

How to type-set logic and natural deductions using GNU troff, pic and eqn

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Fitch is a notation for natural deduction (Pelletier and Hazen, 2024). `troff` is a software system for type-setting using Unix™ and related operating systems (Ossanna and Kernighan, 1994). Brian W. Kernighan was one of the creators of Unix and the C programming language. `pic` (Kernighan, 1982) and `eqn` (Kernighan and Cherry, 1975) are systems for typesetting graphs and mathematics, also created by Brian Kernighan and his friends. This group created a full set of tools for the type-setting of scientific text, graphs and diagrams, mathematics, chemistry, reference management and complex tables. GROFF AKA GNU troff is the implementation I am using (FSF, 1990). There are other competitors, in particular the slightly younger TeX and LaTeX. There are also alternative implementations of troff, but this is the version I use.

The Fitch notations has got its name after its inventor, Fredric Fitch. This notation seems to be a de facto standard: It is used in all the text books I have been able to find electronically, and seems to be taught at logics courses in mathematics as well as philosophy. I wrote this note while learning Fitch; My intention is to demonstrate how to write predicate and propositional logic, and deduction on this platform. I cannot teach you how to format scientific text in general, neither can I give an introduction to logic and natural deduction.

Writing equations

First, we need to be able to write our texts (there is a good tutorial by Kollar and Robinson (2023)) and then we can continue with logical formulas and sentences (Harding, 2011). At a first glance, all of them are “equations”, or anyone who are not familiar with mathematics and logic and the differences between the two will regard them as such. Here is a set of predicate logic sentences, first in eqn source,

```
.EQ (1)
  pile {
    \[fa] x SameSize(x)
    above
    \[fa] x Cube(x) -> Cube(b)
    above
    (Cube(b) \[AN] b=c) -> Small(c)
    above
    (Small(b) \[AN] SameSize(b,c) -> Small(c)
  }
.EN
```

and then formatted, in Equation (1).

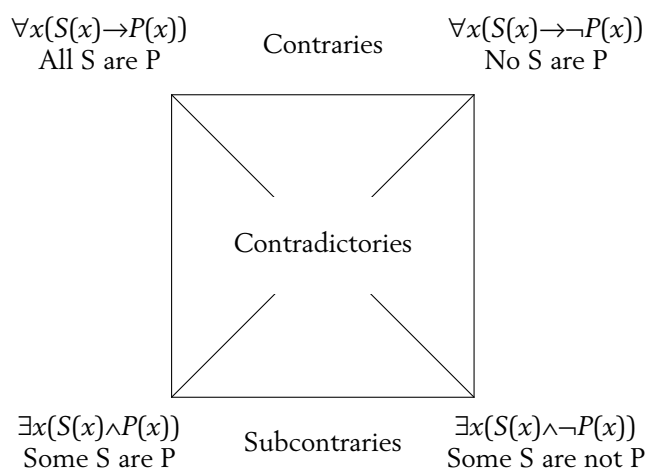
$$\begin{aligned}
& \forall x \text{SameSize}(x) \\
& \forall x \text{Cube}(x) \rightarrow \text{Cube}(b) \\
& (\text{Cube}(b) \wedge b = c) \rightarrow \text{Small}(c) \\
& (\text{Small}(b) \wedge \text{SameSize}(b, c) \rightarrow \text{Small}(c)
\end{aligned} \tag{1}$$

To write these formulas, you need to use either the unicode characters or their Groff names, see Table 1.

Using logics inside graphs

Table 1. Unicode characters for logical signs and operators. On some operating systems you can type them by pressing `ctrl-shift-u` and then the four character code (following `u+`). The Groff name is usually better to use than the Unicode character, but I tend to use the latter.

Unicode	Character	Groff name
u+00AC	¬	\[no]
u+2227	^	\[AN]
u+2228	∨	\[OR]
u+2200	∀	\[fa]
u+2203	∃	\[te]
u+2192	→	\[->]
u+2194	↔	\[<>]
u+22A5	⊥	\[pp]
u+22A2		
u+2261	≡	\[==]
u+25A1	□	\[sq]
u+25C7		
u+2234	∴	\[tf]
U+2208	∈	\[mo]
U+2209	∉	\[nm]
U+2286	⊆	\[ib]



1	$A \vee B$	
2	$\neg A$	
3	A	
4	\perp	\perp Intro: 3,2
5	B	\perp Elim: 4
6	B	
7	B	Reit: 6
8	B	\vee Elim: 6-7,3-5,1

Figure 1. Proof that $A \vee B, \neg A \therefore B$. The line numbering is in the left-most margin. Then there is a vertical line, as long as the proof. The step 1-2 in the proof is where the premises lives. The horizontal line after step 2 is usually referred to as the *fitch line*. The two groups, 3-5 and 3-6 are sub-proofs, with their own premisses, vertical lines and fitch lines

How to write Fitch in troff

Any proof is initialized by calling this macro, which informs all scripts on the number of steps in the proof and its maximum depth, i.e., how deep the hierarchy of proofs is. That is, how many inside proofs, within proofs ... do we have. You better add one or the references at the right will come to close to the logical statements.

```
set_steps_and_depths(8,3)
```

Any proof (the root proof or any sub-proof) starts with the `start_proof()` macro, which also names that proof. After we have started the proof, we add its premises, and end it with `premis_end()`.

```
start_proof(START);
add_premis(START, "A \vee B");
add_premis(START, "\neg A");
premis_end(START);
```

After ending the premises section, we enter the body of our proofs. In this case we start the sub-proofs

```
start_proof(SUB1);
add_premis(SUB1, "A");
premis_end(SUB1);
```

In the body of a proof, we use the `add_step()` macro, which has three argument: (i) the name of the current proof, (ii) the result of the step, and finally (iii) the

references to earlier steps needed for the step.

```
add_step(SUB1, "⊥", "⊥ Intro: 3, 2");
add_step(SUB1, "B", "⊥ Elim: 4");
end_proof(SUB1);
```

We end a proof (be it a sub_proof or a proof) with the `end_proof()` macro, which needs the name of the current proof as an argument. Now we start another subproof.

```
start_proof(SUB2);
add_premis(SUB2, "B");
premis_end(SUB2);
add_step(SUB2, "B", "Reit: 6");
end_proof(SUB2);
```

After we have completed the two sub-proofs, return to the main proof and completes it with a nice \vee elimination.

```
add_step(START, "B", "⋃ Elim: 6-7, 3-5, 1");
end_proof(START)
```

Note that the macros do not check your references. Sanity checks and proof reading is your job.

A QL proof

References

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- Pelletier, Francis Jeffry and Hazen, Allen, "Natural Deduction Systems in Logic" in *The Stanford Encyclopedia of Philosophy (Spring 2024 Edition)*, ed. Zalta, E. N. and U. Nodelman (2024). <https://plato.stanford.edu/archives/spr2024/entries/natural-deduction>.

1	$A \vee B$	
2	$\neg B \vee C$	
3	A	
4	$A \vee C$	\vee Intro:3
5	B	
6	$\neg B$	
7	\perp	\perp Intro:6,5
8	$A \vee C$	\perp Elim:7
9	C	
10	$A \vee C$	\vee Intro:9
11	$A \vee C$	\vee Elim:6-8,9-10,2
12	$A \vee C$	\vee Elim:1,3-4,5-11

Figure 2. A slightly longer example: Prove that $A \vee B, \neg B \vee C \therefore A \vee C$.

1			
2		$\forall x Fx$	
3		$\exists x \neg Fx$	
4		$[a] \neg Fa$	
5		Fa	\forall Elim 2
6		\perp	\perp Intro 4,5
7		\perp	\exists Elim 3,4-6
8		$\neg \exists x \neg Fx$	
9		$\neg \exists x \neg Fx$	
10		$[a] \neg Fa$	
11		$\exists x \neg Fx$	\exists Intro 10
12		\perp	\perp Intro 9,11
13		$\forall x Fx$	\perp Elim 12
14		$\forall x Fx$	\forall Intro 10-13
15		$\forall x Fx \leftrightarrow \neg \exists x \neg Fx$	\leftrightarrow Intro 2-8, 9-14