Introduction

The Espresso Machine

The campus canteen has acquired a new Espresso Machine. The machine has split the opinions of students and faculty into two camps. The reason is that some think that the machine is unreliable; those who purport this have typically put their coins into the machine, which deposited them into its interiors with a clinkety clank. No coffee were delivered. They started bring thermoses with coffee, strong and hot, from home. However, the sometimes machine delivered coffee and good espresso, as a matter of fact a bright student estimated that the proportion p of the cups sold were good.

Let us assume that the there are $n(t)$ at potential customers at time t. They buy Espresso at a rate constant per capita rate a, i.e., customers are removed from the customer class at a rate $an(t)$. However, only a proportion p of the sales actually lead to delivery. Therefore $apn(t)$ of the potential customers are transformed into actual customers, denoted by $y(t)$. They claim that this is a good espresso, although it was produced through an unreliable process. The remaining sales occur a rate $a(1-p)n(t)$ and the affected customers claim that the former are accepting the statement "the espresso is good" is false. Let us assume that at time t there $y(t)$ persons who have actually got espresso out of the machine, and then there are $x(t)$ who have not.

The satisfied customers are rapidly returning to the group of potential customers; they do so at a per capita rate c, i.e., $cy(t)$. Similarly the unsatisfied ones return at a much lower rate (for instance when they forget their thermoses at home). We assume that that is happening at a $bx(t)$.

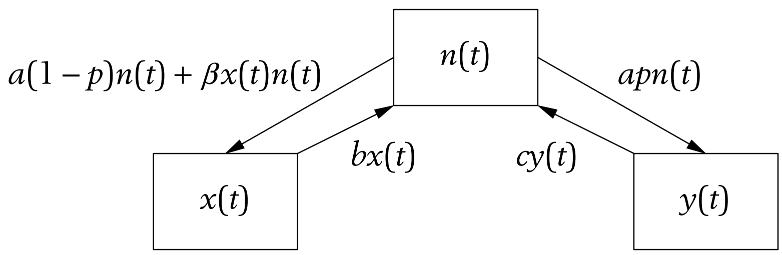


Figure 1.

There arose a new behaviour among the unsatiesfied customers: They started to inform potential customers that the espresso machine was really unreliable. This spread as an epidemic, so we model it as such, see for example Anderson and May (1979). Assume that the contact rate beteen potential customers and unsatiesfied ones is proportional to the product between the two, $\beta x(t)n(t)$ The flows of people between the groups are summarized in Figure 1.

The number of potential customers at time t is $n(t)$

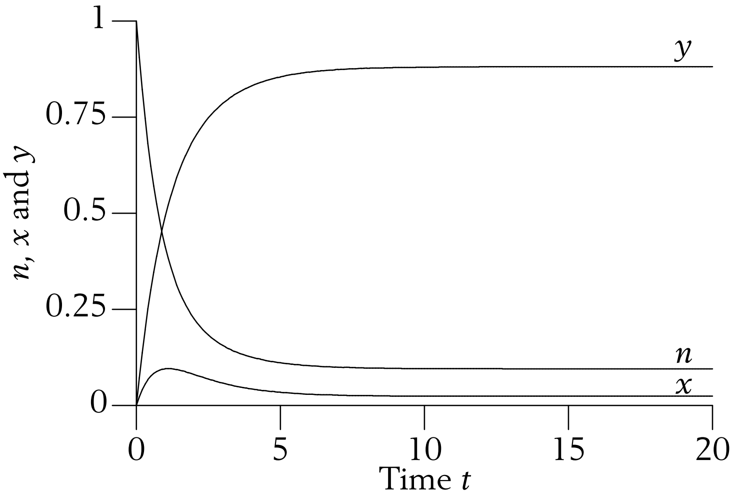
$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = -an(t) + by(t) + cx(t) - \beta nx(t) \tag{1}$$

and $x(t)$ is the number of customers who actually hate the coffee machine,

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = a(1-p)n(t) - cx(t) + \beta nx(t) \tag{2}$$

whereas $y(t)$ is the people who like the coffee machine

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = apn(t) - by(t) \tag{3}$$



Literature

Anderson, R. M. and R. M. May, "Population biology of infectious diseases: Part I." *Nature* **280**, p. 361–367 (1979).

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