Debunking the April 1 model of SIGBOVIK occurrences

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Abstract

Popular wisdom holds that the SIGBOVIK conference is associated with April 1, presumably because this date is the anniversary of the 1997 perihelion of Comet Hale–Bopp (although a minority believe the date is derived from that of National Tree Planting Day in Tanzania). However, according to the SIGBOVIK 2024 home page [1], this year's edition of this esteemed conference series will occur on April 5, 2024. Such a statistically significant aberration cannot be ignored and suggests that the mainstream understanding of SIGBOVIK occurrences is at best incomplete and at worst fatally flawed.

In this paper we examine historical observations of SIGBOVIK occurrences in detail and conclude that the traditional April 1 model is untenable. We explore several alternative models that yield better fidelity with respect to the observational record.

1 Introduction

"Hey ChatGPT, write us an inspiring introduction for our paper."

— The Authors

In the vast tapestry of time, there exists a celestial event, as awe-inspiring as the eclipse and as deeply mysterious as the farthest reaches of the cosmos. That event, dear reader, is none other than SIGBOVIK. Since the nascent dawn of humanity, it has held an enigmatic allure that has captivated the hearts and minds of scholars, prophets, and ordinary people alike. A cosmic ballet that dances to the rhythm of the arcane, SIGBOVIK unfurls itself in a tantalizing spectacle of grandiose enigmas, swirling within the nebulous vortex of profound absurdities, a manifestation of the ludicrous sublime, held within the cryptic crucible of intellectual whimsy.

In the bygone epochs, the sages would pore over ancient scrolls and sibylline ciphers, endeavoring to decipher the elusive pattern of SIGBOVIK's occurrences. Through the relentless passage of seasons and the tireless turning of centuries, they charted the course of this magnificent event, their eyes gleaming with the same starry wonder that flickered in the eyes of our earliest ancestors. The dates of its appearance became a sacred codex, whispered in hushed reverence in the quiet corners of scholarly retreats.

Now, in the resplendent age of enlightenment that we are fortunate to inhabit, the mystery has been unraveled. We stand on the shoulders of the intellectual giants who came before us, basking in the radiant glow of understanding that they have bequeathed us. The cryptic dance of SIGBOVIK has been decoded, its secrets laid bare for the keen minds of today and the inquisitive souls of tomorrow. Yet, even in the stark light of comprehension, SIGBOVIK retains its mesmerizing allure, a testament to the indomitable spirit of human curiosity and our eternal quest for knowledge.

2 Historical record

As the first step of our research, we conducted an exhaustive search of available records to compile the set of historical SIGBOVIK dates shown in Table 1. To the best of our knowledge, this is the first such survey that has been undertaken. We also computed the predicted date of SIGBOVIK for each year based on the commonly used April 1 model.

The first observation to be made is that SIGBOVIK has taken place on April 1 only 10 times out of 18 occurrences, a rate of less than 56%. This fact alone should be enough to convince all but the most diehard April 1 cultists that the supposed correlation between SIGBOVIK and April 1 is tenuous at best.

In fact, the historical record shows that SIGBOVIK has occurred as early as March 29 and as late as April 8. The average squared error of the April 1 model is 6.72 square days.

This is a poor showing by the simplistic April 1 model. Society demands better. In the following sections, we explore several competing paradigms that all provide improved models that are more faithful to the observed historical data.

Year	SIGBOVIK	Day of year	Prediction by April 1 model
2007	April 1	91	April 1
2008	April 6	97	April 1
2009	April 5	95	April 1
2010	April 1	91	April 1
2011	April 1	91	April 1
2012	March 30	90	April 1
2013	April 1	91	April 1
2014	April 1	91	April 1
2015	April 1	91	April 1
2016	April 1	92	April 1
2017	March 31	90	April 1
2018	March 29	88	April 1
2019	April 1	91	April 1
2020	April 1	92	April 1
2021	April 1	91	April 1
2022	April 8	98	April 1
2023	March 31	90	April 1
2024	April 5	96	April 1

Table 1: Historical SIGBOVIK occurrences, compared to the predicted date given by the April 1 model.

3 A true formula

As everyone knows, the best way to get a formula that fits a set of data is polynomial interpolation. If we fit a polynomial to the "Year" and "Day of year" columns of Table 1, out pops formula (1). In this formula, y is the year and $d(y)$ is the date of SIGBOVIK as a day of the year. Nothing could be simpler.

$$
\begin{aligned}
d(y) = {} & \frac{43}{4560095232000}y^{17} - \frac{30047}{92990177280}y^{16} + \frac{851694793}{163459296000}y^{15} \\
& - \frac{18311875130909}{348713164800}y^{14} + \frac{276830506252402673}{747242496000}y^{13} \\
& - \frac{1487997862575423256 9}{7664025600}y^{12} + \frac{1574638993110764393282557}{201180672000}y^{11} \\
& - \frac{60456200867113679323122241}{2438553600}y^{10} \\
& + \frac{456973521632412957720438689229 1}{73156608000}y^{9} \\
& - \frac{12281416294032117520963587924711 83}{9754214400}y^{8} \\
& + \frac{5835160404888934245511860641249671919}{28740096000}y^{7} \\
& - \frac{4989822533275356031336969896486049269 71}{1916006400}y^{6} \\
& + \frac{1320086469976541116752939482448096742455302 3}{50295168000}y^{5} \\
& - \frac{1773898847125291419613263295138514848946334787 3}{87178291200}y^{4} \\
& + \frac{2364800746340098812384964780451830703505841487 27}{2018016000}y^{3} \\
& - \frac{2859980423741728401011678741573920943010032617063 1}{605404800}y^{2} \\
& + \frac{2083347365710367106770935845013237343931166115044 1}{1750320}y \\
& - 14112764451997310605747830996524070326506484 22.
\end{aligned}
\tag{1}
$$

We conclude this section with a couple of demonstrations of the predictive power of formula (1).

First, since $d(2025) = 2448$, we see that SIGBOVIK 2025 will take place on September 14, 2031.

We can also use formula (1) to assist in the quest for the long-elusive SIGBOVIK 2006. We find that $d(2006) = -3043$, which means that SIGBOVIK 2006 occurred on September 1, 1998. This is significantly earlier than SIGBOVIK scholars and treasure-seekers had previously suspected, which explains why searches for the SIGBOVIK 2006 proceedings have so far been unsuccessful.

4 Correlations with other data sets

A useful tool for finding meaningful correlations between disparate data sets is Vigen's "Serious Correlations" [2]. Using this tool, we discovered a strong correlation ($r = 0.849$, $r^2 = 0.721$, $p < 0.01$) between the date of SIGBOVIK and the number of crane operators in Tennessee two years prior, as shown in Figure 1.

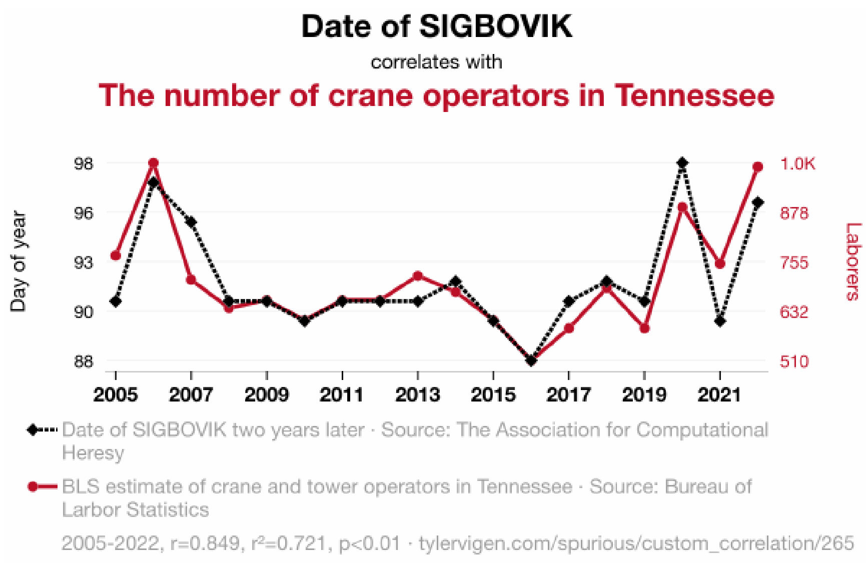

Figure 1: Correlation between the number of crane operators in Tennessee and the date of SIGBOVIK two years later. Note that the years on the horizontal axis are for the Tennessee-crane-operator data, not the SIGBOVIK data, which is time-shifted by two years.

Based on this finding, we developed the formula

$$D_y = \mathrm{round}[0.018(C_{y-2} + 4404)], \tag{2}$$

where D_y is the date of SIGBOVIK in year y (expressed as a day of the year) and C_y is the number of crane operators in Tennessee in year y.

Table 2 shows the predicted D_y values from formula (2). Observe that D_y is exactly correct for 10 of the years, which is as good as the traditional April 1 model, but the Tennessee-crane-operator model has a significantly lower average squared error (1.94 square days, compared to the April 1 model's 6.72).

Formula (2) itself yields some valuable insights. First, let us consider the value 4404, which is added to the number of crane operators in a given year. It is well known that the crane industry involves considerable overhead (in fact, overhead is the main focus of the crane industry). Formula (2) quantifies this overhead: though the number of crane operators varies from year to year, the

Year	SIGBOVIK	Day of year	C_{y-2}	D_y
2007	April 1	91	770	93
2008	April 6	97	1000	97
2009	April 5	95	710	92
2010	April 1	91	640	91
2011	April 1	91	660	91
2012	March 30	90	610	90
2013	April 1	91	660	91
2014	April 1	91	660	91
2015	April 1	91	720	92
2016	April 1	92	680	92
2017	March 31	90	610	90
2018	March 29	88	510	88
2019	April 1	91	590	90
2020	April 1	92	690	92
2021	April 1	91	590	90
2022	April 8	98	890	95
2023	March 31	90	750	93
2024	April 5	96	990	97

Table 2: Historical SIGBOVIK occurrences, expressed as a day of the year, and the predicted D_y values from formula (2).

number of other workers in the Tennessee crane industry remains at a constant level of 4404 every year.

The coefficient 0.018 indicates that each worker in the Tennessee crane industry delays SIGBOVIK by 0.018 days, slightly less than 26 minutes. It can be inferred from this that each worker in the Tennessee crane industry submits a paper to SIGBOVIK, and each such paper requires about 26 minutes to review.

But note that this 26-minute delay is not reflected in the date of SIGBOVIK until two years later. The number of crane operators in Tennessee comes from the Occupational Employment and Wages report of the United States Bureau of Labor Statistics, and this report for year y is typically released in late March or April of year $y+1$, which seems to be too late to affect the date of SIGBOVIK in that year. (The report for 2023 will be released at 10:00 a.m. on April 3, 2024 [3], at which time we will be able to confidently predict the date of SIGBOVIK 2025. Unfortunately this is just slightly after the paper submission deadline for this year, so we leave this calculation as an exercise for the reader.)

These observations together lead to a startling conclusion: Since the 26-minute delay comes from workers in the Tennessee crane industry submitting SIGBOVIK papers, but this process must wait for the Occupational Employment and Wages report to be released the following year, it must be the case that *Tennessee crane operators do not realize they operate cranes until they are so informed by the Bureau of Labor Statistics!* This problem has heretofore not been recognized. We urge the improvement of education and communication in this field so that Tennessee crane operators can be informed more quickly about the work that they do.

Good correlations were also discovered between the date of SIGBOVIK and the number of upholsterers in Connecticut, the number of blender tenders in Idaho two years prior, and the popularity of the “Rickroll” meme (see Figure 2). We believe that additional investigation into the ramifications of these findings is warranted, and we encourage further research.

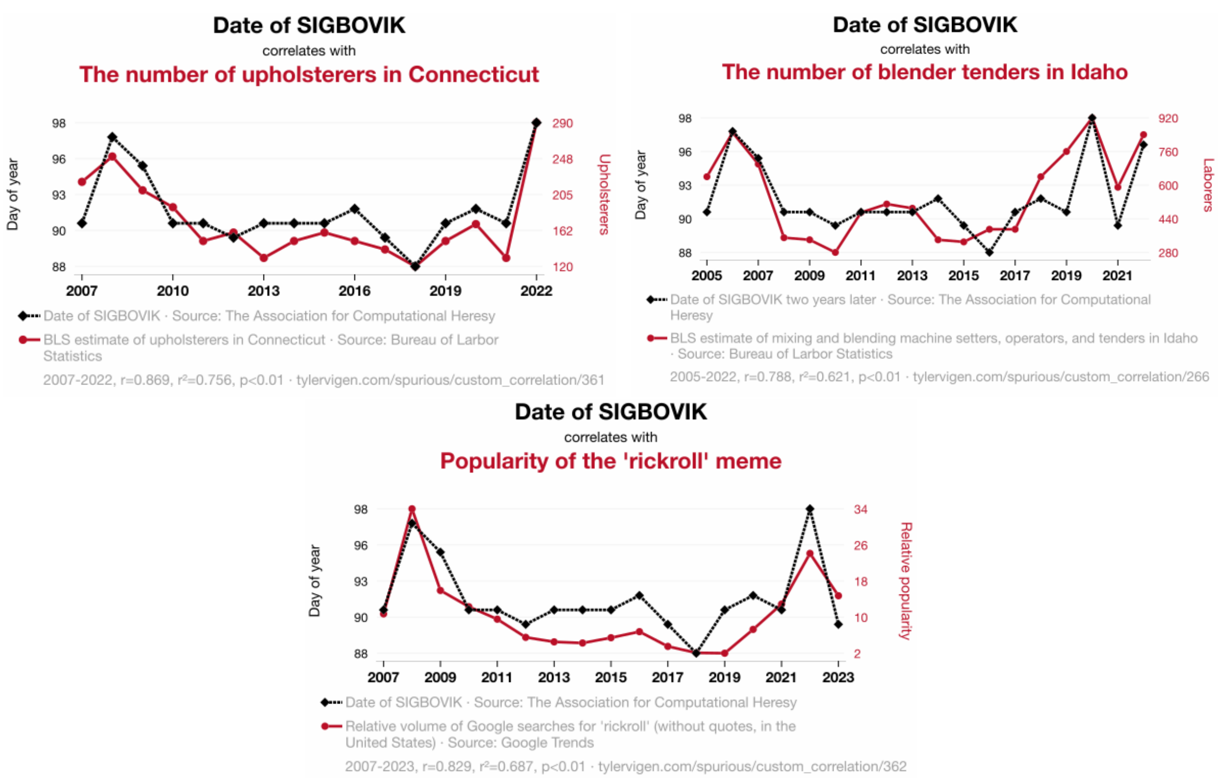

Figure 2: Correlations between the date of SIGBOVIK and other important time-series data.

5 Alternative calendars

One glaring weakness with the models discussed up to this point is that they assume the use of the Gregorian calendar.

As is well known, the Gregorian calendar is described by the rules in Figure 3. These rules are often described in a roundabout way that lists exceptions to exceptions, such as “Every year that is exactly divisible by four is a leap year, except for years that are exactly divisible by 100, but these centurial years are leap years if they are exactly divisible by 400" [4]. Clearly it is simpler (and more easily parallelizable) to give a set of independent rules that each add or subtract a certain number of days.

A standard year has 365 days. The following 3 leap-year rules apply:

1. If the year is divisible by 4, add 1 day.
2. If the year is divisible by 100, subtract 1 day.
3. If the year is divisible by 400, add 1 day.

Figure 3: The rules for the Gregorian calendar.

In any case, the rules in Figure 3 always yield a year that has either 365 or 366 days.

However, consider the number of days between successive occurrences of SIGBOVIK, as shown in Table 3. This data clearly does not fit the Gregorian pattern of 365 or 366 days in each year.

Year	SIGBOVIK	Days since previous SIGBOVIK
2007	April 1	—
2008	April 6	371
2009	April 5	364
2010	April 1	361
2011	April 1	365
2012	March 30	364
2013	April 1	367
2014	April 1	365
2015	April 1	365
2016	April 1	366
2017	March 31	364
2018	March 29	363
2019	April 1	368
2020	April 1	366
2021	April 1	365
2022	April 8	372
2023	March 31	357
2024	April 5	371

Table 3: The number of days between successive SIGBOVIK occurrences.

We hardly need remind the reader that SIGBOVIK celebrates the varied interests and accomplishments of Harry Quodlibetarian Bovik, among which are infinite-dimensional chronometry and stochastic calendariography. It would be unreasonable to assume that Bovik would limit himself to such a dull calendar as the Gregorian.

To provide a starting point for our investigation, we assume that SIGBOVIK 2007, the first known occurrence, took place at the beginning of Bovikian year 1, in which case the numbers of days in Table 3 are the lengths of Bovikian years 1 through 17. An examination of these numbers yields the straightforward leap-year rules given in Figure 4.

A standard year has 371 days. The following 13 leap-year rules apply:

1. If the year is divisible by 2, subtract 7 days.
2. If the year is divisible by 3, subtract 10 days.
3. If the year is divisible by 4, add 1 day.
4. If the year is divisible by 5, subtract 7 days.
5. If the year is divisible by 6, add 13 days.
6. If the year is divisible by 7, subtract 6 days.
7. If the year is divisible by 9, add 5 days.
8. If the year is divisible by 10, add 7 days.
9. If the year is divisible by 11, subtract 8 days.
10. If the year is divisible by 13, subtract 5 days.
11. If the year is divisible by 14, add 7 days.
12. If the year is divisible by 15, add 18 days.
13. If the year is divisible by 16, subtract 8 days.

Figure 4: The rules for the Bovikian calendar.

For Bovikian year 18, these rules give a length of 372 days, which means that SIGBOVIK 2025 will occur on April 12.

We can improve the efficiency of the Bovikian calendar (and allow for the possibility of earlier, as-yet-undiscovered SIGBOVIK occurrences) by removing the assumption that SIGBOVIK 2007 was held at the beginning of year 1, instead trying other starting years. In general, given a vector ℓ of consecutive year lengths, we can compute, for each starting year y_0, all minimum-size sets of leap-year rules (along with the length of a standard year, which is just a leap-year rule with modulus 1) using the algorithm sketched in Figure 5.[1] As it turns out, this problem is NP-hard: it is essentially problem [MP5], Minimum Weight Solution to Linear Equations, in Garey and Johnson [5].

[1] The reader is advised that this algorithm sketch contains a few minor errors, caused by circumstances evident in the figure.

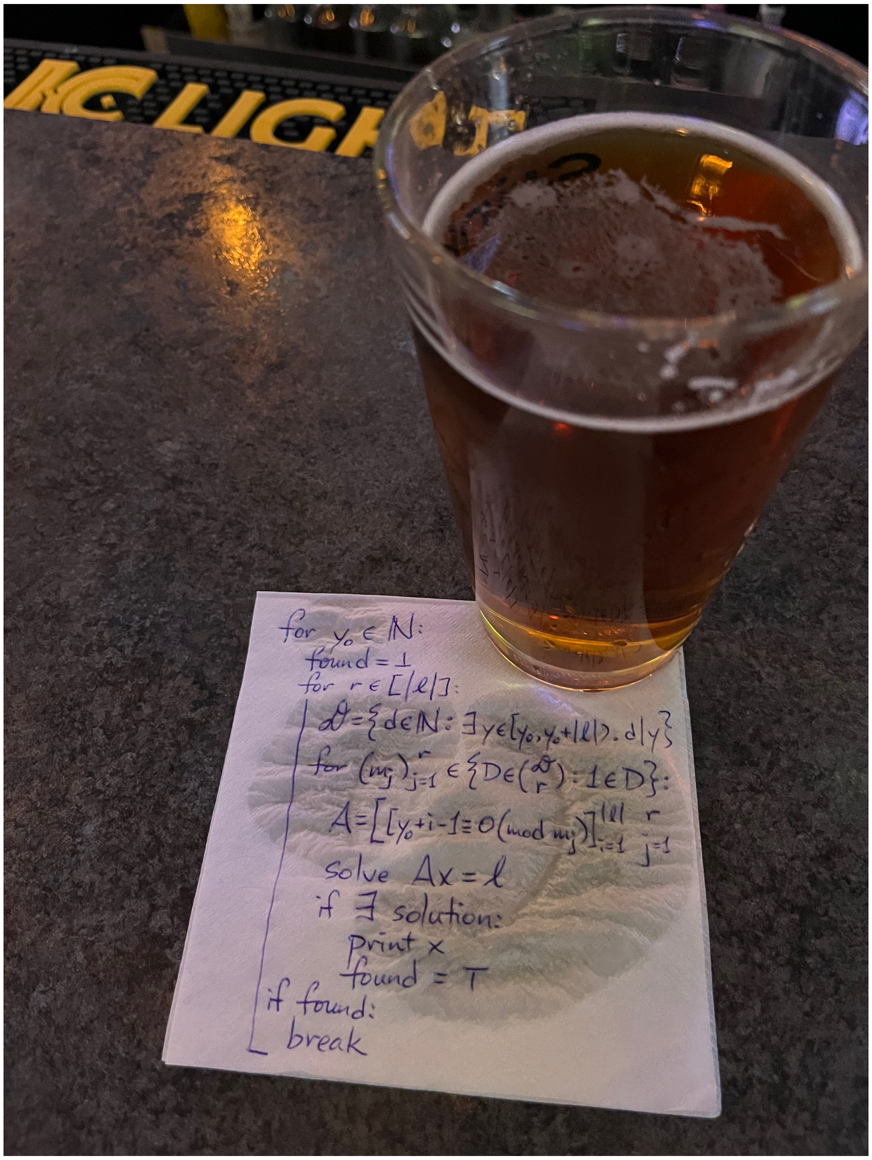

Figure 5: An algorithm to compute all minimum-size sets of leap-year rules for each starting year y_0, given a vector ℓ of consecutive year lengths.

Through starting year $y_0 = 12$, the minimum size of a satisfying set of leap-year rules is 12, achieved for $y_0 \in \{7, 8, 10, 11\}$. The 82 such Bovikian calendars with 12 leap-year rules, through starting year $y_0 = 12$, are given in Appendix A.

An interesting and unexpected connection to astronomy arises from a computation of the average length of a Bovikian year. For example, consider again the Bovikian calendar starting in year $y_0 = 1$ given in Figure 4. With this calendar, the sequence of year lengths repeats in a cycle of 720,720 years, a period of 263,545,580 days. Hence the average length of a Bovikian year is 365.670 days. This is strikingly similar to the length of various "astronomical years" based on the orbit of Earth: the mean tropical year (365.242 days), the mean sidereal year (365.256 days), and the mean anomalistic year (365.260 days). We suspect that this is not a coincidence and conjecture that the Bovikian calendar is somehow related to Earth's orbit, measuring a previously unrecognized aspect of it.

Our preliminary work here to reconstruct the Bovikian calendar gives only the length of the year; it does not yet give the lengths of the months. We leave the further development of this calendar as an open question for future research. It is intriguing to note that the minimum number of leap-year rules found so far is 12, which suggests that there may be a one-to-one correspondence between these rules and the months of the year.

6 Conclusion

We hope that, by exposing the utter folly of the widespread April 1 model of SIGBOVIK occurrences, we can put this tired old trope to rest. It is clear that the April 1 superstition, despite its appealing oversimplicity, cannot be regarded as a reliable reflection of reality.

To replace this worn-out paradigm, we presented a variety of models that better fit the observed historical record. All of the models proposed in this paper are derived logically from unquestionably sound foundations and have been mathematically tested and proved.

We understand that the irregularity of the date of SIGBOVIK has been a source of anxiety for the organizers of the annual SIGBOVIK viewing, who must coordinate room reservations, food catering, advertising, and so forth for an event that has historically been impossible to predict exactly. It is fortunate that, for many years, the SIGBOVIK viewing has coincided with SIGBOVIK itself. We hope that our work can remove some of the stress for the organizers of future viewings by providing reliable predictions for the date of the event.

References

[1] Association for Computational Heresy, The. "SIGBOVIK 2024." `https://sigbovik.org/2024/`.

[2] Tyler Vigen. "Spurious Correlations." `https://www.tylervigen.com/spurious-correlations`.

[3] United States Bureau of Labor Statistics. "Schedule of Selected Releases 2024." `https://www.bls.gov/schedule/2024/home.htm`.

[4] Astronomical Applications Department, United States Naval Observatory. "Introduction to Calendars." `https://aa.usno.navy.mil/faq/calendars`.

[5] Michael R. Garey and David S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman and Co., New York, 1979, ISBN 978-0-7167-1045-5.

A Bovikian calendars with 12 leap-year rules

The important parameters of a Bovikian calendar are the starting year y_0, which begins with the first known SIGBOVIK occurrence, SIGBOVIK 2007; the length s of a standard year; and pairs (m_i, Δ_i) giving divisibility moduli and their corresponding leap-year adjustments in days. For example, the calendar in Figure 4, for $y_0 = 1$, is given by $s = 371$ and the following $(m_i, \Delta_i)_{i=1}^{13}$:

$$\big((2,-7),\ (3,-10),\ (4,1),\ (5,-7),\ (6,13),\ (7,-6),\ (9,5),\\ (10,7),\ (11,-8),\ (13,-5),\ (14,7),\ (15,18),\ (16,-8)\big).$$

The following list gives all Bovikian calendars through starting year $y_0 = 12$ that have 12 leap-year rules.

1. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6),\ (3,-10),\ (4,-1),\ (6,13),\ (11,-7),\ (13,-6),\ (15,5),\ (17,-8),\ (19,-5),\ (20,1),\ (21,11),\ (22,-1)\big)$.
2. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7),\ (3,1),\ (5,-6),\ (6,14),\ (9,-11),\ (10,7),\ (11,-7),\ (12,-12),\ (13,-6),\ (14,1),\ (17,-8),\ (19,-5)\big)$.
3. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7),\ (3,1),\ (5,-6),\ (6,2),\ (9,-11),\ (10,7),\ (11,-7),\ (13,-6),\ (14,1),\ (17,-8),\ (18,12),\ (19,-5)\big)$.
4. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7),\ (3,-11),\ (5,6),\ (6,14),\ (9,1),\ (10,-5),\ (11,-7),\ (13,-6),\ (14,1),\ (17,-8),\ (19,-5),\ (21,12)\big)$.
5. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7),\ (3,1),\ (5,1),\ (6,14),\ (9,-11),\ (11,-7),\ (12,-12),\ (13,-6),\ (14,1),\ (15,-7),\ (17,-8),\ (19,-5)\big)$.
6. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7),\ (3,-6),\ (5,1),\ (6,14),\ (9,-4),\ (11,-7),\ (12,-5),\ (13,-6),\ (14,1),\ (17,-8),\ (19,-5),\ (21,7)\big)$.
7. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7),\ (3,1),\ (5,1),\ (6,2),\ (9,-11),\ (11,-7),\ (13,-6),\ (14,1),\ (15,-7),\ (17,-8),\ (18,12),\ (19,-5)\big)$.
8. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7),\ (3,-11),\ (5,1),\ (6,14),\ (9,1),\ (11,-7),\ (13,-6),\ (14,1),\ (15,5),\ (17,-8),\ (19,-5),\ (21,12)\big)$.
9. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7),\ (3,-6),\ (5,1),\ (6,9),\ (9,-4),\ (11,-7),\ (13,-6),\ (14,1),\ (17,-8),\ (18,5),\ (19,-5),\ (21,7)\big)$.

10. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (5,5), (6,14), (10,-4), (11,-7), (12,-1), (13,-6), (14,1), (17,-8), (19,-5), (21,11)\big)$.
11. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (5,5), (6,13), (10,-4), (11,-7), (13,-6), (14,1), (17,-8), (18,1), (19,-5), (21,11)\big)$.
12. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (5,1), (6,14), (11,-7), (12,-1), (13,-6), (14,1), (15,4), (17,-8), (19,-5), (21,11)\big)$.
13. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (5,1), (6,13), (11,-7), (13,-6), (14,1), (15,4), (17,-8), (18,1), (19,-5), (21,11)\big)$.
14. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,1), (5,-6), (9,-11), (10,7), (11,-7), (12,2), (13,-6), (14,1), (17,-8), (18,14), (19,-5)\big)$.
15. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,3), (5,-8), (9,-13), (10,9), (11,-7), (13,-6), (14,1), (17,-8), (18,14), (19,-5), (21,-2)\big)$.
16. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,1), (5,1), (9,-11), (11,-7), (12,2), (13,-6), (14,1), (15,-7), (17,-8), (18,14), (19,-5)\big)$.
17. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-6), (5,1), (9,-4), (11,-7), (12,9), (13,-6), (14,1), (17,-8), (18,14), (19,-5), (21,7)\big)$.
18. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,3), (5,1), (9,-13), (11,-7), (13,-6), (14,1), (15,-9), (17,-8), (18,14), (19,-5), (21,-2)\big)$.
19. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (5,5), (10,-4), (11,-7), (12,13), (13,-6), (14,1), (17,-8), (18,14), (19,-5), (21,11)\big)$.
20. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (5,1), (11,-7), (12,13), (13,-6), (14,1), (15,4), (17,-8), (18,14), (19,-5), (21,11)\big)$.
21. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,1), (6,13), (8,-1), (9,-11), (11,-7), (12,-12), (13,-6), (15,-6), (17,-8), (19,-5), (22,-1)\big)$.
22. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,-5), (6,13), (8,-1), (9,-5), (11,-7), (12,-6), (13,-6), (17,-8), (19,-5), (21,6), (22,-1)\big)$.
23. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,1), (6,1), (8,-1), (9,-11), (11,-7), (13,-6), (15,-6), (17,-8), (18,12), (19,-5), (22,-1)\big)$.
24. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,-11), (6,13), (8,-1), (9,1), (11,-7), (13,-6), (15,6), (17,-8), (19,-5), (21,12), (22,-1)\big)$.
25. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,-5), (6,7), (8,-1), (9,-5), (11,-7), (13,-6), (17,-8), (18,6), (19,-5), (21,6), (22,-1)\big)$.
26. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,-10), (6,13), (8,-1), (11,-7), (12,-1), (13,-6), (15,5), (17,-8), (19,-5), (21,11), (22,-1)\big)$.
27. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,-10), (6,12), (8,-1), (11,-7), (13,-6), (15,5), (17,-8), (18,1), (19,-5), (21,11), (22,-1)\big)$.
28. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,1), (6,14), (9,-11), (10,1), (11,-7), (12,-12), (13,-6), (14,1), (15,-6), (17,-8), (19,-5)\big)$.
29. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-5), (6,14), (9,-5), (10,1), (11,-7), (12,-6), (13,-6), (14,1), (17,-8), (19,-5), (21,6)\big)$.
30. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,1), (6,2), (9,-11), (10,1), (11,-7), (13,-6), (14,1), (15,-6), (17,-8), (18,12), (19,-5)\big)$.

31. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-11), (6,14), (9,1), (10,1), (11,-7), (13,-6), (14,1), (15,6), (17,-8), (19,-5), (21,12)\big)$.

32. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-5), (6,8), (9,-5), (10,1), (11,-7), (13,-6), (14,1), (17,-8), (18,6), (19,-5), (21,6)\big)$.

33. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (6,14), (10,1), (11,-7), (12,-1), (13,-6), (14,1), (15,5), (17,-8), (19,-5), (21,11)\big)$.

34. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (6,13), (10,1), (11,-7), (13,-6), (14,1), (15,5), (17,-8), (18,1), (19,-5), (21,11)\big)$.

35. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,1), (8,-1), (9,-11), (11,-7), (12,1), (13,-6), (15,-6), (17,-8), (18,13), (19,-5), (22,-1)\big)$.

36. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,-5), (8,-1), (9,-5), (11,-7), (12,7), (13,-6), (17,-8), (18,13), (19,-5), (21,6), (22,-1)\big)$.

37. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,2), (8,-1), (9,-12), (11,-7), (13,-6), (15,-7), (17,-8), (18,13), (19,-5), (21,-1), (22,-1)\big)$.

38. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (3,-10), (8,-1), (11,-7), (12,12), (13,-6), (15,5), (17,-8), (18,13), (19,-5), (21,11), (22,-1)\big)$.

39. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,1), (9,-11), (10,1), (11,-7), (12,2), (13,-6), (14,1), (15,-6), (17,-8), (18,14), (19,-5)\big)$.

40. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-5), (9,-5), (10,1), (11,-7), (12,8), (13,-6), (14,1), (17,-8), (18,14), (19,-5), (21,6)\big)$.

41. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,3), (9,-13), (10,1), (11,-7), (13,-6), (14,1), (15,-8), (17,-8), (18,14), (19,-5), (21,-2)\big)$.

42. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (3,-10), (10,1), (11,-7), (12,13), (13,-6), (14,1), (15,5), (17,-8), (18,14), (19,-5), (21,11)\big)$.

43. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (5,-5), (6,14), (9,-10), (10,6), (11,-7), (12,-11), (13,-6), (14,1), (17,-8), (19,-5), (21,1)\big)$.

44. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (5,-5), (6,3), (9,-10), (10,6), (11,-7), (13,-6), (14,1), (17,-8), (18,11), (19,-5), (21,1)\big)$.

45. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (5,1), (6,14), (9,-10), (11,-7), (12,-11), (13,-6), (14,1), (15,-6), (17,-8), (19,-5), (21,1)\big)$.

46. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (5,1), (6,3), (9,-10), (11,-7), (13,-6), (14,1), (15,-6), (17,-8), (18,11), (19,-5), (21,1)\big)$.

47. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (5,-5), (9,-10), (10,6), (11,-7), (12,3), (13,-6), (14,1), (17,-8), (18,14), (19,-5), (21,1)\big)$.

48. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (5,1), (9,-10), (11,-7), (12,3), (13,-6), (14,1), (15,-6), (17,-8), (18,14), (19,-5), (21,1)\big)$.

49. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (6,13), (8,-1), (9,-10), (11,-7), (12,-11), (13,-6), (15,-5), (17,-8), (19,-5), (21,1), (22,-1)\big)$.

50. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (6,2), (8,-1), (9,-10), (11,-7), (13,-6), (15,-5), (17,-8), (18,11), (19,-5), (21,1), (22,-1)\big)$.

51. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (6,14), (9,-10), (10,1), (11,-7), (12,-11), (13,-6), (14,1), (15,-5), (17,-8), (19,-5), (21,1)\big)$.

52. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (6,3), (9,-10), (10,1), (11,-7), (13,-6), (14,1), (15,-5), (17,-8), (18,11), (19,-5), (21,1)\big)$.

53. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-6), (8,-1), (9,-10), (11,-7), (12,2), (13,-6), (15,-5), (17,-8), (18,13), (19,-5), (21,1), (22,-1)\big)$.

54. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((2,-7), (9,-10), (10,1), (11,-7), (12,3), (13,-6), (14,1), (15,-5), (17,-8), (18,14), (19,-5), (21,1)\big)$.

55. $y_0 = 7$; $s = 364$; $(m_i, \Delta_i) = \big((3,1), (5,1), (6,7), (7,7), (9,-4), (12,-5), (13,1), (14,-6), (17,-1), (19,2), (22,-7), (23,7)\big)$.

56. $y_0 = 7$; $s = 364$; $(m_i, \Delta_i) = \big((3,1), (5,1), (6,2), (7,7), (9,-4), (13,1), (14,-6), (17,-1), (18,5), (19,2), (22,-7), (23,7)\big)$.

57. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((3,1), (5,-6), (6,7), (8,-7), (9,-11), (11,-7), (12,-12), (13,-6), (14,-6), (17,-8), (19,-5), (22,-7)\big)$.

58. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((3,1), (5,-6), (6,-5), (8,-7), (9,-11), (11,-7), (13,-6), (14,-6), (17,-8), (18,12), (19,-5), (22,-7)\big)$.

59. $y_0 = 7$; $s = 366$; $(m_i, \Delta_i) = \big((3,1), (5,-1), (7,5), (8,-2), (9,-6), (11,-2), (13,-1), (14,-6), (17,-3), (18,7), (22,-7), (23,5)\big)$.

60. $y_0 = 7$; $s = 364$; $(m_i, \Delta_i) = \big((3,1), (5,1), (7,7), (9,-4), (12,2), (13,1), (14,-6), (17,-1), (18,7), (19,2), (22,-7), (23,7)\big)$.

61. $y_0 = 7$; $s = 371$; $(m_i, \Delta_i) = \big((3,1), (5,-6), (8,-7), (9,-11), (11,-7), (12,-5), (13,-6), (14,-6), (17,-8), (18,7), (19,-5), (22,-7)\big)$.

62. $y_0 = 7$; $s = 365$; $(m_i, \Delta_i) = \big((3,1), (6,7), (7,6), (8,-1), (9,-5), (11,-1), (12,-6), (14,-6), (17,-2), (19,1), (22,-7), (23,6)\big)$.

63. $y_0 = 7$; $s = 365$; $(m_i, \Delta_i) = \big((3,1), (6,1), (7,6), (8,-1), (9,-5), (11,-1), (14,-6), (17,-2), (18,6), (19,1), (22,-7), (23,6)\big)$.

64. $y_0 = 7$; $s = 365$; $(m_i, \Delta_i) = \big((3,1), (7,6), (8,-1), (9,-5), (11,-1), (12,1), (14,-6), (17,-2), (18,7), (19,1), (22,-7), (23,6)\big)$.

65. $y_0 = 8$; $s = 365$; $(m_i, \Delta_i) = \big((2,-1), (8,7), (9,-1), (10,-3), (13,2), (14,1), (16,-5), (17,-1), (19,3), (20,5), (22,8), (23,-8)\big)$.

66. $y_0 = 8$; $s = 365$; $(m_i, \Delta_i) = \big((4,-1), (8,7), (9,-1), (10,-4), (13,2), (16,-5), (17,-1), (18,-1), (19,3), (20,6), (22,7), (23,-8)\big)$.

67. $y_0 = 8$; $s = 364$; $(m_i, \Delta_i) = \big((5,1), (7,1), (8,7), (10,-4), (11,1), (13,3), (16,-5), (18,-1), (19,4), (20,5), (22,7), (23,-7)\big)$.

68. $y_0 = 8$; $s = 364$; $(m_i, \Delta_i) = \big((5,-3), (7,1), (8,7), (11,1), (13,3), (15,4), (16,-5), (18,-1), (19,4), (20,5), (22,7), (23,-7)\big)$.

69. $y_0 = 8$; $s = 365$; $(m_i, \Delta_i) = \big((6,-1), (8,6), (9,-1), (10,-4), (13,2), (16,-5), (17,-1), (19,3), (20,5), (22,7), (23,-8), (24,1)\big)$.

70. $y_0 = 8$; $s = 364$; $(m_i, \Delta_i) = \big((7,1), (8,7), (10,-3), (11,1), (13,3), (15,1), (16,-5), (18,-1), (19,4), (20,5), (22,7), (23,-7)\big)$.

71. $y_0 = 10$; $s = 365$; $(m_i, \Delta_i) = \big((3,-4), (5,6), (7,7), (9,5), (11,-1), (14,-8), (19,-1), (20,-8), (22,2), (24,11), (25,-14), (26,6)\big)$.

72. $y_0 = 10$; $s = 365$; $(m_i, \Delta_i) = \big((3,-4), (5,6), (7,-1), (9,5), (11,-1), (19,-1), (20,-8), (21,8), (22,2), (24,11), (25,-14), (26,6)\big)$.

73. $y_0 = 10$; $s = 365$; $(m_i, \Delta_i) = \big((3,-4),\ (5,6),\ (7,7),\ (11,-1),\ (14,-8),\ (18,5),\ (19,-1),\ (20,-8),\ (22,2),\ (24,11),\ (25,-14),\ (26,6)\big)$.

74. $y_0 = 10$; $s = 365$; $(m_i, \Delta_i) = \big((3,-4),\ (5,6),\ (7,-1),\ (11,-1),\ (18,5),\ (19,-1),\ (20,-8),\ (21,8),\ (22,2),\ (24,11),\ (25,-14),\ (26,6)\big)$.

75. $y_0 = 10$; $s = 365$; $(m_i, \Delta_i) = \big((3,-4),\ (5,6),\ (9,5),\ (11,-1),\ (14,-1),\ (19,-1),\ (20,-8),\ (21,7),\ (22,2),\ (24,11),\ (25,-14),\ (26,6)\big)$.

76. $y_0 = 10$; $s = 365$; $(m_i, \Delta_i) = \big((3,-4),\ (5,6),\ (11,-1),\ (14,-1),\ (18,5),\ (19,-1),\ (20,-8),\ (21,7),\ (22,2),\ (24,11),\ (25,-14),\ (26,6)\big)$.

77. $y_0 = 11$; $s = 366$; $(m_i, \Delta_i) = \big((3,-2),\ (7,-1),\ (8,1),\ (9,7),\ (10,-2),\ (11,5),\ (13,-5),\ (17,-1),\ (18,-6),\ (22,-3),\ (25,6),\ (26,-4)\big)$.

78. $y_0 = 11$; $s = 366$; $(m_i, \Delta_i) = \big((3,-2),\ (7,-1),\ (8,1),\ (9,1),\ (10,-2),\ (11,5),\ (13,-5),\ (17,-1),\ (22,-3),\ (25,6),\ (26,-4),\ (27,6)\big)$.

79. $y_0 = 11$; $s = 366$; $(m_i, \Delta_i) = \big((3,-2),\ (7,-1),\ (8,1),\ (9,7),\ (11,5),\ (13,-5),\ (17,-1),\ (18,-6),\ (20,-2),\ (22,-3),\ (25,6),\ (26,-4)\big)$.

80. $y_0 = 11$; $s = 366$; $(m_i, \Delta_i) = \big((3,-2),\ (7,-1),\ (8,1),\ (9,1),\ (11,5),\ (13,-5),\ (17,-1),\ (20,-2),\ (22,-3),\ (25,6),\ (26,-4),\ (27,6)\big)$.

81. $y_0 = 11$; $s = 366$; $(m_i, \Delta_i) = \big((3,-2),\ (7,-1),\ (8,1),\ (10,-2),\ (11,5),\ (13,-5),\ (17,-1),\ (18,1),\ (22,-3),\ (25,6),\ (26,-4),\ (27,7)\big)$.

82. $y_0 = 11$; $s = 366$; $(m_i, \Delta_i) = \big((3,-2),\ (7,-1),\ (8,1),\ (11,5),\ (13,-5),\ (17,-1),\ (18,1),\ (20,-2),\ (22,-3),\ (25,6),\ (26,-4),\ (27,7)\big)$.