

How to type-set logic and natural deductions using GNU troff, pic and eqn

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Fitch is a notation for natural deduction (Pelletier and Hazen, 2024). `troff` is a software system for type-setting using Unix™ and related operating systems (Ossanna and Kernighan, 1994). Brian W. Kernighan was one of the creators of Unix and the C programming language. `pic` (Kernighan, 1982) and `eqn` (Kernighan and Cherry, 1975) are systems for typesetting graphs and mathematics, also created by Brian Kernighan and his friends. GROFF AKA GNU troff is the implementation I am using (FSF, 1990). There are other competitors, but this is the version I use.

The Fitch notations has got its name after its inventor, Fredric Fitch. This notation seems to be a de facto standard: It is used in all the text books I have been able to find electronically, and seems to be taught at logics courses in mathematics as well as philosophy. I wrote this note while learning Fitch; I used the writing was a method for learning. My intention is to demonstrate how to format natural deduction on this platform. I cannot teach you how to format scientific text, neither can I give an introduction to natural deduction.

Table 1. Unicode characters for logical signs and operators. On some operating systems you can type them by pressing `ctrl-shift-u` and then the four character code (following `u+`). The Groff name is usually better to use than the Unicode character, but I tend to use the latter.

Unicode	Character	Groff name
u+00AC	¬	\[no]
u+2227	∧	\[AN]
u+2228	∨	\[OR]
u+2200	∀	\[fa]
u+2203	∃	\[te]
u+2192	→	\[->]
u+2194	↔	\[<>]
u+22A5	⊥	\[pp]
u+22A2		
u+2261	≡	\[==]
u+25A1	□	\[sq]
u+25C7		
u+2234	∴	\[tf]
U+2208	∈	\[mo]
U+2209	∉	\[nm]

1		$A \vee B$	
2		$\neg A$	
3		—	
4			A
5			\perp \perp Intro: 3,2
6			B \perp Elim: 4
7			B
8			B Reit: 6
		—	
		B	\vee Elim: 6-7,3-5,1

Figure 1. Proof that $A \vee B, \neg A \therefore B$. The line numbering is in the left-most margin. Then there is a vertical line, as long as the proof. The step 1-2 in the proof is where the premises lives. The horizontal line after step 2 is usually referred to as the *fitch line*. The two groups, 3-5 and 3-6 are sub-proofs, with their own premisses, vertical lines and fitch lines

How to write Fitch in troff

Any proof is initialized by calling this macro, which informs all scripts on the number of steps in the proof and its maximum depth, i.e., how deep the hierarchy of proofs is. That is, how many inside proofs, within proofs ... do we have. You better add one or the references at the right will come to close to the logical statements.

```
set_steps_and_depths(8,3)
```

Any proof (the root proof or any sub-proof) starts with the `start_proof()` macro, which also names that proof. After we have started the proof, we add its premises, and end it with `premis_end()`.

```
start_proof(START);
add_premis(START, "A \vee B");
add_premis(START, "\neg A");
premis_end(START);
```

After ending the premises section, we enter the body of our proofs. In this case we start the sub-proofs

```
start_proof(SUB1);
add_premis(SUB1, "A");
premis_end(SUB1);
```

In the body of a proof, we use the `add_step()` macro, which has three argument: (i) the name of the current proof, (ii) the result of the step, and finally (iii) the

references to earlier steps needed for the step.

```
add_step(SUB1, "⊥", "⊥ Intro: 3, 2");
add_step(SUB1, "B", "⊥ Elim: 4");
end_proof(SUB1);
```

We end a proof (be it a sub-proof or a proof) with the `end_proof()` macro, which needs the name of the current proof as an argument. Now we start another subproof.

```
start_proof(SUB2);
add_premis(SUB2, "B");
premis_end(SUB2);
add_step(SUB2, "B", "Reit: 6");
end_proof(SUB2);
```

After we have completed the two sub-proofs, return to the main proof and completes it with a nice \vee elimination.

```
add_step(START, "B", "⋃ Elim: 6-7, 3-5, 1");
end_proof(START)
```

Note that the macros do not check your references. Sanity checks and proof reading is your job.

References

- FSF, Free Software Foundation, *Groff* (1990). <https://www.gnu.org/software/groff/>.
- Kernighan, Brian W., "PIC — A language for typesetting graphics," *Software: Practice and Experience* **12** (1982). <https://doi.org/10.1002/spe.4380120102>.
- Kernighan, Brian W. and Cherry, Lorinda L., "A System for Typesetting Mathematics," *Scientific Applications* **18** (1975). <https://doi.org/10.1145/360680.360684>.
- Ossanna, Joseph F. and Kernighan, Brian W., "Troff User's Manual," *Computing Science Technical Report* **54** (1994). <https://wolfram.schneider.org/bsd/7thEdMan-Vol2/trofftut/trofftut.pdf>.
- Pelletier, Francis Jeffry and Hazen, Allen, "Natural Deduction Systems in Logic" in *The Stanford Encyclopedia of Philosophy (Spring 2024 Edition)*, ed. Zalta, E. N. and U. Nodelman (2024). <https://plato.stanford.edu/archives/spr2024/entries/natural-deduction>.

1	$A \vee B$	
2	$\neg B \vee C$	
3	A	
4	$A \vee C$	\vee Intro:3
5	B	
6	$\neg B$	
7	\perp	\perp Intro:6,5
8	$A \vee C$	\perp Elim:7
9	C	
10	$A \vee C$	\vee Intro:9
11	$A \vee C$	\vee Elim:6-8,9-10,2
12	$A \vee C$	\vee Elim:1,3-4,5-11

Figure 2. A slightly longer example: Prove that $A \vee B, \neg B \vee C \therefore A \vee C$.