Unit 5

Joint Probability Stochastic Distribution

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1) A fair coin is tossed 3 times $X = 0$ or 1 according to tail or head occurring on the first toss. And Y = no. of tails. Determine the following.
i) Marginal Distributions of x & y.
ii) Joint PDF of x & y. iii) μ_x, μ_y, $E(x, y)$. variance of x, variance of y, co-variance of ~~x~~ XY & corelation of XY.

$\mu_x = E(x)$
$\mu_y = E(Y)$

→ Sample space

$S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$

Marginal distribution of x.

x	0	1
$P(x)$	4/8	4/8

Marginal distribution of y.

y	0	1	2	3
$P(y)$	1/8	3/8	3/8	1/8

JPD

$x \backslash Y$	0	1	2	3	
0	0	1/8	2/8	1/8	4/8
1	1/8	2/8	1/8	0	4/8
	1/8	3/8	3/8	1/8	

$X = \{0, 1\}$ $Y = \{0, 1, 2, 3\}$.

Mean of X $= E(X) = \mu_x$
$= \Sigma x_i p(x_i)$
$= 0(4/8) + 1(4/8) = 4/8$.

Mean of Y $= E(Y) = \mu_Y$
$= \Sigma y_i P(y_i)$
$= 0(1/8) + 1(3/8) + 2(3/8) + 3(1/8)$
$= 3/2$

$$E(XY) = \sum x_i y_i P_{ij}$$
$$= 0(0)(0) + 0(1)(1/8) + 0(2)(2/8) + 0(3)(1/8) + 1(0)(1/8) + 1(1)(2/8) + 1(2)(1/8) + 1(3)(0)$$
$$= \underline{\underline{1/2}}$$

$$\sigma_x^2 = E(x^2) - \mu_x^2$$
$$= \sum x^2 p(x) - \mu_x^2$$
$$= 0 \times 4/8 + 1 \times 4/8 - (4/8)^2$$
$$= \frac{4}{8} - \frac{16}{64} = \frac{1}{4}.$$

$$\sigma^2 y = \sum(Y^2) - \mu_Y^2$$
$$= \sum y^2 p(y) - \mu_Y^2$$
$$= 0 \times 1/8 + 1 \times 3/8 + 4 \times 3/8 + 9 \times 1/8 - \left(\frac{3}{2}\right)^2$$
$$= \frac{3}{8} + \frac{12}{8} + \frac{9}{8} - \left(\frac{9}{4}\right) = \frac{\cancel{24}\,3}{\cancel{8}\,1} - \frac{9}{4} = 3 - \frac{9}{4}$$
$$= \underline{\underline{\frac{3}{4}}}.$$

* Co-variance XY $\sigma_{xy} = E(XY) - E(X) \times E(Y)$
$$= \frac{1}{2} - \frac{\cancel{4}\,1}{\cancel{8}\,2} \times \frac{3}{\cancel{2}\,1} = \underline{\underline{\frac{-1}{4}}}$$

* Corelation of XY (ρ) $= \dfrac{\text{Covariance of } XY}{\sigma X \, \sigma Y}$
$$= \frac{-1/4}{1/2 \times \sqrt{3/4}} = \frac{-\sqrt{3}}{3}$$
$$= \underline{\underline{-0.577}}$$

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→ A Joint PDF is given by, find a) Mean of X, Y b) Mean of Y c) Covariance of X d) " of Y. e) Covariance (X, Y) & f) Corelation

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

X \ Y	-3	2	4	
1	0.1	0.2	0.2	0.5
3	0.3	0.1	0.1	0.5
	0.4	0.3	0.3	

a) Mean of $X = E(X) = \mu_x = \sum x\, p(x)$
$$= 0.5 \times 1 + 0.5 \times 3 = 2$$

b) Mean of $Y = E(Y) = \mu_Y = \sum y\, p(y)$
$$= -3(0.4) + 2 \times (0.3) + 4(0.3)$$
$$= 0.6$$

c) $\sigma^2 x = E(X^2) - \mu_x^2$
$$= \sum x^2 p(x) - \mu_x^2$$
$$= 1 \times 0.5 + 9 \times 0.5 - 4 = \underline{\underline{1}}$$

d) $\sigma^2 y = E(Y^2) - \mu_Y^2$
$$= \sum y^2 p(x) - \mu_Y^2$$
$$= 9 \times 0.4 + 4 \times 0.3 + 16 \times 0.3 - (0.6)^2$$
$$= 9.24$$

e) Co-variance of XY $\sigma^2 xy = E(XY) - E(X) \times E(Y)$
$$= \cancel{9.24}\,0 - 1.2$$
$$= \underline{-1.2}$$

$E(XY) = \sum x_i y_i P_{ij}$
$$= 1 \times -3 \times 0.1 + 1 \times 2 \times 0.2 + 1 \times 4 \times 0.2 + 3 \times -3 \times 0.3 + 3 \times 2 \times 0.1 + 3 \times 4 \times 0.1$$
$$= 0.$$

Corelation $\rho = \frac{\text{covariance of } XY}{\sigma x \, \sigma y}$

$= \frac{-1.2}{1 \times \sqrt{9.24}} = -0.3948$

* Joint Probability Distribution function of 2 variables x & y is given by

$X \backslash Y$	y_1	y_2	....	y_n
x_1	P_{11}	P_{12}	.....	P_{1n}
x_2	P_{21}	P_{22}	.....	P_{2n}
⋮	⋮	⋮	⋱	⋮
x_n	P_{m1}	P_{m2}	....	P_{mn}

Such that i) $0 \le P_{ij} \le 1$ ii $\Sigma P_{ij} = 1$

Formulae :-

1) $E(X) = \mu_X = \Sigma x \, p(x)$
2) $E(Y) = \mu_Y = \Sigma y \, p(y)$
3) $E(XY) = \Sigma x_i y_j P_{ij}$
4) $\sigma^2 x = E(X^2) - \mu_X^2$
5) $\sigma^2 y = E(Y^2) - \mu_Y^2$
6) $cov(XY) = E(XY) - E(X) * E(Y)$
7) Corelation $= \rho = \frac{cov(XY)}{\sigma x \, \sigma y}$

3) The joint PDF of two random variables X Y is

$X \backslash Y$	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

i) Determine the individual distribution of X & Y
ii) verify wether X & Y are independent.

$X \backslash Y$	2	3	4	
1	0.06	0.15	0.09	0.3
2	0.14	0.35	0.21	0.7
	0.2	0.5	0.3	

Marginal X

X	1	2
P(x)	0.3	0.7

Marginal Y

Y	2	3	4
P(y)	0.2	0.5	0.3

ii) $p_1 q_1 = (0.3)(0.2) = 0.06 = P_{11}$
$p_1 q_2 = (0.3)(0.5) = 0.15 = P_{12}$
$p_1 q_3 = (0.3)(0.3) = 0.09 = P_{13}$
$p_2 q_1 = (0.7)(0.2) = 0.14 = P_{21}$
$p_2 q_2 = (0.7)(0.5) = 0.35 = P_{22}$
$p_2 q_3 = (0.7)(0.3) = 0.21 = P_{23}$
$\therefore$ X & Y are independent.

4) The JPD is given as

$X \backslash Y$	0	1
0	0.1	0.2
1	0.4	0.2
2	0.1	0

i) Determine the Marginal Distribution of X & Y
ii) Are X & Y independent
iii) $P(X+Y > 1)$

i) Marginal X

X	0	1	2
P(x)	0.3	0.6	0.1

Marginal Y

Y	0	1
P(y)	0.6	0.4

ii) $p_1 q_1 = (0.3) \times (0.6) = 0.18 \neq P_{11}$
$\therefore$ X & Y are not independent.

iii) $P(X+Y > 1)$
$\{(1,1)\ (2,0)\ (2,1)\}$
$= 0.2 + 0.1 + 0 = \underline{\underline{0.3}}$

5) The Distributions of 2 independent variables X & Y are.

x	0	1
$p(x)$	0.2	0.8

y	1	2	3
$P(y)$	0.1	0.4	0.5

i) Find Joint PDF.

Joint PDF

X \ Y	1	2	3
0	0.02	0.08	0.1
1	0.08	0.32	0.4

$P_{11} = P_1 q_1 = 0.2 \times 0.1 = \underline{0.02}$

① Find the unique fixed probability vector for the following stochastic matrix

→ $A = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$

$VA = V$ $\Rightarrow x+y=1$

$[x, y] \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = [x \ \ y]$

$\left[\frac{3}{4}x + \frac{1}{2}y, \ \frac{1}{4}x + \frac{1}{2}y\right] = [x \ \ y]$

$\frac{3}{4}x + \frac{1}{2}y = x \Rightarrow -\frac{1}{4}x + \frac{1}{2}y = 0$

$\frac{1}{4}x + \frac{1}{2}y = y \Rightarrow \frac{1}{4}x - \frac{1}{2}y = 0$

And $x + y = 1$

$\Rightarrow x = \frac{2}{3} \quad y = \frac{1}{3}.$

2) $A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$

$VA = V \Rightarrow [x \ \ y \ \ z] \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix} = [x \ \ y \ \ z]$

$[\frac{1}{6}y \quad x + \frac{1}{2}y + \frac{2}{3}z \quad \frac{1}{3}y + \frac{1}{3}z] = [x \ y \ z]$

$\frac{1}{6}y = x \quad x + \frac{1}{2}y + \frac{2}{3}z = y \quad \frac{1}{3}y + \frac{1}{3}z = z$

$-x + \frac{1}{6}y = 0 \quad x - \frac{1}{2}y + \frac{2}{3}z = 0 \quad \frac{1}{3}y - \frac{2}{3}z = 0$

And $x + y + z = 1$

$x = 1/10 \quad y = 3/5 \quad z = 3/10.$

③ Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is a regular Stochastic matrix. Also find the associated unique fixed probability vector.

→ $P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$

$P^3 = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$ $\quad P^4 = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.25 & 0.25 & 0.5 \\ 0.25 & 0.5 & 0.25 \end{bmatrix}$

$P^5 = \begin{bmatrix} 0.25 & 0.25 & 0.5 \\ 0.25 & 0.5 & 0.25 \\ 0.125 & 0.375 & 0.5 \end{bmatrix}$

$VA = V \Rightarrow [x \ \ y \ \ z] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = [x \ \ y \ \ z].$

$[\frac{1}{2}z \quad x + \frac{1}{2}z \quad y] = [x, \ y, \ z]$

$\frac{1}{2}z = x \Rightarrow -x + \frac{1}{2}z = 0$.

$x + \frac{1}{2}z = y \Rightarrow x - y + \frac{1}{2}z = 0$.

$y = z \Rightarrow y - z = 0$.

And $x + y + z = 1$

$x = 1/5 \quad y = 2/5 \quad z = 2/5$

4) Prove that the marcochain whose transiation Probability Matrix is, $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducable.

→

$$P^2 = \begin{bmatrix} 0.5 & 0.166 & 0.333 \\ 0.25 & 0.58 & 0.166 \\ 0.25 & 0.33 & 0.416 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1/6 & 1/3 \\ 1/4 & 7/12 & 1/6 \\ 1/4 & 1/3 & 5/12 \end{bmatrix}$$

∴ The matrix is irreducable.

5) A Studentts Study Habits are as follows if he Studies 1 night he is 70% Sure not to Study the next night. On the hand otherhand if he does not Study 1 night he is 60% sure not to study a next night. In long run how often does he study.

→

$$P = \begin{array}{c|cc} & S & NS \\ S & 0.3 & 0.7 \\ NS & 0.4 & 0.6 \end{array}$$

$VP = V$, $x + y = 1$ } In long Run

$$[x, y] \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} = [x, y]$$

$0.3x + 0.4y = x \qquad 0.7x + 0.6y = y$.

$-0.7x + 0.4y = 0 \qquad 0.7x - 0.4y = 0$.

And $x + y = 1 \Rightarrow x = 4/11 \quad y = 7/11$

∴ In long run the probability of Studying is $4/11$ or 36.36% & probability of Not Studying is $7/11$ or 63.63%.

6) A Salesman's terilory Consist of 3 cities A, B & C he never Sales in same city on Succesive days. if he Sells in city A then the next day he sells in city B. However if Sells in either B & C then the next day he is twice as likey he ~~Sales in~~ to sells in city A as in other city. In long run how often does he Sell in each of the city

→

$$P = \begin{array}{c|ccc} & A & B & C \\ A & 0 & 1 & 0 \\ B & 2/3 & 0 & 1/3 \\ C & 2/3 & 1/3 & 0 \end{array}$$

In long run $VP = V$, $x + y + z = 1$.

$$[x, y, z] \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} = [x \; y \; z]$$

$2/3\,y + 2/3\,z = x \qquad x + 1/3\,z = y \qquad 1/3\,y = z$.

$-x + 2/3\,y + 2/3\,z = 0 \qquad x - y + 1/3\,z = 0 \qquad 1/3\,y - z = 0$.

And $x + y + z = 1$

$x = 2/5 \quad y = 9/20 \quad z = 3/20$.

∴ In long run the Salesman ~~sat~~ probability of Salling product in various cities are.

$A = 2/5 = 40\%$

$B = 9/20 = 45\%$

$C = 3/20 = 15\%$

H.W

7) A man's Smoking habits are as follows. If he smokes filter cigarattes one week he switches to non-filter cigarattes next week with probability 0.2. On the other hand if he smokes non-filter cigarattes one week there is probability of 0.7 that he will smoke non-filter cigarattes next week. In the long run how often does he smoke filter cigarattes.

→

$$P = \begin{array}{c|cc} & f & Nf \\ \hline f & 0.8 & 0.2 \\ Nf & 0.3 & 0.7 \end{array}$$

$VP = V$, $x+y=1$

$$[x, y]\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix} = [x \quad y].$$

$0.8x + 0.3y = x$ $\quad$ $0.2x + 0.7y = y$

$-0.2x + 0.3y = 0$ $\quad$ $0.2x - 0.3y = 0$.

And $x+y=1$

⇒ from above eq^ns

$x = 3/5 \quad y = 2/5$

∴ In long run 0.6 probability that he ~~sm~~ smokes filter cigarette.

8) A Habitual Gambler is a member of two clubs A & B. He visits either of the clubs every day for playing cards. He never visits club A on two consecutive days. But if he visits club B on particular day then the next day he is likely to visit club B or club A. i) Find the transition Matrix of Marcochain ii) Show that the matrix is regular stochastic matrix iii) Find the unique fixed probability vector.
iv) If the person had visited club B on Monday find the probability that he visits club ~~B~~ A on Thursday.

→ i)

$$P = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} \end{array}$$

ii) $$P^2 = \begin{bmatrix} 0.5 & 0.5 \\ 0.25 & 0.75 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$$ ∴ Regular.

iii) $VP = V$, $x+y=1$

$$[x, y]\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} = [x \quad y]$$

$\frac{1}{2}y = 2x$ $\quad$ $x + \frac{1}{2}y = y$

$-x + \frac{1}{2}y = 0$ $\quad$ $x - \frac{1}{2}y = 0$ & $x+y=1$

$x = 1/3 \quad y = 2/3$

iv) $$P^{(3)} = P^{(0)} P^3 = [0 \quad 1]\begin{bmatrix} 1/4 & 3/4 \\ 3/8 & 5/8 \end{bmatrix} = [3/8 \quad 5/8]$$

9) Three boys A, B & C are throwing ball to eachother. A also throws ball to B. B always throws ball to C. C is just as likely to throw the ball B as to A. If C was the first person to throw the ball find the probabilities that after 3 throws, (for 4^th throw) i) A has the ball ii) B has the ball iii) C has the ball.

→

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array} \qquad P^{(0)} = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ [0 \quad 0 \quad 1] \end{array}$$

$$P^{(3)} = P^{(0)} P^3 = [0 \ 0 \ 1]\begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} i & ii & iii \end{array} \\ [1/4 \quad 1/4 \quad 1/2] \end{array}$$

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1) If $A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$ is a Stochastic Matrix and $V = [v_1 \;\; v_2]$ is a probability vector. Show that VA is also a probability vector.

→ Since A is stochastic matrix and

$a_1 + a_2 = 1$

$b_1 + b_2 = 1$

∴ Also, Since V is probability vector

$v_1 + v_2 = 1$

$$VA = [v_1 \;\; v_2] \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = [a_1v_1 + b_1v_2 \quad a_2v_1 + b_2v_2]$$

Now

$$(a_1v_1 + b_1v_2) + (a_2v_1 + b_2v_2) = 1$$
$$v_1(a_1 + a_2) + v_2(b_1 + b_2) = 1$$
$$v_1(1) + v_2(1) = 1$$
$$v_1 + v_2 = 1$$
$$1 = 1$$

2) If $P_1 = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$ & $P_2 = \begin{bmatrix} 1-b & b \\ a & 1-a \end{bmatrix}$

Show that P_1, P_2 and P_1P_2 is Stochastic matrix.

→ ~~Sin~~ in P_1 $\quad 1 - \cancel{a} + \cancel{a} = 1$

$1 = 1$

The entries are non-negative

in P_1 $\quad 1 - \cancel{a} + \cancel{a} = 1$, $\quad \cancel{b} + 1 - \cancel{b} = 1$

$1 = 1 \qquad\qquad 1 = 1$

∴ P_1 is Stochastic matrix

in P_2 $\quad 1 - \cancel{b} + \cancel{b} = 1 \qquad\qquad \cancel{a} + 1 - \cancel{a} = 1$

$1 = 1 \qquad\qquad 1 = 1$

∴ P_2 is Stochastic matrix.

∴ P_1

classmate

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$$P_1P_2 = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \begin{bmatrix} 1-b & b \\ a & 1-a \end{bmatrix}$$

$$= \begin{bmatrix} (1-a)(1-b) + a^2 & (1-a)b + a(1-a) \\ b(1-b) + (1-b)a & b^2 + (1-b)(1-a) \end{bmatrix}$$

After Addition

$$\Rightarrow (1-a)(1-b) + a^2 + (1-a)b + a(1-a)$$
$$= 1 - \cancel{b} - \cancel{a} + \cancel{ab} + \cancel{a^2} + \cancel{b} - \cancel{ab} + \cancel{a} - \cancel{a^2}$$
$$= \cancel{ab} - \cancel{b} + 1$$
$$= 1$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

3) If $t = (t_1, t_2 \ldots t_m)$ is a vector & T is square matrix whose rows are each the same vector t. Then $PT = t$ if $P = (P_1, P_2 \ldots P_m)$ is a probability vector.

→

$$T = \begin{bmatrix} t_1 & t_2 & \cdots & t_m \\ t_1 & t_2 & \cdots & t_m \\ \vdots & & & \vdots \\ t_1 & t_2 & \cdots & t_m \end{bmatrix} \qquad P = (P_1, P_2 \ldots P_m)$$

$$P_1 + P_2 + \cdots + P_m = 1$$

$$PT = t$$

$$PT = [P_1 \;\; P_2 \ldots P_m] \begin{bmatrix} t_1 & t_2 & \cdots & t_m \\ t_1 & t_2 & \cdots & t_m \\ \vdots & & & \vdots \\ t_1 & t_2 & \cdots & t_m \end{bmatrix}$$

$$= [P_1t_1 + P_2t_1 \ldots P_mt_1 \quad P_1t_2 + P_2t_2 \ldots P_mt_2 \quad P_1t_m + P_2t_m \ldots P_mt_m]$$

$$= [t_1(P_1 + P_2 \ldots P_m) \quad t_2(P_1 + P_2 + \ldots P_m) \quad t_m(P_1 + P_2 + \ldots P_m)]$$

$$= [t_1 \;\; t_2 \ldots t_m]$$

$$= t.$$

* ~~Joint~~ Conditional Joint Probability Distribution

Conditional probability distribution of Y given that $X = x$ is $h(y/x) = \frac{h(x, y)}{f(x)}$

1) Given the joint distribution
i) Find the Marginal distribution
ii) $P(Y=3 / X=2)$

Y \ X	1	2	3
1	0.05	0.05	0.1
2	0.05	0.1	0.35
3	0	0.2	0.1

Marginal distribution of Y

y	1	2	3
$f(y)$	0.2	0.5	0.3

Marginal distribution of X

x	1	2	3
$f(x)$	0.1	0.35	0.55

$P(Y=3 / X=2) = \frac{0.2}{0.35} = \frac{4}{7} = 0.57$

2) Given the joint probability
i) Find Marginal Distribution of X & Y
ii) Are X & Y independent
iii) Find the conditional probability distribution $h(x/y=1)$

Y \ X	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

i) Marginal of X

x	0	1	2
$f(x)$	0.3	0.6	0.1

~~Magu~~ Marginal of Y

y	0	1
$f(y)$	0.6	0.4

ii) $P_1 q_1 = 0.18 \neq 0.1$
$\therefore X$ & Y are not independent.

iii) $h(x/y=1) \Rightarrow h(0/1), h(1/1), h(2/1)$

$h(0/1) = 0.2/0.4 = 0.5$
$h(1/1) = 0.2/0.4 = 0.5$
$h(2/1) = 0/0.4 = 0$

OBA2:
There are 2 white marbles in box A and 3 red marbles in box B. At each step of the process a marble is selected from each box and 2 marbles selected are interchanged let the state A_i of the system be the number i of the red marbles in box A.
1) Find the transition matrix
2) What is the probability that there are 2 red marbles in box A after 3 steps.
3) In long run what is the probability that there 2 red marbles in box A.

Revision
1) Two cards are selected at random from a box which contains 5 cards numbered 1, 1, 2, 2 and 3. Find the joint probability distribution x denotes the sum y denotes the maximum of the numbers drawn. Also determine Co-variance of xy & correlation.

2) If x & y are independent random variable. x takes the values 2, 5, 7 with probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ and y takes the values 3, 4, 5 with probability $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$.
i) Find the ~~JD~~ JPD ii) Show that covariance = 0.
iii) Find the probability distribution $Z = x+y$

1) Combinations of selecting 2 cards randomly from 1, 1, 2, 2, 3 are.

2	4	3	4	3	5	4	5
(1, 1)	(2, 2)	(1, 2)	(1, 3)	(2, 1)	(2, 3)	(3, 1)	(3, 2)
1	2	2	3	2	3	3	3

X : Sum of NO.s : {2, 3, 4, 5}
Y : Maximum NO : {1, 2, 3}.

Marginal Distribution of X is.

x	2	3	4	5
$f(x)$	$1/8$	$2/8$	$3/8$	$2/8$

Marginal of Y is

y	1	2	3
$f(y)$	$1/8$	$3/8$	$4/8$

$\Rightarrow$ JPD of xy is

X \ Y	1	2	3	
2	$1/64$	$3/64$	$4/64$	$8/64$
3	$2/64$	$6/64$	$8/64$	$16/64$
4	$3/64$	$9/64$	$12/64$	$24/64$
5	$2/64$	$6/64$	$8/64$	$16/64$
	$8/64$	$24/64$	$32/64$	= 1

$E(x) = \mu_x = 2(1/8) + 3(2/8) + 4(3/8) + 5(2/8) = 30/8$.

$E(Y) = \mu_y = 1(1/8) + 2(3/8) + 3(4/8) = 19/8$

$E(XY) = 2 \times 1 \times 1/64 + 12/64 + 24/64 + 6/64 + 36/64 + 72/64 + 12/64 + 72/64 + 144/64 + 10/64 + 60/64 + 120/64$

$= \underline{\underline{\dfrac{510}{64}}}$

Covariance $(XY) = E(XY) - E(X) * E(Y)$

$= \dfrac{510}{64} - \dfrac{30}{8} \times \dfrac{19}{8}$

$= \underline{0}$

Correlation $= \underline{\underline{0}}$ as $\dfrac{0}{\sigma_x \sigma_y} = 0$

2) Marginal of X is:

x	2	5	7
$f(x)$	$1/2$	$1/4$	$1/4$

Marginal of Y is:

y	3	4	5
$f(y)$	$1/3$	$1/3$	$1/3$

i) JPD is

X \ Y	3	4	5	
2	$1/6$	$1/6$	$1/6$	$3/6$
5	$1/12$	$1/12$	$1/12$	$3/12$
7	$1/12$	$1/12$	$1/12$	$3/12$
	$4/12$	$4/12$	$4/12$	= 1

ii) $E(X) = \mu_x = 2 \times (1/2) + 5(1/4) + 7(1/4) = 4$

$E(Y) = \mu_y = 3(1/3) + 4(1/3) + 5(1/3) = 4$

$E(XY) = 2 \times 3 \times 1/6 + 2 \times 4 \times 1/6 + 10/6 + 15/12 + 20/12 + 25/12 + 21/12 + 28/12 + 35/12$

$= 16$

iii) $cov(XY) = E(XY) - E(X) * E(Y) = 16 - 4 \times 4 = 0$

$\Rightarrow$ The X & Y are independent.