

## 匀强磁场中带电小球在圆盘上的运动 (50 分)(命题: LJT)

(1) 解: 按定义

$$\vec{\mu} = \frac{1}{2} \int \rho \vec{r} \times \vec{v} dV \quad (1)$$

$$Q = \frac{4}{3} \pi R^3 \rho \quad (2)$$

得

$$\vec{\mu} = \frac{1}{5} Q \vec{\omega} r^2 \quad (3)$$

(2) 解: 由无相对滑动, 故  $\omega_z = 0$

$$\vec{v}_c + (-r\hat{z}) \times \vec{\omega} = (x\hat{x} + y\hat{y}) \times (\Omega\hat{z}) \quad (4)$$

$$\begin{cases} \dot{x} - \omega_y r = -\Omega y \\ \dot{y} + \omega_x r = \Omega x \end{cases} \quad (5)$$

质心运动定理

$$F_x \hat{x} + F_y \hat{y} + Q \vec{v}_c \times (B\hat{z}) = m(\ddot{x}\hat{x} + \ddot{y}\hat{y}) \quad (7)$$

质心转动定理

$$\frac{2}{5} m r^2 \frac{d\vec{\omega}}{dt} = (-r)\hat{z} \times (F_x \hat{x} + F_y \hat{y}) + \vec{\mu} \times \vec{b} \quad (8)$$

$$\begin{cases} rF_y + \mu_y B = \frac{2}{5} m r^2 \dot{\omega}_x \\ -(rF_x + \mu_x B) = \frac{2}{5} m r^2 \dot{\omega}_y \end{cases} \quad (9)$$

消去  $F_x, F_y, \omega_x, \omega_y$ , 得二阶二元线性微分方程组

$$\begin{cases} \frac{7}{5} m r \ddot{y} + \left( \frac{6}{5} Q B r - \frac{2}{5} m r \Omega \right) \dot{x} + \frac{1}{5} Q r B \Omega y = 0 \\ \frac{7}{5} m r \ddot{x} - \left( \frac{6}{5} Q B r - \frac{2}{5} m r \Omega \right) \dot{y} + \frac{1}{5} Q r B \Omega x = 0 \end{cases} \quad (10)$$

令  $\tilde{\xi} = x + y$ , 将 (10) 式中两式相加, 得

$$\frac{7}{5} m r \ddot{\tilde{\xi}} + \left( \frac{6}{5} Q B r - \frac{2}{5} m r \Omega \right) \dot{\tilde{\xi}} + \frac{1}{5} Q B \Omega r \tilde{\xi} = 0 \quad (11)$$

令  $\tilde{\xi} = e^{\omega t}$ , 带入 (11) 式

$$-\frac{7}{5} m r \omega^2 - \left( \frac{6}{5} Q B r - \frac{2}{5} m r \Omega \right) \omega + \frac{1}{5} Q B \Omega r = 0 \quad (12)$$

得

$$\omega_1 = \frac{3QBr - mr\Omega + \sqrt{(3QBr - mr\Omega)^2 + 7mr^2\omega^2QB\Omega}}{-7mr} \quad (13)$$

$$\omega_2 = \frac{3QBr - mr\Omega - \sqrt{(3QBr - mr\Omega)^2 + 7mr^2\omega^2QB\Omega}}{-7mr} \quad (14)$$

将两解线性叠加，并重新拆分为  $x, y$ , 有

$$x = A \cos(\omega_1 t) + B \cos(\omega_2 t)$$

$$y = A \sin(\omega_1 t) + B \sin(\omega_2 t)$$

带入初值

$$\begin{cases} A + B = r_0 \\ A\omega_1 + A\omega_w = \Omega r_0 \end{cases} \quad (15)$$

解得

$$\begin{cases} A = -\frac{r_0(\omega_2 - \Omega)}{\omega_1 - \omega_2} \\ B = \frac{r_0(\omega_1 - \Omega)}{\omega_1 - \omega_2} \end{cases} \quad (16)$$

评分标准:

共 50 分

(1) 共 5 分 (2) 各 1 分, (1), (3) 各 2 分

(2) 共 45 分 (4), (6) 各 2 分, (8), (12), (13), (14), (15), (16) 各 3 分, (5), (7), (9), (10), (11) 4 分, 指出  $\omega_z = 0$  给 3 分