A perpetual motion machine

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Abstract

In this work, we demonstrate that any sealed container containing some gas is a perpetual motion machine, since the gas molecules are constantly accelerated by Hubble expansion and therefore heated up, and the heat energy can be extracted and converted to work. We give quantitative estimates of the power output of some representative gas containers.

1 Introduction

Perpetual motion machines (PMMs) are machines that can do work forever without energy input[1]. The quest for PMMs has been a popular topic for centuries, since the availability of PMMs would solve the energy crisis problem forever. Although the Sun (and other stars as well, assuming that interstellar travel will become possible) can for all intents and purposes be treated as an effectively inexhaustible energy source in the foreseeable future, this is no longer true if humanity manages to live beyond the lifetimes of the vast majority of stars, or if the energy demand of humankind exceeds the power output of the Sun. Therefore, despite the wide availability of techniques to extract solar energy with little cost and relatively high efficiency, the study of PMMs is still of fundamental importance for the long-term sustainability of humankind.

A frequently quoted argument is that PMMs are theoretically impossible, because they violate the conservation of energy[1]. However, violation of the conservation of energy is not a proof of impossibility, as exemplified by the dark energy[2]. Dark energy is believed to come from nowhere as the universe expands, and is responsible for the accelerated expansion of the universe, therefore in direct conflict with the conservation of energy. Nevertheless, while whether current observational data do support the presence of dark energy is subject to some debate, people generally agree that it is at least not possible to exclude the presence of dark energy with the data we currently have. This immediately suggests that it may be possible to build a PMM taking advantage of the unlimited nature of the dark energy, specifically, using the perpetual expansion of the Universe.

The simplest approach would be to tie two balls to the ends of a very long rope, so that the balls are gradually dragged away by cosmic expansion. The kinetic energies of the balls will eventually be harvested when the rope is straightened, in which case the energy is converted to the tension of the rope, and by attaching the rope to machines we can extract work from it. To allow for indefinite energy output, the rope has to be continuously lengthened; this can be done by using part of the generated energy to create matter and antimatter out of vacuum, and using the matter to lengthen the rope. However, the balls need to be extremely heavy so that the energy output can become greater than the energy spent in matter-antimatter creation. While this may seem to be difficult but not impossible, another issue makes it fundamentally impossible for the method to output energy indefinitely: one of the balls will eventually recede behind the cosmic event horizon of the other ball, at which point the rope will inevitably break.

The present work was inspired by the observation that the balls do not need to separate from each other indefinitely in order for the energy output to last forever. Specifically, the balls still accelerate due to cosmic expansion even if they are allowed to bounce back off a wall. This allows us to put the balls in a closed container, and furthermore replace the balls by gas molecules, so that they will bounce around in the container forever and extract energy from cosmic expansion indefinitely. The extracted energy will be in the form of the kinetic energy of the molecules, which is just thermal energy. A perpetual motion machine can then be built taking advantage of the thermal radiation of the container. In the following section, we prove that the container will indeed generate heat due to cosmic expansion, and provide quantitative estimates of the power output for gas containers of certain sizes.

2 Results and Discussions

Suppose there is a gas molecule M in a rigid container. The molecule moves from a point on the wall of the container, A, with initial velocity v_0, until it hits another point on the container, B. The velocity of M relative to point A increases due to cosmic expansion, as it recedes from point A. The recessional velocity v_r of M at any given time is given by Hubble's law[3]:

$$v_r = HD_{AM} + v_{pec} \tag{1}$$

where $H > 0$ is the Hubble constant, D_{AM} is the distance of M from point A, and v_{pec} is the peculiar velocity of M. Since $v_r = v_0$ when $D_{AM} = 0$, we have

$$v_{pec} = v_0 \tag{2}$$

Thus, the velocity of M relative to point A increases linearly with respect to the distance of M from A.

When the molecule arrives at point B, it bounces off elastically and heads towards point C. From the same analysis as above, we can say that the velocity

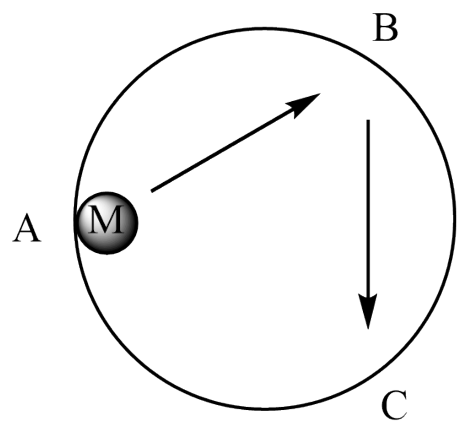

Figure 1: A gas molecule in a rigid container.

of M relative to point B increases linearly as the distance of M from B increases. However, as the container is rigid, point B is stationary relative to point A; therefore, the velocity of M relative to point A also continues to increase even after M has bounced off point B and is not heading away from A anymore. The velocity increment of M is thus the sum of the increment already attained from A *en route* to B (i.e. HD_{AB}), and that attained after the molecule has bounced off point B (HD_{BM}). One can therefore write the general expression of v_r as

$$v_r = Hs_{AM} + v_0 \tag{3}$$

where s_{AM} is the *arc length* of the route travelled by M, starting from A. It is easy to see that the formula is not only applicable when only one bounce has been made, but also to the case of an arbitrary number of bounces. In particular, it is true even until M eventually returns to point A. The important corollary is that, M can depart from point A and return to the same point, yet its velocity increases, and the extra kinetic energy comes solely from cosmic expansion[4]!

Eq. 3 gives the dependence of the velocity of M, i.e. v_r, on the distance travelled by M (s_{AM}). However, a more convenient quantity is the dependence of v_r on time t. Noticing that

$$v_r = \frac{ds_{AM}}{dt} \tag{4}$$

we can solve Eq. 3 as an ordinary differential equation, yielding

$$s_{AM} = \frac{v_0}{H}(e^{Ht} - 1) \tag{5}$$

or

$$v_r = v_0 e^{Ht} \tag{6}$$

Eq. 6 shows that the velocity of M increases exponentially with time. Consequently, the kinetic energy of M also increases exponentially, but with twice the exponent, due to the kinetic energy being proportional to the square of the velocity.

Now, suppose that the container contains not just one gas molecule, but a gas composed of a macroscopic number of molecules. The above derivations are still applicable. Moreover, it is now meaningful to talk about the temperature, T, of the gas. Since the temperature of a gas is proportional to the average kinetic energy of the gas molecules, we have

$$T(t) = T(0)e^{2Ht} \tag{7}$$

Eq. 7 shows that the temperature of a gas in a rigid container increases exponentially, with a time constant of $2H$. Since the current value of H is about 2.27×10^{-18} s^{-1}[5], it follows that the temperature of the gas doubles every 1.53×10^{17} s, or 4.84 billion years. Note that although the Hubble constant changes over time, our qualitative conclusions will remain unchanged as long as the Hubble constant does not approach zero in the long run, which most physicists seem to believe to be the case. Therefore, as long as the container is coated by a sufficiently good thermal insulator, such that its heat loss is on the billion year timescale, the container will become measurably warmer than the outside, and work can be extracted every a few billion years, by temporarily removing a part of the insulating layer and using the resulting heat flow to power a heat pump. This requires the development of extremely good thermal insulators that are well beyond current technology. Alternatively, work can be extracted with a heat pump even without thermal insulators that can work on the billion year timescale; however, the temperature of the container will be only marginally higher than the environment, leading to extremely low Carnot efficiency.

We now estimate the power available for the heat pump. Assuming that the thermal energy $C_V dT(t)$ accumulated over a short period of time dt is converted to work, where C_V is the heat capacity of the gas at constant volume. Then the output power of the container is

$$P = C_V \frac{dT(t)}{dt} = 2C_V HT(0) \tag{8}$$

For example, for a container filled by air ($C_{V,m} = 20.8$ J·mol^{-1}K^{-1})[6] near 298 K, the power output is 4.86×10^{-13} W per kilogram of air, or (at ambient pressure) 5.83×10^{-16} W per liter of air. The numbers may seem negligible, but since there is 5.15×10^{18} kg of air in the world[7], the total power output due to cosmic expansion heating up the atmosphere is 2.50 MW, which is equivalent to 50000 50 W light bulbs. We expect that the figure may not be very accurate since the atmosphere is not contained in a rigid container, but it should have the correct order of magnitude. Also note that cosmic expansion does not seem to be a major contributor to global warming, since the resulting temperature increase is only $\frac{dT(t)}{dt} = 1.35 \times 10^{-15}$ K, tens of magnitudes smaller than the actual global warming rate.

Stars like the Sun are mainly composed of a gas of free protons and electrons, in a 1:1 ratio. Both are single-particle gaseous particles, and thus have $C_{V,m} = 12.5$ J·mol^{-1}K^{-1}[6]. Since the Sun's mass is 1.99×10^{30} kg, and its center temperature is 1.57×10^{7} K[8], the total power output of the Sun due to cosmic expansion is 3.55×10^{24} W. Surprisingly, this is only 2 orders of magnitudes smaller than the total power output of the Sun (3.85×10^{26} W[8]). Although our estimate of the contribution from cosmic expansion is likely an overestimate (due to, for example, using the center temperature of the Sun as the temperature of the whole Sun), we expect that more accurate theoretical models of the Sun may eventually reveal that nuclear fusion does not explain all of the power output of the Sun, and therefore provide an experimental estimate of the cosmic expansion contribution to the power output of the Sun[4].

3 Conclusions

In this work, we show that a gas confined in a rigid container will spontaneously heat up due to cosmic expansion, therefore acting as a PMM, since the heat can be extracted as work by a heat pump. Although the power output of any everyday sized gas container is negligible, it becomes appreciably large when the mass of the gas is comparable to the Earth atmosphere. Even more, we predict that cosmic expansion may have contributed up to 1 % of the total power output of the Sun. Therefore, we propose that the presently mentioned PMM can be built by amassing stellar masses of gases, and collecting the resulting heat radiation. Due to gravity, the gas does not need to be confined within a container anymore, which is especially convenient.

Meanwhile, we point out a few limitations of the current work: we did not take into account the general relativistic corrections due to the gravity of the gas, the fact that there are no infinitely rigid containers, the fact that thermal insulating ability has an upper bound, the finite temperature of the cosmic microwave background, and the breakdown of the non-relativistic velocity-addition formula. These effects are anticipated to be small[4] and will be addressed in future work.

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References

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