Unit - 2

Random Vector

Let S be a sample space of a random experiment. Suppose to each element of S a unique real number X is associated according to some rule then X is called a random variable on sample space S.

Discrete random Variables : A random variable which can take some specified values only.

Ex : Tossing Two coins.
Rolling a die.

Continuous random variable : A random variable which can take any value in specified range.

Ex : Temperature
Speed
Time.

Discrete probability distribution : If for each value x_i of discrete random variable a real number $P(x_i)$ is assigned such that

(i) $P(x_i) \geq 0$

(ii) $\Sigma P(x_i) = 1$

then the $p(x)$-function is called probability density function (PDF).

The distribution function $f(x)$ defined by $f(x) = P(X \leq x) = \sum_{i=1}^{x} P(x_i)$ is called cummulative distribution function.

Note: The mean and the variance of discrete probability distribution is given by

Mean $(\mu) = \sum_i x_i \, p(x_i)$

Variance $(V) = \sum (x_i - \mu)^2 p(x_i)$

$\sum x_i^2 \, p(x_i) - \mu^2$

The set of value $(x_i, P(x_i))$ is called probability distribution.

Q) Find the value of 'k', mean and variance

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	2k	0.3	k

Solⁿ: We know that,

$\sum P(x_i) = 1$

$0.6 + 4k = 1$

$4k = 0.4$

$\boxed{k = 0.1}$

Mean $(\mu) = \sum x_i \, P(x_i)$

$= -2(0.1) + (-1)(0.1) + 1(0.2) + 2(0.3) + 3(0.1)$

$= 0.8$

$\boxed{\mu = 0.8}$

Variance $(V) = \sum x_i^2 \, P(x_i) - \mu^2$

$= 4(0.1) + 1(0.1) + 1(0.2) + 4(0.3) + 9(0.1) - 0.64$

$= 0.4 + 0.1 + 0.2 + 1.2 + 0.9 - 0.64$

$= 2.8 - 0.64$

$\boxed{V = 2.16}$

Q2)

Q3)

Q2) Given

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

find the value of (i) k

(ii) $P(0<x<5)$

Sol^n: We know that

(i) $\Sigma P(x_i) = 1$

$10k^2 + 9k = 1$

$\boxed{k = 0.1}$ $\boxed{k = -1}$

$\therefore k = 0.1$ $\because P(x_i) \geq 0$ since it is PDF

(ii) $P(0<x<5) = P(1) + P(2) + P(3) + P(4)$

$= k + 2k + 2k + 3k$

$= 8k$

$= 8(0.1)$

$= 0.8$

Q3) Find mean and standard deviation of

a)

x	-3	6	9
P(x)	1/6	1/2	1/3

b)

x	8	12	16	20	24
P(x)	1/8	1/6	3/8	1/4	1/12

a) x -3 6 9

$P(x)$ $1/6$ $1/2$ $1/3$

$\mu = \sum x_i\, P(x_i)$

$= -3(1/6) + 6(1/2) + 9(1/3)$

$= -2 + 3 + 3$

$\mu = 4$

$V = \sum (x_i - \mu)^2 P(x_i)$

$= (-7)^2 (1/6) + 2^2 (1/2) + 5(1/3)$

$= \frac{-49}{6} + \frac{4}{2} + 5/3$

$= \frac{-49 + 12 + 10}{6}$

$V = \sum x_i^2 (P(x_i) - \mu^2$

$= 9(1/6) + 36(1/2) + 81(1/3) - 16$

$= 3/2 + 18 + 27 - 16$

$=$

a)

x	-3	6	9
$p(x)$	$1/6$	$1/2$	$1/3$

$\mu = -3(1/6) + 6(1/2) + 9(1/3)$

$= -1/2 + 3 + 3$

$= 11/2$

$\mu = 5.5$

$V = \Sigma x_i^2 P(x_i) - \mu^2$

$= 9(1/6) + 36(1/2) + 81(1/3) - (5.5)^2$

$= 3/2 + 18 + 27 - 30.25$

$v = 16.25$

Standard deviation $= \sqrt{v}$

$= 4.03$

b)

x	8	12	16	20	24
p(x)	1/8	1/6	3/8	1/4	1/12

$\mu = 8(1/8) + 12(1/6) + 16(3/8) + 20(1/4) + 24(1/12)$

$= 1 + 2 + 6 + 5 + 2$

$\mu = 16$

$V = \Sigma x_i^2 P(x_i) - \mu^2$

$= 64(1/8) + 144(1/6) + 256(3/8) + 400(1/4) + 576(1/12)$

$- 256$

$= 8 + 24 + 96 + 100 + 48 - 256$

$= 20$

Standard deviation $= \sqrt{v} = 4.47$

Q. Obtain distribution function for total number of heads occuring in 3 tosses of an unbiased coin.

$$S = \{ \overset{3}{HHH}, \overset{2}{HHT}, \overset{2}{HTH}, \overset{2}{THH}, \overset{1}{TTH}, \overset{1}{THT}, \overset{1}{HTT}, \overset{0}{TTT} \}$$

condition : Total number of heads.

~~$X = \{3, 2, 2, 2, 1, 1, 1, 0\}$~~

$X = \{0, 1, 2, 3\}$

x_i	0	1	2	3
$P(x_i)$	1/8	3/8	3/8	1/8

Q. 3 cards are drawn simultaneously from a well shuffeled deck of 52 cards. Compute the variance for number of aces.

condition : No. of aces. drawing 3 cards.

$X = \{0, 1, 2, 3\}$ [No ace, one ace, two ace, 3 ace]

x_i	0	1	2	3
$P(x_i)$	0.782	0.204	0.013	0.00018

52 → 48 (Non aces), 4 (aces)

$x_i \rightarrow 0 : \dfrac{{}^{48}C_3 \times {}^{4}C_0}{{}^{52}C_3}$ no ace

$1 : \dfrac{{}^{48}C_2 \times {}^{4}C_1}{{}^{52}C_3}$ one ace

$2 : \dfrac{{}^{48}C_1 \times {}^{4}C_2}{{}^{52}C_3}$ two ace

$3 : \dfrac{{}^{48}C_0 \times {}^{4}C_3}{{}^{52}C_3}$ three ace

$\mu = \Sigma x_i P(x_i)$

$= 0 + 1(0.204) + 2(0.013) + 3(0.00018)$

$= 0.204 + 0.026 + 0.00054$

$\mu = 0.23054 = 0.23$

$V = \Sigma x_i^2 P(x_i) - \mu^2$

$= \left(0 + 1(0.204) + 4(0.013) + 9(0.00018)\right) - (0.23)^2$

$x = 0.20472$

$v = 0.204$.

Q. A die is tossed twice. getting a number gretter than 4 is considered a success. Find the variance of probability distribution for no. of successes.

Ans. $P(\text{success}) = 2/6$ $\quad P(\text{failure}) = 1 - P(\text{success}) = 1 - 2/6 = 4/6 = 2/3$

condition : more than 4 is success

$x = \{0, 1, 2\}$

one succes one failure one failure one success

$\frac{2}{6} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{6}$

$\frac{2}{9} + \frac{2}{9}$

$\frac{4}{9}$

x_i $\quad 0 \quad 1 \quad 2$

$P(x_i)$ $\quad \frac{\cancel{4}^2}{\cancel{6}_3} \times \frac{\cancel{4}^2}{\cancel{6}_3} \quad \frac{\cancel{2}}{\cancel{6}_3} \times \frac{\cancel{4}^2}{\cancel{6}_3} \quad \frac{\cancel{2}}{\cancel{6}_3} \times \frac{\cancel{2}}{\cancel{6}_3}$

x_i	0	1	2
$p(x_i)$	4/9	4/9	1/9

$0 \Rightarrow 2/3 \times 2/3 = \frac{4}{8}$

$01 \Rightarrow \frac{2}{6} \times \frac{2}{3} +$

$\frac{4}{18} + \frac{4}{18}$

$\frac{4}{9}$

$\mu = \Sigma x_i P(x_i)$

$= 0 + 1(4/9) + 2(1/9)$

$= \frac{6}{9}$

$\mu = 0.666$

$V = \Sigma x_i^2 P(x_i) - \mu^2$

$= 0 + 1(4/9) + 4(1/9) - (0.666)^2$

$\cancel{V = 0.4625}$

$V = 0.445$

Continuous Probability distribution

If X is continuous random variable, a real number $f(x)$ satisfying the conditions,

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

then $f(x)$ is called probability density function (PDF).

→ A function $F(x)$ defined by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

This $F(x)$ is called cummulative distribution function.

$$\boxed{\frac{d}{dx}\{F(x)\} = f(x)}$$

$$\text{Mean} = \mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$$

$$\text{Variance} = V = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

or

$$\int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

Q) If X is a continuos random variable which with probability density function given by

$$f(x) = \begin{cases} kx & 0 \leq x \leq 2 \\ 2k & 2 \leq x < 4 \\ -kx + 6k & 4 \leq x < 6 \end{cases}$$

Find value of k, mean and variance.

Sol^n : (i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} f(x)dx + \int_{0}^{2} f(x)dx + \int_{2}^{4} f(x)dx + \int_{4}^{6} f(x)dx + \int_{6}^{\infty} f(x)dx = 1$$

$$0 + k\int_0^2 x\,dx + 2k\int_2^4 dx + k\int_4^6 (-x+6)\,dx = 1$$

$$2k + 4k + 2k = 1$$

$$8k = 1$$

$$\boxed{k = 1/8.}$$

mean $= \mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_{-\infty}^{0} x f(x)dx + \int_{0}^{2} x f(x)dx + \int_{2}^{4} x f(x)dx + \int_{4}^{6} x f(x)dx + \int_{6}^{\infty} x f(x)dx$$

$$= 0 + \int_0^2 x(kx)dx + \int_2^4 x(2k)dx + \int_4^6 x(-kx+6k)dx + 0$$

$$= k\int_0^2 x^2dx + 2k\int_2^4 x\,dx - k\int_4^6 (x^2-6x)dx$$

$\left(\frac{x^3}{3} - 3\frac{x^2}{2}\right)_4^6$

$$= \frac{8k}{3} + 12k + \frac{28k}{3}$$

$$= \frac{1}{3} + \frac{4}{3} + \frac{7/2}{3}$$

$$= 3$$

$\mu = 3$

$$\text{Variance} = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{2} x^2 f(x)\,dx + \int_{2}^{4} x^2 f(x)\,dx$$

$$+ \int_{4}^{6} x^2 f(x)\,dx + \int_{6}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= 0 + k\int_{0}^{2} x^3\,dx + 2k\int_{2}^{4} x^2\,dx + k\int_{4}^{6} (-x^3+6x^2)\,dx - \mu^2$$

$$= \frac{1}{8}\cdot 4 + \frac{1}{4}\left(\frac{56}{3}\right) + \frac{1}{8}(44) - \mu^2$$

$$= \frac{1}{2} + \frac{14}{3} + \frac{11}{2} - 9$$

$$V = 5/3$$

Q. Find mean and standard deviation of

$$f(x) = \begin{cases} 0 & x < 2 \\ 1/18\,(2x+3) & 2 \le x \le 4 \\ 0 & x > 4 \end{cases}$$

$$\text{mean} = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{2} x f(x)\,dx + \int_{2}^{4} x f(x)\,dx + \int_{4}^{\infty} x f(x)\,dx$$

$$= 0 + \int_{2}^{4} x\left(\frac{1}{18}(2x+3)\right) + 0$$

$$= \frac{1}{18}\left(\frac{166}{3}\right) \qquad 83$$

$$= \frac{83}{27}$$

$$= 3.07407$$

$-\mu^2$

$$\text{variance} = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$dx - \mu^2$

$$= \int_{-\infty}^{2} x^2 f(x)\,dx + \int_{2}^{4} x^2 f(x)\,dx$$

$$+ \int_{4}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \frac{1}{18} \int_{2}^{4} (2x^3 + 3x^2)\,dx - \mu^2$$

$$= 0.3278$$

$V = 0.323$

Standard deviation $= \sqrt{v} = 0.572$

Discrete probability distribution

1) Binomial distribution
2) poission's distribution.

1) Binomial distribution - The distributions wherein the trials are repititative in nature in which only occurence or non-occurence, true or false, yes or no are of interest.

→ Each trial has two mutually exclusive ~~outcomes~~.

→ The probability of success P and probability failure Q remains constant from trial to trial.

→ Each trial is independent and trials are performed under the same conditions.

→ If a series of independent trials are performed such that for each trial, P is probability of success Q is probability of failure then the probability of x successors in series of n trials is given by

$$\boxed{p(x) = {}^nC_x \, p^x q^{n-x}}$$

n - no. of trials
p - probability of success
q - probability of failure
x - no. of successors.

Mean for binomial distribution

$$\boxed{\mu = np}$$

Variance

$$\boxed{V = npq}$$

Q. The no. of telephone lines busy at an instance of time is a binomial variate which probability 0.1 that a line is busy. If 10 lines are choosen at random what is the probability that

(i) no line is busy
(ii) all lines are busy
(iii) atleast one line is busy
(iv) atmost two line are busy.

Sol: x: no. of lines busy

$p = 0.1$

$q = 0.9$

$n = 10$

$P(x) = {}^nC_x \, p^x q^{n-x}$

(i) no line is busy

$x = 0$

$P(x=0) = {}^{10}C_0 \, (0.1)^0 \, (0.9)^{10-0}$

$= (0.9)^{10}$

$= 0.3486.$

(ii) all lines are busy

$x = 10$

$P(x=10) = {}^{10}C_{10} \, (0.1)^{10} \, (0.9)^0$

$= 10^{-10}$

(iii) atleast one line is busy

$P(x \geq 1) = P(X_1) - P(x=0)$

$= 1 - 0.3486$

$= 0.6514$

(iv) atmost 2 lines are busy.

$P(x \le 2) = P(0) + P(1) + P(2)$

$= 0.3486 + {}^{10}C_1 (0.1)^1 (0.9)^9$

$+ {}^{10}C_2 (0.1)^2 (0.9)^8$

$= \cancel{1.1234}.$

$= 0.9297$

Q. The probability that pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that

(i) exactly 2 will be defective

(ii) atleast 2 will be defective

(iii) none of them will be defective.

Sol: x: no. of defective pieces

$p = 0.1$

$q = 0.9$

$n = 12$

(i) Exactly 2 will be defective

$x = 2$

$P(x=2) = {}^{12}C_2 \; p^2 \; q^{12-2}$

$= {}^{12}C_2 \; (0.1)^2 (0.9)^{10}$

$= 0.23012$

(ii) atleast 2 will be defictive

$P(x \ge 2) = P(x_i) - (P(x=0) + P(x=1))$

$= 1 - (0.2824 + 0.3765)$

$P(X \geq 2) = 0.341$

(iii) none of them are defective

$P(X=0) = {}^{12}C_0 \; p^0 \; q^{n-0}$

$P(X=0) = 0.2824$

~~$= 0.341$~~

Q. If the chance that one telephone line out of 10 telephone lines are busy at an instanle is 0.2.
(i) what is the chance that 5 of the lines are busy
(ii) what is probability that all lines are busy
(iii) what is the most probable number of busy lines and what is the probability of this number.

$p = 0.2$ $\quad$ x: no. of lines busy.
$q = 0.8$
$n = 10$
$P(X) = {}^nC_x \, p^x \, q^{n-x}$

(i) $x = 5$

$P(X=5) = {}^{10}C_5 \; p^5 \; q^{n-5}$

$= {}^{10}C_5 \, (0.2)^5 \, (0.8)^5$

$= 0.02642$

(ii) $x = 10$

$P(X=10) = {}^{10}C_{10} \, (0.2)^{10} \, (0.8)^0$

$= 1.024 \times 10^{-7}$

Note:
mean is going to give probable number, here mean is nothing but average.

(iii) most probable number means here we have to find mean (average)

$$\text{mean } \mu = np$$
$$= 10 \times 0.2$$
$$\mu = 2$$

so $x = 2$

$$p(x=2) = {}^{10}c_2 \, p^2 \, q^{10-2}$$
$$= {}^{10}c_2 \, (0.2)^2 \, (0.8)^8$$
$$= 0.3019$$

Q. Fitch the binomial distribution for the following.

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Note : we have to take value in terms of intervals

$$\text{Mean} = \mu = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{0+14+40+102+88+40}{\Sigma f_i}$$

$$\mu = 2.84$$

$$\mu = np$$

$$2.84 = 5 \times p$$

$$p = 0.568$$

$$q = 0.432$$

don't put in decimal values

$f(x) = N\,p(x)$ $N = \sum f_i$

$= 100\,[{}^nc_x\, p^x\, q^{n-x}]$

$f(0) = 100[{}^5c_0\, p^0\, q^5] = 1.504 \simeq 2$

$f(1) = 100[5c_1\, p^1\, q^4] = 9.89 \simeq 10$

$f(2) = 100[5c_2\, p^2\, q^3] = 26.01 \simeq 26$

$f(3) = 100[{}^5c_3\, p^3\, q^2] = 34.19 \simeq 34$

$f(4) = 100[{}^5c_4\, p^4\, q^1] = 22.48 \simeq 22$

$f(5) = 100[5c_5\, p^5\, q^0] = 5.91 \simeq 6$

$\overline{\overline{100}}$

Q. Fit in a binomial distribution

2.675
0.446
0.554

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

$n = 6$

$\text{mean} = \mu = \frac{\sum f_i x_i}{\sum f_i}$

$= 2.675$

$\mu = np$

$p = \frac{2.675}{6}$

$p = 0.4458 \simeq 0.446$

$q = 0.554$

$$f(x) = N\,p(x)$$
$$= 200[{}^{n}C_x\, p^x\, q^{n-x}]$$

$f(0) = 200[{}^6C_0\, p^0\, q^6] = 5.78 \simeq 6$

$f(1) = 200[{}^6C_1\, p^1\, q^5] = 27.9 \simeq 28$

$f(2) = 200[{}^6C_2\, p^2\, q^4] = 56.21 \simeq 56$

$f(3) = 200[{}^6C_3\, p^3\, q^3] = 60.33 \simeq 60$

$f(4) = 200[{}^6C_4\, p^4\, q^2] = 36.43 \simeq 36$

$f(5) = 200[{}^6C_5\, p^5\, q^1] = 11.73 \simeq 12$

$f(6) = 200[{}^6C_6\, p^6\, q^0] = 1.57 \simeq \underline{2}$

200

Poission's Distribution

It is a limiting case in which n is very large and p is very small. By making $\underbrace{n \times p}_{m}$ fixed.

PDF of poission's is given by

$$p(x) = \frac{e^{-m}\, m^x}{x!} \qquad \frac{e^{-m}\, m^x}{x!}$$

$\boxed{\text{Mean } (M) = m}$ $\{m = np\}$

$\boxed{\text{Variance } (V) = m}$

(i) $p(x) \geq 0$

(ii) $\Sigma p(x) = 1$

x	0	1	2	3
P(x)	$\frac{e^{-m} m^0}{0!}$	$\frac{e^{-m} m^1}{1!}$	$\frac{e^{-m} m^2}{2!}$	$\frac{e^{-m} m^3}{3!}$

$$\Sigma P(x) = e^{-m}\left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots\right)$$

$$= e^{-m}(e^{m})$$

$$= e^{0}$$

$$= 1$$

Q. A car hire firm has 2 cars which it heads day by day, the no. of demands for a car on each day is a poission variate with mean 1.5. Calculate the probability of days

(i) on which there is no demand

(ii) on which demand is refused.

Sol: $m = 1.5$, x: no. of demands.

(i) $P(x=0) = \frac{e^{-m} m^x}{x!}$

$$= \frac{e^{-1.5} m^0}{0!}$$

$$= 0.223$$

$P(x=0) = 0.2231$

(ii) $P(x>2)$ (when the demand is more than available products then owner will refuse to not give)

$$p(x>2) = \frac{e^{-1.5} m^3}{3!} \quad 1 - \frac{e^{-1.5} m^0}{0!} - \frac{e^{-1.5} m^1}{1!} - \frac{e^{-1.5} m^2}{2!}$$

$$= 1 - 0.2231 - 0.33465 - 0.2509$$

$$P(x>2) = 0.19135$$

Q. In a certain factory turning out ~~blasour~~ ~~grad~~ razer there is a small chance of 0.002 for any blade to be defective. The blades are supplied in a packets of 10. Calculate the approximate no. of packets containing

(i) no. defective

(ii) ~~one~~ defective

(iii) ~~three~~ defective Blades in a consignment of 10000 packets.

Sol: $m = np$

$n = 10$ [we always say n is very ~~small~~ but here we consider 10 because $p = 0.002$ is comparitively very small for 10]

$p = 0.002$

$m = np$

$= 10(0.002)$

$m = 0.02$

x: no of defective blades.

there are 10 blades in one packet,

$p(x=0)$, $p(x=1)$, $p(x=2)$ we get probability of no. of defective pieces containing 10 blades

but w.r.to 10000 we need to multiply by 10000.

(i) $P(X=0) = \frac{e^{-m} m^x}{x!} = \frac{e^{-0.02} (0.02)^0}{0!} = 0.98019$

10000 packets, no. of packets containing defective blades $= P(X=0) \times 10000$

$= 0.98019 \times 10000$

$= 9802$

(ii) $P(X=1) = \frac{e^{-0.02} (0.02)^1}{1} = 0.01960$

among 10000 packets, no of packets containg defective blades $= 0.01960 \times 10000$

$= 196$

(iii) $P(X=2) = \frac{e^{-0.02} (0.02)^2}{2!} = \cancel{0.0}\ 1.96 \times 10^{-4}$

among 10000 packets, no of packets containing defective blades.

$= 1.96 \times 10^{-4} \times 10000$

$= 1.96$

≈ 2

$$\boxed{\text{Mean} = \mu = \frac{1}{\alpha}}$$

$$\boxed{\text{Variance} = V = \frac{1}{\alpha^2}}$$

Q. For the exponential variate x with mean 5, evaluate

(i) $P(0 < x < 1)$

(ii) $P(-\infty < x < 10)$

$$-\infty \underbrace{\qquad\qquad}_{0} 0 \underbrace{\qquad\qquad}_{\alpha e^{-\alpha x}} \infty$$

Sol: $\mu = 5$

$\mu = 1/\alpha$

$\alpha = 1/5$

(i) $P(0 < x < 1) = \int_0^1 f(x)dx$

$= \int_0^1 \alpha e^{-\alpha x} dx$

$= 0.1812$

(ii) $P(-\infty < x < 10) = \int_{-\infty}^{0} f(x)dx + \int_0^{10} f(x)dx$

$= 0 + \int_0^{10} \alpha e^{-\alpha x} dx$

$= 0.8646.$

Q. The duration of telephone conversation has been found to have an exponential distribution with mean 3 mins. Find the probability that the conversation will last

(i) more than one minute

(ii) less than 3 minute

Sol: mean $= \mu = 3$

$\frac{1}{\alpha} = 3$

$\alpha = 1/3$

(i) $P(x>1) = \int_1^\infty f(x)dx$

$= \int_1^\infty \alpha e^{-\alpha x} dx$

$= 1 - \int_0^1 \alpha e^{-\alpha x} dx$

$= 1 - 0.2834$

$= 0.7165$

$\frac{1}{3} \frac{e^{-1/3x}}{-1/3}$

$-\left[e^{-1/3} - e^{1/3}\right]$

(ii) $P(x<3) = \int_{-\infty}^0 f(x)dx + \int_0^3 f(x)dx$

$= 0 + \int_0^3 \frac{1}{3} e^{-1/3x} dx$

$= 0.6321$

Q. In a certain town the duration of shower is exponentially distributed with mean 5 mins. What is the probability that the shower will last for

(i) 10 min or more   0.1353

(ii) less than 10 min   0.8647

(iii) b/n 10 to 12 min   0.0446

Sol: mean $= \mu = 5$

$1/\alpha = 5$

$\alpha = 1/5$

(i) $P(x \geq 10) = \int_{10}^\infty \alpha e^{-\alpha x} dx$

$= \int_{10}^\infty \frac{1}{5} e^{-1/5x} dx$

Q. A certain screw making machine produce on average of 2 defective screws out of 100 and packs them in a packet of box of 500. Find the probability that box contains 15 defective screws.

Ans: [on an average of 2 i.e $m=2$ for 100 i.e $n=100$ so for 500 m value is $m=10$].

For 500, $m=10$

x : no. of defective screws.

$$p(x=15) = \frac{e^{-10}(10)^{15}}{15!}$$

$= 0.0347$

$= 0.035.$

Q. Fit a poissions distribution for the following data.

last

x	0	1	2	3	4
f(y)	46	38	22	9	1

$y = f(x)$

Sol: $$p(x) = \frac{e^{-m} m^x}{x!}$$

$$\text{mean} = m = \frac{\Sigma f_i x_i}{\Sigma f_i} = 0.974$$

$$f(x) = N\, p(x) \quad \{N = \Sigma f_i\}$$

$$f(0) = N\, p(x=0) = 116 \frac{e^{-0.974}(0.974)^0}{0!}$$

$= 43.97 \doteq 44$

$f(1) = N(P(X=1)) = 42.65 \simeq 43$

$f(2) = N(P(x=2)) = 20.68 \simeq 21$

$f(3) = N\ P(x=3) = 6.68 \simeq 7$

$f(4) = N\ P(X=4) = 1.622 \simeq 2$

$\overline{117}$

Continuous Probability distribution.

1) Exponential distribution
2) Normal distribution

1) Exponential distribution :

Let α be a constant greater than 0 then the continuous probability distribution taking the PDF of the form

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$ is exponential distribution.

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx$$

$$= 0 + \int_{0}^{\infty} \alpha e^{-\alpha x}\,dx$$

$$= \alpha \left(\frac{e^{-\alpha x}}{-\alpha}\right)_0^{\infty}$$

$$= -\left[e^{-\infty} - e^{0}\right]$$

$$= 1$$

$$= \frac{1}{5}\left[\frac{e^{-1/5x}}{-1/5}\right]_{10}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-2}\right]$$

$$= 0.1353$$

(ii) $P(X<10) = \int_{-\infty}^{10} f(x)dx$ $= 1 - P(X \geq 10)$

$$= \int_{-\infty}^{0} f(x)dx + \int_{0}^{10} f(x)dx$$ or $= 1 - 0.1353$

$= 0.8647$

$$= \frac{1}{5}\int_{0}^{10} e^{-1/5x}dx$$

$$= 0.8647$$

(iii) $P(10<x<12) = \int_{10}^{12} f(x)\,dx$

$$= \frac{1}{5}\int_{10}^{12} \frac{1}{5} e^{-1/5x} dx$$

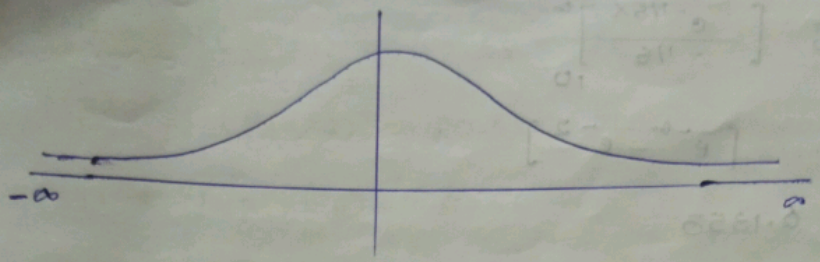
$$= 0.04461.$$

2. <u>Normal distribution.</u>

The PDF of normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The graph of normal distribution is called the normal curve. It is bell shaped and symmetric about the mean μ. The two tails of the curve extend towards positive and negative direction of the x-axis never ever meeting the x-axis.



The area under the normal curve above the x-axis = 1.

Mean = μ and variance = σ^2 = variance

<u>Standard</u> <u>Normal</u> <u>distribution</u> : The normal distribution for which the <u>mean</u> is <u>zero</u> and <u>standard</u> <u>deviation</u> is <u>one</u>. is called standard normal distribution.

$$g(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

This is PDF for Standard Normal distribution.

$z = \frac{x-\mu}{\sigma}$ will help us to shift from Normal to Standard Normal variable.

Q. For the standard normal distribution evaluate the following.

(i) $P(0 \le z \le 1.45)$

(ii) $P(-2.60 \le z \le 0)$

(iii) $P(-3.40 \le z \le 2.65)$

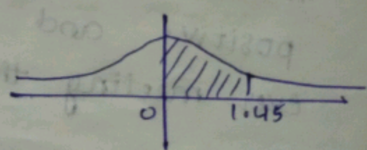
(iv) $P(z \ge 1.7)$

(v) $P(z \le -3.35)$

(i) $P(0 \le z \le 1.45)$

$= A(1.45)$ (in calculator)

$= 0.4265$



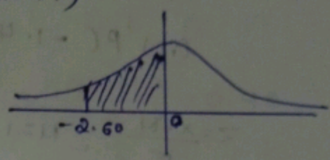
(ii) $P(-2.60 \le z \le 0)$

$= A(-2.60)$ (but dont prefer this although we get same answer because Area is not negative)

or

$= A(2.60)$

$= 0.49534$



(iii) $P(-3.40 \le z \le 2.65)$

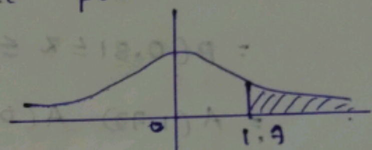
$= A(3.40) + A(2.65)$

$= 0.9956$

(iv) $P(z \ge 1.7)$

$= 0.5 - A(1.7)$ (Here we are only considering area after 0 i.e positive x-axis)

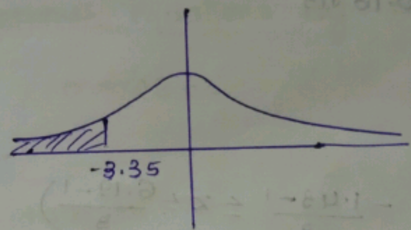
$= 0.04457$



(v) $P(z \le -3.35)$

$= 0.5 - A(3.35)$

$= 0.0004$



Negative values i.e limits we can take as positive side because curve is symmetric if we fold we get same area.

mode setup : Stat mode

clear all : shift +1

5: Distib (② → Area

Note:

whenever we are finding area it has to start from zero.

Q. If x is a normal variate with mean 1 and standard deviation 3. Find

(i) $P(3.43 \le x \le 6.19)$

(ii) $P(-1.43 \le x \le 6.19)$

$Z = \frac{X-\mu}{\sigma}$. $\mu = 1$, $\sigma = 3$

(i) $P(3.43 \le x \le 6.19)$

$= P\left(\frac{3.43-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{6.19-\mu}{\sigma}\right)$

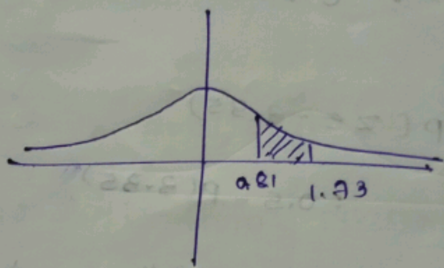
$= P\left(\frac{3.43-1}{3} \le z \le \frac{6.19-1}{3}\right)$

$= P(0.81 \le z \le 1.73)$

$= A(1.73) - A(0.81)$

$= 0.45818 - 0.29103$

$= 0.16715$



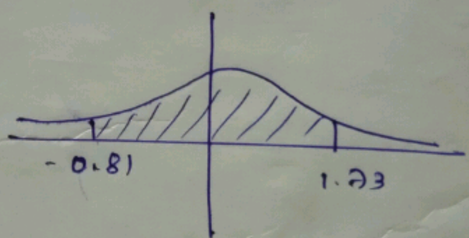
(ii) $P\left(-\frac{1.43-1}{3} \le z \le \frac{6.19-1}{3}\right)$

$= P(-0.81 \le z \le 1.73)$

$= A(0.81) + A(1.73)$

$= 0.29103 + 0.45818$

$= 0.74921$



Q. In a certain examination percentage of candidates passing and getting distinctions were 45 and 9 respectively. Estimate the average marks obtained by the candidate if the minimum pass and distinction marks are being 40 and 75 respectively.

$p(x \geq 75) = 0.09$

$p(x \geq 40) = 0.45$ $\qquad z = \frac{x-\mu}{\sigma}$

$P\left(\frac{x-\mu}{\sigma} \geq \frac{75-\mu}{\sigma}\right) = 0.09$ $\qquad P\left(\frac{x-\mu}{\sigma} \geq \frac{40-\mu}{\sigma}\right) = 0.45$

$P\left(z \geq \frac{75-\mu}{\sigma}\right) = 0.09$ $\qquad P\left(z \geq \frac{40-\mu}{\sigma}\right) = 0.45$

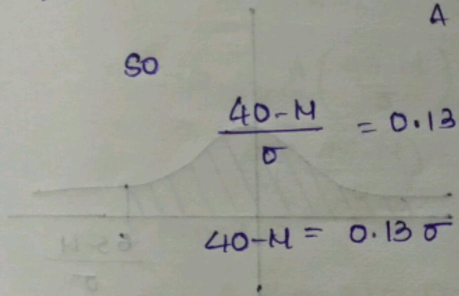
$0.5 - A\left(\frac{75-\mu}{\sigma}\right) = 0.09$ $\qquad 0.5 - A\left(\frac{40-\mu}{\sigma}\right) = 0.45$

$A\left(\frac{75-\mu}{\sigma}\right) = 0.41$ $\qquad A\left(\frac{40-\mu}{\sigma}\right) = 0.05$

we have $A(0.13) = 0.05$

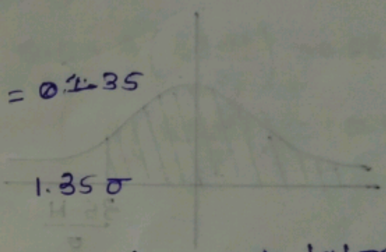
$A(1.35) = 0.41$

so



$\frac{40-\mu}{\sigma} = 0.13$ $\qquad \frac{75-\mu}{\sigma} = 1.35$

$40-\mu = 0.13\sigma$ $\qquad 75-\mu = 1.35\sigma$



By solving simultaneous equations by calculator

$\mu = 36.27$ $\qquad \sigma = 28.68$

Q. The mean height of 500 students is 151cm and the standard deviation is 15cm. Assuming heights to be normally distributed, find how many students heights lie between 120 to 155 cm.

$\mu = 151cm$ $\qquad \sigma = 15cm$

$P(120 < x < 155)$

$z = \frac{x-\mu}{\sigma}$

$P\left(\frac{120-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{155-\mu}{\sigma}\right)$

$P(-2.067 < z < 0.267)$

$= A(2.067) + A(0.267)$

$= 0.48068 + 0.10526$

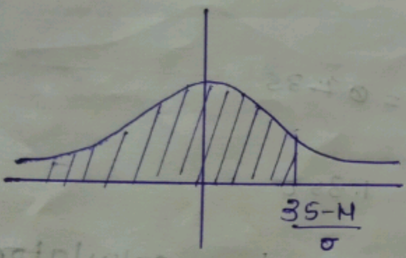
$= 0.58689.$

Q. In a normal distribution, 7% of items are under 35 and 89% are under 63. Find the mean and standard deviation given that $A(1.23) = 0.39$ and $A(1.48) = 0.43$

$P(X<35) = 0.07$

$P\left(\frac{X-\mu}{\sigma} < \frac{35-\mu}{\sigma}\right) = 0.07$

$P\left(z < \frac{35-\mu}{\sigma}\right) = 0.07$



$0.5 + A\left(\frac{35-\mu}{\sigma}\right) = 0.07$

$A\left(\frac{35-\mu}{\sigma}\right) = -0.43$

$A(1.48) = 0.43$

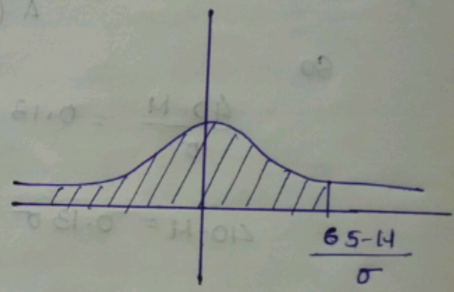
$\frac{35-\mu}{\sigma} = -1.48$

$35 - \mu = -1.48\sigma$

$P(x<63) = 0.89$

$P\left(\frac{x-\mu}{\sigma} < \frac{63-\mu}{\sigma}\right) = 0.89$

$P\left(z < \frac{63-\mu}{\sigma}\right) = 0.89$



$0.5 + A\left(\frac{63-\mu}{\sigma}\right) = 0.89$

$A\left(\frac{63-\mu}{\sigma}\right) = 0.39$

$\frac{63-\mu}{\sigma} = 1.23$

$63 - \mu = 1.23\sigma$

$\mu = 50.29$

$\sigma = 10.332$

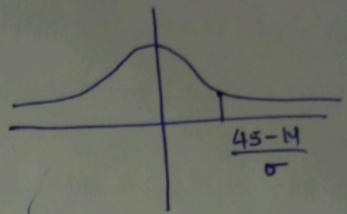
Q. In a normal distribution 31% of items are under 45 and 8% over 64. Find the mean and standard deviation.

$P(x < 45) = 0.31$ $\quad$ $P(x > 64) = 0.08$

$$z = \frac{x-\mu}{\sigma}$$

$$P\left(\frac{x-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.31$$

P(x



A

$$P\left(\frac{x-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.31 \qquad P\left(\frac{x-\mu}{\sigma} > \frac{64-\mu}{\sigma}\right) = 0.08$$

$$P\left(z < \frac{45-\mu}{\sigma}\right) = 0.31 \qquad P\left(z > \frac{64-\mu}{\sigma}\right) = 0.08$$

$$0.5 + A\left(\frac{45-\mu}{\sigma}\right) = 0.31 \qquad 0.5 - A\left(\frac{64-\mu}{\sigma}\right) = 0.08$$

$$A\left(\frac{45-\mu}{\sigma}\right) = -0.19 \qquad A\left(\frac{64-\mu}{\sigma}\right) = 0.42.$$

we have

$A(0.5) = 0.19$

$A(1.4) = 0.42$

so

$$\frac{45-\mu}{\sigma} = -0.5 \qquad \frac{64-\mu}{\sigma} = 1.4$$

$$45-\mu = -0.5\sigma \qquad 64-\mu = 1.4\sigma$$

$\mu = 50 \quad \sigma = 10$