

4. Sampling Distribution

Suppose we have different samples of size n drawn from the population and for each and every sample of size n compute the means. Obviously these means will not be same.

If we group these characteristics according to its frequency the frequency distribution so generated is called sampling distribution of means.

Note: 1) The sampling distribution of the large samples is assumed to be normal distribution.

2) The standard deviation of sampling distribution is also called standard error.

case 1: Sampling with replacement.

Here the items drawn are put back to the population before the next draw. If N is the size of the population and n is size of the sample then we have N^n number of samples.

And $H_{\bar{x}}$ of the frequency distribution of the sample mean

will be equal to the population mean H . ($H_{\bar{x}} = H$)

The variance $\sigma_{\bar{x}}^2$ of the frequency distribution of the sample means will be equal to $\frac{\sigma^2}{n}$, where σ^2 is variance of population

$$\text{i.e. } \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

case 2: Sampling without replacement.

Here the items are drawn and are not put back to the population before the next draw. In this case N^n samples can be collected

$$\text{mean : } H_{\bar{x}} = H$$

$$\text{variance : } \sigma_{\bar{x}}^2 = \left(\frac{N-n}{N-1} \right) \frac{\sigma^2}{n}$$

correcting factor $\left(\frac{N-n}{N-1} \right) \neq 1$ for large samples
(with replacement)

$$\sigma = \sqrt{\sigma_{\bar{x}}^2} = \sigma_{\bar{x}}$$

- ① A population consists of 5 numbers 2, 3, 6, 8, 11. All possible samples of size 2 which can be drawn with replacement from this population. Find

 - Find mean and standard deviation of population
 - The mean and standard deviation of sampling distribution of means.
 - Considering samples without replacements find mean and standard deviation of sampling distribution of means.

$$\text{Sol}^n : N = 5$$

$$n=2$$

$$\text{mean}(H) = \frac{2+3+6+8+11}{5} = 6$$

$$S.D (\sigma) = 3.28$$

$$\sqrt{\frac{\sum (x_i - \bar{X})^2}{n}}$$

(iii) Sample with replacement are soft in E. a

$$(2,1) \quad (2,3) \quad (2,6) \quad (2,8) \quad (2,11)$$

(3,2) (3,3) (3,6) (3,8) (3,11) report on

$(6,2)(6,3)(6,6)(6,8)(6,11)$

(8,3) (8,3) (8,6) (8,8) (8,11) mit 10

(11,2) (11,3) (11,6) (11,8) (11,9) large ad 11

..... at each sample's

Now individual means of each samples

2 2.5 4 5 6.5 ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~ ~~39~~ ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ ~~53~~ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ ~~59~~ ~~60~~ ~~61~~ ~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ ~~67~~ ~~68~~ ~~69~~ ~~70~~ ~~71~~ ~~72~~ ~~73~~ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ ~~79~~ ~~80~~ ~~81~~ ~~82~~ ~~83~~ ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ ~~89~~ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ ~~97~~ ~~98~~ ~~99~~ ~~100~~ ~~101~~ ~~102~~ ~~103~~ ~~104~~ ~~105~~ ~~106~~ ~~107~~ ~~108~~ ~~109~~ ~~110~~ ~~111~~ ~~112~~ ~~113~~ ~~114~~ ~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~ ~~124~~ ~~125~~ ~~126~~ ~~127~~ ~~128~~ ~~129~~ ~~130~~ ~~131~~ ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ ~~137~~ ~~138~~ ~~139~~ ~~140~~ ~~141~~ ~~142~~ ~~143~~ ~~144~~ ~~145~~ ~~146~~ ~~147~~ ~~148~~ ~~149~~ ~~150~~ ~~151~~ ~~152~~ ~~153~~ ~~154~~ ~~155~~ ~~156~~ ~~157~~ ~~158~~ ~~159~~ ~~160~~ ~~161~~ ~~162~~ ~~163~~ ~~164~~ ~~165~~ ~~166~~ ~~167~~ ~~168~~ ~~169~~ ~~170~~ ~~171~~ ~~172~~ ~~173~~ ~~174~~ ~~175~~ ~~176~~ ~~177~~ ~~178~~ ~~179~~ ~~180~~ ~~181~~ ~~182~~ ~~183~~ ~~184~~ ~~185~~ ~~186~~ ~~187~~ ~~188~~ ~~189~~ ~~190~~ ~~191~~ ~~192~~ ~~193~~ ~~194~~ ~~195~~ ~~196~~ ~~197~~ ~~198~~ ~~199~~ ~~200~~ ~~201~~ ~~202~~ ~~203~~ ~~204~~ ~~205~~ ~~206~~ ~~207~~ ~~208~~ ~~209~~ ~~210~~ ~~211~~ ~~212~~ ~~213~~ ~~214~~ ~~215~~ ~~216~~ ~~217~~ ~~218~~ ~~219~~ ~~220~~ ~~221~~ ~~222~~ ~~223~~ ~~224~~ ~~225~~ ~~226~~ ~~227~~ ~~228~~ ~~229~~ ~~230~~ ~~231~~ ~~232~~ ~~233~~ ~~234~~ ~~235~~ ~~236~~ ~~237~~ ~~238~~ ~~239~~ ~~240~~ ~~241~~ ~~242~~ ~~243~~ ~~244~~ ~~245~~ ~~246~~ ~~247~~ ~~248~~ ~~249~~ ~~250~~ ~~251~~ ~~252~~ ~~253~~ ~~254~~ ~~255~~ ~~256~~ ~~257~~ ~~258~~ ~~259~~ ~~260~~ ~~261~~ ~~262~~ ~~263~~ ~~264~~ ~~265~~ ~~266~~ ~~267~~ ~~268~~ ~~269~~ ~~270~~ ~~271~~ ~~272~~ ~~273~~ ~~274~~ ~~275~~ ~~276~~ ~~277~~ ~~278~~ ~~279~~ ~~280~~ ~~281~~ ~~282~~ ~~283~~ ~~284~~ ~~285~~ ~~286~~ ~~287~~ ~~288~~ ~~289~~ ~~290~~ ~~291~~ ~~292~~ ~~293~~ ~~294~~ ~~295~~ ~~296~~ ~~297~~ ~~298~~ ~~299~~ ~~300~~ ~~301~~ ~~302~~ ~~303~~ ~~304~~ ~~305~~ ~~306~~ ~~307~~ ~~308~~ ~~309~~ ~~310~~ ~~311~~ ~~312~~ ~~313~~ ~~314~~ ~~315~~ ~~316~~ ~~317~~ ~~318~~ ~~319~~ ~~320~~ ~~321~~ ~~322~~ ~~323~~ ~~324~~ ~~325~~ ~~326~~ ~~327~~ ~~328~~ ~~329~~ ~~330~~ ~~331~~ ~~332~~ ~~333~~ ~~334~~ ~~335~~ ~~336~~ ~~337~~ ~~338~~ ~~339~~ ~~340~~ ~~341~~ ~~342~~ ~~343~~ ~~344~~ ~~345~~ ~~346~~ ~~347~~ ~~348~~ ~~349~~ ~~350~~ ~~351~~ ~~352~~ ~~353~~ ~~354~~ ~~355~~ ~~356~~ ~~357~~ ~~358~~ ~~359~~ ~~360~~ ~~361~~ ~~362~~ ~~363~~ ~~364~~ ~~365~~ ~~366~~ ~~367~~ ~~368~~ ~~369~~ ~~370~~ ~~371~~ ~~372~~ ~~373~~ ~~374~~ ~~375~~ ~~376~~ ~~377~~ ~~378~~ ~~379~~ ~~380~~ ~~381~~ ~~382~~ ~~383~~ ~~384~~ ~~385~~ ~~386~~ ~~387~~ ~~388~~ ~~389~~ ~~390~~ ~~391~~ ~~392~~ ~~393~~ ~~394~~ ~~395~~ ~~396~~ ~~397~~ ~~398~~ ~~399~~ ~~400~~ ~~401~~ ~~402~~ ~~403~~ ~~404~~ ~~405~~ ~~406~~ ~~407~~ ~~408~~ ~~409~~ ~~410~~ ~~411~~ ~~412~~ ~~413~~ ~~414~~ ~~415~~ ~~416~~ ~~417~~ ~~418~~ ~~419~~ ~~420~~ ~~421~~ ~~422~~ ~~423~~ ~~424~~ ~~425~~ ~~426~~ ~~427~~ ~~428~~ ~~429~~ ~~430~~ ~~431~~ ~~432~~ ~~433~~ ~~434~~ ~~435~~ ~~436~~ ~~437~~ ~~438~~ ~~439~~ ~~440~~ ~~441~~ ~~442~~ ~~443~~ ~~444~~ ~~445~~ ~~446~~ ~~447~~ ~~448~~ ~~449~~ ~~450~~ ~~451~~ ~~452~~ ~~453~~ ~~454~~ ~~455~~ ~~456~~ ~~457~~ ~~458~~ ~~459~~ ~~460~~ ~~461~~ ~~462~~ ~~463~~ ~~464~~ ~~465~~ ~~466~~ ~~467~~ ~~468~~ ~~469~~ ~~470~~ ~~471~~ ~~472~~ ~~473~~ ~~474~~ ~~475~~ ~~476~~ ~~477~~ ~~478~~ ~~479~~ ~~480~~ ~~481~~ ~~482~~ ~~483~~ ~~484~~ ~~485~~ ~~486~~ ~~487~~ ~~488~~ ~~489~~ ~~490~~ ~~491~~ ~~492~~ ~~493~~ ~~494~~ ~~495~~ ~~496~~ ~~497~~ ~~498~~ ~~499~~ ~~500~~ ~~501~~ ~~502~~ ~~503~~ ~~504~~ ~~505~~ ~~506~~ ~~507~~ ~~508~~ ~~509~~ ~~510~~ ~~511~~ ~~512~~ ~~513~~ ~~514~~ ~~515~~ ~~516~~ ~~517~~ ~~518~~ ~~519~~ ~~520~~ ~~521~~ ~~522~~ ~~523~~ ~~524~~ ~~525~~ ~~526~~ ~~527~~ ~~528~~ ~~529~~ ~~530~~ ~~531~~ ~~532~~ ~~533~~ ~~534~~ ~~535~~ ~~536~~ ~~537~~ ~~538~~ ~~539~~ ~~540~~ ~~541~~ ~~542~~ ~~543~~ ~~544~~ ~~545~~ ~~546~~ ~~547~~ ~~548~~ ~~549~~ ~~550~~ ~~551~~ ~~552~~ ~~553~~ ~~554~~ ~~555~~ ~~556~~ ~~557~~ ~~558~~ ~~559~~ ~~560~~ ~~561~~ ~~562~~ ~~563~~ ~~564~~ ~~565~~ ~~566~~ ~~567~~ ~~568~~ ~~569~~ ~~570~~ ~~571~~ ~~572~~ ~~573~~ ~~574~~ ~~575~~ ~~576~~ ~~577~~ ~~578~~ ~~579~~ ~~580~~ ~~581~~ ~~582~~ ~~583~~ ~~584~~ ~~585~~ ~~586~~ ~~587~~ ~~588~~ ~~589~~ ~~590~~ ~~591~~ ~~592~~ ~~593~~ ~~594~~ ~~595~~ ~~596~~ ~~597~~ ~~598~~ ~~599~~ ~~600~~ ~~601~~ ~~602~~ ~~603~~ ~~604~~ ~~605~~ ~~606~~ ~~607~~ ~~608~~ ~~609~~ ~~610~~ ~~611~~ ~~612~~ ~~613~~ ~~614~~ ~~615~~ ~~616~~ ~~617~~ ~~618~~ ~~619~~ ~~620~~ ~~621~~ ~~622~~ ~~623~~ ~~624~~ ~~625~~ ~~626~~ ~~627~~ ~~628~~ ~~629~~ ~~630~~ ~~631~~ ~~632~~ ~~633~~ ~~634~~ ~~635~~ ~~636~~ ~~637~~ ~~638~~ ~~639~~ ~~640~~ ~~641~~ ~~642~~ ~~643~~ ~~644~~ ~~645~~ ~~646~~ ~~647~~ ~~648~~ ~~649~~ ~~650~~ ~~651~~ ~~652~~ ~~653~~ ~~654~~ ~~655~~ ~~656~~ ~~657~~ ~~658~~ ~~659~~ ~~660~~ ~~661~~ ~~662~~ ~~663~~ ~~664~~ ~~665~~ ~~666~~ ~~667~~ ~~668~~ ~~669~~ ~~670~~ ~~671~~ ~~672~~ ~~673~~ ~~674~~ ~~675~~ ~~676~~ ~~677~~ ~~678~~ ~~679~~ ~~680~~ ~~681~~ ~~682~~ ~~683~~ ~~684~~ ~~685~~ ~~686~~ ~~687~~ ~~688~~ ~~689~~ ~~690~~ ~~691~~ ~~692~~ ~~693~~ ~~694~~ ~~695~~ ~~696~~ ~~697~~ ~~698~~ ~~699~~ ~~700~~ ~~701~~ ~~702~~ ~~703~~ ~~704~~ ~~705~~ ~~706~~ ~~707~~ ~~708~~ ~~709~~ ~~710~~ ~~711~~ ~~712~~ ~~713~~ ~~714~~ ~~715~~ ~~716~~ ~~717~~ ~~718~~ ~~719~~ ~~720~~ ~~721~~ ~~722~~ ~~723~~ ~~724~~ ~~725~~ ~~726~~ ~~727~~ ~~728~~ ~~729~~ ~~730~~ ~~731~~ ~~732~~ ~~733~~ ~~734~~ ~~735~~ ~~736~~ ~~737~~ ~~738~~ ~~739~~ ~~740~~ ~~741~~ ~~742~~ ~~743~~ ~~744~~ ~~745~~ ~~746~~ ~~747~~ ~~748~~ ~~749~~ ~~750~~ ~~751~~ ~~752~~ ~~753~~ ~~754~~ ~~755~~ ~~756~~ ~~757~~ ~~758~~ ~~759~~ ~~760~~ ~~761~~ ~~762~~ ~~763~~ ~~764~~ ~~765~~ ~~766~~ ~~767~~ ~~768~~ ~~769~~ ~~770~~ ~~771~~ ~~772~~ ~~773~~ ~~774~~ ~~775~~ ~~776~~ ~~777~~ ~~778~~ ~~779~~ ~~780~~ ~~781~~ ~~782~~ ~~783~~ ~~784~~ ~~785~~ ~~786~~ ~~787~~ ~~788~~ ~~789~~ ~~790~~ ~~791~~ ~~792~~ ~~793~~ ~~794~~ ~~795~~ ~~796~~ ~~797~~ ~~798~~ ~~799~~ ~~800~~ ~~801~~ ~~802~~ ~~803~~ ~~804~~ ~~805~~ ~~806~~ ~~807~~ ~~808~~ ~~809~~ ~~8010~~ ~~8011~~ ~~8012~~ ~~8013~~ ~~8014~~ ~~8015~~ ~~8016~~ ~~8017~~ ~~8018~~ ~~8019~~ ~~8020~~ ~~8021~~ ~~8022~~ ~~8023~~ ~~8024~~ ~~8025~~ ~~8026~~ ~~8027~~ ~~8028~~ ~~8029~~ ~~8030~~ ~~8031~~ ~~8032~~ ~~8033~~ ~~8034~~ ~~8035~~ ~~8036~~ ~~8037~~ ~~8038~~ ~~8039~~ ~~8040~~ ~~8041~~ ~~8042~~ ~~8043~~ ~~8044~~ ~~8045~~ ~~8046~~ ~~8047~~ ~~8048~~ ~~8049~~ ~~8050~~ ~~8051~~ ~~8052~~ ~~8053~~ ~~8054~~ ~~8055~~ ~~8056~~ ~~8057~~ ~~8058~~ ~~8059~~ ~~8060~~ ~~8061~~ ~~8062~~ ~~8063~~ ~~8064~~ ~~8065~~ ~~8066~~ ~~8067~~ ~~8068~~ ~~8069~~ ~~8070~~ ~~8071~~ ~~8072~~ ~~8073~~ ~~8074~~ ~~8075~~ ~~8076~~ ~~8077~~ ~~8078~~ ~~8079~~ ~~8080~~ ~~8081~~ ~~8082~~ ~~8083~~ ~~8084~~ ~~8085~~ ~~8086~~ ~~8087~~ ~~8088~~ ~~8089~~ ~~8090~~ ~~8091~~ ~~8092~~ ~~8093~~ ~~8094~~ ~~8095~~ ~~8096~~ ~~8097~~ ~~8098~~ ~~8099~~ ~~80100~~ ~~80101~~ ~~80102~~ ~~80103~~ ~~80104~~ ~~80105~~ ~~80106~~ ~~80107~~ ~~80108~~ ~~80109~~ ~~80110~~ ~~80111~~ ~~80112~~ ~~80113~~ ~~80114~~ ~~80115~~ ~~80116~~ ~~80117~~ ~~80118~~ ~~80119~~ ~~80120~~ ~~80121~~ ~~80122~~ ~~80123~~ ~~80124~~ ~~80125~~ ~~80126~~ ~~80127~~ ~~80128~~ ~~80129~~ ~~80130~~ ~~80131~~ ~~80132~~ ~~80133~~ ~~80134~~ ~~80135~~ ~~80136~~ ~~80137~~ ~~80138~~ ~~80139~~ ~~80140~~ ~~80141~~ ~~80142~~ ~~80143~~ ~~80144~~ ~~80145~~ ~~80146~~ ~~80147~~ ~~80148~~ ~~80149~~ ~~80150~~ ~~80151~~ ~~80152~~ ~~80153~~ ~~80154~~ ~~80155~~ ~~80156~~ ~~80157~~ ~~80158~~ ~~80159~~ ~~80160~~ ~~80161~~ ~~80162~~ ~~80163~~ ~~80164~~ ~~80165~~ ~~80166~~ ~~80167~~ ~~80168~~ ~~80169~~ ~~80170~~ ~~80171~~ ~~80172~~ ~~80173~~ ~~80174~~ ~~80175~~ ~~80176~~ ~~80177~~ ~~80178~~ ~~80179~~ ~~80180~~ ~~80181~~ ~~80182~~ ~~80183~~ ~~80184~~ ~~80185~~ ~~80186~~ ~~80187~~ ~~80188~~ ~~80189~~ ~~80190~~ ~~80191~~ ~~80192~~ ~~80193~~ ~~80194~~ ~~80195~~ ~~80196~~ ~~80197~~ ~~80198~~ ~~80199~~ ~~80200~~ ~~80201~~ ~~80202~~ ~~80203~~ ~~80204~~ ~~80205~~ ~~80206~~ ~~80207~~ ~~80208~~ ~~80209~~ ~~80210~~ ~~80211~~ ~~80212~~ ~~80213~~ ~~80214~~ ~~80215~~ ~~80216~~ ~~80217~~ ~~80218~~ ~~80219~~ ~~80220~~ ~~80221~~ ~~80222~~ ~~80223~~ ~~80224~~ ~~80225~~ ~~80226~~ ~~80227~~ ~~80228~~ ~~80229~~ ~~80230~~ ~~80231~~ ~~80232~~ ~~80233~~ ~~80234~~ ~~80235~~ ~~80236~~ ~~80237~~ ~~80238~~ ~~80239~~ ~~80240~~ ~~80241~~ ~~80242~~ ~~80243~~ ~~80244~~ ~~80245~~ ~~80246~~ ~~80247~~ ~~80248~~ ~~80249~~ ~~80250~~ ~~80251~~ ~~80252~~ ~~80253~~ ~~80254~~ ~~80255~~ ~~80256~~ ~~80257~~ ~~80258~~ ~~80259~~ ~~80260~~ ~~80261~~ ~~80262~~ ~~80263~~ ~~80264~~ ~~80265~~ ~~80266~~ ~~80267~~ ~~80268~~ ~~80269~~ ~~80270~~ ~~80271~~ ~~80272~~ ~~80273~~ ~~80274~~ ~~80275~~ ~~80276~~ ~~80277~~ ~~80278~~ ~~80279~~ ~~80280~~ ~~80281~~ ~~80282~~ ~~80283~~ ~~80284~~ ~~80285~~ ~~80286~~ ~~80287~~ ~~80288~~ ~~80289~~ ~~80290~~ ~~80291~~ ~~80292~~ ~~80293~~ ~~80294~~ ~~80295~~ ~~80296~~ ~~80297~~ ~~80298~~ ~~80299~~ ~~80300~~ ~~80301~~ ~~80302~~ ~~80303~~ ~~80304~~ ~~80305~~ ~~80306~~ ~~80307~~ ~~80308~~ ~~80309~~ ~~80310~~ ~~80311~~ ~~80312~~ ~~80313~~ ~~80314~~ ~~80315~~ ~~80316~~ ~~80317~~ ~~80318~~ ~~80319~~ ~~80320~~ ~~80321~~ ~~80322~~ ~~80323~~ ~~80324~~ ~~80325~~ ~~80326~~ ~~80327~~ ~~80328~~ ~~80329~~ ~~80330~~ ~~80331~~ ~~80332~~ ~~80333~~ ~~80334~~ ~~80335~~ ~~80336~~ ~~80337~~ ~~80338~~ ~~80339~~ ~~80340~~ ~~80341~~ ~~80342~~ ~~80343~~ ~~80344~~ ~~80345~~ ~~80346~~ ~~80347~~ ~~80348~~ ~~80349~~ ~~80350~~ ~~80351~~ ~~80352~~ ~~80353~~ ~~80354~~ ~~80355~~ ~~80356~~ ~~80357~~ ~~80358~~ ~~80359~~ ~~80360~~ ~~80361~~ ~~80362~~ ~~80363~~ ~~80364~~ ~~80365~~ ~~80366~~ ~~80367~~ ~~80368~~ ~~80369~~ ~~80370~~ ~~80371~~ ~~80372~~ ~~80373~~ ~~80374~~ ~~80375~~ ~~80376~~ ~~80377~~ ~~80378~~ ~~80379~~ ~~80380~~ ~~80381~~ ~~80382~~ ~~80383~~ ~~80384~~ ~~80385~~ ~~80386~~ ~~80387~~ ~~80388~~ ~~80389~~ ~~80390~~ ~~80391~~ ~~80392~~ ~~80393~~ ~~80394~~ ~~80395~~ ~~80396~~ ~~80397~~ ~~80398~~ ~~80399~~ ~~80400~~ ~~80401~~ ~~80402~~ ~~80403~~ ~~80404~~ ~~80405~~ ~~80406~~ ~~80407~~ ~~80408~~ ~~80409~~ ~~80410~~ ~~80411~~ ~~80412~~ ~~80413~~ ~~80414~~ ~~80415~~ ~~80416~~ ~~80417~~ ~~80418~~ ~~80419~~ ~~80420~~ ~~80421~~ ~~80422~~ ~~80423~~ ~~80424~~ ~~80425~~ ~~80426~~ ~~80427~~ ~~80428~~ ~~80429~~ ~~80430~~ ~~80431~~ ~~80432~~ ~~80433~~ ~~80434~~ ~~80435~~ ~~80436~~ ~~80437~~ ~~80438~~ ~~80439~~ ~~80440~~ ~~80441~~ ~~80442~~ ~~80443~~ ~~80444~~ ~~80445~~ ~~80446~~ ~~80447~~ ~~80448~~ ~~80449~~ ~~80450~~ ~~80451~~ ~~80452~~ ~~80453~~ ~~80454~~ ~~80455~~ ~~80456~~ ~~80457~~ ~~80458~~ ~~80459~~ ~~80460~~ ~~80461~~ ~~80462~~ ~~80463~~ ~~80464~~ ~~80465~~ ~~80466~~ ~~80467~~ ~~80468~~ ~~80469~~ ~~80470~~ ~~80471~~ ~~80472~~ ~~80473~~ ~~80474~~ ~~80475~~ ~~80476~~ ~~80477~~ ~~80478~~ ~~80479~~ ~~80480~~ ~~80481~~ ~~80482~~ ~~80483~~ ~~80484~~ ~~80485~~ ~~80486~~ ~~80487~~ ~~80488~~ ~~80~~

soft off road 25 f37 104.5-105.5⁷ no work no amni

3600 8101 01-0476 8.5
45 81 XMAS soft notes

4 4.5 5 6 6.5

5 5.5 7 8 9

6.5 7 8.5 9.5 11

• $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

— : 2 2.5 3 4 4.5 5 5.5 6 6.5

2 2 2 2 2 2 2 2 2

$\{ \text{ } : \text{ } x \text{ } = \text{ } 2 \text{ } \} \text{ } \cap \text{ } \{ \text{ } : \text{ } t \text{ } = \text{ } 4 \text{ } \}$

$$M \bar{x} = \frac{\sum \bar{x}_f}{\bar{z}_f} = 6$$

standard deviation = $\sigma_{\bar{x}} = 2.32$

$$\left\{ \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \Rightarrow 5.38 = 5.38 \right\}$$

iii) without replacement samples are

$$N_{cn} = 5C_2 = 10 \text{ samples}$$

(2, 3) (2, 6) (2, 8) (2, 11) (3, 6) (3, 8) (3, 11) (6, 8) (6, 11) (8, 11)

Now individual means are

$$2.5 \quad 4 \quad 5 \quad 6.5 \quad 4.5 \quad 5.5 \quad 7 \quad 7 \quad 8.5 \quad 9.5$$

$$H_{\bar{x}} = H = 6$$

$$\sigma_{\bar{x}}^2 = \left(\frac{N-n}{N-1} \right) \frac{\sigma^2}{n} = \left(\frac{5-2}{5-1} \right) \left(\frac{10.76}{2} \right) = 4.035$$

Q) Certain tubes manufactured by a company have mean life time of 800 hrs and standard deviation 60 hrs. Find the probability that a sample of 16 tubes taken from the group will have a mean lifetime

(i) between 790 hrs to 810 hrs

(ii) less than 785 hrs

(iii) more than 820 hrs

Given : For population $H_{\bar{x}} = 800$, $\sigma_{\bar{x}} = 60$, $n = 16$

converting population data to sample

$$Z = \frac{\bar{x} - H_{\bar{x}}}{\sigma_{\bar{x}}} \quad \left\{ \begin{array}{l} H_{\bar{x}} = H \\ \sigma_{\bar{x}}^2 = \sigma^2/n \end{array} \right.$$

$$= \frac{\bar{x} - H}{(\frac{\sigma}{\sqrt{n}})}$$

$$Z = \frac{\bar{x} - 800}{60/4} = \frac{\bar{x} - 800}{15}$$

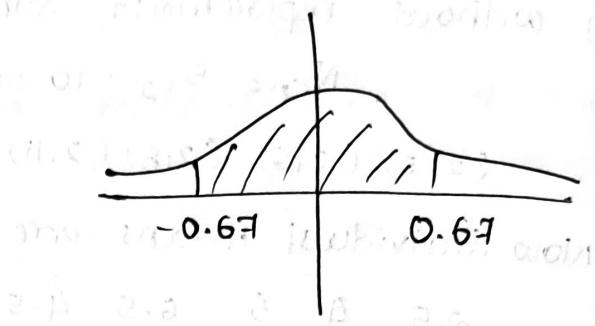
$$(i) P(-90 < \bar{x} \leq 810)$$

$$= P\left(\frac{-90 - 800}{15} < \frac{\bar{x} - 800}{15} \leq \frac{810 - 800}{15}\right)$$

$$= P(-0.67 < z < 0.67)$$

$$= 2A(0.67)$$

$$= 0.497$$



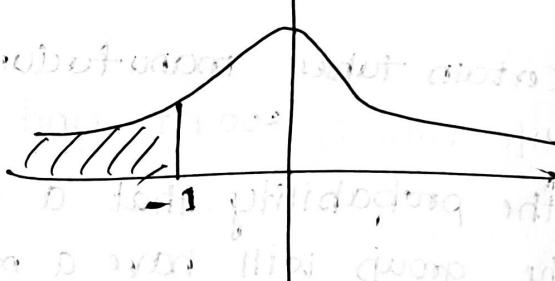
$$(ii) P(\bar{x} < 785)$$

$$= P\left(\frac{\bar{x} - 800}{15} < \frac{785 - 800}{15}\right) = \frac{1}{2} \left(1 - A\left(\frac{15}{15}\right)\right) = \frac{1}{2}$$

$$= P(z < -1)$$

$$= 1/2 - A(-1)$$

$$= 0.158$$



$$(iii) P(\bar{x} > 820)$$

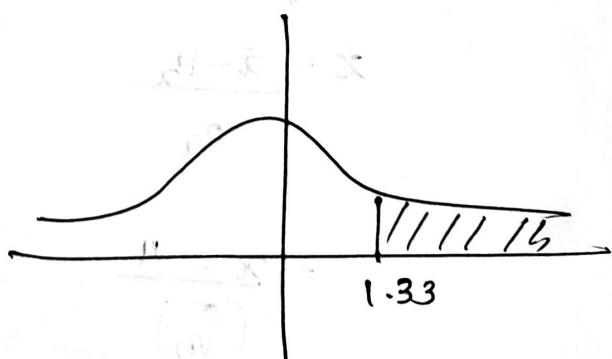
$$= P\left(\frac{\bar{x} - 800}{15} > \frac{820 - 800}{15}\right)$$

signals of other meteorological parameters

$$= P(z > 1.33)$$

$$= 1/2 - A(1.33)$$

$$= 0.091$$



Q) If the mean population is 575 with standard deviation 8.3. How large a sample must be in order that there will be one chance in 100 that the mean sample is less than 572.

population: $N = 575$ $\sigma = 8.3$ $n = ?$

$\bar{x} < 572$ which requires a large sample size.

for sample

$$z = \frac{\bar{x} - H}{\sigma_{\bar{x}}} \quad \left. \begin{array}{l} H_{\bar{x}} = H \\ \sigma_{\bar{x}} = \sigma / \sqrt{n} \end{array} \right\}$$

$$z = \frac{\bar{x} - H}{\sigma / \sqrt{n}} = \frac{\bar{x} - 575}{8.3 / \sqrt{n}}$$

For $\bar{x} = 572$

$$z = \frac{572 - 575}{8.3 / \sqrt{n}} = -0.36 / \sqrt{n}$$

Given, $P(\bar{x} < 572) = 0.01$

$$P(z < -0.36 \sqrt{n}) = 0.01$$

From int. $1/2 - A(0.36 \sqrt{n}) = 0.01$

$0.49 = A(0.36 \sqrt{n})$

$A(0.35) = A(0.36 \sqrt{n})$

by data $A(0.35) = 0.49$

$$0.35 = 0.36 \sqrt{n}$$

$$n = 40.6$$

$$\boxed{n = 43}$$

* Test of significance of small samples

26/07/23

Student t-distribution (t-test)

1) t-test of significance of mean of a random sample

Under the null hypothesis that there is no significance difference between the sample mean \bar{x} and population mean H .

The test statistic is given by

$$t = \frac{\bar{x} - H}{S/\sqrt{n}}$$

$$\text{where } S^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

With degree of freedom $n-1$ (D.F.)

Here the formula is used when the population standard deviation is not known

The confidence limit of population mean is given by

$$\bar{x} - t_{\alpha/2} (S/\sqrt{n}) < H < \bar{x} + t_{\alpha/2} (S/\sqrt{n})$$

- 1) A random sample of size 16 has 53 as mean. The sum of the squares of the standard deviation from mean is 135. Can this sample be regarded as taken from the population having 56 as the mean. Also obtain 95% and 99% confidence limit of the mean of the population.

$$n = 16$$

$$\bar{x} = 53$$

$$\sum (x - \bar{x})^2 = 135$$

$$H = 56$$

Null hypothesis $H_0: H = 56$

alternative hypothesis $H_1: H \neq 56$ / two tailed test)

test statistics

$$t \text{ or } z = \frac{\bar{x} - H}{(s/\sqrt{n})}$$

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1} = \frac{185}{15} = 12.33$$

$$s = 3$$

$$t \text{ or } z = \frac{(53 - 56)}{(3/\sqrt{15})}$$

$$z = -4$$

$|z| = 4 > 2.13$ at 5% level of significance

and $n=15$ as degree of freedom

(check under 0.05 at 15 D.F. freedom
in given table)

$|z| = 4 > 2.13$ so H_0 is rejected. (at 5% level of significance).

~~$|z| = 4 > 2.95$ at 1% level of significance and
15 as degree of freedom.~~

~~so H_0 is rejected at 1% significant level~~

~~but $2.95 < 4 < 4.5$ at 0.95% level of significance~~

$$\bar{x} - t_{\alpha/2} (s/\sqrt{n}) < H < \bar{x} + t_{\alpha/2} (s/\sqrt{n})$$

$$\text{at } 95\% : \bar{x} \pm t_{0.05} (s/\sqrt{n})$$

$$53 \pm 2.13 (3/\sqrt{15})$$

$$53 \pm 1.5975$$

$$\Rightarrow [54.59 \quad 51.4023]$$

$$\text{at } 99\%: \bar{x} \pm t_{0.01}(s/\sqrt{n})$$

$$\Rightarrow 53 \pm 2.95(3/4)$$

$$\Rightarrow 53 \pm 2.2125$$

$$[55.2125 \quad 50.7875]$$

2) 9 items of the sample have following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these values differ significantly from assumed mean of 47.5.

$$n=9 \quad H=47.5 \quad d.f = 9-1=8 \quad \bar{x}=49.11 \text{ (calculator)}$$

$$H_0: H=47.5$$

$$H_1: H \neq 47.5 \text{ (two-tailed test)}$$

test-statistics

$$t \text{ or } z = \frac{\bar{x}-H}{(s/\sqrt{n})}$$

$$\text{bmas: } s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

$$\begin{array}{cccccccccc} x & 45 & 47 & 50 & 52 & 48 & 47 & 49 & 53 & 51 \\ (x-\bar{x}) & -4.1 & -2.1 & 0.9 & 2.9 & -1.1 & -2.1 & -0.1 & 3.9 & 1.9 \end{array}$$

$$(x-\bar{x})^2 \quad 16.81 \quad 4.41 \quad 0.81 \quad 8.41 \quad 1.21 \quad 4.41 \quad 0.01 \quad 15.21 \quad 3.61$$

$$\sum (x-\bar{x})^2 = 54.89$$

$$s^2 = \frac{54.89}{8} = 6.86$$

$$s = 2.619$$

assumed mean is nothing but population mean

$$t \text{ or } z = \frac{\bar{x} - H}{S/\sqrt{n}} = \frac{49.1 - 47.5}{(2.619/\sqrt{8})}$$

$z = 1.838 < 2.31$ at 5% level of significance
at d.f = 8 as degree of freedom.

$\therefore H_0$ is accepted.

- 3) A sample of 20 items has mean 42 units and standard deviation 5 units. Test the hypothesis that it is random sample from the normal population with mean 45 units.

$$n = 20$$

$$S = 5$$

$$H_0 : H = 45$$

$$\bar{x} = 42$$

$$D.O.F = 19$$

$$H_1 : H \neq 45 \text{ (two tailed test)}$$

$$H = 45$$

t-test

$$t = \frac{\bar{x} - H}{S/\sqrt{n-1}} \rightarrow \text{(Here } T \text{ instead of } S \text{ we take 's' because S.D is mentioned)}$$

$$= \frac{42 - 45}{5/\sqrt{20-1}}$$

$$t = -2.68$$

$|t| = 2.68 > 2.09$ at 5% level of significance.

as 19 as degree of freedom

H_0 is rejected.

* t-test for diff of means of two samples (Small)

This test is used to check whether the two samples x_1, x_2, \dots, x_n , and y_1, y_2, \dots, y_{n_2} which are of the sizes n_1 and n_2 respectively, and are drawn from two normal populations with mean H_1 and H_2 under the assumption that population variance are equal (i.e $\sigma_1^2 = \sigma_2^2 = \sigma^2$).

Under the null hypothesis that the samples are drawn from the normal population with mean H_1 and H_2

$$t = \frac{\bar{x} - \bar{y}}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where \bar{x} and \bar{y} are means of the samples and degree of freedom is $n_1 + n_2 - 2$.

Note: 1) If the two samples standard deviation s_1 and s_2 are given $S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

2) If s_1 and s_2 are not given

$$S^2 = \frac{\sum (x - \bar{x})^2 + \sum (y - \bar{y})^2}{n_1 + n_2 - 2}$$

- 1) Two samples of sodium vapour bulbs were tested for length of life and the following results were obtained.

Type	Sample size	Sample mean	Sample S.D
Type 1	8	1234 hrs	36 hrs

Type 2	7	1036 hrs	40 hrs
--------	---	----------	--------

is the difference in the mean significant to generalise that Type 1 is superior to Type 2, regarding length of life.

$$n_1 = 8 \quad n_2 = 7$$

$$\bar{x} = 1234 \quad \bar{y} = 1036$$

$$S_1 = 36 \quad S_2 = 40$$

$$H_0: H_1 = H_2$$

$$H_1: H_1 > H_2 \text{ (One tailed test)}$$

$$t = \frac{\bar{x} - \bar{y}}{S \sqrt{1/n_1 + 1/n_2}}$$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$S = 40.73$$

$$t = 9.392$$

$$t = 2 \times 9.39$$

$t = 18.79$ (here we are considering single tailed test and formula is for two tailed test so we have to double it)

$t = 18.79 > 1.77$ at $2\alpha = 2 \times 0.05 = 0.1$ significance level at df 13.

$\therefore H_0$ is rejected.

- 2) Two horses A and B were tested according to time to run a particular race with the following results

Horse A	28	30	32	33	33	29	34	32
Horse B	29	30	30	24	27	29	31	28

Test whether you can discriminate b/w two horses.

$$n_1 = 7 \quad \bar{x} = 31.28 \quad \text{to } 0.68 \text{ p.p. = } 3$$

$$n_2 = 6 \quad \bar{y} = 28.16 \quad \text{to } 0.68 \text{ p.p. = } 3$$

$$H_0: H_1 = H_2$$

$$H_1: H_1 \neq H_2 \text{ (two tailed test)}$$

$$t = \frac{\bar{x} - \bar{y}}{S \sqrt{1/n_1 + 1/n_2}}$$

$$S^2 = \frac{\sum (x - \bar{x})^2 + \sum (y - \bar{y})^2}{n_1 + n_2 - 2}$$

\bar{x}	28	30	32	33	33	29	34
$(\bar{x}-\bar{x})$	-3.28	-1.28	0.72	1.42	1.72	-2.28	2.72
$(\bar{x}-\bar{x})^2$	10.75	1.63	0.51	2.95	2.95	5.19	7.39

$$\sum (x-\bar{x})^2 = 31.37$$

\bar{y}	29	30	30	24	27	29
$(\bar{y}-\bar{y})$	0.84	1.84	1.84	-4.16	-1.16	0.84
$(\bar{y}-\bar{y})^2$	0.70	3.38	3.38	17.3	1.34	0.70

$$\sum (\bar{y}-\bar{y})^2 = 26.8$$

$$S^2 = \frac{\sum (x-\bar{x})^2 + \sum (\bar{y}-\bar{y})^2}{n_1+n_2-2}$$

$$= \frac{31.37 + 26.8}{11} = 5.8$$

$$t = \frac{\bar{x} - \bar{y}}{S} = \frac{31.28 - 28.16}{2.29} = 1.44$$

degrees of freedom = 11

t = 2.44 > 2.20 at 5% significance level

of degree of freedom 11

$\therefore H_0$ is rejected.