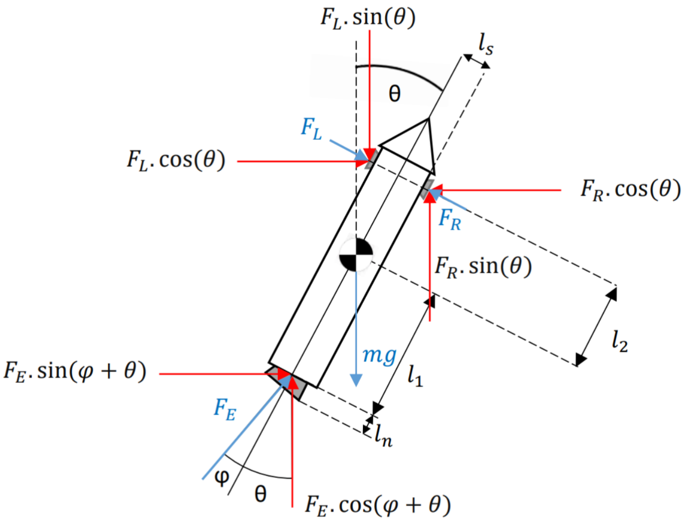

Figure 12 Free body diagram showing all forces considered.

F_E = Main Thruster Force
F_R = Right Thruster Force
F_L = Left Thruster Force
$F_S = F_L - F_R$
θ = Angle between the $z-$axis and the longitudinal axis of the rocket
φ = Angle between the Nozzle and the longitudinal axis of the rocket
l_1 = Londigutdinal length between the Center of Gravity (COG) and F_E
l_2 = Longitudinal length between the COG and F_R, F_L
l_n = Nozzle length
m = Rocket Dry Mass + Fuel Mass
x = Horizontal Position of the Rocket
z = Vertical Position of the Rocket
α = Real Constant

Inputs: $[F_E, \overbrace{F_L, F_R}^{F_S}, \varphi] = [F_E, F_S, \varphi] =: u$

States: $[x, z, \theta, \dot{x}, \dot{z}, \dot{\theta}] =: x$

$$\ddot{x} = \frac{1}{m}\left(F_E \cdot \sin(\varphi+\theta) + F_S \cdot \cos(\theta)\right) \qquad =: f_4(x,u)$$

$$\ddot{z} = \frac{1}{m}\left(F_E \cos(\varphi+\theta) - F_S \sin(\theta) - mg\right) \qquad =: f_5$$

$$\ddot{\theta} = \frac{1}{J}\left(-F_E \sin(\varphi) \cdot (l_1 + l_n \cos(\varphi)) + F_S\, l_2\right) =: f_6 \quad \text{with moment of inertia } J$$

$$\dot{x} = f(x,u) =: \begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{\theta} \\ \frac{1}{m}\left(F_E \cdot \sin(\varphi+\theta) + F_S \cdot \cos(\theta)\right) \\ \frac{1}{m}\left(F_E \cos(\varphi+\theta) - F_S \sin(\theta) - mg\right) \\ \frac{1}{J}\left(-F_E \sin(\varphi) \cdot (l_1 + l_n\cos(\varphi)) + F_S\, l_2\right) \end{bmatrix} = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ \frac{1}{m}\left(u_1 \sin(u_3+x_3) + u_2 \cos(x_3)\right) \\ \frac{1}{m}\left(u_1 \cos(u_3+x_3) - u_2 \sin(x_3) - mg\right) \\ \frac{1}{J}\left(-u_1 \sin(x_3)(l_1 + l_n \cos(u_3)) + u_2 l_2\right) \end{bmatrix}$$

Jacobians:

$$D_x f = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x & z & \theta & \dot{x} & \dot{z} & \dot{\theta} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{m}\left(F_E\cos(\varphi+\theta) - F_S \sin(\theta)\right) & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{m}\left(F_E \sin(\varphi+\theta) + F_S \cos(\theta)\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$x = \begin{bmatrix} x \\ z \\ \theta \\ \dot{x} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} \qquad u = \begin{bmatrix} F_E \\ F_S \\ \varphi \end{bmatrix}$$

$$D_u f = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{array} \begin{array}{c} \begin{array}{ccc} F_E & F_S & \varphi \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m}\sin(\varphi+\theta) & \frac{1}{m}\cos\theta & \frac{1}{m}F_E\cos(\varphi+\theta) \\ \frac{1}{m}\cos(\varphi+\theta) & -\frac{1}{m}\sin\theta & -\frac{1}{m}F_E \sin(\varphi+\theta) \\ -\frac{1}{J}\sin(\varphi)(l_1 + l_n\cos\varphi) & \frac{1}{J}l_2 & \frac{1}{J}F_E\left(-\cos(\varphi)(l_1 + l_n\cos\varphi) + l_n(\sin\varphi)^2\right) \end{bmatrix} \end{array}$$

Equilibria: $f(x_e, u_e) \overset{!}{=} 0 \Rightarrow x_e = 0 \in \mathbb{R}^6, \quad u_e = \begin{pmatrix} mg \\ 0 \\ 0 \end{pmatrix}$

Linearization at $x_e = 0, \quad u_e = [mg, 0, 0]$:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & mg & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/m & g \\ 1/m & 0 & 0 \\ 0 & l_2/J & -\frac{1}{J} mg(l_1 + l_n) \end{bmatrix}$$

Model taken from

"A Robust Control Approach for Rocket Landing"
by Reuben Ferrante, 2017, University of Edinborough.