Unit - 2

Random Vector

Date 24/05/23.

Let S be a sample space of a random experiment. Suppose to each element of S a unique real number X is associated according some rule then X is called random variable on Sample space S

* Descrete Random Variable: A random variable which can take some specified values only. eg:- USN of Students, Tossing a coin.

* Continuous Random Variable: A random variable which can take any value in specified range. eg:- temperature, Speed, time etc.

* Descrete Probability Distribution :-

If for each value x_i of discrete random variable a real number $p(x_i)$ is assigned Such that, i) $P(x_i) \geq 0$
ii) $\Sigma p(x_i) = 1$ then the function $p(x)$ is called probability Density function (PDF)

The Distribution function $f(x)$ defined by $f(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$ is called Cummulative Distribution function

* Note: The mean & the variance of discrete probability Distribution is given by

$$Mean(\mu) = \Sigma x_i p(x_i)$$

$$Variance(V) = \Sigma (x_i - \mu)^2 p(x_i)$$
$$= \Sigma x_i^2 p(x_i) - \mu^2.$$

The set of values (x_i and $p(x_i)$) is called probability Distribution

* Problems:-

1) Find the value of k, mean & variance.

x_i:	-2	-1	0	1	2	3
$p(x)$:	0.1	k	0.2	2k	0.3	k

→ $p(x_i) \geq 0$

$\Sigma p(x_i) = 1$

$0.6 + 4k = 1 \Rightarrow 4k = 1 - 0.6 = 0.4$

$\Rightarrow k = \frac{0.4}{4} = \boxed{0.1}$

ii) Mean $(\mu) = \Sigma x_i p(x_i)$

$= -2(0.1) + (-1)(0.1) + 0(0.2) + 1\times(0.2) + 2(0.3) + 3(0.1)$

$\boxed{\mu = 0.8}$

iii) Variance $(V) = \Sigma (x_i - \mu)^2 p(x_i)$

$= (-2-.8)^2 \times .1 + (-1-0.8)^2 \times 0.1 + (0-0.8)^2 \times 0.2 + (1-0.8)^2 \times 0.2 + (2-0.8)^2 \times 0.3 + (3-0.8)^2 \times 0.1$

$= 0.784 + 0.324 + (0.128) + 0.008 + 0.432 + 0.484$

$= 2.16$

2) Given x values are below. find k ii) $P(0 < x < 5)$

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

→ $P(x_i) \geq 0$

$\Sigma P(x_i) = 1$

$10k^2 + 9k = 1$

$\Rightarrow k = 0.1$ or $k = -1$

i) $k = 0.1$ [$\because$ the $p(x) \geq 0$].

ii) $P(0 < x < 5)$

$= P(1) + P(2) + P(3) + P(4)$

$= k + 2k + 3k + 4k = 8k = 8 \times 0.1 = 0.8$

3) Find mean & standard deviation of

a)

x:	-3	6	9
p(x):	1/6	1/2	1/3

Mean $(\mu) = \Sigma x_i p(x_i) = -3(1/6) + 6(1/2) + 9(1/3)$

$= 5.5$

Std deviation $= \sqrt{\text{Variance}}$

$= \sqrt{\Sigma x_i^2 p(x_i) - \mu^2}$

$= \sqrt{16.25} = 4.0311$

$= 3.24$.

b)

25/05

4) Obtain Distribution function for total number of heads occuring in 3 tosses of an unbiased coin.

→ Sample space = S = {HHH HHT HTH HTT THH THT TTH TTT}

total number of heads

$\therefore x = \{0, 1, 2, 3\}$

x_i the entries are: 0, 1, 2, 3

$p(x_i)$: 1/8 3/8 3/8 1/8.

5) Three cards are drawn Simultaneously from well shuffled deck of 52 cards. Compute the variance for the number of ~~cards~~, aces.

→ Condition: Number of Aces

$52 \to 48,\ 4\text{(Aces)}$

$X = \{0, 1, 2, 3\}$

x_i	0	1	2	3
$p(x_i)$	0.7826	0.2041	0.013	1.809×10^{-4}

$0 \to \dfrac{{}^{48}C_3 \times {}^{4}C_0}{{}^{52}C_3}$

$1 \to \dfrac{{}^{48}C_2 \times {}^{4}C_1}{{}^{52}C_3}$

$2 \to \dfrac{{}^{48}C_1 \times {}^{4}C_2}{{}^{52}C_3}$

$3 \to \dfrac{{}^{48}C_0 \times {}^{4}C_3}{{}^{52}C_3}$

Mean $= \Sigma\, x_i\, p(x_i)$

$= 0 \times 0.7826 + 1 \times 0.2041 + 2 \times 0.013 + 3 \times 1.809 \times 10^{-4}$

$\mu = 0.2306$

Variance $= \Sigma (x_i - \mu)^2 \times p(x_i)$

$= (-.23)^2 \times .7826 + (1 - .23)^2 \times .2041) + (2 - .23)^2 \times .013 + (3 - .23)^2 \times 1.809 \times 10^{-4}$

$= 0.2047$

6) A dice is tossed twice getting a number greater than 4 is considered a success. Find the variance of probability distribution for no. of successes.

→ Condition: More than 4 is Success.

$X = \{0, 1, 2\}$

x_i	0	1	2
$p(x_i)$	$4/9$	$4/9$	$1/9$
	$(2/3 \times 2/3)$	$(2/3 \times 2/3)$	$(1/3 \times 1/3)$

$p(\text{no success}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$p(\text{both success}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$p(\text{Success}) = \frac{2}{6} = \frac{1}{3}$, $p(\text{failure}) = 1 - 2/6 = \frac{4}{6} = \frac{2}{3}$

$p(\text{one Success}) = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{2}{9} + \frac{2}{9} = 4/9$

Mean $(\mu) = \Sigma\, p(x_i)\, x_i$

$= 0 \times 4/9 + 1 \times 4/9 + 2 \times 1/9$

$= 0.666$

Variance $= \Sigma (x_i - \mu)^2 p(x_i)$

$= ~~2.137~~\ 0.44$

* Continuous Probability Distribution

If X is a continuous random variable a real number $f(x)$ satisfying the conditions i) $f(x) \geq 0$ ii) $\int_{-\infty}^{+\infty} f(x)\,dx = 1$ then $f(x)$ is called Probability Density function (PDF)

* A function $F(x)$ defined by $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$. This $f(x)$ is called Cummulative distributⁿ fuⁿ.

$$\frac{d}{dx}[F(x)] = f(x).$$

Mean $(\mu) = \int_{-\infty}^{+\infty} x f(x)\,dx$.

Variance(V) $= \int_{-\infty}^{+\infty} (x - \mu)^2 f(x)\,dx$

$= \int_{-\infty}^{+\infty} x^2 f(x)\,dx - \mu^2$

* Problems:

1) If X is a continuous random variable with probability density function given by ~~f(x) is equal~~

$$f(x) = \begin{cases} kx, & 0 \leq x < 2 \\ 2k, & 2 \leq x < 4 \\ -kx + 6k, & 4 \leq x < 6 \end{cases}$$

Find the value of k, mean & variance.

→ Con^n i) $f(x) \geq 0$ ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$= \int_{-\infty}^{0} f(x)dx + \int_{0}^{2} f(x)dx + \int_{2}^{4} f(x)dx + \int_{4}^{6} f(x)dx + \int_{6}^{\infty} f(x)dx = 1$

$= 0 + \int_{0}^{2} kx\,dx + \int_{2}^{4} 2k\,dx + \int_{4}^{6} (-kx+6k)\,dx + 0 = 1$

$k\int_{0}^{2} x\,dx + 2k\int_{2}^{4} 1\,dx + k\int_{4}^{6} (-x+6)\,dx = 1$

$2k + 2k(2) + k(2) = 1$

$8k = 1$

$k = 1/8$.

Mean $(\mu) = \int_{-\infty}^{\infty} x f(x)\,dx$.

$= \int_{-\infty}^{0} x f(x)dx + \int_{0}^{2} x f(x)dx + \int_{2}^{4} x f(x)dx + \int_{4}^{6} x f(x)dx + \int_{6}^{\infty} x f(x)dx$

$= 0 + \int_{0}^{2} x(kx)dx + \int_{2}^{4} x(2k)dx + \int_{4}^{6} x(-kx+6k)dx + 0$

$= k\int_{0}^{2} x^2 dx + k\int_{2}^{4} 2x\,dx + k\int_{4}^{6} -x^2+6x\,dx$.

$= k\left(\frac{8}{3}\right) + k(12) + k\left(\frac{28}{3}\right)$

$= k(24) = \frac{1}{8}(24)$

$= \underline{\underline{3}}$

Variance$(V) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,dx$.

$= \int_{-\infty}^{0} (x-3)^2 f(x)\,dx + \int_{0}^{2} (x-3)^2 kx\,dx + \int_{2}^{4} (x-3)^2 2k\,dx$

$+ \int_{4}^{6} (x-3)^2 (-kx+6k)\,dx + \int_{6}^{\infty} (x-3)^2 f(x)\,dx$.

$= 0 + k\int_{0}^{2} x(x-3)^2 dx + 2k\int_{2}^{4} (x-3)^2 dx + k\int_{4}^{6} (x-3)^2(-x+6)\,dx$

$= 0 + 6k + 2k\left(\frac{1}{3}\right)^2 + 6k$

$= 12k + \frac{4}{3}k = 12\times\frac{1}{8} + \frac{4}{3}\times\frac{1}{8} = \underline{\underline{\frac{5}{3}}}$

2) Find mean & Std deviation of f

$$f(x) = \begin{cases} 0 & x<2 \\ 1/18\,(2x+3) & 2 \leq x \leq 4. \\ 0 & x>4. \end{cases}$$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow$ mean $(\mu) = \int_{-\infty}^{\infty} x f(x)\,dx$.

$= \int_{-\infty}^{2} x f(x)dx + \int_{2}^{4} x f(x)dx + \int_{4}^{6} x f(x)dx + \int_{6}^{\infty} x f(x)dx$

$= 0 + \int_{2}^{4} x(0)\,dx + \int_{2}^{4} x\left(\frac{1}{18}(2x+3)\right) + \int_{4}^{6} x(0)\,dx$

$= 0 + \frac{83}{27}$

$\Rightarrow$ SD $= \sqrt{\text{variance}}$ 0.32

$= \sqrt{\int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2}$ 0.57

Variance: $\int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$

$= -\int_{0}^{2}\mu^2 + \int_{2}^{4} x^2 (1/18)(2x+3)\,dx - \mu^2 - \int \mu^2$

$= -\left(\frac{83}{27}\right)^2 \times 2 + \frac{88}{9} - \left(\frac{83}{27}\right)^2$

$= -\left(\frac{83}{27}\right)^2 \times 3 + \frac{88}{9}$

1st formula: $\int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,dx$.

$\int_{2}^{4} (x - 83/27)^2 \times \frac{1}{18}(2x+3)$

Variance $= 1/18 \times \frac{478}{81} = 0.3278$.

$SD = \sqrt{variance}$
$= \sqrt{0.3278}$
$= 0.5725$.

* Discrete Probability Distribution

1) Binomial Distribution
2) Poission Distribution

1) Binomial Distribution : The distributions wherein the triels are repeatative in nature in which only Occurence or non-occurence true or false, Yes or NO are of interest. Each trial ~~to~~ has two mutually exclusive Outcomes. The Probability of Success P & Probability of failure (Q) from trial to trial. Each trial is independent & the trials are performed under the Same Conditions.

If a Series Independent trials are performed such that for each trial P is the probability of Success, Q is the probability of failure then probability of x Success in series of P trials is given by:

$P(x) = {}^nC_x p^x q^{n-x}$

$x: 0 \quad 1 \quad 2 \quad 3 \quad \ldots \quad n$

$p(x): {}^nC_0 p^0 q^{n-0} \quad {}^nC_1 p^1 q^{n-1} \quad \ldots \quad {}^nC_n p^n q^0$

$q^n + {}^nC_1 p^1 q^{n-1} \ldots \ldots p^n$
$= (P+Q)^n = 1^n = 1$

Mean $(\mu) = np$
Variance $(V) = npq$

Problems :-

1) The number of Telephone lines busy at an instant of time is a binomial variest. ~~which~~ with probability 0.1 that the line is busy: If 10 lines are choosen at random what is the probability that
i) No line is busy, ii) All lines are busy
iii) At least 1 line is busy.
iv) At most 2 lines are busy.

$\Rightarrow$ x: No. of lines busy. $\{P+Q=1\}$
$P = 0.1, \quad q = 0.9 \quad n = 10$
$P(x) = {}^nC_x p^x q^{n-x}$
$\Rightarrow P(x) = 10C_x (0.1)^x (0.9)^{10-x}$

i) $P(x=0) = 10C_0 (0.1)^0 (0.9)^{10-0} = 0.3486$.

ii) $P(x=10) = 10C_{10} (0.1)^{10} (0.9)^0 = 10^{-10}$

iii) $P(x \geq 1) = P(1) + P(2) + P(3) + P(4) + P(5) + \ldots P(10)$
$= 1 - P(0) \qquad p(0) + p(1) \ldots + p(10) = 1$
$= 1 - 0.3486 \qquad p(1) \ldots p(10) = 1 - p(0)$
$= 0.6514$

iv) $P(x \leq 2) = p(0) + p(1) + p(2)$
$= 0.3486 + 0.3874 + 0.1937$
$= 0.9297$

* The probabity that a pen manufactured by a Company will be defective is 1/10. If 12 such pens are manufactured find the probabity that i) Exaitly 2 will be defective ii) Atleast two will be defective.

iii) None of them will be defective

i) $p(x=2) = {}^{12}C_2 (0.1)^2 (0.9)^{10} = \cancel{4.6025}$
$= 0.2301$

iii) $p(0) = 0.2824$.

iv) $= P(2) + P(3) + \dots P(12)$
$= 1 - (P(0) + P(1))$
$= 1 - (0.2824 + 0.3765)$
$= \underline{0.3411}$

3) If the chance that 1 telephone line out of 10 telephones lines is busy at an instant is 0.2. i) What is the chance that five of the lines are busy.
ii) What is the probability that all lines are busy.
iii) What is a most probable no. of busy lines & what is probability of this number. (hint: Mean = np).

$10C_x \times (p)^x \times (q)^{n-x}$.

iii) $n = 10 \quad p = 0.2$
Mean $= np = 10 \times 0.2 = 2$
or

$p(2) = p(0) + p(1) + p(2)$
$= 0.1073 + 0.2684 + 0.3019$
$p(2) = \underline{\underline{0.6776}} \times$

$p(2) = \underline{\underline{0.3019}}$ ✓.

i) $p(x=5) = 0.0264$

ii) $p(x=10) = 1.024 \times 10^{-7}$

* Fit the Binomial Distribution for the following

x:	0	1	2	3	4	5
f:	2	14	20	34	22	8

$p(x) = nC_x p^x q^{n-x}$ $\quad n = 5, \; p = ? \; q = ?$

Mean $(\mu) = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = 2.84$.

$\mu = 2.84$
$np = 2.84$
$p = \dfrac{2.84}{5} = 0.568$

$q = 1 - p = 1 - 0.568 = 0.432$

$N = \Sigma f_i$

$f(x) = N\,p(x)$
$= 100 \left[5C_x (0.568)^x (0.432)^{5-x}\right]$.

$f(0) = 1.5045 \quad f(2) = 26.01 \quad f(1) = 9.8912$
$f(3) = 34.198 \quad f(4) = 22.48 \quad f(5) = 5.912$

$f(0) = 2 \quad f(1) = 10 \quad f(2) = 26 \quad f(3) = 34$
$f(4) = 22 \quad f(5) = 6$

2) Fit in the Binomial Distribution

x:	0	1	2	3	4	5	6
f:	13	25	52	58	32	16	4

$p(x) = nC_x p^x q^{n-x}$ $\quad n = 6 \quad p = ? \quad q = ?$

Mean $(\mu) = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = 2.675$ $\quad 2.675$

$\mu = 2.675$
$np = 2.675 \Rightarrow p = 0.445$
$q = 1 - p = 0.555$

$f(0) = 8 \quad f(1) = 28 \quad f(2) = 56 \quad f(3) = 60$
$f(4) = 38 \quad f(5) = 12 \quad f(6) = 2$

* Poisson Distribution :-

It is a limiting case in which 'n' is very large and 'p' is very small. By making $n \times p$ fixed.

PDF of poisson is given by : $p(x) = \dfrac{e^{-m} m^x}{x!}$

Mean $(\mu) = m \quad \{m = np\}$.
Variance $(V) = m$

i) $p(x) \geq 0$
ii) $\Sigma p(x) = 1$

x :	0	1	2	3 .....
$p(x)$:	$\dfrac{e^{-m} m^0}{0!}$	$\dfrac{e^{-m} m^1}{1!}$	$\dfrac{e^{-m} m^2}{2!}$	

$$e^{-m} + \frac{e^{-m} m}{1!} + \frac{e^{-m} m^2}{2!} + \dots$$
$$e^{-m} \left\{ 1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots \right\}$$
$$e^{-m} \{ e^{m} \} = e^0 = 1$$

* Problems :

1) A Car hire farm has two Cars which it has day by day. the number of demands for a car on each day is poisson variant with mean 1.5. Calculate the probability of days i) On which there is no demand
ii) On which the demand is refused.

$m = 1.5 \quad x$: No. of demands.

→ i) $P(x=0) = \dfrac{e^{-1.5} \times (1.5)^0}{0!} = 0.223$.

ii) $P(x>2) = P(3) + P(4) + \dots$
$= \dfrac{e^{-1.5} \times (1.5)^3}{3!} + \dfrac{e^{-1.5} \times (1.5)^4}{4!} + \dots$
$= 1 - P(0) - P(1) - P(2)$
$= 1 - 0.22 - 0.33 - 0.25$
$= 0.2$

2) In a certain factory turning out razer blades. There is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate no. of packets containing i) No defective ii) One defective iii) 2 defective blades Consignment of 10,000 packets

→ $m = np, \quad n = 10 \quad p = 0.002 \quad P(x) = \dfrac{e^{-m} m^x}{x!}$
$= 10 \times (0.002)$
$\boxed{m = 0.02}$

x : No. of defective blades.
$p(x=0) = 0.98$
$p(x=1) = 0.0196$
$p(x=2) = 1.96 \times 10^{-4}$

$f(0) = 0.98 \times 10{,}000 \quad f(1) = 196$
$= 9800$

$f(2) = 2$

3) A certain screw making machine produces on an averages two defective screws out of 1000 & packs them in a box of 500. find the probability that box contain 15 defective screws

→ On an average $m=2$ for $n=100$
& $m=10$ for $n=500$ (packet)
x: No. of defective Screws.

$$p(15) = \frac{e^{-10} \times (10)^{15}}{15!} = 0.0347 = \underline{\underline{0.035}}$$

4) Fit a poisson distribution for the following data

x:	0	1	2	3	4
f:	48	38	22	9	1

We know $p(x) = \dfrac{e^{-m} m^x}{x!}$

$$\text{Mean}(\mu) = m = \frac{\sum f_i x_i}{\sum f_i} = 0.9741$$

$$f(x) = Np(x) = 118 \times \left(\frac{e^{-0.97} \times (0.97)^x}{x!} \right)$$

$f(0) = 43.9 = 44$ $\quad$ $f(3) = 6.68 \simeq 7$
$f(1) = 42.6 = 43$ $\quad$ $f(4) = 1.62 = 2$
$f(2) = 20.6 = 21$

* Continuous Probability Distributions :

1) Exponential Distribution
2) Normal :

1) Exponential Distribution : Let α be a constant greater than 0 then Continuous probability distribution taking PDF of the form $f(x) = \begin{cases} \alpha e^{-\alpha x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$
is exponential Distribution

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx.$$
$$= 0 + \int_0^\infty \alpha e^{-\alpha x}\,dx.$$
$$= \alpha \left[\frac{e^{-\alpha x}}{-\alpha} \right]_0^\infty = -(0-1) = 1$$

Mean $(\mu) = \dfrac{1}{\alpha}$ $\qquad$ **Variance** $(V) = \dfrac{1}{\alpha^2}$

* Problems :

1) For the exponential variant x with mean 5 evaluate i) $P(0 < x < 1)$ ii) $P(-\infty < x < 10)$

→ i) $p(0 < x < 1) = \int_0^1 \alpha e^{-\alpha x}\,dx$

$\alpha e^{-\alpha x}$ $\qquad$ $= \int_0^1 \frac{1}{5} e^{-\frac{1}{5}x}\,dx$

$-\infty$ $\quad$ 0 $\quad$ 0 $\quad$ ∞

$= 1 - e^{-0.2}$ $\qquad$ 0.1812

$= 0.1812$

ii) $p(-\infty < x < 10) = \int_{-\infty}^{0} p(x)\,dx + \int_0^{10} p(x)\,dx$

$= 0 + \int_0^{10} \frac{1}{5} e^{-\frac{1}{5}x}\,dx$

$= 1 - e^{-2}$

$= \underline{0.8646}$

2) The Duration a telephone conversation has been found to have an exponential distribut^n with mean 3 minutes. Find the probability that the Conversion last i) ~~On~~ more than 1 min ii) less than 3 min.

→ μ = mean = 3 min

$-\infty$ $\quad$ 1 $\quad$ ∞ $\qquad$ i) $p(x > 1)$ ii) $p(x < 3)$

$-\infty \quad 0 \quad 1 \quad \infty$

$$p(x>1) = \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

$\alpha = 1/3$

$$P(x>1) = \int_1^\infty \alpha e^{-\alpha x}\,dx$$

$$= \alpha \left[\frac{e^{-\alpha x}}{-\alpha}\right]_1^\infty$$

$$= -e^{-\infty} + e^{-1/3} = \boxed{0.716}$$

$$\int_1^\infty \alpha \cdot e^{-\alpha x}\,dx$$

$$\alpha \left[\frac{e^{-\alpha x}}{\alpha}\right]_1^\infty$$

$$e^{-\infty} - e^{-1}$$

$$1 - 0.367$$

$$P(x<3) = \int_{-\infty}^{3} p(x)\,dx$$

$$= \int_{-\infty}^{0} p(x)\,dx + \int_0^3 p(x)\,dx$$

$$0 + \int_0^3 \alpha e^{-\alpha x}\,dx$$

$$= \underline{0.6321}$$

0.1353
0.8647
0.0440

In a Certain town the duration of shower is expontentially distributed with mean 5. What is the probability that the shower will last for i) 10 min or more ii) less than 10 min iii) bet^n 10 to 12 min

→ Mean (μ) = 5 min. $\alpha = \frac{1}{5}$

i) $$p(x \geq 10) = \int_{10}^{\infty} p(x)\,dx$$

$$= \int_{10}^{\infty} \alpha e^{-\alpha x}\,dx$$

$$= \alpha \left[\frac{e^{-\alpha x}}{-\alpha}\right]_{10}^{\infty}$$

$$= -e^{-\infty} + e^{-1/5 \times 10}$$

$$= 0 + e^{-2}$$

$$= 0.1353$$

ii) $$p(x<10) = \int_{-\infty}^{10} p(x)\,dx$$

$$= \int_{-\infty}^{0} p(x)\,dx + \int_0^{10} p(x)\,dx$$

$$= 0 + \int_0^{10} \alpha e^{-\alpha x}\,dx$$

$$= 0.8646$$

iii) $$p(10 < x < 12) = \int_{10}^{12} \alpha e^{-\alpha x}\,dx$$

$$= 0.0446$$

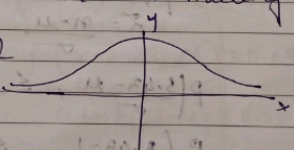
* Normal Distribution.

The pdf of normal distribution is given by $F(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The graph of the normal distribution is called the normal curve it is bel shaped & symmetric about the mean(μ). The 2 tails of the curve extend towards +ve & -ve direction of x-axis never ever meeting the x-axis.

The area under the normal curve above the x-axis is equal to 1



Mean = μ & Variance = V

* Standard Normal Distribution:-

The normal distribution for which the mean is 0 & the std-deviation is 1 is called Standard Normal Distribution

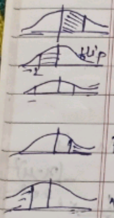
$$g(x) = \frac{1}{\sqrt{2\pi}} \times e^{-x^2/2} \quad \Big| \quad g(z) = \frac{1}{\sqrt{2\pi}} \times e^{-z^2/2}$$

// Area must start from 0

$z = \frac{x-\mu}{\sigma}$ will helped to have Std normal variate.

* Problems.

1) For the Std normal distribution evaluate the following. i) $P(0 \le z \le 1.45)$
ii) $P(-2.60 \le z \le 0)$
iii) $P(-3.40 \le z \le 2.65)$ iv) $P(z \ge 1.7)$ v) $P(z \le -3.35)$



i) $P(0 \le z \le 1.45) = A(1.45) = 0.4265$

ii) $P(-2.60 \le z \le 0) = A(2.60) = 0.49534$.

iii) $P(-3.40 \le z \le 2.65) = A(3.40) + A(2.65) =$
$= 0.49966 + 0.49598 = 0.9956$

iv) $P(z \ge 1.7) = 0.5 - A(1.7)$
$= 0.04457$

v) $P(z \le -3.35) = 0.5 - (3.35) = 4 \times 10^{-4}$.

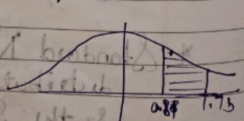
0.164 2) If x is normal variate with mean 1 & Std-deviation 3 find
i) $P(3.43 \le x \le 6.19)$
ii) $P(-1.43 \le x \le 6.19)$.

→ i) $P(3.43 \le x \le 6.19)$
$z = \frac{x-\mu}{\sigma}$, $\mu = 1$ $\sigma = 3$.

$P\left(\frac{3.43-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{6.19-\mu}{\sigma}\right)$

$P\left(\frac{3.43-1}{3} \le z \le \frac{6.19-1}{3}\right)$

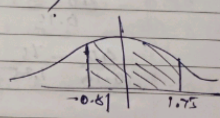
$P(0.81 \le z \le 1.73)$
$= A(1.73) - A(0.81)$
$= 0.45818 - 0.29103$
$= 0.16715$.



ii) $P(-1.43 \le x \le 6.19)$
$z = \frac{x-\mu}{\sigma}$ $\mu = 1$ $\sigma = 3$

$P\left(\frac{-1.43-1}{3} \le z \le \frac{6.19-1}{3}\right)$

$P(-0.81 \le z \le 1.73)$



$= A(1.73) + A(0.81)$
$= 0.45818 + 0.29103$
$= \cancel{0.75097}$ 0.74921

4.14 3) In a Certain Examination % of Candidate passing & getting distinction were 45 (%) & 9(%) respectively. estimate the average marks obtained by the Candidates. if the minimum pass & distinction marks are being 40 & 75 respectively.

$\cancel{P(x \le 40) = 0.45}$ $\cancel{P(x \le 75) = 0.09}$
$P(x \ge 40) = 0.45$ $P(x \ge 75) = 0.09$.

$P\left(\frac{x-\mu}{\sigma} \ge \frac{40-\mu}{\sigma}\right) = 0.45$ $P\left(\frac{x-\mu}{\sigma} \ge \frac{75-\mu}{\sigma}\right) = 0.09$

$P\left(z \ge \frac{40-\mu}{\sigma}\right) = 0.45$ $P\left(z \ge \frac{75-\mu}{\sigma}\right) = 0.09$

$0.5 - A\left(\frac{40-\mu}{\sigma}\right) = 0.45$ $0.5 - A\left(\frac{75-\mu}{\sigma}\right) = 0.09$

$A\left(\frac{40-\mu}{\sigma}\right) = 0.5 - 0.45$ $A\left(\frac{75-\mu}{\sigma}\right) = 0.5 - 0.09$

$= 0.05$ $= 0.41$.

We have
$A(0.13) = 0.05$
$A(1.35) = 0.41$.

$\frac{40-\mu}{\sigma} = 0.13$ $\qquad$ $\frac{75-\mu}{\sigma} = 1.35$

$\frac{40}{\sigma} - \mu$ $\qquad$ $40-\mu = 0.13\sigma$ $\qquad$ $75-\mu = 1.35\sigma$

$\frac{40-\mu}{0.13} = \frac{75-\mu}{1.35}$ $\qquad$ $\mu + 0.13\sigma = 40$

$\mu + 1.35\sigma = 75$

$\frac{40}{0.13} - \frac{75}{1.35} = -\frac{\mu}{1.35} + \frac{\mu}{0.13}$

$\boxed{\mu = 36.27}$

$\boxed{\sigma = 28.68}$

4) The mean ~~of~~ height of 500 Students is 151 cm & the Standard deviation is 15 cm. Assuming heights to be normally distributed find how many students height lie between 120 to 155 cm.

→ Given: $\mu = 151$ $\sigma = 15$.

$p(120 \le x \le 155)$

$p\left(\frac{120-151}{15} \le \frac{x-\mu}{\sigma} \le \frac{155-151}{15}\right)$

$P(-0.62 \le Z \le 0.08)$ $\qquad$ $P(-2.066 \le Z \le 0.266)$

$= A(0.08) + A(0.62)$ $\qquad$ $= A(0.266) + A(2.066)$

$= 0.03188 + 0.2323$ $\qquad$ $= 0.1048 + 0.48059$

$= 0.26425$ $\qquad$ $= 0.585$

$= 500 \times (P -0.62 \le Z \le 0.08)$

$= 132.125$

$= \underline{\underline{132}}$

$= 500 \times 0.585$

$= 292.6 \approx \underline{\underline{293}}$

5) In a normal distribution 7% of items are under 35 89% are under 63 find the mean & the standard deviation given that $A(1.23) = 0.39$ & $A(1.48) = 0.43$.

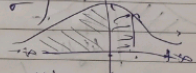
→ $p(x \le 35) = 0.07$ $\qquad$ $P(x \le 63) = 0.89$

$p\left(\left(\frac{x-\mu}{\sigma}\right) \le \frac{35-\mu}{\sigma}\right) = 0.07$ $\qquad$ $p\left(\frac{x-\mu}{\sigma} \le \frac{63-\mu}{\sigma}\right) = 0.89$

$p\left(z \le \frac{35-\mu}{\sigma}\right) = 0.07$ $\qquad$ $p\left(z \le \frac{63-\mu}{\sigma}\right) = 0.89$

$0.5 + A\left(\frac{35-\mu}{\sigma}\right) = 0.07$ $\qquad$ $A\left(\frac{63-\mu}{\sigma}\right) = 0.89$

$A\left(\frac{35-\mu}{\sigma}\right) = -0.5 + 0.07$ $\qquad$ $A\left(\frac{63-\mu}{\sigma}\right) = -0.5 + 0.89$



$A\left(\frac{35-\mu}{\sigma}\right) = -0.43$

$A\left(\frac{35-\mu}{\sigma}\right) = -0.43$ $\qquad$ $A\left(\frac{63-\mu}{\sigma}\right) = +0.39$

We $A(1.48) = 0.43$.

& $A(1.23) = 0.39$.

$\frac{35-\mu}{\sigma} = -1.48$ $\qquad$ $\frac{63-\mu}{\sigma} = +1.23$

$35-\mu = -1.48\sigma$ $\qquad$ $63-\mu = -1.23\sigma$

$\mu + (-1.48\sigma) = 35$ $\qquad$ $\mu + (+1.23\sigma) = 63$

$\mu + 1.48\sigma = 35$ $\qquad$ $\mu + 1.23\sigma = 63$

~~$\mu = 271.8$~~

$\sigma =$

~~$\mu = 271.8$~~ $\qquad$ ~~$\sigma = \sqrt{60}$~~

$\mu = 50.29$ $\qquad$ $\sigma = 10.33$

6) In a normal distribution 31% of items are under 45 & 8% over 64 find the mean & std deviation.

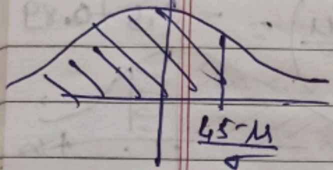
→

$p(x \leq 45) = 0.31$

$p\left(\frac{x-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.31$

$p\left(z < \frac{45-\mu}{\sigma}\right) = 0.31$

$0.5 + A\left(\frac{45-\mu}{\sigma}\right) = 0.31$



$A\left(\frac{45-\mu}{\sigma}\right) = -0.19$

We have.

$A(0.5) = 0.19$

$\frac{45-\mu}{\sigma} = -0.5$

$45 - \mu = -0.5\sigma$

$\cancel{\mu - 0.19\sigma = 45}$

$\mu - 0.5\sigma = 45$

$\mu = 50$

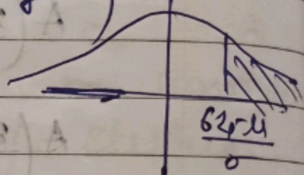
$\sigma = 10$

$p(x > 64) = 0.08$

$p\left(\frac{x-\mu}{\sigma} > \frac{64-\mu}{\sigma}\right) = 0.08$

$p\left(z > \frac{64-\mu}{\sigma}\right) = 0.08$

$0.5 - A\left(\frac{64-\mu}{\sigma}\right) = 0.08$



$A\left(\frac{64-\mu}{\sigma}\right) = 0.5 - 0.08 = 0.42$

$A(1.4) = 0.42$

$\frac{64-\mu}{\sigma} = 1.4$

$64 - \mu = 1.4\sigma$

$\mu + 0.42\sigma = 64$

$\mu + 1.4\sigma = 64$

$\cancel{\sigma = 3.14}$