Introduction

The Espresso Machine

The campus canteen has acquired a new Espresso Machine. The machine has split the opinions of students as well as faculty into two camps. The reason for this is that some think that the machine is unreliable; those who purport this has typically put their coins into the machine which deposited into its inner parts with a clink clonk kind of sound, but no coffee are delivered. They started bring thermoses with coffee, strong and hot, from home.

However, the machine delivered coffee, sometimes at least. Let us assume that the there are $n(t)$ at potential customers any given time t, and that they buy Espresso at a rate constant per capita rate a, i.e., customers are removed from the customer class at a rate $an(t)$. However, only a proportion p of the sales actually lead to delivery. Therefore $apn(t)$ of the potential customers are transformed into actual customers, denoted by $y(t)$. They claim that a good espresso is good even if it was produced in an unreliable process. The remaining of sales occur a $a(1-p)n(t)$ and the affected customers claim that the former are adopting a false theory, and they claim that the espresso machine actually is a thief. Let us assume that at time t there $y(t)$ persons who have actually got espresso out of the machine, and then there are $x(t)$ who have not.

As mentioned, the behaviours of the to categories of customers. The satisfied ones are rapidly returning to the potential customers group of students and faculty, they do so at a per capita rate c, i.e., $cy(t)$. Similarly the unsatisfied ones return to that at a much lower rate (for instance when they forget their thermoses at home). We assume that that is happening at a $bx(t)$.

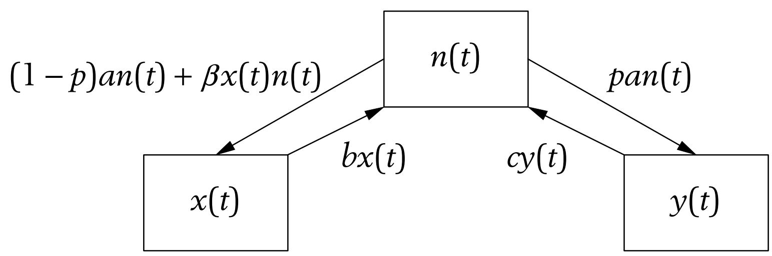


Figure 1.

There arose a new behaviour among the unsatiesfied customers: They started to inform potential customers that the espresso machine was really unreliable. This spread as an epidemic, so we model it as such, see for example Anderson and May (1979). Assume that the contact rate beteen potential customers and unsatiesfied ones is proportional to the product between the two, $\beta x(t)n(t)$ The flows of people between the groups are summarized in Figure 1.

The number of potential customers at time t is $n(t)$

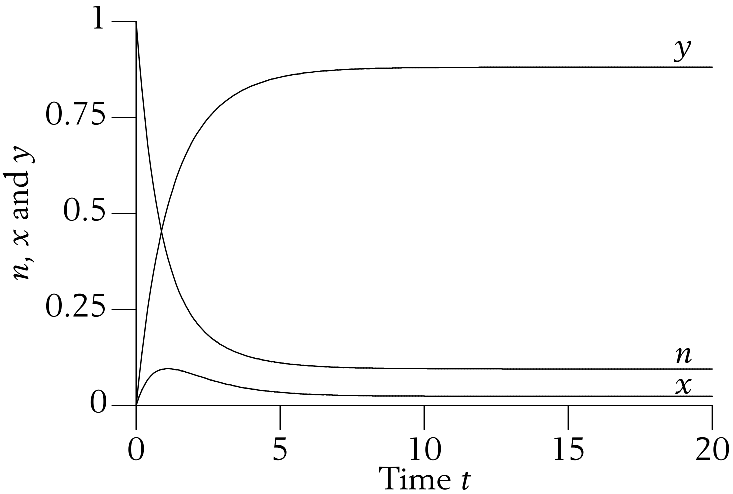
$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = -an(t) + by(t) + cx(t) - \beta nx(t) \tag{1}$$

and $x(t)$ is the number of customers who actually hate the coffee machine,

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = a(1-p)n(t) - cx(t) + \beta nx(t) \tag{2}$$

whereas $y(t)$ is the people who like the coffee machine

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = apn(t) - by(t) \tag{3}$$



Literature

Anderson, R. M. and R. M. May, "Population biology of infectious diseases: Part I." *Nature* **280**, p. 361–367 (1979).

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