Modality and dynamical systems theory

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These are my points of departure. The document is about what I believe my spring 2026 essay will be about. It takes the form of a handful of theses that I may use in the forthcoming text.

1. Modal reasoning appears everywhere in science, but usually not as modal logic

According to the writings of Williamson (2018) on modal logic and dynamic systems.[1] much theory in the natural sciences is modal, albeit they do not use modal logic *per se.* Maudlin (2020) shares Williamson's views on modality in the sciences, but does so from the point of view of the philosophy of physics.

Williamson introduces modality by listing modal auxiliary verbs like

a. can, could

b. may, might, must

c. able to, has to, needs to

All of them are related to possibilities, contingencies and necessities. Williamson introduces further distinctions:

metaphysical or *objective modality:* things that are necessary or possible in the real world.

nomic modality: a special case of metaphysical modality which is related to what necessities and contingencies is entailed from the laws of nature.

dynamical, epistemic or *subjective modality:* related to whether we know something for sure or just believe it. Subjective modality seems to be a special case of epistemic modality related to the strengths of beliefs. Dynamic epistemic logic requires special operators (Baltag and Renne, 2016).

Objective and subjective modalities are Williamson's choice of terminology. He argues that this mirrors the vocabulary used for probability, where objective probability refers to truly stochastic processes and subjective probability refers to the strength of a conviction. I will follow him on this in the following, using objective and subjective as determinants for both probabilities and modalities.

Williamson spends a whole section on scepticism about objective modality. Early on that scepticism was due to the resistance against Aristotelian essentialism. That is, the idea that things have some inherent unchangeable essence that they cannot exist without, like the object o is essentially p *iff* o is necessarily p. Without p it would not exist. The scepticism of any kind of essentialism this goes from Hume to Quine and beyond. It extends to virtually all kinds of objective modality, and, according to Williamson, it is not saved by nomic necessity and contingency. That is, one would need to show that the laws of nature entail full blown nomic modalities. This is not made easier by the fact that scientists seldom use formal logic.

Nomic modality is an obvious case where science is modal, but it is not really tractable for modal reasoning.

[1] The original lecture can be found on YouTube https://youtu.be/ZfaHf_TESEw

2. Dynamical systems provide examples of modality

The dynamical systems theory is a branch of mathematics widely used in the sciences: physics, chemistry and biology but also in social sciences and economics and more recently even in the humanities (for example in history). The study of chaos is a part of this theory. I used it extensively when working as researcher in population biology during the eighties and nineties.

A dynamical system consists of

1. a set of state variables, i.e., the variables needed to define the system such as the weights, positions and velocities of the planets in the solar system. The set of state variables is often referred to as a state space.
2. a set of equations defining the change in the state variables.
3. the set of state variables form sets referred to as orbits or trajectories defining the states a system can be reached given its initial condition.

I'll give a number of concrete examples below.

3. An example where sustainability is necessarily impossible

One of the simplest system with nonlinear dynamics is the logistic equation. It is one-dimensional equation, and in continuous time it has no complex dynamics but it is capable of demonstrating nomic necessity and possibility.

The examples below come from Clark (1976). We have three versions (Eq 1a – c), the first of which has solutions showing the typical sigmoid form where at low population densities the growth is exponential but at high it approaches the equilibrium $\hat{N} = K$ (Figure 1 a – b). Sustainability is possible.

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1 - \frac{N}{K}\right) \tag{1a}$$

With harvesting, the logistic equation becomes more interesting. In Eq. 1b the harvesting rate is proportional to the population density

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1 - \frac{N}{K}\right) - hN \tag{1b}$$

which has a single stable equilibrium at $\hat{N} = (r - h)K/r$. A wild population can possibly be sustained at this rate as long as $r > h$. Eq. 1c shows a population which is harvested at a constant rate independent of the population density.

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1 - \frac{N}{K}\right) - h \tag{1c}$$

That equation has two equilibria, one (the higher) is stable, and the other (the lower) which is unstable.[2] Below the lower equilibrium sustainability is necessarily impossible whereas at the higher one sustainability is possible (Figure 1e – f).

We have seen cases where dynamical systems can elucidate possibility and necessity in biological system. In particular we have seen one case where a harvesting strategy is capable driving a species to extinction (Eq. 1b and Figure 1 c – d)

[2] The to equilibria are given by the quadratic equation

$$rN - \frac{rN^2}{K} = h$$

which gives

$$\hat{N}_{1,2} = \frac{K}{2} \pm \sqrt{\frac{K^2}{4} + \frac{hK}{r}}$$

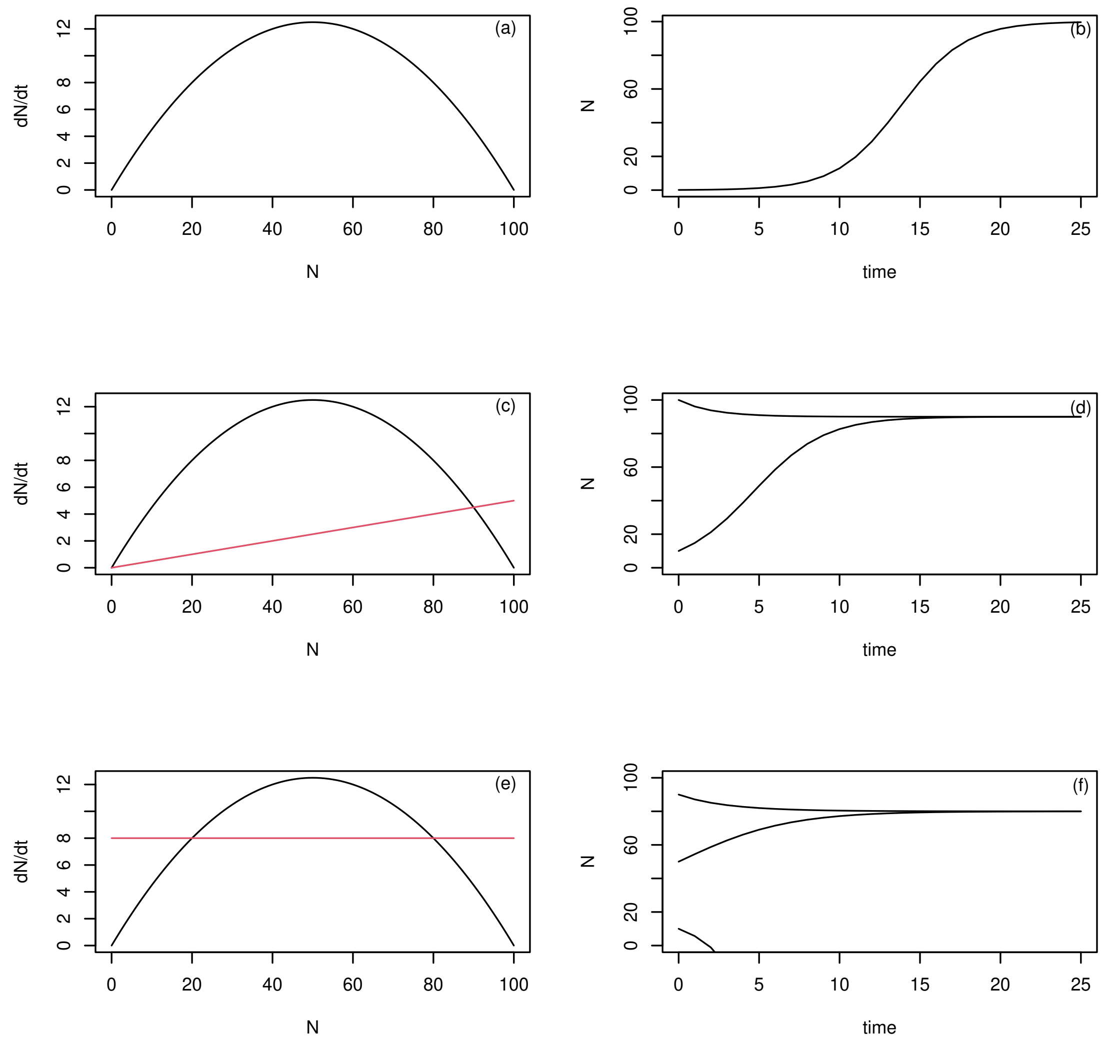

Figure 1. Graph (1a – b) depicts the ordinary differential equation Eq. 1a and its solution, respectively. The population grows asymptotically towards the equilibrium $N = K$.

Graph (1c – f) adds harvesting (such as a fishery) to the population dynamics. Graph 1c shows the situation when the harvesting strategy is a constant per capita harvesting rate (i.e., the rate is proportional to the population density). The curves in Graph 1d show how the population could approach the equlibrium from below and above.

The curve in Graph 1e is population growth (i.e., the logistic equation the same as in Graph 1a) and the line represents a constant harvesting rate, which is assumed to be independent of the population density. The harvesting line intersects twice with the curve giving two equilibria. The lower one is unstable and be low that the population will necessarily go extinct. Graph 1d shows three solutions approaching the equilibrium from above and below, and one where the population rapidly disappears because the initial population density is below the lower equilibrium. I solved the differential equations using Rscript (Development Core Team, 2010) with the package deSolve (Soetaert *et al.*, 2021).

4. Modal operators for dynamical systems are possible

A list extracted from Williamson (2018)

- sometime in the future
- always in the future

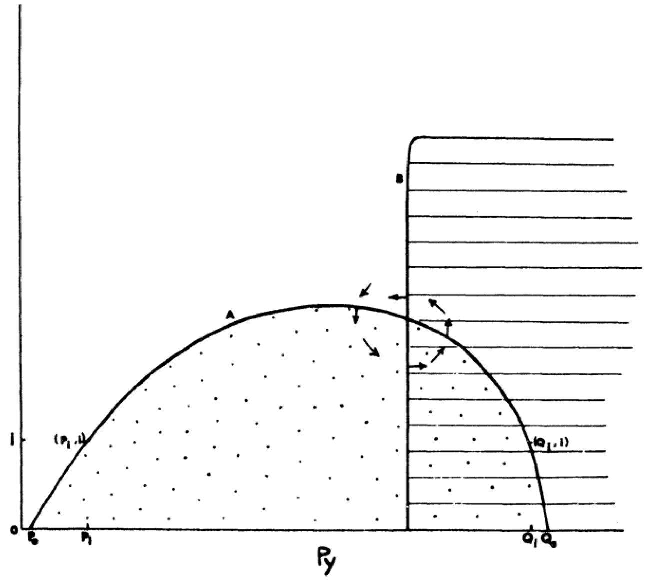

FIGURE 1. The instantaneous model of the interaction of a food-limited predator and its prey. Py = prey density; Pd = predator density. Line A is the prey isocline, that is, the set of all points for which $\frac{dPy}{dt} = 0$; line B is the predator isocline. Py increases in the dotted area only; Pd increases only in the shaded area. The vectors are the instantaneous (general) direction of change of the community at eight qualitatively-different points in the graph.

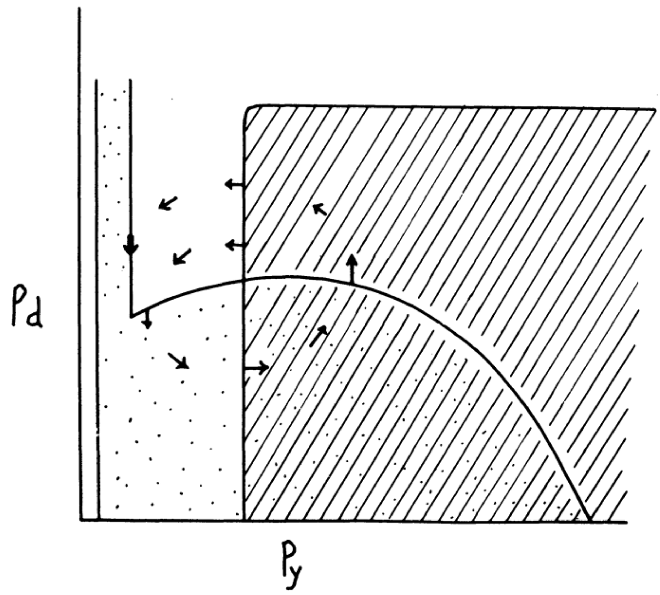

FIGURE 5. The interaction of a predator and its prey where the predator is unable to eat the prey at every prey density. Isoclines, coordinates, and markings as in figure 1. In this case, growing oscillations do not necessarily grow until one or both species becomes extinct, for a limiting oscillation may be reached.

Figure 2. Two models of predator–prey-relations (Rosenzweig and MacArthur, 1963).

- sometime in the past
- always in the past

Things to consider: These are meant to be operators for objective or metaphysical modality, not epistemic ones. Still they are close to the historical predictions so vehemently objected to by Karl Popper, which I believe are more epistemic (Popper, 1941; Popper, 1957).

5. Snowshoe hare and lynx are in a Nietzschean eternal recurrence

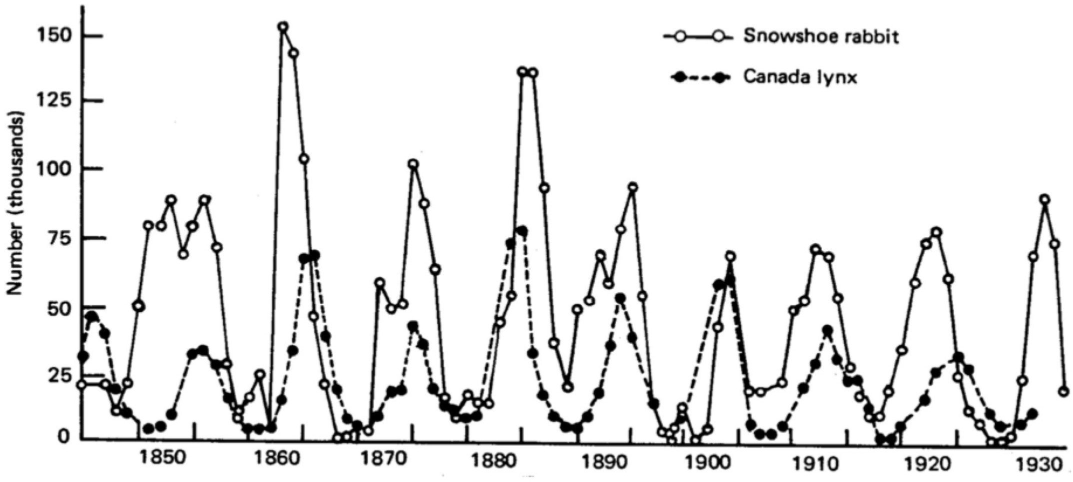

Figure 3. Fluctuations in Canadian populations of snowshoe hare and lynx (Udell, 2016).

6. Herd immunity has an effect on the spread of infections

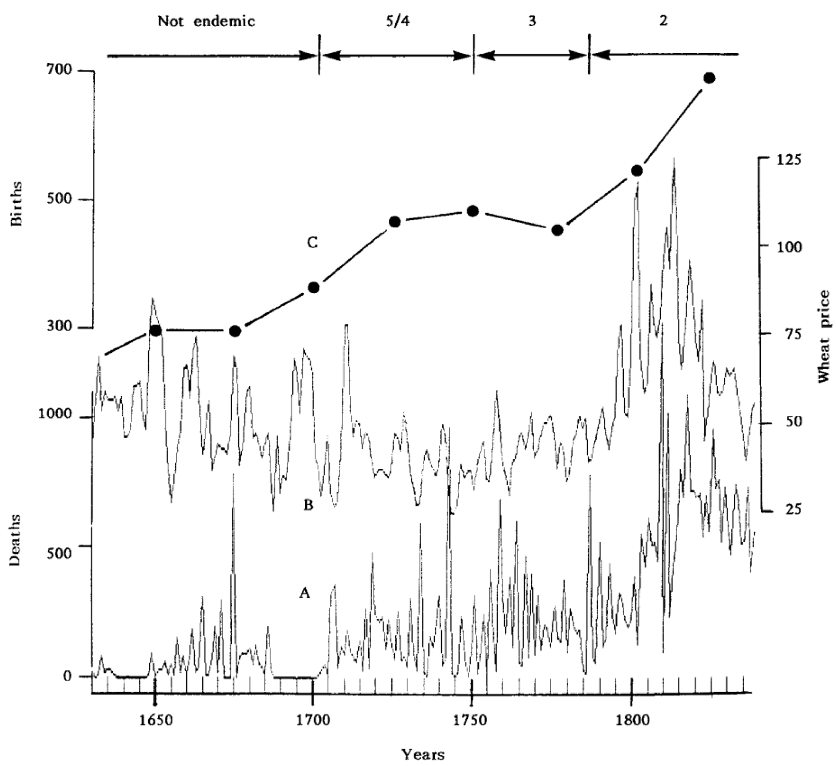

FIG. 1. Measles in London, 1629–1837. A: Annual measles deaths. B: Annual wheat price (shillings). C: Cumulative number of baptisms in the preceding 25 years (thousands). Above: Division into different interepidemic periods (years).

Figure 4. Historical records of outbreak of measels in London.

In the absence of herd immunity, there will be rapid outbreaks of measles.

The typical situation (before the mass immunisations programs) used to be like this:

1. There is an outbreak of the disease.
2. The outbreak lasts until the proportion of immune people is enough to cause herd immunity, then the incidence of infected people declines.

3. The number of susceptible children increases through birth such that the herd immunity weakens which will eventually lead to a new outbreak. Return to 1 and repeat.

In England and Wales, the period between the outbreaks (from step 1 in one cycle to step 1 in the next, see above) decreased from a 5-yearly cycle 1647 to a 2-yearly one around 1800. The reason being the population growth in London and other cities (Duncan *et al.*, 1997). From 1800 onwards it was stable 2 year cycle until the onset of vaccination 1969. Then the period started to increase again, with the mass immunisation programs (Anderson *et al.*, 1984).

7. Dynamical systems can lead to chaotic behaviour

The Lorenz equations

$$\frac{\mathrm{d}X}{\mathrm{d}t} = aX + YZ$$
$$\frac{\mathrm{d}Y}{\mathrm{d}t} = b(Y - Z)$$
$$\frac{\mathrm{d}Z}{\mathrm{d}t} = -XY + cY - Z$$

See Figure 5.

8. Philosophy is the study of philosophical problem

Philosophers claim, perhaps starting with Bertrand Russell, that philosophy is defined by its problems. Russell gives an example in the first paragraph in the first page of the first chapter of his book *Problems of philosophy*

> Is there any knowledge in the world which is so certain that no reasonable man could doubt it? This question, which at first sight might not seem difficult, is really one of the most difficult that can be asked. When we have realized the obstacles in the way of a straightforward and confident answer, we shall be well launched on the study of philosophy—for philosophy is merely the attempt to answer such ultimate questions, not carelessly and dogmatically, as we do in ordinary life and even in the sciences, but critically, after exploring all that makes such questions puzzling, and after realizing all the vagueness and confusion that underlie our ordinary ideas (Russell, 1912, p. 1).

About a century later Floridi (2013) spends an entire essay on just this statement. In particular,

> The result is a definition of philosophical questions as questions whose answers are in principle open to informed, rational, and honest disagreement, ultimate but not absolute, closed under further questioning, possibly constrained by empirical and logico-mathematical resources, but requiring noetic resources to be answered.

9. The law of cause and effect are not a law of nature

10. Metaphysical grounding and ontological commitment

Quine and more

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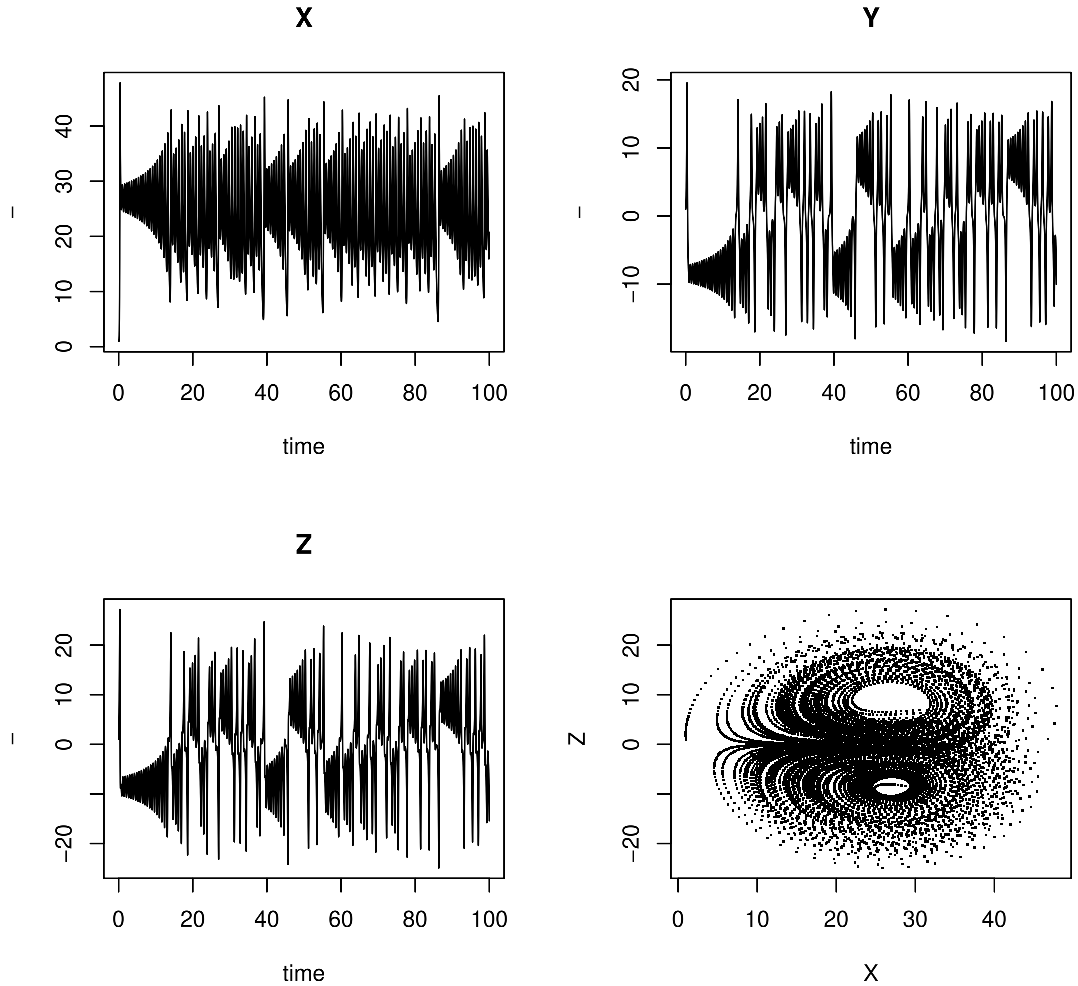

Figure 5. The famous Lorenz attractor which demonstrates chaotic behaviours of weather systems. $a = -8/3$, $b = -10$ and $c = 28$. I solved the differential equations using Rscript (Development Core Team, 2010) with the package deSolve (Soetaert *et al.*, 2021).

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Github project

Scan QR to get project at
https://github.com/siglun/term-paper-spring-2026/blob/main/initial-theses.pdf