SIGIL

Sovereign Identity-Gated Interaction Layer

A Mathematical Companion for Bachelor Students in *Wirtschaftsmathematik*

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26 February 2026

German Utility Model (Gebrauchsmuster) GBM-0–GBM-5
Patent Pending (DPMA 2026-02-23/25) · EUPL-1.2 Open Source

"Letter, not a Ledger — Trust is a Protocol."

Contents

Abstract

We introduce **SIGIL** — the *Sovereign Identity-Gated Interaction Layer* — a modular, open cryptographic protocol for identity-bound, atomically settled value transfers between parties identified by W3C Decentralised Identifiers (DIDs). Unlike blockchain-based settlement frameworks, SIGIL operates as a *protocol layer* rather than a *ledger*: it attaches cryptographic non-repudiability and tamper-evident audit trails to existing financial infrastructure (ISO 20022, SEPA, SWIFT, T2-RTGS) without requiring consensus mechanisms, native tokens, or permissioned node sets.

The protocol family consists of five interdependent components: the identity-and-audit core (GBM-0), a crypto-agility layer enabling drop-in migration to post-quantum signatures (NIST FIPS 204/205/206, GBM-1), a universal asset-agnostic transfer primitive based on Hash Time-Locked Contracts (GBM-2), an eIDAS 2.0-compliant payment gateway (GBM-3), a multi-hop foreign exchange routing protocol with cryptographically liable route attestation (GBM-4), and a milestone-based service escrow with deterministic arbitration (GBM-5).

We establish the *atomicity theorem* for the SIGIL HTLC primitive, formalise the tamper-evidence property of the HMAC audit chain, and present an empirical performance analysis demonstrating that a commodity single-core server processes 2,300 transactions per second at a data-availability layer cost below €50 per annum.

Keywords: SIGIL, post-quantum cryptography, HTLC, atomic settlement, eIDAS, W3C DID, HMAC audit chain, crypto-agility, DA-layer, FX market microstructure, EUPL.

Public Good Statement. SIGIL is released under the EUPL-1.2 open-source licence. Access is free of charge, permanently and unconditionally, for all central banks, national banks, individuals, academic institutions, and NGOs. A perpetual Celestia endowment funds data-availability costs for all free-tier participants for a projected period exceeding 100 years.

1 Introduction

1.1 Motivation

Financial infrastructure is characterised by a paradox: modern cryptography offers tools for provably-atomic, provably-attributed transactions, yet interbank settlement still relies on bilateral trust relationships, proprietary messaging protocols (SWIFT FIN), and settlement lag (T+1 to T+2). The introduction of the *blockchain* paradigm promised to resolve this paradox. It did not: most distributed-ledger deployments merely relocated the trust requirement from bilateral banking relationships to consensus-mechanism governance or validator-set membership.

SIGIL takes a different position, captured in its guiding maxim:

> *"Letter, not a Ledger — Trust is a Protocol."*

A *letter* carries its authentication intrinsically (the signature of the sender) and is self-contained. A *ledger* requires a shared write-authority. SIGIL designs trust at the protocol level: every interaction is signed, self-describing, and independently verifiable.

1.2 Contributions

This paper makes the following contributions:

C1. Formal atomicity theorem for the SIGIL HTLC primitive, with proof by reduction from SHA-256 preimage resistance (Theorem 6.3).

C2. Tamper-evidence formalisation of the HMAC audit chain (Theorem 4.3).

C3. Crypto-agility architecture enabling post-quantum migration (GBM-1) as a drop-in upgrade.

C4. Economic welfare analysis of SIGIL deployment on global FX markets using realised-volatility methodology (Section 8).

C5. Live empirical evidence: full-stack deployment on commodity hardware with on-chain Merkle anchoring (Celestia Mocha testnet, Block 10,221,745, 2026-02-24).

2 Mathematical Preliminaries

2.1 Hash Functions

Definition 2.1 (Cryptographic Hash Function)**.** A function $H: \{0,1\}^* \to \{0,1\}^n$ is a *cryptographic hash function* if:

1. **Preimage resistance:** For any $h \in \{0,1\}^n$, no PPT algorithm $\mathcal{A}$ satisfies $\Pr[\mathcal{A}(h) = m \mid H(m) = h] \geq \text{negl}(n)$.
2. **Second-preimage resistance:** Given m, finding $m' \neq m$ with $H(m) = H(m')$ is infeasible.
3. **Collision resistance:** Finding any pair (m, m') with $m \neq m'$ and $H(m) = H(m')$ is infeasible.

SIGIL uses SHA256 throughout ($n = 256$, security level 128 bits).

Worked Example

Toy hash. Define $h: \{0, \ldots, 15\} \to \{0,1,2,3\}$ by $h(x) = x \bmod 4$:

Input x	$h(x)$
5	1
7	3
11	3

Here $h(7) = h(11) = 3$: a collision. SHA-256 makes collisions computationally infeasible on 2^{256} outputs. Changing even one character in the input ("SIGIL" vs. "sIGIL") produces a completely different output — the *avalanche effect*.

2.2 Digital Signatures

Definition 2.2 (Digital Signature Scheme)**.** A triple of PPT algorithms (KeyGen, Sign, Verify) such that:

- $\text{KeyGen}(1^\lambda) \to (sk, vk)$
- $\text{Sign}(sk, m) \to \sigma$
- $\text{Verify}(vk, m, \sigma) \to \{0,1\}$

satisfying correctness ($\text{Verify}(vk, m, \text{Sign}(sk, m)) = 1$ always) and EU-CMA security.

SIGIL uses **Ed25519** (RFC 8032), with $\lambda = 128$, 32-byte public keys, and 64-byte signatures. Post-quantum replacements are introduced in Section 5.

Worked Example

Geometric intuition in $\mathbb{R}$. Let G be a fixed generator. Alice's secret key is a scalar $sk = 5$; her public key is $vk = 5 \cdot G$. To sign message m: compute $r = H(m)$, output $\sigma = r + sk$. Bob verifies: $\sigma \cdot G \stackrel{?}{=} r \cdot G + vk$. The security rests on the Elliptic Curve Discrete Logarithm Problem (ECDLP): given $vk = 5 \cdot G$, recovering 5 is computationally infeasible in a finite group.

2.3 Message Authentication Codes (HMAC)

Definition 2.3 (HMAC)**.** Let H be a cryptographic hash function with block size B. For key k and message m:

$$\mathrm{HMAC}(k, m) = H((k \oplus \mathsf{opad}) \,\|\, H((k \oplus \mathsf{ipad}) \,\|\, m))$$

where $\mathsf{opad} = \mathtt{0x5c}^B$ and $\mathsf{ipad} = \mathtt{0x36}^B$.

Under the PRF-security of H, HMAC is itself a PRF: an adversary without k cannot distinguish $\mathrm{HMAC}(k, m)$ from a random value.

3 Background

3.1 W3C Decentralised Identifiers

DIDs provide a globally unique, cryptographically resolvable identity of the form `did:method:identifier` (e.g. `did:sigil:0x4a7b...c3f2`). A DID Document associates the identifier with a verification key. SIGIL uses DIDs as the canonical party-identity type across all layers.

3.2 Hash Time-Locked Contracts

The HTLC core invariant is:

> Settlement is reachable if and only if the party knows a preimage s such that $\mathrm{SHA256}(s) = h$.

The timeout τ ensures refund if the party does not act in time. SIGIL generalises this to arbitrary asset classes (Section 6).

3.3 Existing Settlement Infrastructure

System	Limitation
SWIFT FIN	No party-carried cryptographic signatures; post-hoc reconciliation
SEPA Instant	T+0, but no tamper-evident audit trail without operator cooperation
CLS (FX)	PvP settlement, but no eIDAS or post-quantum support
Ethereum/DLT	Requires shared consensus ledger and native token; GDPR-incompatible
Lightning Network	BTC-only, no asset generalisation, no regulatory compliance layer

Table 1: Limitations of existing settlement infrastructure. SIGIL addresses all five gaps.

4 GBM-0: Identity and Audit Core

4.1 The SIGIL Envelope

Definition 4.1 (SIGIL Envelope)**.** A SIGIL Envelope E is a tuple

$$E = (did,\ payload\_hash,\ timestamp,\ algorithm,\ \sigma)$$

where:

- $did \in \mathcal{D}$ is a W3C DID identifying the sender
- $payload\_hash = \mathrm{SHA256}(payload)$
- $timestamp \in \mathbb{Z}_{\geq 0}$ is a Unix timestamp
- $algorithm \in \mathcal{A} = \{\texttt{Ed25519}, \texttt{ML-DSA-65}, \texttt{SLH-DSA-SHA2-128s}\}$
- $\sigma = \mathrm{Sign}(sk,\ payload\_hash \| timestamp \| did)$

The verification procedure is:

1. Recompute $h' \leftarrow \mathrm{SHA256}(payload)$; check $h' = E.payload\_hash$.
2. Reconstruct $m \leftarrow E.payload\_hash \| E.timestamp \| E.did$.
3. Return $\mathrm{Verify}_{algorithm}(vk, m, E.\sigma)$.

Worked Example

Alice sends €15.00 to Bob. The JSON payload is:

{"from":"did:sigil:alice","to":"did:sigil:bob","amount":1500}

Step 1. $payload\_hash = \mathrm{SHA256}(payload) = \texttt{3a7c4f} \ldots \texttt{d8b2e1}$.
Step 2. $m = \texttt{3a7c4f} \ldots \| \texttt{0x67C12100} \| \texttt{did} : \texttt{sigil} : \texttt{alice}$.
Step 3. $\sigma = \texttt{Ed25519.Sign}(sk_{\text{Alice}}, m)$ (64 bytes).
Tampering with "amount" changes $\mathrm{SHA256}(payload')$, causing step 1 to fail. Changing the hash without a valid σ causes step 3 to fail.

4.2 HMAC Audit Chain

Definition 4.2 (HMAC Audit Chain)**.** An HMAC Audit Chain is a sequence $(a_0, a_1, \ldots, a_n)$ where:

$$a_0 = (seq{=}0,\ \texttt{genesis},\ h_0 = \mathrm{HMAC}(k, 0 \| \texttt{genesis}))$$
$$a_i = (seq{=}i,\ event_i,\ h_i = \mathrm{HMAC}(k, h_{i-1} \| i \| \tau_i \| B_i \| tag_i))$$

with $B_i = \mathrm{SHA256}(payload_i)$ and secret key k.

Worked Example

Three-entry chain (abbreviated HMAC values shown in hex):

$h_0 = \mathrm{HMAC}(k, \text{"}\texttt{0}\text{"} \| \text{"}\texttt{genesis}\text{"}) \approx \texttt{f3a7c211}$

Entry 1 (€15.00, Alice $\rightarrow$ Bob): $B_1 = \mathrm{SHA256}(\cdot) \approx \texttt{3a7c4fbd}$, $h_1 = \mathrm{HMAC}(k, h_0 \| \texttt{1} \| \tau_1 \| B_1 \| \texttt{payment}) \approx \texttt{8b1d9e32}$

Entry 2 (€100.00, Carol $\rightarrow$ Dave): $B_2 \approx \texttt{c48fa1e7}$, $h_2 \approx \texttt{21f4a730}$

Tamper test: Change amount in Entry 1 from 1,500 to 150,000 cents. Then $B_1' \neq B_1$, so $h_1' \neq$ `8b1d9e32`, and verification fails immediately at $i = 1$.

4.3 Theorem: Tamper Evidence

Theorem 4.3 (Tamper Evidence)**.** *Under the PRF-security of* HMAC-*SHA-256, modification of any single field in entry* a_j *(*$0 \leq j \leq n$*) is detected by the verifier with probability at least* $1 - (n - j + 1) \cdot \varepsilon_{\mathrm{PRF}}$*, where* $\varepsilon_{\mathrm{PRF}}$ *is negligible in the key length.*

Proof. By induction on the suffix $[j, n]$.

Base case. If a_j is modified, the HMAC input

$$msg_j' = h_{j-1} \| j \| \tau_j' \| B_j' \| tag_j' \neq msg_j.$$

By PRF-security, $\Pr[\mathrm{HMAC}(k, msg_j') = \mathrm{HMAC}(k, msg_j)] \leq \varepsilon_{\mathrm{PRF}}$. The verifier's check at position j fails with probability $\geq 1 - \varepsilon_{\mathrm{PRF}}$.

Student note: The adversary cannot "patch up" h_j' to equal the stored h_j without knowing k, since doing so requires finding a preimage under the PRF.

Inductive step. A corrupted h_j' propagates: $h_{j+1}' = \mathrm{HMAC}(k, h_j' \| \ldots) \neq h_{j+1}$ with probability $\geq 1 - \varepsilon_{\mathrm{PRF}}$.

Union bound. The adversary must pass $n - j + 1$ checks; by union bound the probability of all checks passing despite tampering is at most $(n - j + 1) \cdot \varepsilon_{\mathrm{PRF}}$. □ □

Numeric illustration. For $\varepsilon_{\mathrm{PRF}} = 2^{-128}$ and $n = 10{,}000$, the detection probability exceeds $1 - 10{,}001 \cdot 2^{-128} \approx 1 - 2.96 \times 10^{-35}$.

5 GBM-1: Crypto-Agility

5.1 Motivation

Current public-key cryptography (Ed25519, RSA) is vulnerable to *cryptographically relevant quantum computers* (CRQCs) via Shor's algorithm, which solves ECDLP and factorisation in polynomial quantum time. NIST finalised three post-quantum standards in 2024:

- **FIPS 204** (ML-DSA / Dilithium): lattice-based signatures
- **FIPS 205** (SLH-DSA / SPHINCS+): hash-based signatures
- **FIPS 206** (ML-KEM): lattice-based key encapsulation

SIGIL's crypto-agility layer provides drop-in migration to all three. Every signed record carries a self-describing `algorithm` field (see Definition 4.1), so existing Ed25519 signatures remain valid alongside new ML-DSA signatures.

Algorithm	PK size	Sig size	PQ level	Standard
Ed25519	32 B	64 B	No	RFC 8032
ML-DSA-65	1,952 B	3,293 B	NIST 3	FIPS 204
SLH-DSA-SHA2-128s	32 B	7,856 B	NIST 1	FIPS 205
ML-KEM-768	1,184 B	(KEM)	NIST 3	FIPS 206

Table 2: Algorithm comparison. SIGIL uses ML-DSA-65 as default post-quantum signature.

5.2 Lattice Geometry: Intuition for ML-DSA

Definition 5.1 (Lattice)**.** Let $\mathbf{b}_1, \dots, \mathbf{b}_n \in \mathbb{R}^n$ be linearly independent. The *lattice* is

$$\Lambda = \Big\{ \sum_i z_i \mathbf{b}_i \;\Big|\; z_i \in \mathbb{Z} \Big\}.$$

Worked Example

Let $\mathbf{b}_1 = (1, 0)$, $\mathbf{b}_2 = (0.5, 0.866)$. Then Λ is the triangular lattice in $\mathbb{R}^2$ containing $(0, 0)$, $(1, 0)$, $(0.5, 0.866)$, etc. The *Shortest Vector Problem* (SVP) asks to find the shortest non-zero lattice point. In high dimension ($n = 1024$ for ML-DSA), SVP is believed to be intractable even for quantum computers — unlike ECDLP, it has no hidden periodic structure exploitable by quantum Fourier transforms.

5.3 The Learning With Errors Problem (LWE)

ML-DSA rests on the hardness of Module-LWE: given samples

$$(\mathbf{a}_i, b_i) \text{ where } b_i = \mathbf{a}_i^\top \mathbf{s} + e_i \pmod q,$$

with small errors $e_i \sim \chi$, recover the secret $\mathbf{s}$.

Worked Example

Toy LWE in $\mathbb{Z}_7^3$. Secret $\mathbf{s} = (2, 5, 1)$:

$\mathbf{a}_i$	True $\mathbf{a}_i^\top \mathbf{s} \bmod 7$	e_i	Observed b_i
$(3, 1, 4)$	$15 \equiv 1$	$+1$	2
$(2, 6, 0)$	$34 \equiv 6$	-1	5
$(1, 1, 6)$	$13 \equiv 6$	$+1$	0

Without e_i this is a linear system solvable by Gaussian elimination. The small errors destroy this structure, making recovery infeasible.

6 GBM-2: Bridge Core — HTLC Atomicity

6.1 Polymorphic Asset Type

Definition 6.1 (Polymorphic Asset)**.** $\mathcal{V}$ is a closed sum type over tagged variants:

$$\mathcal{V} \;::=\; \texttt{Currency}(c, q) \mid \texttt{Security}(\mathit{isin}, q) \mid \texttt{Token}(\mathit{contract}, \mathit{chain}, q) \mid \cdots$$

with $c \in \text{ISO 4217}$, $q \in \mathbb{Z}_{\geq 0}$ (amounts in minimal units to avoid floating-point non-associativity).

Worked Example

$\texttt{Currency}(\texttt{EUR}, 1500) \mapsto$ £15.00; $\texttt{Security}(\text{"US0231351067"}, 100) \mapsto$ 100 shares of Apple Inc.

6.2 BridgeIntent and HTLC State Machine

Definition 6.2 (BridgeIntent)**.** $\mathcal{B} = (h, v, \mathit{did}_A, \mathit{did}_B, \tau, \sigma)$ where $h = \text{SHA256}(s_0)$ for secret $s_0 \in \{0, 1\}^{256}$, $v \in \mathcal{V}$, τ is a Unix timeout, and σ is the sender's signature.

The state machine transitions are:

$$\underbrace{\texttt{Pending}}_{\text{initial}} \xrightarrow{\text{SHA256}(s)=h} \underbrace{\texttt{Settled}}_{\text{B receives } v} \qquad \underbrace{\texttt{Pending}}_{\text{initial}} \xrightarrow{t>\tau} \underbrace{\texttt{Expired}}_{\text{A refunded}}$$

Worked Example

Two-party HTLC. Alice wants to send €15.00 to Bob.

1. Alice samples $s_0 \leftarrow \{0,1\}^{256}$ (kept secret) and publishes $h = \text{SHA256}(s_0)$.
2. Alice creates $\mathcal{B} = (h, \texttt{Currency}(\texttt{EUR}, 1500), did_A, did_B, \tau, \sigma)$.
3. Bob, seeing h and v, waits. He trusts: reveal $s_0 \Rightarrow$ receive €15.00.
4. Alice reveals s_0 to Bob off-band. Bob submits. Gateway verifies $\text{SHA256}(s_0) = h$. State $\rightarrow$ `Settled`.
5. If timeout: State $\rightarrow$ `Expired`, Alice is refunded.

6.3 HTLC Atomicity Theorem

Theorem 6.3 (HTLC Atomicity). *Let $\mathcal{B}$ be a BridgeIntent with preimage hash $h = \text{SHA256}(s_0)$. There exists no* PPT *execution of the settlement protocol in which party B receives asset v without party A having previously observed s_0.*

Proof. By reduction to SHA-256 preimage resistance.

Step 1. Assume adversary $\mathcal{A}$ breaks atomicity with non-negligible probability ε.

Step 2. Construct inverter $\mathcal{I}(h^*)$: embed h^* as the lock hash in a fresh $\mathcal{B}^*$ and run $\mathcal{A}$. If $\mathcal{A}$ achieves `Settled`, extract the submitted preimage s^* and output it.

Step 3. The gateway transition to `Settled` is gated *exclusively* on the check $\text{SHA256}(s) = h$:

```python
def attempt_settle(intent, preimage):
    if SHA256(preimage) == intent.h:    # sole gate condition
        pay(intent.did_B, intent.v)
        return Settled
    return Failed
```

Hence if $\mathcal{A}$ reaches `Settled`, it must have supplied s^* with $\text{SHA256}(s^*) = h^*$.

Step 4. Therefore $\mathcal{I}$ inverts SHA256 on h^* with probability ε. By preimage resistance, $\varepsilon \leq \text{negl}(\lambda)$ — contradiction. □ □

6.4 Multi-Hop Timeout Chain

Proposition 6.4 (Timeout Chain Invariant). *Let $(\mathcal{B}_1, \ldots, \mathcal{B}_n)$ share preimage hash h, with gaps $\tau_i - \tau_{i+1} \geq \delta > 0$ and network latency $\ell < \delta$. Then the preimage reveal propagates backwards through the entire chain before any intermediate contract expires.*

Proof. By induction. Available window at hop $n-1$ after reveal at time $t_n \leq \tau_n$:

$$\tau_{n-1} - t_n \geq \tau_{n-1} - \tau_n \geq \delta > \ell. \quad \square$$

□

Worked Example

$n = 3$ hops (EUR $\to$ USD $\to$ JPY), $\delta = 60$ min, $\ell = 2$ s.
$\tau_3 = t + 60$ min, $\tau_2 = t + 120$ min, $\tau_1 = t + 180$ min. Even if Bob reveals at $t + 59$ min 58 s, intermediaries still have ≥ 2 s to act.

7 Application Layers (GBM-3 to GBM-5)

7.1 GBM-3: SIGIL-EURO — eIDAS Payment Protocol

The `PaymentIntent` extends `BridgeIntent` with:

- `trust_level` $\in \{$`Low`, `Substantial`, `High`$\}$: a bijection to eIDAS 2.0 assurance levels.
- `recipient_hash` $= \text{SHA256}(did_B)$: GDPR Article 5(1)(c) data minimisation as a type-level invariant.
- `aml_scan`(&`self`, &`str`) $\to$ `Vec` $<$ `AmlFlag` $>$: pure function by Rust's type system (no side effects).

Three-layer audit.

1. Layer 1: Per-entry HMAC chain — $O(1)$ verification (operator).
2. Layer 2: Hourly Merkle tree over HMAC values — $O(\log n)$ membership proof (any party with root).
3. Layer 3: Merkle root on Celestia DA layer — $O(1)$ global lookup (anyone, operator-independent).

Live evidence. Celestia Mocha, Block 10,221,745, 2026-02-24. Merkle root `0xfb19a5ff...0517c64`, reference `sigileuro-20260224-512a1bcc`.

7.2 GBM-4: SIGIL-FXBridge

Each FX hop carries context $\mathcal{F} = (src, dst, r, \rho, t_r, t_{\text{exp}})$ with $r \in \mathbb{Q}^+$ (decimal string, no floating-point). Effective transfer over route of length n:

$$V_{\text{out}} = V_{\text{in}} \cdot \prod_{i=1}^{n} r_i.$$

7.3 GBM-5: ServiceBridge — Deterministic Escrow

A DFA $\mathcal{M} = (S, \Sigma, \delta, \texttt{Pending}, F)$ with $F = \{\texttt{Settled}, \texttt{Refunded}, \texttt{Expired}\}$ governs milestone-based service delivery. The arbitrator DID is committed at inception:

$$\texttt{ServiceIntent.arbitrator\_hash} = \text{SHA256}(did_{\text{arb}})$$

and is immutable thereafter, ensuring cryptographic legal attributability.

8 Econometric Welfare Analysis

8.1 Model Setup

Let S_t be the spot exchange rate. Model log-price $X_t = \ln S_t$ as geometric Brownian motion:

$$dX_t = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma\, dW_t.$$

Under traditional T+2 settlement, a firm entering an obligation at t bears uncompensated exposure over lag $\Delta t = 2$ days.

8.2 Pre-Settlement VaR

Variance of log-return: $\mathrm{Var}(X_{t+\Delta t} - X_t) = \sigma^2 \Delta t$. The 99% Value at Risk for notional N is:

$$\mathrm{VaR}_{0.99} \approx N \cdot 2.33\, \sigma \sqrt{\Delta t}.$$

Theorem 8.1 (Asymptotic Elimination of Pre-Settlement Risk)**.** *As* $\Delta t \to 0$, $\mathrm{VaR}_{1-\alpha}(\Delta t) \to 0$.

Proof. $\mathrm{VaR}_{1-\alpha}(\Delta t) = N \cdot z_{1-\alpha}\, \sigma \sqrt{\Delta t} \to 0$ since $\sqrt{\Delta t} \to 0$. □ □

8.3 Capital Cost Calculation

Worked Example

$N = £1{,}000{,}000$, $\sigma = 10\%$ p.a., cost of capital $\kappa = 8\%$, multiplier $\gamma = 3$, $\Delta t = 2/365$ yr.

$$\begin{aligned} \text{2-day std} &= 0.1 \times \sqrt{2/365} \approx 0.74\% \\ \mathrm{VaR}_{0.99} &= 1{,}000{,}000 \times 2.33 \times 0.0074 \approx £17{,}242 \\ C &= 3 \times 17{,}242 \approx £51{,}726 \\ \text{2-day cost} &= 51{,}726 \times 0.08 \times \tfrac{2}{365} \approx £22.67 \end{aligned}$$

SIGIL eliminates Δt, driving this €23 cost to zero for every €1M transferred.

9 Empirical Evidence

Metric	**Value**
Throughput (commodity VPS, 1 core)	2,300 TX/s
DA-layer cost	$<$ £50/yr
Settlement finality	$<$ 1 s
Registry response time (p99)	$<$ 50 ms
PQ key size overhead vs. Ed25519	+5.1 kB/TX
Additional bandwidth at 2,300 TX/s	$\approx$ 12 MB/s ($<$ 1% of 10G NIC)

Table 3: Empirical performance on a commodity VPS (2 vCPU, 2 GB RAM).

10 Conclusion

The SIGIL Protocol demonstrates that modern cryptographic machinery — formally verified post-quantum signatures, deterministic typed state machines, and hash-chain audit logs — can be applied directly to financial routing infrastructure without inventing a new currency or consensus ledger.

By separating the **Identity Layer** (W3C DIDs), the **Execution Layer** (Atomic HTLCs), and the **Data Availability Layer** (Celestia), SIGIL achieves high throughput with negligible operational cost.

For the mathematical economist, SIGIL transforms the trust required for settlement from an *ex-post institutional probability* (will the counterparty default?) into an *ex-ante cryptographic certainty*: it is computationally infeasible to break SHA-256 preimage resistance.