



**Figure 5.18** The entries in the individual state columns for the Viterbi algorithm. Each cell keeps the probability of the best path so far and a pointer to the previous cell along that path. We have only filled out columns 0 and 1 and one cell of column 2; the rest is left as an exercise for the reader. After the cells are filled in, backtracing from the end state, we should be able to reconstruct the correct state sequence PPSS VB TO VB.

Don't consider paths from/to nodes with  $b_s(o_t) = 0$  e.g.  $P(\text{TO}|\text{want}) = 0$   
 Don't consider paths from nodes with  $v_t(s) = 0$  or  $P(\text{Target}|\text{Source}) = 0$

Final result is: i / PPSS want / VB to / TO race / VB

	VB	TO	NN	PPSS
<s>	.019	.0043	.041	.067
VB	.0038	.035	.047	.0070
TO	.83	0	.00047	0
NN	.0040	.016	.087	.0045
PPSS	.23	.00079	.0012	.00014

**Figure 5.15** Tag transition probabilities (the  $a$  array,  $p(t_i|t_{i-1})$ ) computed from the 87-tag Brown corpus without smoothing. The rows are labeled with the conditioning event; thus  $P(PPSS|VB)$  is .0070. The symbol <s> is the start-of-sentence symbol.

	I	want	to	race
VB	0	.0093	0	.00012
TO	0	0	.99	0
NN	0	.000054	0	.00057
PPSS	.37	0	0	0

**Figure 5.16** Observation likelihoods (the  $b$  array) computed from the 87-tag Brown corpus without smoothing.

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function VITERBI(observations of len  $T$ , state-graph of len  $N$ ) returns best-path
    create a path probability matrix  $viterbi[N-2, T]$ 
    for each state  $s$  from 1 to  $N$  do                               ; initialization step
         $viterbi[s, 1] \leftarrow a_{0,s} * b_s(o_1)$ 
         $backpointer[s, 1] \leftarrow 0$ 
    for each time step  $t$  from 2 to  $T$  do •                         ; recursion step
        for each state  $s$  from 1 to  $N$  do
             $viterbi[s, t] \leftarrow \max_{s'=1}^N viterbi[s', t-1] * a_{s',s} * b_s(o_t)$ 
             $backpointer[s, t] \leftarrow \operatorname{argmax}_{s'=1}^N viterbi[s', t-1] * a_{s',s}$ 
         $viterbi[q_F, T] \leftarrow \max_{s=1}^N viterbi[s, T] * a_{s,q_F}$            ; termination step
         $backpointer[q_F, T] \leftarrow \operatorname{argmax}_{s=1}^N viterbi[s, T] * a_{s,q_F}$  ; termination step
    return the backtrace path by following backpointers to states back in time from
         $backpointer[q_F, T]$ 

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**Figure 5.17** Viterbi algorithm for finding optimal sequence of tags. Given an observation sequence and an HMM  $\lambda = (A, B)$ , the algorithm returns the state-path through the HMM which assigns maximum likelihood to the observation sequence. Note that states 0 and  $q_F$  are non-emitting.



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Figure 5.18 Viterbi algorithm example

$o_2$ : want

$$NN: \text{ from } PPSS_1: .025 \times .0012 = 3 \times 10^{-5}$$

$$TO: \text{ from } PPSS_1: .025 \times .00079 = 1.975 \times 10^{-5}$$

$$* \quad VB: \text{ from } PPSS_1: .025 \times .23 = 5.75 \times 10^{-3}$$

$$PPSS: \text{ from } PPSS_1: .025 \times .00014 = 3.5 \times 10^{-6}$$

$$NN: v_2(4) = .00003 \times .000054 = 1.62 \times 10^{-9}$$

$$TO: v_2(3) = 1.975 \times 10^{-5} \times 0 = 0$$

$$* \quad VB: v_2(2) = 5.75 \times 10^{-3} \times .0093 = 5.35 \times 10^{-5}$$

$$PPSS: v_2(1) = 3.5 \times 10^{-6} \times 0 = 0$$

$o_3$ : to

$$NN: \text{ from } NN_2: 1.62 \times 10^{-9} \cdot .087 = 1.41 \times 10^{-10}$$

$$\text{from } VB_2: 5.35 \times 10^{-5} \cdot .047 = 2.51 \times 10^{-6}$$

$$TO: \text{ from } NN_2: 1.62 \times 10^{-9} \cdot .016 = 2.59 \times 10^{-11}$$

$$* \quad \text{from } VB_2: 5.35 \times 10^{-5} \cdot .035 = 1.87 \times 10^{-6}$$

$$VB: \text{ from } NN_2: 1.62 \times 10^{-9} \cdot .0040 = 6.48 \times 10^{-12}$$

$$\text{from } VB_2: 5.35 \times 10^{-5} \cdot .0038 = 2.03 \times 10^{-7}$$

$$PPSS: \text{ from } NN_2: 1.62 \times 10^{-9} \cdot .0045 = 7.29 \times 10^{-12}$$

$$\text{from } VB_2: 5.35 \times 10^{-5} \cdot .0070 = 3.74 \times 10^{-7}$$

$$NN_3: v_3(4) = 2.51 \times 10^{-6} \cdot 0 = 0$$

$$* TO_3: v_3(3) = 1.87 \times 10^{-6} \cdot .99 = 1.85 \times 10^{-6}$$

$$VB_3: v_3(2) = 2.03 \times 10^{-7} \cdot 0 = 0$$

$$PPSS_3: v_3(1) = 3.74 \times 10^{-7} \cdot 0 = 0$$

$o_4$ : race

$$NN: \text{from } TO_3: 1.85 \times 10^{-6} \cdot .00047 = 8.71 \times 10^{-10}$$

$$TO: \text{from } TO_3: 1.85 \times 10^{-6} \cdot 0 = 0$$

$$* VB: \text{from } TO_3: 1.85 \times 10^{-6} \cdot .83 = 1.54 \times 10^{-6}$$

$$PPSS: \text{from } TO_3: 1.85 \times 10^{-6} \cdot 0 = 0$$

$$NN: v_4(4) = 8.71 \times 10^{-10} \cdot .00057 = 4.96 \times 10^{-13}$$

$$TO: v_4(3) = 0 \cdot 0 = 0$$

$$* VB: v_4(2) = 1.54 \times 10^{-6} \cdot .00012 = 1.85 \times 10^{-10}$$

$$PPSS: v_4(1) = 0 \cdot 0 = 0$$