How to type-set logic and natural deductions using GNU **troff**, **pic** and **eqn**

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Fitch is a notation for natural deduction (Pelletier and Hazen, 2024). `troff` is a software system for type-setting using Unix™ and related operating systems (Ossanna and Kernighan, 1994). Brian W. Kernighan was one of the creators of Unix and the C programming language. pic (Kernighan, 1982) and `eqn` (Kernighan and Cherry, 1975) are systems for typesetting graphs and mathematics, also created by Brian Kernighan and his friends. This group created a full set of tools for the type-setting of scientific text, graphs and diagrams, mathematics, chemistry, reference management and complex tables. GROFF aka GNU `troff` is the implementation I am using (FSF, 1990). There are other competitors, in particular the slightly younger TeX and LaTeX. There are also alternative implementations of troff, but this is the version I use.

The Fitch notations has got its name after its inventor, Fredric Fitch. This notation seems to be a de facto standard: It is used in all the text books I have been able to find electronically, and seems to be taught at logics courses in mathematics as well as philosophy. I wrote this note while learning Fitch; My intention is to demonstrate how to write predicate and propositional logic, and deduction on this platform. I cannot teach you how to format scientific text in general, neither can I give an introduction to logic and natural deduction.

Writing equations

First, we need to be able to write our texts (there is a good tutorial by Kollar and Robinson (2023)) and then we can continue with formulas and sentences using `eqn` (Harding, 2011). At a first glance, all of them are are "equations", or anyone who are not familiar with mathematics and logic and the differences between the two will regard them as such. Logic is actually a special genre of its own when it comes to formulas or equations. Here is a set of predicate logic sentences, first in `eqn` source,

```
.EQ (1)
    pile {
     forall(x) SameSize(x)
     above
     forall(x) Cube(x) implies Cube(b)
     above
     (Cube(b) and b=c) implies Small(c)
     above
     (Small(b) and SameSize(b,c) implies Small(c)
   }
.EN
```

and then formatted, in Equation (1).

$$
\begin{gathered}
\forall\, x\; SameSize(x) \\
\forall\, x\; Cube(x) \rightarrow Cube(b) \\
(Cube(b) \wedge b = c) \rightarrow Small(c) \\
(Small(b) \wedge SameSize(b, c) \rightarrow Small(c)
\end{gathered}
\tag{1}
$$

To write these formulas, you need to use either the unicode characters, their Groff names or macros I have defined in order to simplify typing. See Table 1.

Table 1. Unicode characters for logical signs and operators. On some operating systems you can type them by pressing `ctrl-shift-u` and then the four character code (following `u+`). The Groff name is usually better to use than the Unicode character, but takes a long time to type. The `eqn` macros are for easier typing and I have tried to adjust spacings for a nicer look.

Unicode	**Character**	**Groff name**	**eqn macro**
U+00AC	¬	\[no]	not
U+2227	∧	\[AN]	and
U+2228	∨	\[OR]	or
U+2200	∀	\[fa]	any
U+2200	∀	\[fa]	forall(x)
U+2203	∃	\[te]	some
U+2203	∃	\[te]	exists(x)
U+2192	→	\[->]	implies
U+2194	↔	\[<>]	iff
U+2194	↔	\[<>]	equiv
U+21D4	⇔	\[hA]	
U+22A5	⊥	\[pp]	falsum
U+22A2		not available	
U+22A8		not available	
U+2261	≡	\[==]	identicalto
U+25A1	□	\[sq]	nece
U+25A1	□	\[sq]	necessarily
U+25C7	◊	\[lz]	possi
U+25C7	◊	\[lz]	possibly
U+2234	∴	\[tf]	therefore
U+2205	∅	\[es]	empty
U+2208	∈	\[mo]	member
U+2209	∉	\[nm]	notmember
U+2286	⊆	\[ib]	subset
U+2118	℘	\[wp]	powerset

Using logics in tables

Table 2. Some useful equivalents if you are doing logic. They are presented here as an example how you can embed formulas in a table.

De Morgan's Laws (Predicate Logic)	$\neg \forall\, x\, A \leftrightarrow \exists\, x \neg A$	$\neg \exists\, x\, A \leftrightarrow \forall\, x \neg A$

Using logic in graphs

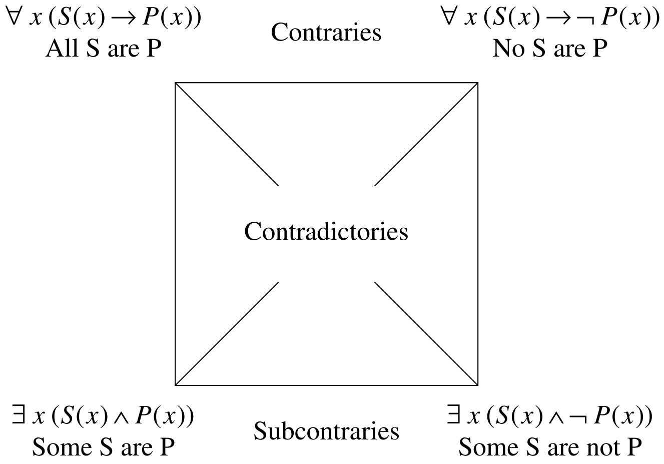

Figure 1. The traditional aristotelian syllogisms.

Writing Fitch arguments

1	$A \vee B$	
2	$\neg A$	
3	$\quad A$	
4	$\quad \bot$	$\bot$ Intro: 3,2
5	$\quad B$	$\bot$ Elim: 4
6	$\quad B$	
7	$\quad B$	Reit: 6
8	B	$\vee$ Elim: 6-7,3-5,1

Figure 1. Proof that $A \vee B, \neg A \therefore B$. The line numbering is in the left-most margin. Then there is a vertical line, as long as the proof. The step 1-2 in the proof is where the premises lives. The horisontal line after step 2 is usually referred to as the *fitch line.* The two groups, 3–5 and 3–6 are subproofs, with their own premisses, vertical lines and fitch lines

How to write Fitch in troff

Any proof is initialized by calling this macro, which informs all scripts on the number of steps in the proof and its maxiumum depth, i.e., how deep the hierarchy of proofs is. That is, how many inside proofs, whitin proofs ... do we have. You better add one or the references at the right will come to close to the logical statements.

```
set_steps_and_depths(8,3)
```

Any proof (the root proof or any sub-proof) starts with the start_proof() macro, which also names that proof. After we have started the proof, we add its premises, and end it with `premis_end()`.

```
start_proof(START);
add_premis(START,"$A or B$");
add_premis(START,"$not A$");
premis_end(START);
```

After ending the premiws section, we enter the body of our proofs. In this case we start the sub-proofs

```
start_proof(SUB1);
add_premis(SUB1,"$A$");
premis_end(SUB1);
```

In the body of a proof, we use the `add_step()` macro, which has three argument: (i) the name of the current proof, (ii) the result of the step, and finally (iii) the references to earlier steps needed for the step.

```
add_step(SUB1,"$falsum$","⊥ Intro: 3,2");
add_step(SUB1,"$B$","⊥ Elim: 4");
end_proof(SUB1);
```

We end a proof (be it a sub_proof or a proof) with the `end_proof()` macro, which needs the name of the current proof as an argument. Now we start another subproof.

```
start_proof(SUB2);
add_premis(SUB2,"$B$");
premis_end(SUB2);
add_step(SUB2,"$B$","Reit: 6");
end_proof(SUB2);
```

After we have completed the two sub-proofs, return to the main proof and completes it with a nice $\vee$ elimination.

```
add_step(START,"$B$","∨ Elim: 6−7,3−5,1");
end_proof(START)
```

Note that the macros do not check your references. Sanity checks and proof reading is your job.

References

FSF, Free Software Foundation, *Groff* (1990). https://www.gnu.org/software/groff/.

Harding, Ted, *A Guide to Typesetting Mathematics using GNU eqn* (2011). https://lists.gnu.org/archive/html/groff/2013-10/pdfTyBN2VWR1c.pdf.

1	$A \vee B$			
2	$\neg B \vee C$			
3		A		
4		$A \vee C$		$\vee$ Intro:3
5		B		
6			$\neg B$	
7			$\bot$	$\bot$ Intro:6,5
8			$A \vee C$	$\bot$ Elim:7
9			C	
10			$A \vee C$	$\vee$ Intro:9
11		$A \vee C$		$\vee$ Elim:6-8,9-10,2
12	$A \vee C$			$\vee$ Elim:1,3-4,5-11

Figure 2. A slightly longer example: Prove that $A \vee B, \neg B \vee C \therefore A \vee C$.

Kernighan, Brian W., “PIC — A language for typesetting graphics,” *Software: Practice and Experience* **12** (1982). https://doi.org/10.1002/spe.4380120102.

Kernighan, Brian W. and Cherry, Lorinda L., “A System for Typesetting Mathematics,” *Scientific Applications* **18** (1975). https://doi.org/10.1145/360680.360684.

Kollar, Larry and Robinson, G. Branden, *Using groff with the ms Macro Package* (2023). https://lists.gnu.org/r/groff/2023-04/pdfkxOTeCk24w.pdf.

Ossanna, Joseph F. and Kernighan, Brian W., “Troff User’s Manual,” *Computing Science Technical Report* **54** (1994). https://wolfram.schneider.org/bsd/7thEdManVol2/trofftut/trofftut.pdf.

Pelletier, Francis Jeffry and Hazen, Allen, “Natural Deduction Systems in Logic” in *The Stanford Encyclopedia of Philosophy (Spring 2024 Edition),* ed. Zalta, E. N. and U. Nodelman (2024). https://stanford.io/4jpc5KF.

1		
2	$\forall x\, Fx$	
3	$\exists x\, \neg Fx$	
4	$[a]\, \neg Fa$	
5	Fa	$\forall$ Elim 2
6	$\perp$	$\perp$ Intro 4,5
7	$\perp$	$\exists$ Elim 3,4-6
8	$\neg\exists x\, \neg Fx$	
9	$\neg\exists x\, \neg Fx$	
10	$[a]\, \neg Fa$	
11	$\exists x \neg Fx$	$\exists$ Intro 10
12	$\perp$	$\perp$ Intro 9,11
13	$\forall x\, Fx$	$\perp$ Elim 12
14	$\forall x\, Fx$	$\forall$ Intro 10-13
15	$\forall x\, Fx \leftrightarrow \neg\exists x\, \neg Fx$	$\leftrightarrow$ Intro 2-8, 9-14

Figure 3. A proof using predicate logic.