

# How to type-set logic and natural deductions using GNU **t**roff, **p**ic and **e**qn

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`t`roff is a software system for type-setting using Unix™ and related operating systems (Ossanna and Kernighan, 1994). Brian W. Kernighan was one of the creators of Unix and the C programming language. `p`ic (Kernighan, 1982) and `e`qn (Kernighan and Cherry, 1975) are systems for typesetting graphs and mathematics, also created by Brian Kernighan and his friends. This group created a full set of tools for the type-setting of scientific text, graphs and diagrams, mathematics, chemistry, reference management and complex tables. GROFF aka GNU `t`roff is the implementation I am using (FSF, 1990). There are other competitors, in particular the slightly younger TeX and LaTeX. There are also alternative implementations of troff, but this is the version I use.

Fitch is a notation for natural deduction (Pelletier and Hazen, 2024). It has got its name after its inventor, Fredric Fitch. This notation seems to be a de facto standard: It is used in all the text books I have been able to find electronically, and seems to be taught at logics courses in mathematics as well as philosophy. I wrote this note while learning Fitch; My intention is to demonstrate how to write predicate and propositional logic, and deduction on this platform. I cannot teach you how to format scientific text in general, neither can I give an introduction to logic and natural deduction.

## Writing equations

First, we need to be able to write our texts (there is a good tutorial by Kollar and Robinson (2023)) and then we can continue with formulas and sentences using `e`qn (Harding, 2011). At a first glance, all of them are “equations”, or anyone who are not familiar with mathematics and logic and the differences between the two will regard them as such. Logic is actually a special genre of its own when it comes to formulas or equations. Here is a set of predicate logic sentences, first in `e`qn source,

```
.EQ (1)
  pile {
    forall(x) SameSize(x)
    above
    forall(x) Cube(x) implies Cube(b)
    above
    (Cube(b) and b=c) implies Small(c)
    above
    (Small(b) and SameSize(b,c) implies Small(c)
  }
.EN
```

and then formatted, in Equation (1).

$$\begin{aligned}
& \forall x \text{ SameSize}(x) \\
& \forall x \text{ Cube}(x) \rightarrow \text{Cube}(b) \\
& (\text{Cube}(b) \wedge b = c) \rightarrow \text{Small}(c) \\
& (\text{Small}(b) \wedge \text{SameSize}(b, c) \rightarrow \text{Small}(c)
\end{aligned} \tag{1}$$

To write these formulas, you need to use either the unicode characters, their Groff names or macros I have defined in order to simplify typing. See Table 1.

Table 1. Unicode characters for logical signs and operators. On some operating systems you can type them by pressing `ctrl-shift-u` and then the four character code (following `u`). The Groff name is usually better to use than the Unicode character, but takes a long time to type. The eqn macros are for easier typing and I have tried to adjust spacings for a nicer look.

Unicode	Character	Groff name	eqn macro	Comment
U+00AC	¬	\[no]	not	
U+2227	∧	\[AN]	and	
U+2228	∨	\[OR]	or	
U+2200	∀	\[fa]	any	
U+2200	∀	\[fa]	forall(x)	
U+2203	∃	\[te]	some	
U+2203	∃	\[te]	exists(x)	
U+2192	→	\[->]	implies	
U+2194	↔	\[<>]	iff	
U+2194	↔	\[<>]	equiv	
U+21D4	↔	\[hA]		
U+22A5	⊥	\[pp]	falsum	
U+22A2		not available		syntactic consequence turnstile
U+22A8		not available		semantic consequence double turnstile
U+2261	≡	\[==]	identicalto	
U+25A1	□	\[sq]	nece	
U+25A1	□	\[sq]	necessarily	
U+25C7	◇	\[lz]	possi	
U+25C7	◇	\[lz]	possibly	
U+2234	∴	\[tf]	therefore	
U+2205	∅	\[es]	empty	
U+2208	∈	\[mo]	member	
U+2209	∉	\[nm]	notmember	
U+2286	⊆	\[ib]	subset	
U+2118	℘	\[wp]	powerset	

## Using logics in tables

Table 2. Some useful equivalents if you are doing logic. They are presented here as an example how you can embed formulas in a table.

Assertion of Universality	$\forall x Ax \leftrightarrow \neg \exists x \neg Ax$	If everything is, there exists nothing that is not.
Denial of Universality	$\neg \forall x Ax \leftrightarrow \exists x \neg Ax$	If not everything is, there exists something that is not.
Denial of Existence	$\forall x \neg Ax \leftrightarrow \neg \exists x Ax$	If everything is not, there exists nothing that is.
Assertion of Existence	$\neg \forall x \neg Ax \leftrightarrow \exists x Ax$	If not everything is not, there exists something that is.

### Using logic in graphs

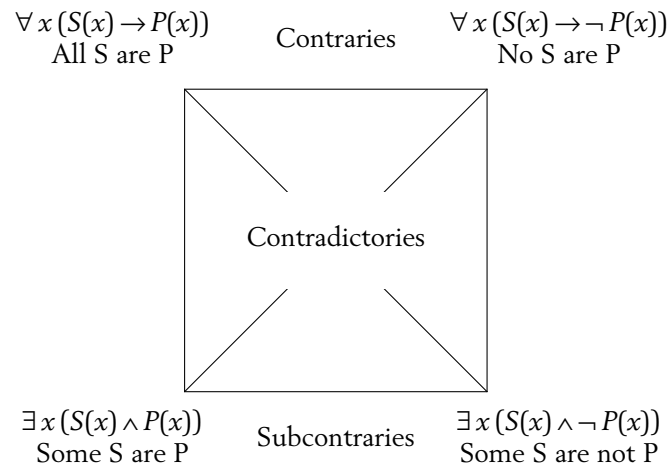


Figure 1. The traditional aristotelian syllogisms.

### Writing Fitch arguments

Any proof is initialized by calling this macro, which informs all scripts on the number of steps in the proof and its maximum depth, i.e., how deep the hierarchy of proofs is. That is, how many inside proofs, within proofs ... do we have. You better add one or the references at the right will come to close to the logical statements.

```
set_steps_and_depths (8, 3)
```

Any proof (the root proof or any sub-proof) starts with the `start_proof()` macro, which also names that proof. After we have started the proof, we add its premises, and end it with `premis_end()`.

```
start_proof (START);
add_premis (START, "$A or B$");
add_premis (START, "$not A$");
```

1		$A \vee B$	
2		$\neg A$	
3			
4			$\perp$ Intro: 3,2
5			$B$ $\perp$ Elim: 4
6			$B$
7			$B$ Reit: 6
8		$B$	$\vee$ Elim: 6-7,3-5,1

Figure 1. Proof that  $A \vee B, \neg A \therefore B$ . The line numbering is in the left-most margin. Then there is a vertical line, as long as the proof. The step 1-2 in the proof is where the premises lives. The horizontal line after step 2 is usually referred to as the *fitch line*. The two groups, 3-5 and 3-6 are sub-proofs, with their own premisses, vertical lines and fitch lines

```
premis_end (START);
```

After ending the premiws section, we enter the body of our proofs. In this case we start the sub-proofs

```
start_proof (SUB1);
add_premis (SUB1, "$A$");
premis_end (SUB1);
```

In the body of a proof, we use the `add_step()` macro, which has three argument: (i) the name of the current proof, (ii) the result of the step, and finally (iii) the references to earlier steps needed for the step.

```
add_step (SUB1, "$falsum$", "\perp Intro: 3,2");
add_step (SUB1, "$B$", "\perp Elim: 4");
end_proof (SUB1);
```

We end a proof (be it a sub\_proof or a proof) with the `end_proof()` macro, which needs the name of the current proof as an argument. Now we start another subproof.

```
start_proof (SUB2);
add_premis (SUB2, "$B$");
premis_end (SUB2);
add_step (SUB2, "$B$", "Reit: 6");
end_proof (SUB2);
```

After we have completed the two sub-proofs, return to the main proof and completes it with a nice  $\vee$  elimination.

```
add_step(START, "$B$", "\vee Elim: 6-7, 3-5, 1");
end_proof(START)
```

Note that the macros do not check your references. Sanity checks and proof reading is your job.

1	$A \vee B$	
2	$\neg B \vee C$	
3	$A$	
4	$A \vee C$	$\vee$ Intro:3
5	$B$	
6	$\neg B$	
7	$\perp$	$\perp$ Intro:6,5
8	$A \vee C$	$\perp$ Elim:7
9	$C$	
10	$A \vee C$	$\vee$ Intro:9
11	$A \vee C$	$\vee$ Elim:2,6-8,9-10
12	$A \vee C$	$\vee$ Elim:1,3-4,5-11

Figure 2. A slightly longer example: Prove that  $A \vee B, \neg B \vee C \therefore A \vee C$ .

## References

- FSF, Free Software Foundation, *Groff* (1990). <https://www.gnu.org/software/groff/>.
- Harding, Ted, *A Guide to Typesetting Mathematics using GNU eqn* (2011). <https://lists.gnu.org/archive/html/groff/2013-10/pdf/TyBN2VWR1c.pdf>.
- Kernighan, Brian W., "PIC — A language for typesetting graphics," *Software: Practice and Experience* **12** (1982). <https://doi.org/10.1002/spe.4380120102>.

1					
2		$\forall x Fx$			
3			$\exists x \neg Fx$		
4				$[a] \neg Fa$	
5				$Fa$	$\forall$ Elim 2
6				$\perp$	$\perp$ Intro 4,5
7			$\perp$	$\exists$ Elim 3,4-6	
8		$\neg \exists x \neg Fx$			
9			$\neg \exists x \neg Fx$		
10				$[a] \neg Fa$	
11				$\exists x \neg Fx$	$\exists$ Intro 10
12				$\perp$	$\perp$ Intro 9,11
13				$\forall x Fx$	$\perp$ Elim 12
14		$\forall x Fx$		$\forall$ Intro 10-13	
15		$\forall x Fx \leftrightarrow \neg \exists x \neg Fx$		$\leftrightarrow$ Intro 2-8, 9-14	

Figure 3. A proof using predicate logic. It proves without premisses one of the de Morgan formulas:  $\forall x Fx \leftrightarrow \neg \exists x \neg Fx$ , All  $x$  are  $F$  if and only if there is no  $x$  which is not  $F$ .

Kernighan, Brian W. and Cherry, Lorinda L., "A System for Typesetting Mathematics," *Scientific Applications* **18** (1975). <https://doi.org/10.1145/360680.360684>.

Kollar, Larry and Robinson, G. Branden, *Using groff with the ms Macro Package* (2023). <https://lists.gnu.org/r/groff/2023-04/pdfkxOTeCk24w.pdf>.

Ossanna, Joseph F. and Kernighan, Brian W., "Troff User's Manual," *Computing Science Technical Report* **54** (1994). <https://wolfram.schneider.org/bsd/7thEdManVol2/trofftut/trofftut.pdf>.

Pelletier, Francis Jeffry and Hazen, Allen, “Natural Deduction Systems in Logic” in *The Stanford Encyclopedia of Philosophy (Spring 2024 Edition)*, ed. Zalta, E. N. and U. Nodelman (2024). <https://stanford.io/4jpc5KF>.