Multidiscipline Elo - A complex analysis

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Abstract
Elo has been used as a common method for ranking the comparative strengths of competitors in a wide variety of fields. Since its introduction in 1960 for chess[1], it has since been used for numerous other sporting events, to mixed levels of success. Even with the ubiquity, one problem is that these Elo ratings are largely independent of each other. For instance, at time of writing, Magnus Carlsen has a chess Elo of 2852[2], Jauny has a League Elo of 2007[3], and the Boston Celtics have an NBA Elo of 1672[4]. However, these cannot tell us what to expect if Magnus Carlsen was put into League of Legends, and Jauny played as him in a match against the Boston Celtics. This is *obviously* a major flaw, and this paper will seek to address this issue.

Keywords
Puyo Puyo — Tetris — Elo — Maximum Likelihood Estimation

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Introduction

Unfortunately, there is some limitations as to what fits within the scope of this paper. For instance, the NBA for some reason failed to respond to a request to play hundreds of games of various types and record the results based solely on my whims. As such, this will only focus on a scale more easily performed by a graduate student: Puyo Puyo Tetris 2.

Puyo Puyo is a competitive Japanese puzzle game, wherein players try to cause their opponents to run out of space to place new pieces. To do this, players match up four or more puyos of the same color. As the puyos are affected by gravity, this can cause a chain reaction, and the larger the chain, the more nuisance puyo that are dropped on the opponent's playfield, taking up space. However, the opponent can create a chain of their own during this time to neutralise the incoming nuisance. One of the interesting factors of this game was that the different characters had different play-styles. For instance, some of them fill up the edge columns immediately before playing slowly in the center to develop chains, others will make use of quick and/or hard drop (if available) to build up their potential chains faster, and still others *don't realise that the pieces can be rotated*[5].

Tetris is, as we all know, an inventory-management survival-horror game[6]. Of the three main flavors of Tetris, modern Tetris adds a competitive element as well, where when one clears lines on their playfield, a number of lines of garbage missing a single square are sent to push up the bottom of the opponent's playfield, depending on the number of lines cleared at once, the number of consecutive pieces dropped which cleared a piece, and whether or not this or the previous line clear were Tetrises or T-spins.

Puyo Puyo Tetris (and its sequel), thus, is the puzzle game equivalent of Avengers: Endgame. It takes a bunch of the most popular Puyo characters, and adds in anthropomorphised versions of the Tetris pieces (and Sonic the Hedgehog for some reason). The Tetris characters received AIs from some of the Puyo characters that weren't in the first game (but not Ai, who was one of the Tetris characters added), and everyone got an AI for Tetris. And since the crossover aspect was highly emphasised, one can play Puyo against Tetris. Characters were split into those who preferred to play Puyo and those who preferred to play Tetris, with all the Tetris characters joining some Puyo characters (usually villainous, because nobody good would dare do a center four-wide well[7]) in preferring Tetris.

Because both Tetris and Puyo have a skill component, and this game provides twenty-two characters out of the box (up to twenty-eight after beating the game, and forty with all the DLC), this is a perfect medium for the creation of two separate Elo scales, and then looking at how their combination interacts.

1. Methodology

Given time constraints of having to watch all the AI matches manually, rather than thousands of simulations between the forty different characters in the game, a subset of eight characters were chosen for a proof-of-concept, evenly taken from those who specialise in Puyo and in Tetris:

1. Arle - *The protagonist of the entire Puyo Puyo series, Puyo*
2. Ringo - *One of the two protagonists of the Puyo Puyo Tetris games, Puyo*
3. Tee - *The other protagonist, who uses similar AI to Ringo, Tetris*
4. Squares - *The final boss of Puyo Puyo Tetris 2 (not sorry for spoilers), Tetris*
5. Ecolo - *Final boss of Puyo Puyo 7, considered to have one of the best Puyo AIs, Puyo*
6. Lemres - *Considered to have one of the best Tetris AIs, and also my favorite character, Tetris*
7. Draco - *Has a specifically different Puyo AI, in that she cannot rotate the pieces, Puyo*
8. Jay and Elle - *I needed another Tetris main, and the fact that there's two characters underscores the symbolism of this projects scope, Tetris*

These characters were then ran through three experiments to develop a basis for multidimensional Elo:

1.1 Baseline calculation

Each of the characters played five games against each of the other characters in both Puyo Puyo and Tetris. Puyo Puyo games were just two character matches, with player 1 set to CPU controlled.

This was originally going to be done the same for Tetris, but after the Arle and Ringo match took over an hour, and one specific game between Arle and Jay/Elle went half an hour without any sign of finishing, these were switched to three character matches, with player 1 being the author playing Witch in Puyo, and the other two being the Tetris CPUs. This is because Puyo Puyo matches have margin time, wherein after a set amount of time (96 seconds on default), all garbage sent is multplied by 4/3, and this continues until someone wins. This is missing from pure Tetris matches, but by throwing a Puyo character in (and immediately topping them out so as to not influence the results), margin time remained active for the Tetris match and reduced the games to only being *reasonably* slow.

For both of these, the results were used to derive the Elo for each game separately. Characters were given a base Elo equal to 40 points per win above 17.5. As the expected score from Elo are only based on difference between Elo rankings, the same results would be derived from a match between a 100 Elo and a -100 Elo player as would from a 39642 Elo and a 39442 Elo player. For ease of facilitating complex numbers, the Elo was set to average to 0.

From these base Elo values, the expected score of a single game was calculated. The binomial odds of seeing the results was calculated, and the Elo values were then modified through maximum likelihood estimation to find an estimate for the actual Elo values of the characters. To account for a scenario in which one group of characters always beat another group, a penalty was added to the log likelihood of one-two hundredth of the consecutive Elo differences.

1.2 Cross-game comparison

As mentioned when talking about Tetris above, Puyo Puyo Tetris allows matches to be played between Puyo and Tetris. For this, each character played three games as Puyo against every other character as Tetris.

These results were then compared with the Elo values calculated in the first experiment to determine if the results were consistent with these prior values. As there were prior Elo values for this, these were updated with the results to create the final Elo for the final experiment. Once again, maximum likelihood estimation was used to find the correct K-value for updating the scores.

1.3 Combined games and generalised complex Elo

Puyo Puyo Tetris also has two game modes where Puyo and Tetris are played at the same time. First up is Swap, where players start in one of the games, and after twenty-five seconds, move to the other game, where whoever is first to win in either game wins the match, and the garbage/nuisance to be sent to the other player carries over and stacks on top of each other, with bonuses for sending over both of them at the same time. The second game mode is Fusion, which *sucks*[8], and will have no further discussion in this paper.

As such, each of the characters played three games of Swap with each of the other characters. From this, these data were combined with the other results to create a method for using two disparate Elo values for a combined prediction and comparison. This is where complex numbers mentioned in the title are brought in, as an easy way to have two distinct Elo scales in one number not directly affecting each other. Maximum likelihood estimation was used yet again, this time to figure out Swap's complex game value (a notion that will be introduced in Section 4).

2. Experiment 1: Baseline Puyo and Tetris Elo

2.1 Puyo Puyo

The Puyo section of games showed some potential flaws that were not initially accounted for. The original plan was to use the characters' core AI (an advanced version that plays at a

Table 1. Puyo match results

	Arle	Ringo	Tee	Squares	Jay/Elle	Lemres	Ecolo	Draco
Arle		5	4	0	2	0	0	3
Ringo	0		2	0	1	0	0	2
Tee	1	3		0	0	0	0	2
Squares	5	5	5		5	4	3	5
Jay/Elle	3	4	5	0		2	0	5
Lemres	5	5	5	1	3		4	5
Ecolo	5	5	5	2	5	1		5
Draco	2	3	3	0	0	0	0	

much higher level than the normal ones). However, after a couple of matches, there was only a slight difference between the characters performances; everyone was quickly dropping their puyos and making six-plus chains. As such, this trial was restarted fairly early with the normal AIs.

The normal AIs did show quite a large difference between how they performed, almost to a degree of extreme separation. Apparently, the choice of the eight characters didn't *quite* span the spectrum as much as expected, or rather, they spanned a fairly large degree, but instead of being quite separated, they were clumped into three groups: To no surprise, Ecolo, Lemres, and Squares were significantly better performers than the rest of the characters, and Ringo and Tee were significantly worse. Surprisingly enough, Draco was an admirable performer.

2.2 Tetris

As noted in the methodology, Tetris had an initial difficulty in that, without margin time, matches kept on dragging on and on. Even with figuring out a way to introduce the margin timing, these matches were still by far the lengthiest to run.

The largest problem with the Tetris matches were that the games were heavily based on the skill, and as such, the divisions noted before were more extreme. While Ringo and Tee were 6-54 against the other characters in Puyo, and Squares, Lemres, and Ecolo were 73-2, these were a perfect 0-60 and 75-0. This is why the additional weighting for the separation was added to prevent the Elo scores from going to infinity when solved for with maximum likelihood estimation.

2.2.1 MLE problems with perfect records

As a refresher, if player A has an Elo rating of R_A and player B likewise has rating R_B, the expected value of player A over a single match is

$$E_A = \frac{1}{1+10^{\frac{R_B-R_A}{400}}} = \frac{10^{R_A/400}}{10^{R_A/400}+10^{R_B/400}}$$

If player A won every game in a match of n games, then from a binomial distribution, the odds of that happening are ${E_A}^n$. Defining $R_A = 0$ for ease of calculations (seeing that there is only one equation and two variables), the log likelihood is

$$-n\ln\left(1+10^{R_B/400}\right) \tag{1}$$

This can be made arbitrarily close to zero by making R_B increasingly negative.

If there is a single loss, however, the odds of this result occuring are $n\,{E_A}^{n-1}(1-E_A)$, leading to a log likelihood of

$$\ln n - n\ln\left(1+10^{R_B/400}\right) + \frac{\ln 10}{400}R_B \tag{2}$$

In this case, as R_B is made increasingly negative, the first term is a constant and the second tends to zero, but the third decreases without limit, providing a bound to the gap.

2.3 Elo calculations

The data were entered into Excel, and the default solver was used to find the maximum value of the likelihood. To ensure a somewhat consistent starting value, the initial values of Elo were set to $40(w-17.5)$ for a character with w wins for everyone but the last character, and the last character was set to the negative sum of the other characters to ensure the total Elo was zero.

Table 2. Tetris match results

	Arle	Ringo	Tee	Squares	Jay/Elle	Lemres	Ecolo	Draco
Arle		5	5	0	1	0	0	1
Ringo	0		3	0	0	0	0	0
Tee	0	2		0	0	0	0	0
Squares	5	5	5		5	4	3	5
Jay/Elle	4	5	5	0		0	0	3
Lemres	5	5	5	1	5		1	5
Ecolo	5	5	5	2	5	4		5
Draco	4	5	5	0	2	0	0	

To set up the log likelihood, the formula

$$= \mathtt{LN(BINOM.DIST(C4,5,1/(1+10\hat{}((C\$2-\$B4)/400)),0))} \quad (3)$$

was used, where `C4` was the square containing the number of wins said character had, `$B4` was the chosen character's Elo (mirrored from the vertical column that had the modified values), and `C$2` was the Elo of the other character. These values were added up (only once for each match between characters), as well as adding $1/200$ of the largest gap between consecutive Elo ranks.

Table 3. Elo calculations, experiment 1

	Puyo Elo	Tetris Elo
Squares	497	642
Ecolo	371	625
Lemres	373	449
Jay/Elle	38	-19
Arle	-138	-229
Draco	-332	-56
Ringo	-407	-697
Tee	-402	-715
Log Likelihood	-28.886	-19.712

As seen from the maximum likelihood, the Elo values are sensible from what was seen before. Characters who had similar number of wins had similar Elo values, and more wins always resulted in a higher Elo. The log likelihood of the Tetris was much higher, as a result of having the splits between the characters, and there being more matches that were decided 5-0

3. Experiment 2: Puyo vs Tetris and Elo updates

Of course, the main selling point of Puyo Puyo Tetris is, of course, the ability to play Puyo and Tetris *against* each other. As such, any attempt of creating an Elo with both of them should be able to look between the two.

In effect, pitting the two against each other dealt with multiple issues that the separate trials from experiment one had. First up, the problem of groups always or never winning has more chances to happen (and thankfully, it did happen). Because of this, for the final maximum likelihood estimation, the additional term for distance was removed, as there is no worry about the values spreading to infinity.

The second potential result is a well-known meme in Puyo Puyo Tetris, that the Tetris player is at a slight disadvantage[9]. Normally taken as a joke for when a Tetris player pulls out a ridiculous combo and recovers to win, but the analysis was from mid-level players where just the different playstyles supposedly leads to a slight advantage to Puyo. By pitting the two against each other, one can see if such an advantage really exists.

The third and final result is that since maximum likelihood estimation takes quite a bit of time to run and set up, being able to know the K-factor for incremental updates to the Elo, and after the fact, can be calculated for each character individually rather than needing to solve for everyone at once. Unfortunately, this does need one last MLE run to get the K-value, but in future works for Puyo Puyo Tetris Elo, these results will graciously be available to save everyone involved some time.

Please note that in Table 4, the results only list the number of games that the Puyo player (in row) won, and the number of wins by the Tetris player (in column) is just three minus that number.

Table 5. Puyo vs Tetris relative performance

	Puyo		Tetris	
	Expected	Actual	Expected	Actual
Arle	8.11	11	7.09	6
Ringo	4.03	3	0.97	3
Tee	4.04	3	0.89	3
Squares	17.36	20	19.80	14
Jay/Elle	10.73	17	10.70	8
Lemres	15.47	18	17.69	14
Ecolo	16.06	17	19.30	16
Draco	6.92	7	8.83	7

As expected from the meme, generally the Puyo character tended to win slightly more often, going 97-71 in the 168 matches. As such, the prior thought fact that the Tetris player being at a slight disadvantage is actually true (or at least, for the specific chosen characters). This ends up being equivalent to a 52 point Elo advantage for the Puyo characters.

Table 4. Puyo results vs Tetris opponents

	Arle	**Ringo**	**Tee**	**Squares**	**Jay/Elle**	**Lemres**	**Ecolo**	**Draco**
Arle		3	3	0	2	0	0	3
Ringo	1		2	0	0	0	0	0
Tee	1	2		0	0	0	0	0
Squares	3	3	3		3	3	2	3
Jay/Elle	3	3	3	3		1	2	3
Lemres	3	3	3	2	3		1	3
Ecolo	2	3	3	1	3	3		2
Draco	2	1	2	0	2	0	0	

To estimate the K-value for these games, once again, maximum likelihood was set up with the same formula as before. However, rather than solving for all the Elo values, the only thing modified was a multiplier on the difference between the expected and actual values from table 5. The value of log likelihood used was the sum of the all three trials from the final Elo calculated, rather than just the Puyo vs Tetris trial.

After calculating, the K-value calculated was 24.95, for a log likelihood of -25.31 for this trial and an average log likelihood of -24.96. Since this value was close to 25, which is a somewhat commonly used K-value and a lot easier to calculate with, this value was used instead, as the only difference between it and the calculated value was Jay and Elle gained one more point in Puyo (which, as it was not counteracted by a point loss anywhere else, was just ignored to keep the average score equal to zero).

Table 6. Elo calculations, experiment 2

	Puyo Elo	Tetris Elo
Squares	563	522
Ecolo	394	543
Lemres	436	357
Jay/Elle	219	-86
Arle	-66	-256
Draco	-330	-102
Ringo	-433	-646
Tee	-428	-687

4. Experiment 3: Swap and Generalised Complex Elo

4.1 Elo combination

At this point, keeping the two Elo values separately is getting kind of annoying for bookkeeping purposes. As such, since the Puyo and Tetris values are well defined for each character, we just need some form of number such that it can keep both parts together. This is a perfect situation to bring in everyone's favorite part of algebra: Complex numbers.

Definition 1 *If a competitor has an Elo rating of R_A in game A and R_B in game B, then the* ***complex Elo*** *Z for those two games is $R_A + R_B i$.*

For the purposes of this paper, analyses will be done with $Z = R_P + R_T i$, that is, the real portion will be the Puyo Elo and the imaginary portion will be the Tetris Elo. This is for multiple reasons: The games are set in the Puyo universe so they should receive top billing; Tetris deals with squares and if you square an imaginary number you get a real number, while if you circle an imaginary number you just get $\textcircled{i}$; and the names of the Tetris characters are so ridiculously uncreative they *can't* be real.

Although easier for manipulation, this does bring up some problems with respect to the classic Elo formulae. Firstly, since an exponent of a complex number results in a rotation, this would make the probability calculated for E_A have an imaginary component, and could also result in a case with the real part outside of $[0,1]$. For instance, if a player had complex Elo of $400+400i$ against someone with an Elo of 0, this would naïvely appear to result in an expected value of $\frac{1}{1+10^{-1-i}} \approx 1.065 + .085i$. Secondly, this would also imply that how good one is at Puyo would affect the odds of winning a match of pure Tetris, and vice versa. Thirdly, this also cannot distinguish between Tetris or Puyo matches, and would give the same result in any case.

To solve the problems in reverse order, we thus define these terms:

Definition 2 *A game's* ***complex value*** *$\check{g}$ is a unit Chebyshev[10] complex number (that is, of the form $\pm 1 + bi$ or $a \pm i$ for $|a|, |b| \leq 1$) that indicated the relatve weights of the complex Elo components of the game.*

Definition 3 *A* ***complex game vector*** *$\mathcal{G}$ is a vector $< \check{g}_A, \check{g}_B >$ consisting of the values of the games played by competitor A and competitor B*

While experimenting for the definition of a complex game value, tests were performed for both numbers on the unit circle and on the unit square. As will be seen later when actually detailing the Swap games, the unit square values ended up being more accurate, despite some details that will be discussed further.

The game vector helps to separate the different games that can be played. Overall, there are four broad classes that the game vector can take, three of which are looked at in this paper:

Case 0 *Pure game* - Both players are playing the same game that makes up one of the components of their Elo. In this case, $\mathcal{G} =< 1,1 >$ or $< i,i >$, but the equation results exactly the same as in normal Elo.

Case 1 *Mixed game* - The players are playing the two different components of their Elo. In this case, $\mathcal{G} =< 1,i >$ or $< i,1 >$, depending on which player is playing which game.

Case 2 *Combined game* - The players are playing the same game, but it is a mix of the two components of their Elo. In this case, $\mathcal{G} =< \check{g}, \check{g} >$.

Case 3 *Generalised game* - This is the catch-all collection for everything else. Most uses for generalised complex Elo will be either cases 1 or 2, but this is the broadest sense and the only one in which no simplifications can be made.

From a given game value, a characters skill with respect to that value can be found by dividing their complex Elo with the value. The real component of this result corresponds to how much of their Elo is in this direction, and the imaginary component is an orthogonal addition to reach the full value. Thus by only looking at the real component of this division, all three problems are solved: there's no imaginary component

Table 7. Swap results

	Arle	Ringo	Tee	Squares	Jay/Elle	Lemres	Ecolo	Draco
Arle		3	2	0	0	0	0	2
Ringo	0		0	0	0	0	0	0
Tee	1	3		0	0	0	0	3
Squares	3	3	3		3	2	0	3
Jay/Elle	3	3	3	0		0	1	3
Lemres	3	3	3	1	3		1	3
Ecolo	3	3	3	3	2	2		3
Draco	1	3	0	0	0	0	0	

to give rotation, the game can be uniquely identified, and the skill in a game not being played only adds to the imaginary component, and does not effect the calculated value. As such, the expected score for generalised complex Elo is

$$E_{A,\mathcal{G}} = \frac{1}{1+10^{\frac{1}{400}\Re\left(\frac{Z_B}{\breve{g}_B}-\frac{Z_A}{\breve{g}_A}\right)}} \tag{4}$$

One may wonder what value the imaginary component of the difference in the exponent corresponds to. It probably deals with some measure of the uncertainty, as it shows how off the main game value, however, this was not explored due to ~~Big Elo not sufficiently paying off the authors of this paper~~ time constraints.

Similarly, the formula for updating complex Elo for a player receiving a score of S_A is only slightly modified from the normal Elo formula, only adding a factor of $\breve{g}_A$ to turn the outcome back into complex values.

$$Z'_A = Z_A + K\breve{g}_A(S_A - E_A) \tag{5}$$

4.2 Puyo Puyo Tetris Swap

The last set of games played was, as noted in the methodology, three sets of Puyo Puyo Tetris Swap. The goal of this was to find the correct game value for Swap. Additionally, this could also be used to see if the game value should be of magnitude one, or have maximum value one (*i.e.* if they are of the set $\|z\|_2 = 1$ or $\|z\|_\infty = 1$). Both of these are simple norms, and in the limits of a pure Puyo or Tetris matchup, both are either $\breve{g} = 1$ or i respectively.

For those of you foolhardy enough to try and enter this into your Excel file to solve, the equation for the log likelihood[11] used here is

$$\begin{aligned}&= \texttt{LN(BINOM.DIST(C4,3,1/(1+10\^{}((IMREAL(}\\&\texttt{IMDIV(IMSUB(C\$2,\$B4),\$B\$12)))/400)),0))}\end{aligned} \tag{6}$$

The cells mentioned before are the same as in Equation 3, with the addition of `$B$12` as the game value, which is what is going to be solved for. For common sense reasons, the maximum likelihood was chosen within the range 0 and $\pi/2$, and then raising e to i times this power (as this has both components greater than zero, that is, if one gets better at either Puyo or Tetris, then they have a higher effective rating). To check the correct norm, this was run a second time, multiplying the game value by the smaller of the secant or cosecant of the angle.

The maximum norm, having a larger magnitude, results in smaller relative differences in Elo, and thus, would increase the probability of seeing a split match rather than a sweep. Because of all the matches that were won 3-0, this seemed to rule out that norm, however, the likelihood was actually higher for this norm by about 50%. The Euclidian norm found a game value for Swap of $.893 + .450i$ with a log likelihood of -20.17, while the maximum norm found a game value of $1 + .667i$ and a higher likelihood of -19.77. Of note is that these are not in the same direction (the Euclidian norm was at angle .467rad while the maximum norm is at angle .588rad).

Both methods struggled with the fact that Tee, heretofore the combined worst player, swept Draco in the three games played. Because Draco was only slightly better in Puyo, but significantly better in Tetris, the original maximum likelihood in both cases was extremely close to just being pure Puyo, with less than 1% of the total performance attributed to Tetris in either case. Thus, the maximum likelihood was rerun with ignoring this one specific match as an outlier to give the results presented above.

Additionally, since this has additional data from the swap games, the Elo can be incremented, using Equation 5 and the results acertained from the previous experiments. The biggest changes from this were that Tee improved his Puyo score with his very successful Swap run, even surpassing Draco for 6th in Puyo and easily jumping past Ringo to not be the worst overall, and Ecolo jumped ahead with their Tetris score, but not enough to get into first overall.

Table 8. Elo calculations, experiment 3

	Complex Elo
Squares	$515+490i$
Ecolo	$433+569i$
Lemres	$445+363i$
Jay/Elle	$248-67i$
Arle	$-96-276i$
Draco	$-395-146i$
Tee	$-305-605i$
Ringo	$-489-684i$

5. Conclusion and further study

The purported purpose of this paper was to develop a groundwork for the combination of Elo between multiple different competitions. Overall, especially with the successful application on Puyo Puyo Tetris, this has been (in the author's biased opinion) an unqualified success.

One of the assumptions used in the MLE for Swap was that getting better at either of the components would help in winning the game. A future diversion might look into a combined game with one of the components allowing misère play, to see how the derived results hold up to a system in which this seemingly obvious assumption is violated.

The results from this paper also offer an obvious extension into quaternions for three or four different games being combined into one, or even octonions for up to eight games. As sedenions have zero divisors[12], the division required for the expected score fails to have a unique solution, but surely the combination of more than eight games at once is a fanciful endeavor that would never exist in real life[13].

Additional trials to acquire further data for the Puyo Puyo Tetris Elo would always be helpful to get a fuller picture of the AIs in this game. Only a fifth of the characters in the game had any research done on their strengths, and so further studies, especially to look at cases of Swap in which there is a serious difference in Puyo and Tetris abilities.

Not mentioned before, but the Puyo and Tetris values being fairly close was a bit of a problem, as with possibly the exception of Draco, Tetris ability was fairly well predicted by Puyo ability. A linear regression run on the Elo scores gave an *r* value of .921, with the calculated best-fit line being $R_T = 1.09R_P - 92.75$. This high correlation meant that the Swap matches already had a pretty solid guess beforehand which of the AIs would end up being better in the match, and so the winner was fairly predictable ahead of time. If further research found more characters with large differences between the Puyo and Tetris abilities, perhaps the imaginary portion of the expected score formula could be better explored.

And if anything should be taken from this paper, the author hopes that at least one person sees this, decides to pick up Puyo Puyo Tetris (the first game), and encounters the dialogue of Chapter 9.

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Lastly, she would like to thank the investigators reading this paper in the distant future, when its seminal value to the field of predictive forecasting is well known and inevitably there is a scandal involving an NBA team cheating at League of Legends through the use of vibrating devices[14], and this paper is looked back on to see how this became a problem.

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