

$$A \in \mathbb{R}^{m \times n}$$

$$\mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$\mathbf{C}(A^T)$$
  
 $\dim = r$

row space  
all  $yA$



perpendicular

$$\mathbb{R}^n$$

nullspace  
 $Ax = 0$

$$\mathbf{N}(A)$$
  
 $\dim = n - r$

$$\mathbb{R}^n = \mathbf{N}(A) + \mathbf{C}(A^T)$$
  

$$\mathbf{N}(A) \perp \mathbf{C}(A^T)$$

$$\mathbf{C}(A)$$
  
 $\dim = r$

column space  
all  $Ax$



perpendicular

$$\mathbb{R}^m$$

left nullspace  
 $yA = 0$

$$\mathbf{N}(A^T)$$
  
 $\dim = m - r$

$$\mathbb{R}^m = \mathbf{C}(A) + \mathbf{N}(A^T)$$
  

$$\mathbf{C}(A) \perp \mathbf{N}(A^T)$$