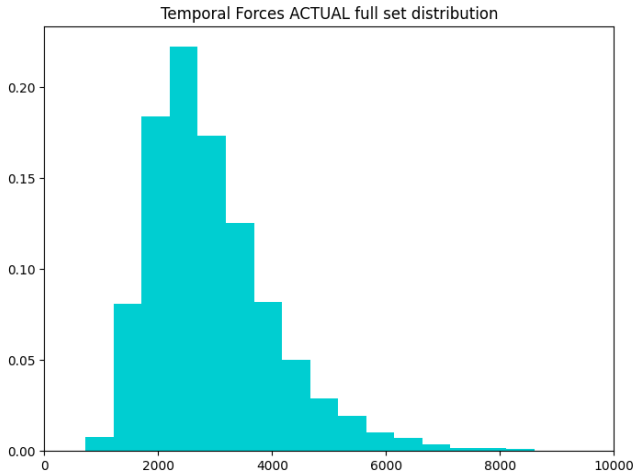

Figure 7. Simulation distributions for 10,000 simulations of *Temporal Forces* with the real data. Four simulations were stopped early after failing to collect all the cards in 10,000 packs.

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