Arago 圆盘 (40 分)(命题人：HJH)

(1)

$$\vec{B}=\frac{\mu_0}{4\pi}\frac{3\left(\vec{\mu}\cdot\hat{r}\right)\hat{r}-\vec{\mu}}{r^3} \tag{1}$$

$$\begin{aligned}\vec{F}&=q\vec{v}\times\vec{B}=q(\vec{\omega}\times\vec{\rho})\times\vec{B}\\&=q\left[\left(\vec{\omega}\cdot\vec{B}\right)\vec{\rho}-\left(\vec{\omega}\cdot\vec{\rho}\right)\vec{B}\right]\\&=q\left(\vec{\omega}\cdot\vec{B}\right)\vec{\rho}\end{aligned} \tag{2}$$

$$\vec{\mu}=\mu\left(\cos\varphi,\sin\varphi,0\right) \tag{3}$$

$$\vec{r}=\left(\rho\cos\theta,\rho\sin\theta,-h\right) \tag{4}$$

$$B_z=\frac{\mu_0}{4\pi}\frac{3\mu\rho\cos(\varphi-\theta)(-h)}{r^5} \tag{5}$$

(2) 认为 $\vec{E}=\vec{v}\times\vec{B}$ 为等效电场

$$\begin{aligned}\vec{j}&=\sigma\vec{E}=\sigma(\vec{\omega}\cdot\vec{B})\vec{r}\\&=\frac{\sigma\mu_0\omega3\mu\rho\cos(\varphi-\theta)(-5)}{u\pi r^5}(\rho\cos\theta,\rho\sin\rho,0)\end{aligned} \tag{6}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\int\frac{\vec{j}\cdot\mathrm{d}V\times(-\vec{r})}{r^3} \tag{7}$$

积分得

$$\vec{B}=\left(\frac{\mu_0}{4\pi}\right)^2\frac{3\sigma\omega\mu h^2\pi}{2}\left[\frac{1}{2}\left(\frac{1}{h^4}-\frac{1}{\left(R^2+h^2\right)^2}\right)+\frac{h^2}{3}\left(-\frac{1}{h^6}+\frac{1}{\left(R^2+h^2\right)^3}\right)\right]\left(-\sin\varphi\hat{x}+\cos\varphi\hat{y}\right) \tag{8}$$

令

$$k=\left(\frac{\mu_0}{4\pi}\right)^2\frac{3\sigma\mu h^2\pi}{2}\left[\frac{1}{2}\left(\frac{1}{h^4}-\frac{1}{\left(R^2+h^2\right)^2}\right)+\frac{h^2}{3}\left(-\frac{1}{h^6}+\frac{1}{\left(R^2+h^2\right)^3}\right)\right] \tag{9}$$

由力矩公式

$$\vec{M}=\vec{v}\times\vec{B}=k\mu\omega\hat{z} \tag{10}$$

评分标准:

共 40 分

(1) 共 10 分 (1), (2), (3), (4), (5) 各 2 分

(2) 共 30 分 (6), (7), (8), (9), (10) 各 6 分