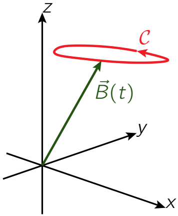
Floquet theory for open quantum systems

Sigmund Kohler

Instituto de Ciencia de Materiales de Madrid, CSIC



- **I.** Floquet theory for Schrödinger equation
- **II.** Dissipation & transport
- **III.** Miscellaneous topics — LZSM interference, time-dependent Liouvillians, hidden symmetries, Floquet-Gibbs states, bichromatic driving, scattering theory, adiabatic Floquet theory, . . .

https://sigmundkohler.github.io/download/FloquetTutorial.pdf

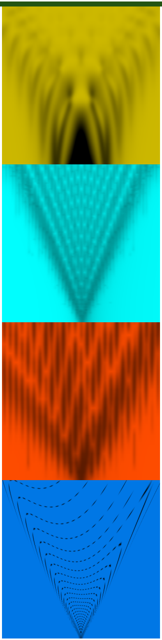
Laser physics since the 1970s

- strong ac fields acting on atoms
  - treatment beyond linear response
- Floquet theory for Schrödinger equation
  - discrete time translation and Floquet ansatz
  - properties of Floquet states
  - relation to Berry phase
  - analytical and numerical techniques
- Floquet-Bloch-Redfield theory
  - dissipation & decoherence
  - Floquet state population
  - efficient computation

Since $\sim$ 2000: qubits + microwaves

Floquet theory for open quantum systems

1. **Floquet & Schrödinger equation**
   - Geometric phases
   - Time-periodicity, Floquet ansatz, and all that
2. **Quantum dissipation and transport**
   - System-bath model
   - Floquet-Bloch-Redfield formalism
3. **Applications & miscellaneous topics**
   - LZSM Interference
   - Time-dependent Liouvillians
   - Floquet scattering theory
   - Adiabatic Floquet theory
   - Hidden symmetries
   - Quantum chaos and dissipation
   - Floquet-Gibbs states
   - Two-color Floquet theory



- Time evolution of an eigenstate ($\hbar = 1$)

$$|\psi(t)\rangle = e^{-iE_n t}|\phi_n\rangle$$

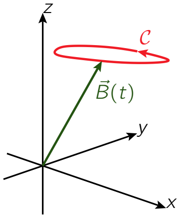
Notation:
ψ: solution of Schrödinger equation
ϕ: other state vector, e.g., eigenstate

- energy $\leftrightarrow$ phase

$\boxed{\text{for (periodically) time-dependent system ?}}$

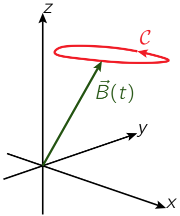
- Spin in magnetic field $\vec{B}(t) = \vec{B}(t+T)$:

$$H(t) = \frac{1}{2}\vec{B}(t) \cdot \vec{\sigma}$$



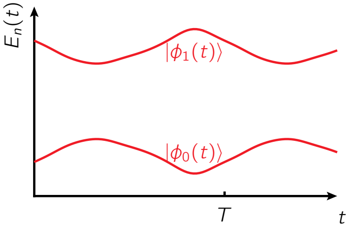
- Spin in magnetic field
  $\vec{B}(t) = \vec{B}(t+T)$:

$$H(t) = \frac{1}{2}\vec{B}(t)\cdot\vec{\sigma}$$



- Quantum dynamics for $\dot{B} \ll B^2$:
  state follows the eigenstate adiabatically

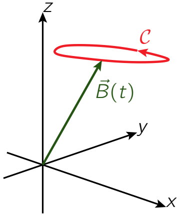
$$|\psi(t)\rangle \propto |\phi_n(t)\rangle$$



➔ $|\psi(t)\rangle$ determined up to phase factor

After one period: $|\psi(T)\rangle = e^{i\varphi}|\psi(0)\rangle$

$$\varphi = -\int_0^T dt\, E_n(t) + \gamma_{\mathcal{C}}$$

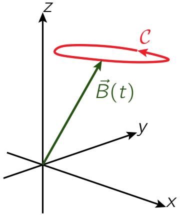


- dynamical phase $\leftrightarrow$ mean energy
- Berry phase $\gamma_{\mathcal{C}}$
  - depends only on closed curve $\mathcal{C}$ in parameter space

M. Berry, Proc. Roy. Soc. London, Ser. A **392**, 45 (1984)

After one period: $|\psi(T)\rangle = e^{i\varphi}|\psi(0)\rangle$

$$\varphi = -\int_0^T dt\, E_n(t) + \gamma_{\mathcal{C}}$$



- dynamical phase $\leftrightarrow$ mean energy
- Berry phase $\gamma_{\mathcal{C}}$
  - depends only on closed curve $\mathcal{C}$ in parameter space

M. Berry, Proc. Roy. Soc. London, Ser. A **392**, 45 (1984)

- Assumptions:
  1. $\vec{B}(t)$ changes adiabatically slowly
  2. initial state: eigenstate $|\phi_n(0)\rangle$

Non-adiabatic generalization

Different perspective:

State vector undergoes periodic time-evolution

- $|\psi(T)\rangle = e^{i\varphi}|\psi(0)\rangle$
- dynamics $|\psi(t)\rangle$ induced by some Hamiltonian $H(t)$

Remarks:

- no adiabatic condition
- $|\psi(t)\rangle$ need not be an eigenstate of $H(t)$
- $H(t)$ is not unique

- only condition: cyclic time-evolution in Hilbert space

$$|\psi(t)\rangle = e^{if(t)}|\phi(t)\rangle, \qquad \phi(t) = \phi(t+T)$$

Non-adiabatic generalization

→ Phase acquired during cyclic evolution: $\varphi = f(T) - f(0)$

$$\frac{df}{dt} = -\langle\phi|H(t)|\phi\rangle + \langle\phi|i\frac{d}{dt}|\phi\rangle \quad \Longrightarrow \quad \boxed{\varphi = \gamma_{\mathrm{dyn}} + \gamma}$$

Non-adiabatic generalization

➔ Phase acquired during cyclic evolution: $\varphi = f(T) - f(0)$

$$\frac{df}{dt} = -\langle\phi|H(t)|\phi\rangle + \langle\phi|i\frac{d}{dt}|\phi\rangle \quad \Longrightarrow \quad \boxed{\varphi = \gamma_{\rm dyn} + \gamma}$$

Dynamical phase $\gamma_{\rm dyn} = -\int_0^T dt\, \langle\phi(t)|H(t)|\phi(t)\rangle$

– depends on choice of $H(t)$
– reflects mean energy

Non-adiabatic generalization

➔ Phase acquired during cyclic evolution: $\varphi = f(T) - f(0)$

$$\frac{df}{dt} = -\langle\phi|H(t)|\phi\rangle + \langle\phi|i\frac{d}{dt}|\phi\rangle \quad \Longrightarrow \quad \boxed{\varphi = \gamma_{\text{dyn}} + \gamma}$$

Dynamical phase $\gamma_{\text{dyn}} = -\int_0^T dt\, \langle\phi(t)|H(t)|\phi(t)\rangle$

- depends on choice of $H(t)$
- reflects mean energy

Aharonov-Anandan phase ("non-adiabatic Berry phase")

$$\gamma = \int_0^T dt\, \langle\phi|i\frac{d}{dt}|\phi\rangle$$

- depends only on trajectory in Hilbert space — not in parameter space!
- adiabatic limit: $\gamma = \gamma_{\mathcal{C}}$

Aharonov & Anandan, PRL 1987

Goal: propagator $U(t, t')$

- Time-independent system: diagonalize Hamiltonian ➔ $|\phi_n\rangle$, E_n

$$U(t, t') = U(t - t') = \sum_n e^{-iE_n(t-t')}|\phi_n\rangle\langle\phi_n|$$

Goal: propagator $U(t,t')$

- Time-independent system: diagonalize Hamiltonian ➔ $|\phi_n\rangle$, E_n

$$U(t,t') = U(t-t') = \sum_n e^{-iE_n(t-t')} |\phi_n\rangle\langle\phi_n|$$

- Driven system:

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle \quad \text{➔ numerical integration}$$

problem 1: time-integration not efficient for long times

problem 2: no information about structure of U

Time-dependent Schrödinger equation

Goal: propagator $U(t, t')$

- Time-independent system: diagonalize Hamiltonian ➔ $|\phi_n\rangle$, E_n

$$U(t, t') = U(t - t') = \sum_n e^{-iE_n(t-t')} |\phi_n\rangle\langle\phi_n|$$

- Driven system:

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle$$ ➔ numerical integration

problem 1: time-integration not efficient for long times

problem 2: no information about structure of U

- Solution for $H(t) = H(t + T)$: "Bloch theory in time"
  cf. $H(x)|\phi\rangle = \epsilon|\phi\rangle$ with $H(x) = H(x + a)$
  ➔ Bloch waves $\phi(x) = e^{iqx}\varphi(x)$, where $\varphi(x)$ is a-periodic

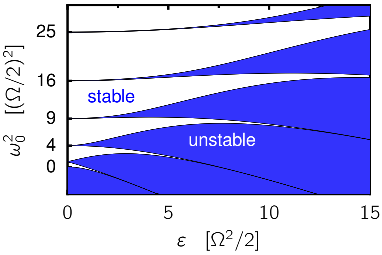
F. Bloch, Z. Phys. A **52**, 555 (1928)

Gaston Floquet (1883):

Ann. de l'Ecole Norm. Sup. **12**, 47 (1883)

Parametric oscillator (cf. Paul trap)

$$\ddot{x} + (\omega_0^2 + \epsilon \cos \Omega t)x = 0$$



Floquet theorem:
solutions have the structure

$$x(t) = e^{\pm i\mu t}\xi(t)$$

where $\xi(t) = \xi(t + 2\pi/\Omega)$

(undriven limit: $\mu = \omega_0$, $\xi = \text{const}$)

μ real
➔ oscillating solutions

μ imaginary
➔ one solution unstable

- $H(t) = H(t+T)$
  ➔ $t \to t+T$ is symmetry operation
  ➔ solutions of Schrödinger equation with $|\psi(t{+}T)\rangle = e^{i\varphi}|\psi(t)\rangle$
- Floquet ansatz

$$|\psi(t)\rangle = e^{-i\epsilon t}|\phi(t)\rangle = e^{-i\epsilon t}\sum_k e^{-ik\Omega t}|\phi_k\rangle$$

  - ϵ quasienergy (cf. quasi momentum) ➔ long-time dynamics
  - $|\phi(t)\rangle = |\phi(t+T)\rangle$, Floquet state ➔ within driving period
- Floquet theorem: $H(t)$ has a complete set of Floquet solutions

- Schrödinger equation $i\partial_t|\psi\rangle = H(t)|\psi\rangle$ yields

$$\boxed{\left(H(t) - i\frac{d}{dt}\right)|\phi(t)\rangle = \epsilon|\phi(t)\rangle}$$

Brillouin zone structure

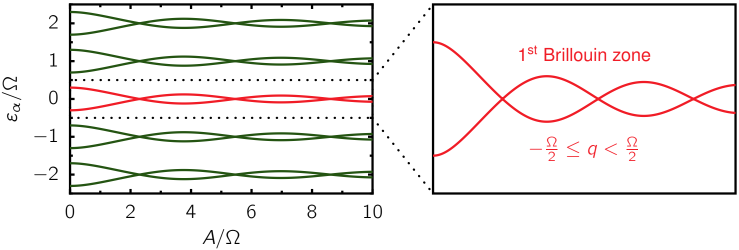
$|\phi(t)\rangle$ Floquet state with quasienergy ϵ

- ➔ $e^{ik\Omega t}|\phi(t)\rangle$ Floquet state with $\epsilon + k\Omega$
- ➔ Equivalent states: all have the same $|\psi(t)\rangle$

Brillouin zone structure

$|\phi(t)\rangle$ Floquet state with quasienergy ϵ

- ➔ $e^{ik\Omega t}|\phi(t)\rangle$ Floquet state with $\epsilon + k\Omega$
- ➔ Equivalent states: all have the same $|\psi(t)\rangle$



- all Brillouin zones equivalent, choice arbitrary
- ➔ quasienergies cannot serve for ordering!

- Physical quantity / observable: mean energy

$$E = \frac{1}{T}\int_0^T dt\,\langle\psi(t)|H(t)|\psi(t)\rangle = \frac{1}{T}\int_0^T dt\,\langle\phi(t)|H(t)|\phi(t)\rangle$$

- All equivalent states have the same mean energy [proof: insert $e^{ik\Omega t}|\phi(t)\rangle$]

➔ Floquet states can be ordered by their mean energy

- Mean energy

$$E = \frac{1}{T}\int_0^T dt\, \langle\phi(t)|\{H(t) - i\partial_t + i\partial_t\}|\phi(t)\rangle$$

where $(H - i\partial_t)|\phi(t)\rangle = \epsilon|\phi(t)\rangle$

$$E = \epsilon + \frac{1}{T}\int_0^T dt\, \langle\phi(t)|i\partial_t|\phi(t)\rangle$$

- Mean energy

$$E = \frac{1}{T}\int_0^T dt\,\langle\phi(t)|\{H(t) - i\partial_t + i\partial_t\}|\phi(t)\rangle$$

where $(H - i\partial_t)|\phi(t)\rangle = \epsilon|\phi(t)\rangle$

$$-\epsilon = -E + \frac{1}{T}\int_0^T dt\,\langle\phi(t)|\, i\partial_t\,|\phi(t)\rangle$$

- Compare to $\qquad \varphi = \gamma_{\mathrm{dyn}} + \gamma$

- Mean energy

$$E = \frac{1}{T}\int_0^T dt\, \langle \phi(t)|\left\{H(t) - i\partial_t + i\partial_t\right\}|\phi(t)\rangle$$

where $(H - i\partial_t)|\phi(t)\rangle = \epsilon|\phi(t)\rangle$

$$-\epsilon = -E + \frac{1}{T}\int_0^T dt\, \langle \phi(t)|\, i\partial_t\, |\phi(t)\rangle$$

- Compare to $\varphi = \gamma_{\text{dyn}} + \gamma$

$$\boxed{(E - \epsilon)T \text{ is a geometric phase}}$$

Floquet equation as function of Ω and Ωt:

$$\epsilon(\Omega)\phi(\Omega t) = \left[H(\Omega t) - i\Omega\frac{\partial}{\partial \Omega t}\right]\phi(\Omega t)$$

Floquet equation as function of Ω and Ωt:

$$\epsilon(\Omega)\phi(\Omega t) = \left[H(\Omega t) - i\Omega\frac{\partial}{\partial \Omega t}\right]\phi(\Omega t)$$

Compute derivative $(\partial/\partial\Omega)\big|_{\Omega t}$ and apply $\int \frac{dt}{T}\,\phi^+$ to obtain

$$\frac{\partial\epsilon}{\partial\Omega} = -\frac{\gamma}{2\pi} = \frac{\epsilon - E}{\Omega} \quad \rightarrow \quad \boxed{E = \epsilon - \Omega\frac{\partial\epsilon}{\partial\Omega}}$$

Floquet equation as function of Ω and Ωt:

$$\epsilon(\Omega)\phi(\Omega t) = \left[H(\Omega t) - i\Omega\frac{\partial}{\partial\Omega t}\right]\phi(\Omega t)$$

Compute derivative $(\partial/\partial\Omega)\big|_{\Omega t}$ and apply $\int \frac{dt}{T}\,\phi^{+}$ to obtain

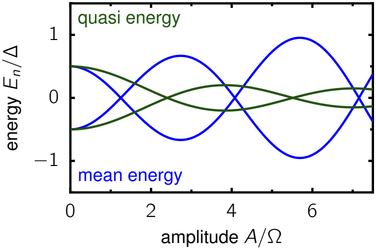
$$\frac{\partial\epsilon}{\partial\Omega} = -\frac{\gamma}{2\pi} = \frac{\epsilon - E}{\Omega} \quad \rightarrow \quad \boxed{E = \epsilon - \Omega\frac{\partial\epsilon}{\partial\Omega}}$$

E.g. for $\gamma = \gamma_{\mathrm{ad}} + \mathcal{O}(\Omega)$

→ $\epsilon = \mathrm{const} - (\gamma_{\mathrm{ad}}/2\pi)\Omega + \ldots$ → $E = E_{\mathrm{ad}} + \mathcal{O}(\Omega^2)$

Fainshtein, Manakov, Rapoport, J. Phys. B (1978)

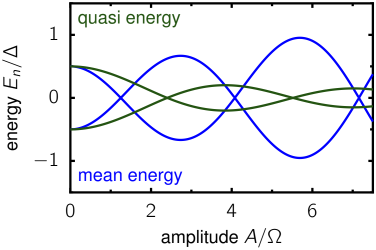
Mean energies — two-level system



Driven undetuned two-level system

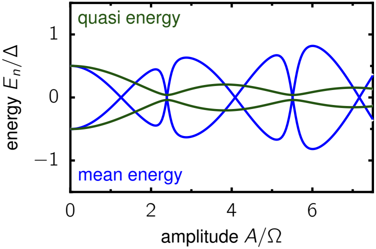
- exact crossings
  (consequence of a symmetry)

Mean energies — two-level system



Driven undetuned two-level system

- exact crossings
  (consequence of a symmetry)



. . . with small detuning

- quasi energies
  - avoided crossings
- mean energies
  - exact crossings remain
  - additional crossings

  ➔ do not follow from any eigenvalue equation

Goal: more formal treatment of $H(t) - i\partial_t$

- $|\phi(t)\rangle \in \mathcal{R} \otimes \mathcal{T}$ composite Hilbert space / Sambe space

  Shirley, PR **138**, B979 (1965), Sambe, PRA **7**, 2203 (1973)

  $\mathcal{T}$: Hilbert space of T-periodic functions with inner product

$$\langle f|g\rangle = \int_0^T f(t)^*\, g(t) \frac{dt}{T} = \sum_k f_k^* g_k$$

- extended Dirac notation:
  - $|\phi(t)\rangle = \langle t|\phi\rangle\rangle$
  - Fourier coefficient $|\phi_k\rangle = \langle k|\phi\rangle\rangle$

e.g.: $|\phi(t)\rangle = \langle t|\phi\rangle\rangle = \sum_k \langle t|k\rangle\langle k|\phi\rangle\rangle = \sum_k e^{-ik\Omega t}|\phi_k\rangle$

Completeness and Orthogonality

- $H - i\partial_t$ is hermitian (in Sambe space!)

→ Floquet states $|\phi_\alpha\rangle$ orthonormal and complete in $\mathcal{R} \otimes \mathcal{T}$

$$\langle\langle \phi_\alpha^{(k)} | \phi_\beta^{(k')} \rangle\rangle = \delta_{\alpha\beta}\delta_{kk'}$$

? but in $\mathcal{R}$?

- $H - i\partial_t$ is hermitian (in Sambe space!)

→ Floquet states $|\phi_\alpha\rangle$ orthonormal and complete in $\mathcal{R} \otimes \mathcal{T}$

$$\langle\langle \phi_\alpha^{(k)} | \phi_\beta^{(k')} \rangle\rangle = \delta_{\alpha\beta}\delta_{kk'}$$

? but in $\mathcal{R}$?

- Consider $\langle \phi_\alpha(t) | \phi_\beta(t) \rangle = \sum_k \lambda_k e^{-ik\Omega t}$ since T-periodic

  with the Fourier coefficient

$$\lambda_k = \frac{1}{T}\int_0^T dt\, e^{ik\Omega t} \langle \phi_\alpha(t) | \phi_\beta(t) \rangle = \langle\langle \phi_\alpha | \phi_\beta^{(k)} \rangle\rangle = \delta_{\alpha\beta}\delta_{k,0}$$

→ Floquet states orthogonal at equal times

- propagator in terms of Floquet states

$$U(t,t') = \sum_\alpha |\psi_\alpha(t)\rangle\langle\psi_\alpha(t')| = \sum_\alpha e^{-i\epsilon_\alpha(t-t')}|\phi_\alpha(t)\rangle\langle\phi_\alpha(t')|$$

  – long-time dynamics (depends on $t - t'$)
  – dynamics within driving period (“micro motion”, depends on t and t')

Propagator

- propagator in terms of Floquet states

$$U(t,t') = \sum_\alpha |\psi_\alpha(t)\rangle\langle\psi_\alpha(t')| = \sum_\alpha e^{-i\epsilon_\alpha(t-t')} |\phi_\alpha(t)\rangle\langle\phi_\alpha(t')|$$

  - long-time dynamics (depends on $t - t'$)
  - dynamics within driving period ("micro motion", depends on t and t')

- one-period propagator for kicked systems

$$H(t) = H_0 + K \sum_n \delta(t - nT)$$

→ $U(T) = e^{-iH_0 T} e^{-iK}$

✓ easy to compute
✓ provides quasienergies
✗ only long-time dynamics (stroboscopic)

Solve eigenvalue problem

$$\left\{H(t) - i\partial_t\right\}|\phi\rangle\rangle = \epsilon|\phi\rangle\rangle$$

Solve eigenvalue problem

$$\left\{H(t) - i\partial_t\right\}|\phi\rangle\rangle = \epsilon|\phi\rangle\rangle$$

Straightforward in Fourier representation (“Floquet matrix”)

$$H_0 + H_1\cos(\Omega t) - i\frac{d}{dt} \leftrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & H_0 + 2\Omega & \frac{1}{2}H_1 & 0 & 0 & 0 & \cdots \\ \cdots & \frac{1}{2}H_1 & H_0 + \Omega & \frac{1}{2}H_1 & 0 & 0 & \cdots \\ \cdots & 0 & \frac{1}{2}H_1 & H_0 & \frac{1}{2}H_1 & 0 & \cdots \\ \cdots & 0 & 0 & \frac{1}{2}H_1 & H_0 - \Omega & \frac{1}{2}H_1 & \cdots \\ \cdots & 0 & 0 & 0 & \frac{1}{2}H_1 & H_0 - 2\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Computation of Floquet states

1. direct diagonalization of $H(t) - i\partial_t$
   - conceptually simple ➔ first choice
   - increasingly difficult with smaller frequency
2. analytical tool: perturbation theory
   strong driving: $H_1 \cos(\Omega t) - i\partial_t$ as zeroth order

Computation of Floquet states

1. direct diagonalization of $H(t) - i\partial_t$
   - conceptually simple ➔ first choice
   - increasingly difficult with smaller frequency
2. analytical tool: perturbation theory
   strong driving: $H_1 \cos(\Omega t) - i\partial_t$ as zeroth order

3. numerical propagation: $U(T, 0)$ ➔ $e^{-i\epsilon T}$ and $|\phi(0)\rangle$ ➔ $|\phi(t)\rangle$
4. matrix-continued fractions (convenient for time-dep. Liouvillians)
5. (t, t') formalism (very efficient propagation scheme)

Floquet theory as semiclassical limit

Rabi-like Hamiltonian

$$H = \Omega a^\dagger a + g(a^\dagger + a)X + \dots$$

Rabi-like Hamiltonian

$$H = \Omega a^\dagger a + g(a^\dagger + a)X + \dots$$

- Fock basis $|n_0 - k\rangle$ with $n_0 \approx \langle a^\dagger a\rangle$
- for $k \ll n_0$:

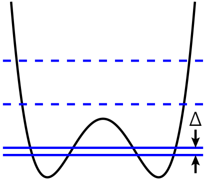
$$a \longrightarrow \sqrt{n_0}\,\delta_{k,k'-1}$$
$$H \longrightarrow (n_0 - k)\Omega\delta_{k,k'} + 2g\sqrt{n_0}\,\delta_{k,k'\pm 1}X + \dots$$

➔ Floquet Hamiltonian with $A = 2g\sqrt{n_0}$

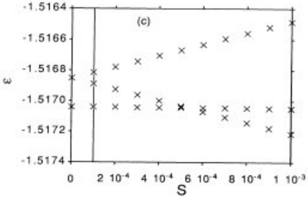
Example I: Coherent destruction of tunneling

1. role of quasienergy crossings
2. perturbation theory (two-level approximation)

Driven double-well potential $H(t) = H_{\mathrm{DW}} + Sx\cos(\Omega t)$



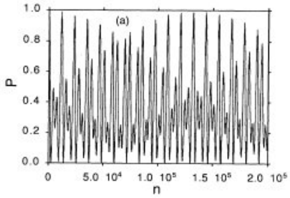
quasienergies $\epsilon_\alpha(S)$

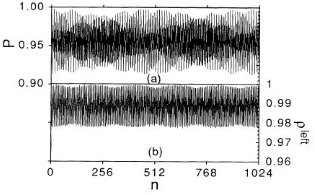


? tunnel oscillations influenced by driving

? dynamics at quasienergy crossing

Occupation $P_{\text{left}}(nT)$





far from crossing:

- tunnel oscillations

at crossing:

- particle stays in left well

➔ "coherent destruction of tunneling" by ac field

Grossmann *et al.*, PRL 1991

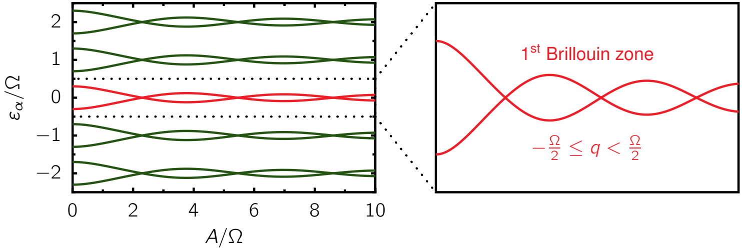
Analytical understanding ➔ two-level approximation

Example I: CDT — two-level approach

Driven two-level system

$$H(t) = -\frac{\Delta}{2}\sigma_x + \frac{A}{2}\cos(\Omega t)\sigma_z$$

quasienergy spectrum



Example I: CDT — perturbation theory

Analytical approach for $\Delta \ll \Omega$: high-frequency limit

$$H(t) = -\frac{\Delta}{2}\sigma_x + \frac{A}{2}\cos(\Omega t)\sigma_z$$

- zeroth order propagator

$$U_{\mathrm{dr}}(t,0) = \exp\left(-\frac{iA}{2\Omega}\sin(\Omega t)\sigma_z\right)$$

- transformation and rotating-wave approximation (i.e. time-average)

$$H(t) \longrightarrow -\frac{\Delta}{2}U_{\mathrm{dr}}^{\dagger}(t,0)\sigma_x U_{\mathrm{dr}}(t,0) \longrightarrow -\frac{\Delta}{2}J_0(A/\Omega)\sigma_x$$

➔ tunnel-matrix element renormalized by Bessel function J_0

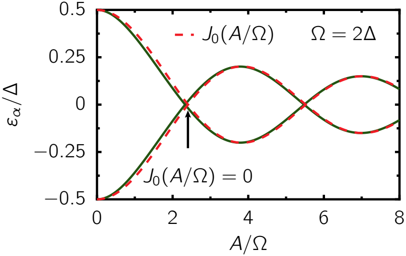
Bessel function $J_n(x)$: nth Fourier coefficent of $e^{-ix\sin(\Omega t)}$

Example I: CDT — perturbation theory

$$H_{\text{eff}} = \frac{\tilde{\Delta}}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

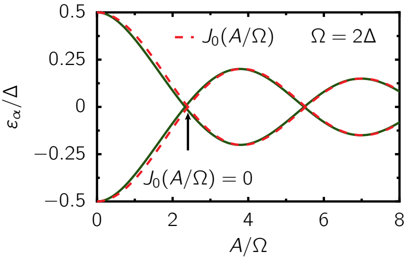
- quasienergies

$$\epsilon_{\pm} = \pm \frac{\Delta}{2} J_0(A/\Omega)$$



Example I: CDT — perturbation theory

$$H_{\text{eff}} = \frac{\tilde{\Delta}}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



- quasienergies

$$\epsilon_\pm = \pm\frac{\Delta}{2} J_0(A/\Omega)$$

- mean energies $E = \epsilon - \Omega(\partial\epsilon/\partial\Omega)$

$$E_\pm = \pm\frac{\Delta}{2}\left[J_0(A/\Omega) - \frac{A}{\Omega}J_1(A/\Omega)\right]$$

no need to compute Floquet states!

Bessel function $J_1(x) = \frac{d}{dx}J_0(x)$

$$H(t) = -\frac{\Delta}{2}\sigma_z + \frac{A}{2}\sigma_x \cos(\Omega t)$$

- close to resonance: $\delta = \Delta - \Omega \ll \Delta$, small amplitude: $A \ll \Delta$

Example II: Rabi problem beyond RWA

$$H(t) = -\frac{\Delta}{2}\sigma_z + \frac{A}{2}\sigma_x \cos(\Omega t)$$

- close to resonance: $\delta = \Delta - \Omega \ll \Delta$, small amplitude: $A \ll \Delta$

$$\mathcal{H}_0 = \frac{\Omega}{2}\sigma_z - i\frac{\partial}{\partial t} \qquad \mathcal{H}_1 = \frac{\delta}{2}\sigma_z + \frac{A}{2}\cos(\Omega t)\sigma_x$$

- 2nd order perturbation theory

$$\mathcal{H} \approx \frac{1}{2}\begin{pmatrix} \delta + A^2/16\Omega & A/2 \\ A/2 & -\delta - A^2/16\Omega \end{pmatrix}$$

Rabi Hamiltonian beyond RWA

- absorption maximum at avoided crossing: $\Delta - \Omega_{\text{res}} + A^2/16\Omega_{\text{res}} = 0$:

$$\Omega_{\text{res}} \approx \Delta + \frac{A^2}{16\Delta}$$

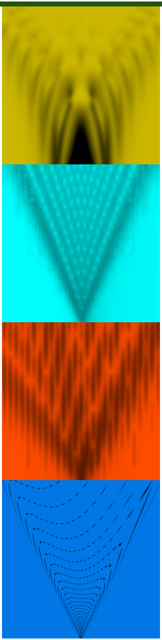
Bloch-Siegert shift

Some standard references

- Classic work:
  - Shirley, Phys. Rev. 138, B979 (1965)
  - Sambe, Phys. Rev. A 7, 2203 (1973)
- Reviews:
  - Grifoni, Hänggi, Phys. Rep. 304, 229 (1998)
  - Hänggi, Chap.5 of “Quantum transport and dissipation” (1998)
    http://www.physik.uni-augsburg.de/theo1/hanggi/Papers/Chapter5.pdf
  - Eckardt, Rev. Mod. Phys. 89, 011004 (2017)

1. Compute numerically the quasienergies of the driven TLS
2. Perform the corresponding perturbation theory for $\Delta \ll \Omega$
3. Derive the Rapoport relation $E = \epsilon - \Omega \frac{\partial \epsilon}{\partial \Omega}$
4. Compute the stability borders of the Mathieu equation
   Hint: assume integer μ and compute the corresponding ω_0^2, why does this work?
5. Given $H(t)$ and an adiabatic eigenstate $|u(t)\rangle$ with energy $E(t)$.
   Write the ansatz for adiabatic following as Floquet state.
   Which gauge transformations are still allowed?

1. **Floquet & Schrödinger equation**
   - Geometric phases
   - Time-periodicity, Floquet ansatz, and all that
2. **Quantum dissipation and transport**
   - System-bath model
   - Floquet-Bloch-Redfield formalism
3. **Applications & miscellaneous topics**
   - LZSM Interference
   - Time-dependent Liouvillians
   - Floquet scattering theory
   - Adiabatic Floquet theory
   - Hidden symmetries
   - Quantum chaos and dissipation
   - Floquet-Gibbs states
   - Two-color Floquet theory



Heuristic approach

coupling of qubit to electromagnetic environment ➔ sponaneous decay

$$|\psi\rangle \longrightarrow \begin{cases} \sigma_-|\psi\rangle & \text{decay with probability } \alpha \ll 1 \\ |\psi\rangle + |\delta\psi\rangle & \text{no decay, probability } 1-\alpha \end{cases}$$

- normalization requires $|\delta\psi\rangle = \frac{\alpha}{2}\sigma_+\sigma_-|\psi\rangle$

Heuristic approach

coupling of qubit to electromagnetic environment ➔ sponaneous decay

$$|\psi\rangle \longrightarrow \begin{cases} \sigma_-|\psi\rangle & \text{decay with probability } \alpha \ll 1 \\ |\psi\rangle + |\delta\psi\rangle & \text{no decay, probability } 1-\alpha \end{cases}$$

- normalization requires $|\delta\psi\rangle = \frac{\alpha}{2}\sigma_+\sigma_-|\psi\rangle$
- corresponding density operator

$$\rho \longrightarrow \rho + \frac{\alpha}{2}\left(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\right)$$

- continuous limit with coherent time-evolution ➔ master equation

$$\frac{d}{dt}\rho = -i[H,\rho] + \frac{\gamma}{2}\left(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\right)$$

Lindblad form

Time evolution must conserve

- hermiticity and trace of ρ
- positivity (all eigenvalues of $\rho \geq 0$)

Time evolution must conserve

- hermiticity and trace of ρ
- positivity (all eigenvalues of $\rho \geq 0$)

Fulfilled by a Markovian master equation iff of “Lindblad form”

$$\frac{d}{dt}\rho = -i[H,\rho] + \sum_n \gamma_n \left(2Q_n\rho Q_n^\dagger - Q_n^\dagger Q_n\rho - \rho Q_n^\dagger Q_n\right)$$

G. Lindblad, Comm. Math. Phys. **48**, 119 (1976)

V. Gorini, J. Math. Phys. **17**, 821 (1976)

- Interpretation: incoherent transitions $|\psi\rangle \to Q_n|\psi\rangle$

Lindblad form

Time evolution must conserve

- hermiticity and trace of ρ
- positivity (all eigenvalues of $\rho \geq 0$)

Fulfilled by a Markovian master equation iff of "Lindblad form"

$$\frac{d}{dt}\rho = -i[H,\rho] + \sum_n \gamma_n \left(2Q_n \rho Q_n^\dagger - Q_n^\dagger Q_n \rho - \rho Q_n^\dagger Q_n \right)$$

G. Lindblad, Comm. Math. Phys. **48**, 119 (1976)

V. Gorini, J. Math. Phys. **17**, 821 (1976)

- Interpretation: incoherent transitions $|\psi\rangle \to Q_n|\psi\rangle$

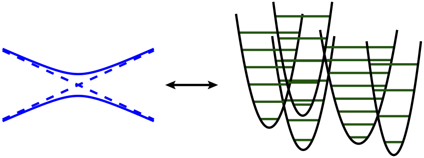
✗ Critique

- request for Markovian evolution unphysical
- axiomatic, not based on physical model
- high-temperature limit typically wrong
  i.e. not the Klein-Kramers or the Smoluchowski equation

Caldeira-Leggett model

Magalinskii 1959; Caldeira, Leggett 1981

Coupling of a system to bath of harmonic oscillators



$$H = H_{\text{system}}(t) + X\sum_\nu \gamma_\nu (b_\nu^\dagger + b_\nu) + \sum_\nu \omega_\nu b_\nu^\dagger b_\nu$$

- ➔ eliminate bath
- ➔ equation of motion for reduced density operator
- ■ interpretation: bath “measures” system operator X

Total density operator $R \approx \rho \otimes \rho_{\text{bath,eq}}$

$$\dot{R} = -i[H_{\text{total}}, R]$$

2nd order perturbation theory in system-bath coupling

$$\frac{d}{dt}\rho = -i[H_{\text{sys}}, \rho] - i\int_0^{(t-t_0)\to\infty} d\tau\,\mathcal{A}(\tau)[X, [\tilde{X}(-\tau), \rho(t-\tau)]_+] - \int_0^{(t-t_0)\to\infty} d\tau\,\mathcal{S}(\tau)[X, [\tilde{X}(-\tau), \rho(t-\tau)]]$$

- Heisenberg operator $\tilde{X}(-\tau) = U(\tau)\,X\,U^\dagger(\tau)$
- bath correlation functions $\mathcal{A}$, $\mathcal{S}$
- non-Markovian
- short system-bath correlation time: Markov approximation

- anti-symmetric correlation function

$$\mathcal{A}(\tau) = -i\langle[\xi(\tau), \xi(0)]\rangle$$

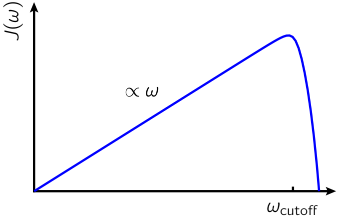
- Fourier transformed: spectral density $\longrightarrow$ continuum limit

$$\mathcal{A}(\omega) = \pi \sum_{\nu} |\gamma_\nu|^2 \delta(\omega - \omega_\nu) \longrightarrow J(\omega)$$

  - here: Ohmic with cutoff

$$J(\omega) = 2\pi\alpha\omega e^{-\omega/\omega_{\mathrm{cutoff}}}$$

  - dimensionless dissipation strength α

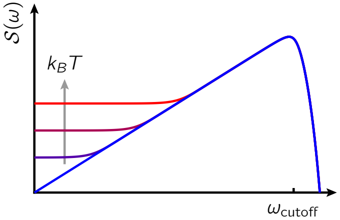


- symmetric bath correlation function

$$\mathcal{S}(\tau) = \frac{1}{2}\langle[\xi(\tau),\xi(0)]_+\rangle$$

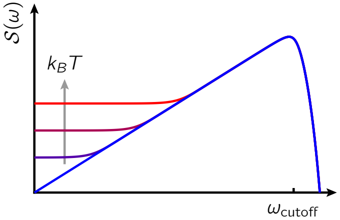
$$\mathcal{S}(\omega) = J(\omega)\coth\left(\frac{\omega}{2k_BT}\right)$$

$$= \begin{cases} 4\pi\alpha k_\mathrm{B}T & \text{high } k_BT \\ 2\pi\alpha\omega & \text{low } k_BT \end{cases}$$



- symmetric bath correlation function

$$\mathcal{S}(\tau) = \frac{1}{2}\langle[\xi(\tau), \xi(0)]_+\rangle$$
$$\mathcal{S}(\omega) = J(\omega)\coth\left(\frac{\omega}{2k_B T}\right)$$
$$= \begin{cases} 4\pi\alpha k_\mathrm{B} T & \text{high } k_B T \\ 2\pi\alpha\omega & \text{low } k_B T \end{cases}$$



- $\mathcal{S}(\omega)$ evaluated at transition frequencies

➔ dissipation strength depends on spectrum / coherent dynamics

- Ohmic, short memory times (e.g. for $\gamma < k_{\mathrm{B}} T$)
  ➔ Bloch-Redfield master equation

$$\dot{\rho} = -i[H_{\mathrm{S}}, \rho] + i\gamma[X, \{[H_{\mathrm{S}}, X], \rho\}] - [X, [Q, \rho]]$$

coherent dynamics dissipation decoherence

coherent dynamics enters via $Q = \int_0^\infty d\tau\, \mathcal{S}(\tau)\, \tilde{X}(-\tau)$

- Ohmic, short memory times (e.g. for $\gamma < k_\mathrm{B} T$)
  ➔ Bloch-Redfield master equation

$$\dot{\rho} = -i[H_\mathrm{S}, \rho] + i\gamma[X, \{[H_\mathrm{S}, X], \rho\}] - [X, [Q, \rho]]$$

coherent dynamics dissipation decoherence

coherent dynamics enters via $Q = \int_0^\infty d\tau\, \mathcal{S}(\tau)\, \tilde{X}(-\tau)$

- not of Lindblad form
  - ✗ positivity might be violated
  - ✓ happens only on unphysically small time scales
- high-temperature limit: Fokker-Planck equation

Pauli master equation

- Decomposition into energy basis and rotating-wave approximation

➔ rate equation for the populations (Pauli master equation)

$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\alpha'} \left[w_{\alpha\leftarrow\alpha'}\,\rho_{\alpha'\alpha'} - w_{\alpha'\leftarrow\alpha}\,\rho_{\alpha\alpha} \right]$$

with the golden-rule rates

$$w_{\alpha\leftarrow\alpha'} = J(E_\alpha - E_{\alpha'})\,\left|\langle\phi_\alpha|X|\phi_{\alpha'}\rangle\right|^2 n_{\text{th}}(E_\alpha - E_{\alpha'})$$

– notice: $-n_{\text{th}}(-\omega) = n_{\text{th}}(\omega)+1$

Pauli master equation

- Decomposition into energy basis and rotating-wave approximation

➔ rate equation for the populations (Pauli master equation)

$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\alpha'} \left[w_{\alpha\leftarrow\alpha'}\, \rho_{\alpha'\alpha'} - w_{\alpha'\leftarrow\alpha}\, \rho_{\alpha\alpha} \right]$$

with the golden-rule rates

$$w_{\alpha\leftarrow\alpha'} = J(E_\alpha - E_{\alpha'})\, \left| \langle \phi_\alpha | X | \phi_{\alpha'} \rangle \right|^2 n_{\mathrm{th}}(E_\alpha - E_{\alpha'})$$

- notice: $-n_{\mathrm{th}}(-\omega) = n_{\mathrm{th}}(\omega) + 1$

✓ detailed balance: $\dfrac{w_{\alpha\leftarrow\alpha'}}{w_{\alpha'\leftarrow\alpha}} = e^{-(E_\alpha - E_{\alpha'})/k_B T}$

✓ Lindblad form

✗ high-temperature limit typically wrong

full Bloch-Redfield: golden rule for non-diagonal $\rho_{\alpha\beta}$

Driven system ➔ noise term becomes time-dependent

$$\dot{\rho} = \ldots - [X, [Q(t), \rho]], \quad Q(t) = \int_0^\infty d\tau\, \mathcal{S}(\tau)\, \tilde{X}(t-\tau, t)$$

Driven system ➔ noise term becomes time-dependent

$$\dot{\rho} = \ldots - [X, [Q(t), \rho]], \quad Q(t) = \int_0^\infty d\tau\, \mathcal{S}(\tau)\, \tilde{X}(t-\tau, t)$$

Central idea:

1. adapted basis: Floquet states $|\phi_\alpha(t)\rangle$ ➔ captures coherent dynamics
2. master equation in Floquet basis

$$\frac{d}{dt}\rho_{\alpha\beta} = -i(\epsilon_\alpha - \epsilon_\beta)\rho_{\alpha\beta} + \sum_{\alpha'\beta'} \mathcal{L}_{\alpha\beta,\alpha'\beta'}(t)\, \rho_{\alpha'\beta'}$$

   where $\mathcal{L}(t) = \mathcal{L}(t+T)$
3. rotating-wave approximation

Blümel *et al.*, PRA 1991; SK, Dittrich, Hänggi, PRE 1997

Rotating-wave approximation*

$$\frac{d}{dt}\rho_{\alpha\beta} = -i(\epsilon_\alpha - \epsilon_\beta)\rho_{\alpha\beta} + \sum_{\alpha'\beta'} \mathcal{L}_{\alpha\beta,\alpha'\beta'}(t)\, \rho_{\alpha'\beta'}$$

$$i(\epsilon_\alpha - \epsilon_\beta - k\Omega)\rho_{\alpha\beta}^{(k)} = \sum_{\alpha',\beta',k'} \mathcal{L}_{\alpha\beta,\alpha'\beta'}^{(k-k')}\, \rho_{\alpha'\beta'}^{(k')} \quad \text{(if } \rho \text{ is } T\text{-periodic)}$$

	Liouvillian	Density operator	Remark
Full RWA	$\mathcal{L}^{(0)}_{\alpha\alpha,\alpha'\alpha'}$	populations only	efficient, Pauli type, analytic calculations
Moderate RWA	$\mathcal{L}^{(0)}_{\alpha\beta,\alpha\beta}$	transients	coherent dynamics, depends on BZ (!)
Convolution	full $\mathcal{L}$	time periodic	best Markovian long-time solution

* for sufficiently weak dissipation

➔ full RWA (Pauli master equation)

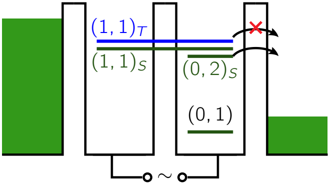
$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\alpha'} w_{\alpha\leftarrow\alpha'}\ \rho_{\alpha'\alpha'} - \sum_{\alpha} w_{\alpha'\leftarrow\alpha}\ \rho_{\alpha\alpha}$$

with the golden-rule rates

$$w_{\alpha\leftarrow\alpha'} = \sum_k J(\epsilon_\alpha - \epsilon_{\alpha'} + k\Omega) \left| \sum_{k'} \langle \phi_{\alpha,k+k'} | X | \phi_{\alpha',k} \rangle \right|^2 n_{\text{th}}(\epsilon_\alpha - \epsilon_{\alpha'} + k\Omega)$$

- sidebands contribute to $w_{\alpha\leftarrow\alpha'}$

  ... but NOT as independent states!
- no detailed balance between forward / backward rates



- (many-body) Floquet states
- evaluate rates $w_{\alpha\leftarrow\alpha'}$

→ dc current, counting statistics

	Dissipation	Transport
Environment	harmonic oscillators	electron source / drain
Coupling of mode ν	$X(a_\nu^\dagger + a_\nu)$	$c^\dagger c_\nu + c_\nu^\dagger c$
Absorption / tunnel in	$n_{\mathrm{th}}(\omega)$	$f(\epsilon-\mu)$
Emission / tunnel out	$1+n_{\mathrm{th}}(\omega)$	$1-f(\epsilon-\mu)$
"Ohmic"	$J(\omega)\propto\omega$	$\Gamma(\omega)=\mathrm{const}$

Floquet theory for open quantum systems

1 Floquet & Schrödinger equation

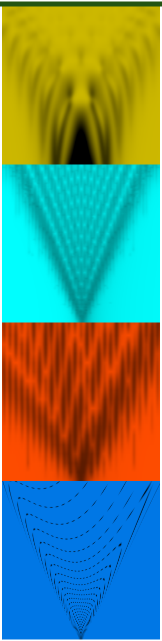
- Geometric phases
- Time-periodicity, Floquet ansatz, and all that

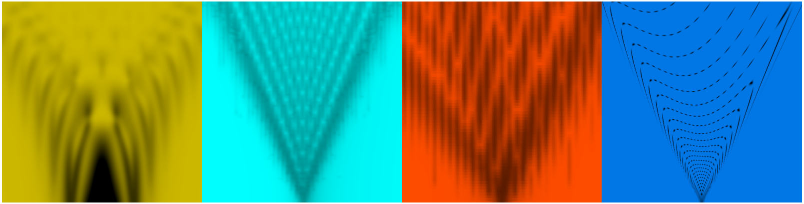
2 Quantum dissipation and transport

- System-bath model
- Floquet-Bloch-Redfield formalism

3 Applications & miscellaneous topics

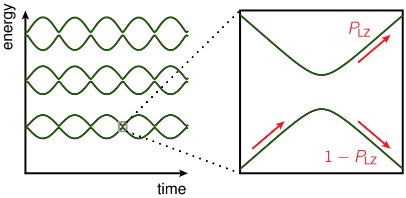
- LZSM Interference
- Time-dependent Liouvillians
- Floquet scattering theory
- Adiabatic Floquet theory
- Hidden symmetries
- Quantum chaos and dissipation
- Floquet-Gibbs states
- Two-color Floquet theory





Experiments: Stefan Ludwig (PDI Berlin)
Jason Petta (UCLA)
Gang Cao & Guo-Ping Guo (Hefei)

Quantum system in AC-field, $H(t)$



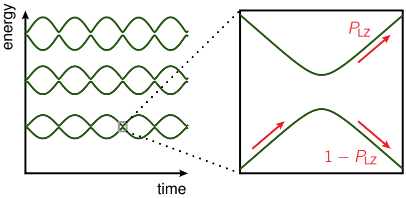
- non-adiabatic transition probability

$$P_{\mathrm{LZ}} = e^{-\pi\Delta^2/2\hbar v}$$

Landau, Zener, Stückelberg, Majorana, 1932

AC-driving and Landau-Zener transitions

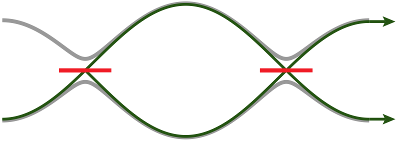
Quantum system in AC-field, $H(t)$



- non-adiabatic transition probability

$$P_{LZ} = e^{-\pi\Delta^2/2\hbar v}$$

Landau, Zener, Stückelberg, Majorana, 1932



➔ beam splitter, interference

➔ Landau-Zener-(Stückelberg-Majorana) interferometry

Interference condition

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{g(t)}{2}\sigma_z \qquad g(t) = \epsilon + A\cos(\Omega t)$$

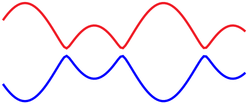
1. (avoided) crossing requires $A > |\epsilon|$

Interference condition

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{g(t)}{2}\sigma_z \qquad g(t) = \epsilon + A\cos(\Omega t)$$

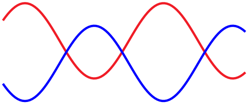
1. (avoided) crossing requires $A > |\epsilon|$
2. relative phase between dominant paths

adiabatic: $P_{\mathrm{LZ}} \ll 1$



$$\varphi(T) = \int_0^T dt\, |g(t)|$$

diabatic: $1 - P_{\mathrm{LZ}} \ll 1$



$$\varphi(T) = \int_0^T dt\, g(t)$$

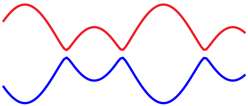
➔ fringes for $\varphi(T) = 2\pi k$

Interference condition

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{g(t)}{2}\sigma_z \qquad g(t) = \epsilon + A\cos(\Omega t)$$

1. (avoided) crossing requires $A > |\epsilon|$
2. relative phase between dominant paths

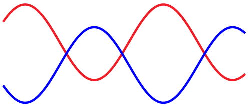
adiabatic: $P_{\mathrm{LZ}} \ll 1$



$$\varphi(T) = \int_0^T dt\, |g(t)|$$

→ fringes for $\varphi(T) = 2\pi k$

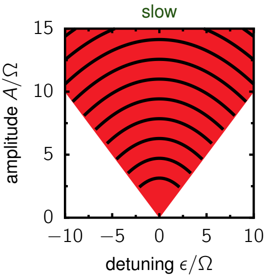
diabatic: $1 - P_{\mathrm{LZ}} \ll 1$

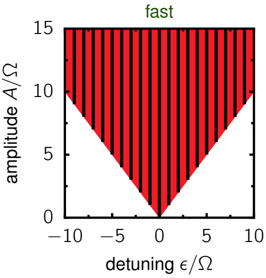


$$\varphi(T) = \int_0^T dt\, g(t) = \epsilon T$$

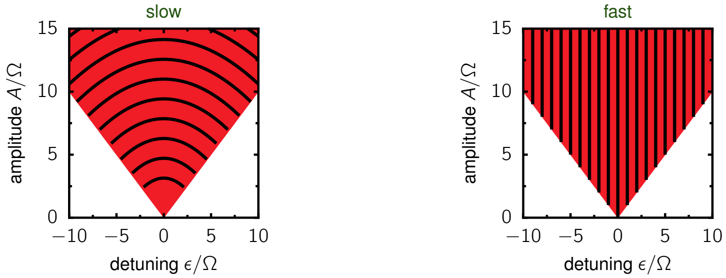
$\epsilon = k\Omega$ "k-photon resonance"

Patterns for two-level systems

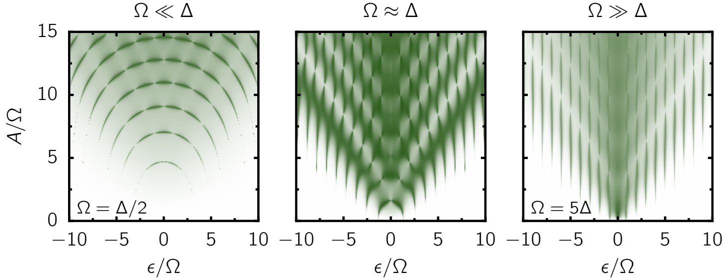




Patterns for two-level systems



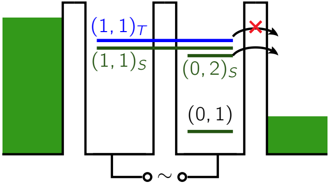
E.g. excitation probability of a TLS:

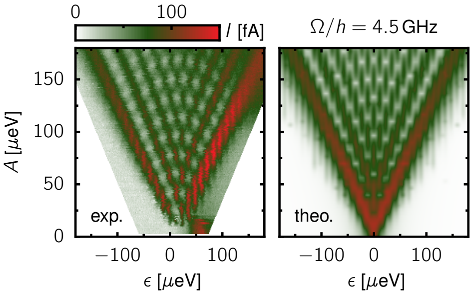


Measurement I: Transport

DC current:

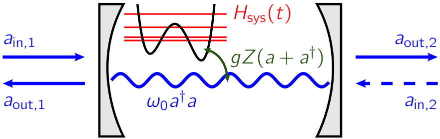
$$I(t) = e_0 \Gamma_R \langle n_R(t) \rangle \quad \rightarrow \quad \overline{\langle n_R(t) \rangle}^T$$





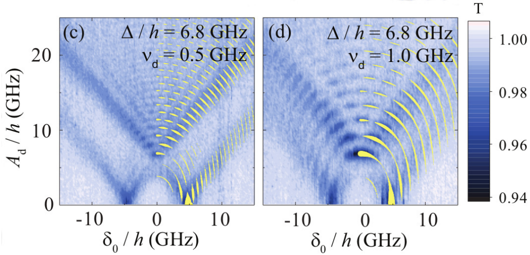
Forster *et al.*, PRL 2014

Measurement II: Cavity transmission



Dispersive frame:
effective cavity frequency

$$\omega_0 \longrightarrow \omega_0 + \frac{g^2}{\epsilon_{qb} - \omega_0}\sigma_z$$

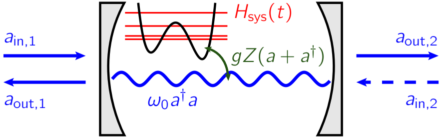


Theory for readout of driven system?

Koski *et al.*, PRL 2018

Dispersive frequency shift

Qubit-cavity Hamiltonian



$$H = H_{\rm sys}(t) + gZ(a^\dagger + a) + \omega_0 a^\dagger a$$

- Backaction: cavity ➔ qubit ➔ cavity
- Cavity equation (input/output formalism)

$$\frac{d}{dt}a = -i\omega_0 a - \frac{\kappa}{2}a - \sum_{\nu=1,2}\sqrt{\kappa_\nu}a_{{\rm in},\nu} - igZ$$

- (non equilibrium) Kubo formula $Z(t) = g \int dt' \, \chi(t-t') a(t')$

  with the response function (may depend on the initial state!)

$$\chi(t) = -i \langle [Z(t), Z] \rangle \theta(t-t')$$

➔ $-i\omega a = -i\big(\omega_0 + g^2 \chi(\omega)\big) a - \frac{\kappa}{2} a - \sum_{\nu=1,2} \sqrt{\kappa_\nu} a_{\mathrm{in},\nu}$

➔ measured quantity: (non-equilibrium) susceptibility

Response of periodically driven system

$$\chi(t, t') = -i\langle [Z(t), Z(t')] \rangle_{\text{non-eq}} = \chi(t+T, t'+T)$$

such that

$$\chi(t, t-\tau) = \sum_k e^{-ik\Omega t} \int d\omega \, e^{-i\omega\tau} \chi^{(k)}(\omega)$$

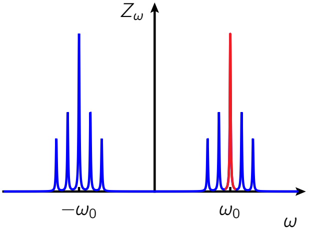
Response of periodically driven system

$$\chi(t,t') = -i\langle [Z(t), Z(t')]\rangle_{\text{non-eq}} = \chi(t+T, t'+T)$$

such that

$$\chi(t, t-\tau) = \sum_k e^{-ik\Omega t} \int d\omega\, e^{-i\omega\tau}\chi^{(k)}(\omega)$$

- resonant cavity driving, $\omega = \omega_0$
- response $Z(t)$ acquires sidebands
- good cavity limit, $\kappa \ll \omega_0, \Omega$



Response function for driven system

Relevant component:

$$\chi^{(0)}(\omega_0) = \sum_{\beta,\alpha,k} \frac{(p_\alpha - p_\beta)|Z_{\beta\alpha,k}|^2}{\epsilon_\alpha - \epsilon_\beta + \omega_0 + k\Omega + i\gamma/2}$$

- Floquet theory ➔ quasi-energies ϵ_α
- Floquet-Bloch-Redfield ➔ populations p_α

Response function for driven system

Relevant component:

$$\chi^{(0)}(\omega_0) = \sum_{\beta,\alpha,k} \frac{(p_\alpha - p_\beta)|Z_{\beta\alpha,k}|^2}{\epsilon_\alpha - \epsilon_\beta + \omega_0 + k\Omega + i\gamma/2}$$

- Floquet theory ➔ quasi-energies ϵ_α
- Floquet-Bloch-Redfield ➔ populations p_α

Resonance conditions

cavity response:

$$\Delta\epsilon = \omega_0 + k\Omega$$

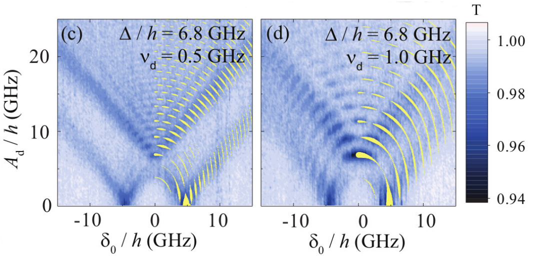
cf. population:

$$\Delta\epsilon = k\Omega$$

e.g. Ivakhnenko *et al.*, Phys.Rep. 2023

➔ Agree only for low-frequency oscillator!

GaAs double quantum dots

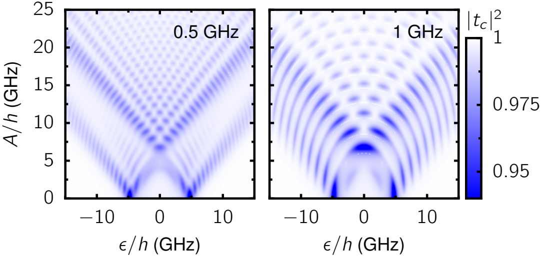


Double QD in GaAs

➔ two-level system

Koski *at al.*, PRL 2018

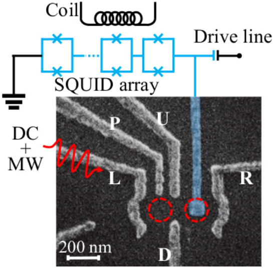
(yellow: heuristic theory)



Present theory

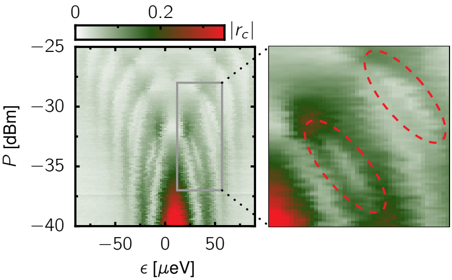
SK, PRA 2018

Readout of Floquet state population



Chen *et al.*, Phys. Rev. B **103**, 205428 (2021)

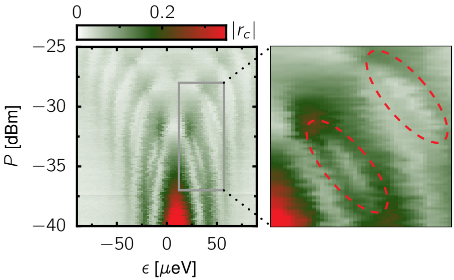
Motivation: Holes in interference fringes



Experiment (Cao & Guo, Hefei)

- holes in LZSM pattern
- GaAs DQD ➔ two-level sys.

Motivation: Holes in interference fringes



Experiment (Cao & Guo, Hefei)

- holes in LZSM pattern
- GaAs DQD ➔ two-level sys.

Recap: susceptibility (two-level system)

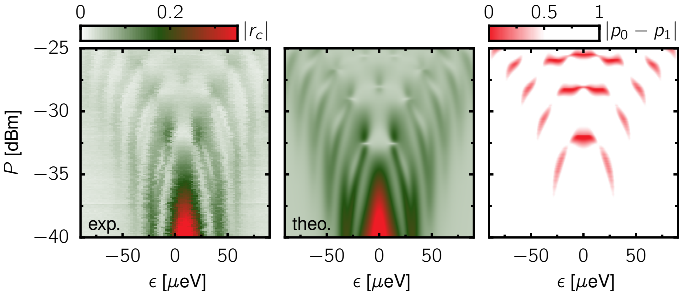
$$\chi^{(0)}(\omega_0) = (p_0 - p_1) \sum_k \frac{|Z_{10,k}|^2}{\epsilon_1 - \epsilon_0 + \omega_0 + k\Omega + i\gamma/2}$$

response determined by

– resonance condition for cavity signal

– Floquet state population

Experiment *vs.* Theory



- competing resonance conditions
- holes in fringes when $p_0 \approx p_1 \approx 1/2$

➔ cavity response provides information about Floquet state population

Chen *et al.*, PRB 2021

Periodically time-dependent Liouvillians

$$\frac{d}{dt}P = L(t)P$$

Periodically time-dependent Liouvillians

Master equation of type

$$\frac{d}{dt}P = L(t)P$$

- Floquet-Bloch-Redfield beyond moderate RWA
- time-dependent system with Lindblad dissipator

$$\dot{\rho} = -i[H(t), \rho] + \gamma(2a^\dagger \rho a - a^\dagger a \rho - \rho a^\dagger a)$$

  - very weak dissipation
  - transport problem with large bias

➔ long-time solution T-periodic

➔ Floquet ansatz with "quasienergy" zero

$$P(t) = \sum_k e^{-ik\Omega t} p_k$$

$\frac{d}{dt}P = L(t)P$ with

$$L(t) = L_0 + 2L_1\cos(\Omega t)$$

➔ kernel of tridiagonal Floquet matrix

$$L_0 + 2L_1\cos(\Omega t) - \partial_t \leftrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & L_0 + 2i\Omega & L_1 & 0 & 0 & 0 & \cdots \\ \cdots & L_1 & L_0 + i\Omega & L_1 & 0 & 0 & \cdots \\ \cdots & 0 & L_1 & L_0 & L_1 & 0 & \cdots \\ \cdots & 0 & 0 & L_1 & L_0 - i\Omega & L_1 & \cdots \\ \cdots & 0 & 0 & 0 & L_1 & L_0 - 2i\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Matrix-continued fractions

- ansatz $P(t) = \sum_k e^{-ik\Omega t} p_k$ yields

$$L_1 p_{k-1} + (L_0 + ik\Omega) p_k + L_1 p_{k+1} = 0$$

- idea: truncate and iterate $p_{k-1} = -L_1^{-1}\left\{(L_0 - ik\Omega)p_k + L_1 p_{k+1}\right\}$

Matrix-continued fractions

- ansatz $P(t) = \sum_k e^{-ik\Omega t} p_k$ yields

$$L_1 p_{k-1} + (L_0 + ik\Omega) p_k + L_1 p_{k+1} = 0$$

- idea: truncate and iterate $p_{k-1} = -L_1^{-1}\left\{(L_0 - ik\Omega)p_k + L_1 p_{k+1}\right\}$
  - ✗ fails, L_1 generally singular

Matrix-continued fractions

- ansatz $P(t) = \sum_k e^{-ik\Omega t} p_k$ yields

$$L_1 p_{k-1} + (L_0 + ik\Omega) p_k + L_1 p_{k+1} = 0$$

- idea: truncate and iterate $p_{k-1} = -L_1^{-1}\left\{(L_0 - ik\Omega)p_k + L_1 p_{k+1}\right\}$
  - ✗ fails, L_1 generally singular
- solution: ansatz $p_k = S_k L_1 p_{k\mp 1}$ ($k \gtrless 0$) leads to

$$S_k = -\left(L_0 + ik\Omega + L_1 S_{k\pm 1} L_1\right)^{-1} \longrightarrow S_{\pm 1} \tag{1}$$

$$0 = \left(L_1 S_{-1} L_1 + L_0 + L_1 S_1 L_1\right) p_0 \tag{2}$$

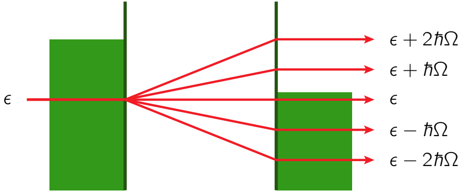
➔ truncate at $\pm k_0$, iterate (1), and solve (2)

➔ time-averaged $P(t) = p_0$ ➔ time-averaged expectation values

Risken, "The Fokker-Planck Equation"

Appendix of Forster *et al.*, PRB 2015

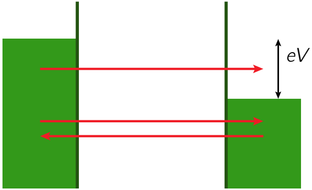
Floquet scattering theory



SK, Lehmann, Hänggi, Phys. Rep. **406**, 379 (2005)

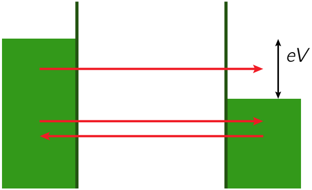
Landauer-Büttiker formula

- Landauer (1957): "conductance is transmission"



Landauer-Büttiker formula

- Landauer (1957): “conductance is transmission”



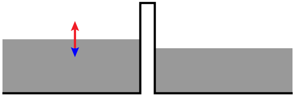
- current $\boxed{I = \frac{e}{h} \int dE \, T(E) \left[f(E+eV) - f(E) \right]}$
- transmission of an electron with energy E

$$T(E) = \Gamma_L \Gamma_R \left| \langle 1 | G(E) | N \rangle \right|^2$$

Fischer, Lee, PRB 1981; Meir, Wingreen, PRL 1992

- ac bias voltage:

$$V_0 \longrightarrow V_0 + V_{ac}\cos(\Omega t)$$

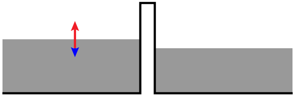


- time-dependent energy shift by $eV_{ac}\cos(\Omega t)$

$$e^{-iEt} \longrightarrow \exp\left(-iEt - i\frac{eV_{ac}}{\Omega}\sin(\Omega t)\right)$$

- ac bias voltage:

$$V_0 \longrightarrow V_0 + V_{ac}\cos(\Omega t)$$



- time-dependent energy shift by $eV_{ac}\cos(\Omega t)$

$$e^{-iEt} \longrightarrow \exp\left(-iEt - i\frac{eV_{ac}}{\Omega}\sin(\Omega t)\right) = \sum_k J_k(eV_{ac}/\Omega)e^{-i(E+k\Omega)t}$$

  - sidebands occupied with probability $J_k^2(\ldots)$
  - energy $k\Omega$ corresponds to additonal DC bias voltage $k\Omega/e$

$$\boxed{I(V_0, V_{ac}) = \sum_k J_k^2\left(\frac{eV_{ac}}{\Omega}\right) I_0(V_0 + k\Omega/e)}$$

DC conductivity determines the current !

Tucker, Feldmann, Rev. Mod. Phys. 1985

Tien-Gordon theory

- Approach rather heuristic

➔ Rigorous derivation?

➔ When is Tien-Gordon theory applicable?

Transport and driving:

Green's function and Landauer formula for time-dependent situation

Transport and driving:

Green's function and Landauer formula for time-dependent situation

- Floquet equation

  with self-energy $\Sigma = |1\rangle \frac{i\Gamma_L}{2} \langle 1| + |N\rangle \frac{i\Gamma_R}{2} \langle N|$

$$\left(H(t) + \Sigma - i\frac{d}{dt}\right)|\varphi_\alpha(t)\rangle = (\epsilon_\alpha - i\gamma_\alpha)|\varphi_\alpha(t)\rangle$$

- propagator in the presence of the contacts

$$G(t, t-\tau) = \sum_k e^{ik\Omega t} \int d\epsilon \, e^{-i\epsilon\tau} \underbrace{\sum_{\alpha,k'} \frac{|\varphi_{\alpha,k+k'}\rangle\langle\varphi_{\alpha,k'}|}{\epsilon - (\epsilon_\alpha + k'\Omega - i\gamma_\alpha)}}_{G^{(k)}(\epsilon)}$$

propagation under absorption/emission of $|k|$ photons

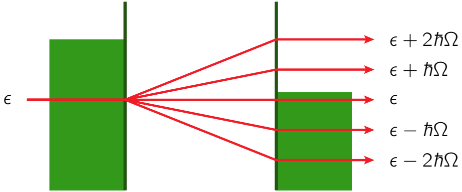
Floquet transport theory: current

- dc current

$$I = \frac{e}{h}\sum_k \int d\epsilon \left\{ T_{LR}^{(k)}(\epsilon) f(\epsilon-\mu_L) - T_{RL}^{(k)}(\epsilon) f(\epsilon-\mu_R) \right\}$$

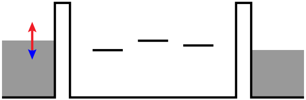
- transmission under absorption of k photons

$$T_{LR}^{(k)}(\epsilon) = \Gamma_L \Gamma_R \left| \langle 1 | G^{(k)}(\epsilon) | N \rangle \right|^2 \neq T_{RL}^{(\pm k)}(\epsilon \pm k\Omega)$$



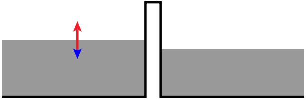
Camalet, Lehmann, SK, Hänggi, PRL 2003

When is Tien-Gordon theory applicable ?

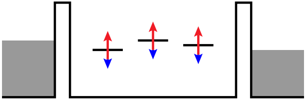


applicable for

- AC bias voltage

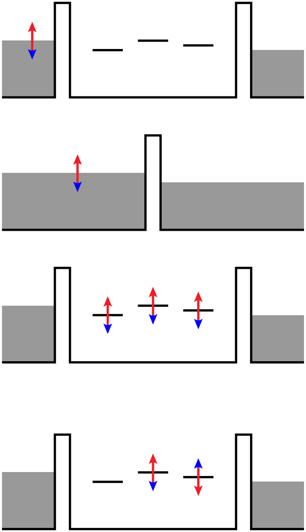


- tunnel barriers
  (studied by Tien & Gordon)



- uniform AC gate voltage

When is Tien-Gordon theory applicable ?



applicable for

- AC bias voltage
- tunnel barriers
  (studied by Tien & Gordon)
- uniform AC gate voltage

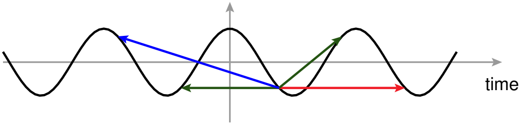
but not for

- non-uniform gating
- dipole force

Camalet, SK, Hänggi, PRB 2004

Spatio-temporal symmetries

$H_{\text{dipole}} \propto x \cos(\Omega t)$



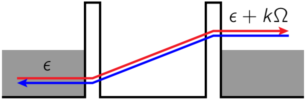
1. time periodicity $t \longrightarrow t + T$ ➔ Floquet theory applicable
2. time reversal $t \longrightarrow -t$ ➔ Floquet states real
3. generalized parity $(x, t) \longrightarrow (-x, t + T/2)$
   e.g. symmetric potential with dipole driving
   ➔ Floquet states even/odd
4. time-reversal parity $(x, t - T/4) \longrightarrow (-x, T/4 - t)$
   - combination of the other three

Consequences for scattering probabilities ?

Symmetry-related processes have the same probability

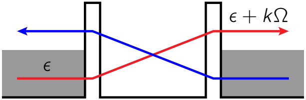
time-reversal

$t \to -t$



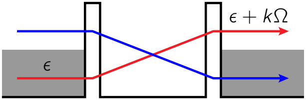
generalized parity

$(x, t \to -x, t + \frac{T}{2})$

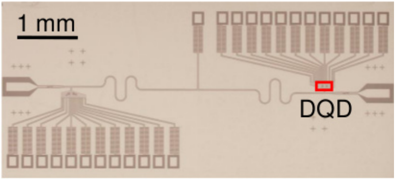


time-reversal parity

$(x, t \to -x, -t)$

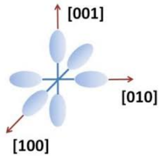


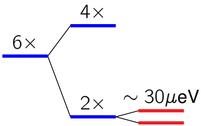
Dispersive readout of LZSM patterns in Si/SiGe DQDs (under low-frequency driving)



Mi, SK, Petta, Phys. Rev. B **98**, 161404(R) (2018)

Motivation: LZSM with silicon double quantum dots

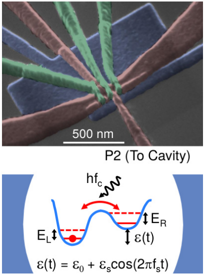




Level structure

- strain, confinement
- valley splitting

➔ valley qubit



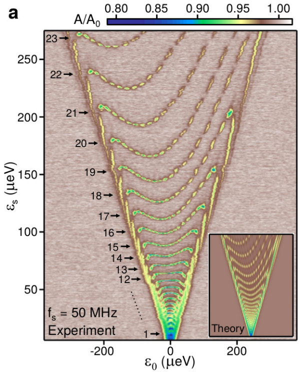
J. Petta, U Princeton

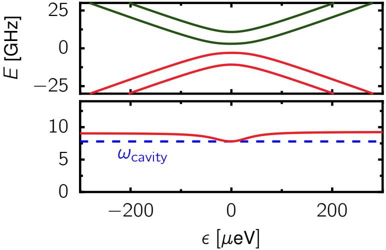
Si double quantum dots

- position, spin, valley
- coupling to cavity

➔ “dispersive readout”

Motivation: LZSM with silicon double quantum dots





Experiment (J.R. Petta, Princeton)

- very slow driving: 50–100 MHz

  ➔ non-eq. population tiny

- heuristic resonance condition:

$$\bar{E}_1 - \bar{E}_0 = \omega_{\text{cavity}} + k\Omega$$

(mean energies!)

Computational problem:

- amplitude: up to 300 GHz

➔ Floquet theory requires $\gtrsim 10^4$ sidebands

- adiabatic eigenstates $H(t)|u(t)\rangle = \big(\bar{E} + \delta E(t)\big)|u(t)\rangle$

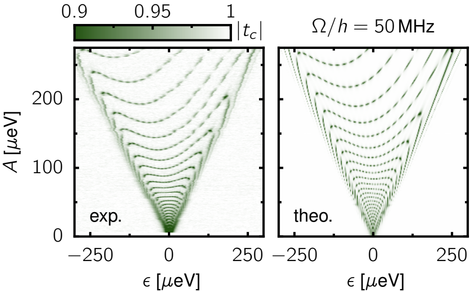
➔ $|\psi(t)\rangle = e^{-i\bar{E}t} e^{i\varphi(t)}|u(t)\rangle$

➔ adiabatic Floquet states

$$|\phi(t)\rangle = e^{i\varphi(t)}|u(t)\rangle \quad \text{with} \quad \varphi(t) = -\int_0^t dt'\, \delta E(t') \qquad T \text{ periodic!}$$

with quasienergies $\epsilon = \bar{E} = \frac{1}{T}\int_0^T dt\, E(t)$

[here: time-reversal symmetry ➔ $\langle u|\partial_t|u\rangle = 0$, *i.e.*, no Berry phase]



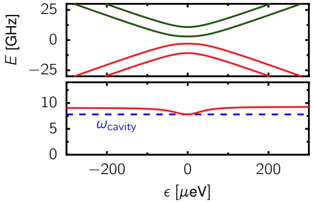
Dressed-state interpretation

- LZSM between states

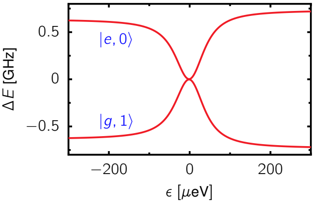
$$|g, 1\rangle, \quad |e, 0\rangle$$

→ cavity-assisted LZSM

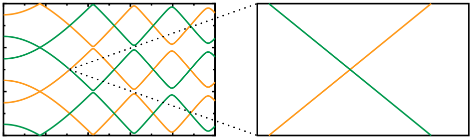
Mi, SK, Petta, PRB 2018



→



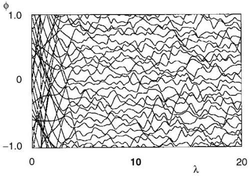
Hidden Symmetries in Floquet Systems



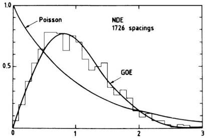
in collaboration with J. Casado-Pascual, U Sevilla

Level repulsion

- Spectra of classically chaotic systems



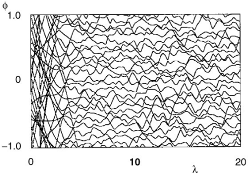
Haake, book on Q. Chaos



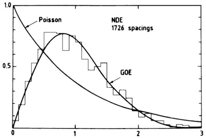
Bohigas 1983

➔ level repulsion ➔ anti crossings

- Spectra of classically chaotic systems



Haake, book on Q. Chaos



Bohigas 1983

➔ level repulsion ➔ anti crossings

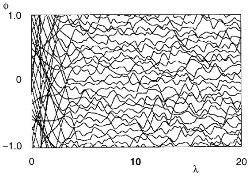
- symmetry: Hamiltonian block diagonal (here: even/odd)

$$H = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}$$

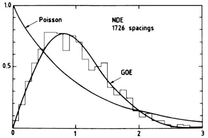
➔ allows exact level crossings

Level repulsion

- Spectra of classically chaotic systems



Haake, book on Q. Chaos



Bohigas 1983

➔ level repulsion ➔ anti crossings

- symmetry: Hamiltonian block diagonal (here: even/odd)

$$H = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}$$

➔ allows exact level crossings

➔ Exact level crossings indicate symmetries / integrals of motion

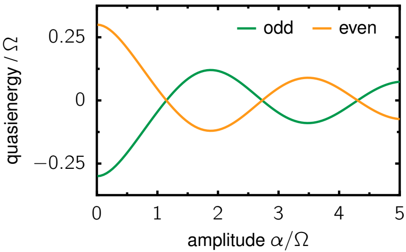
TLS Hamiltonian

$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t)$$

generalized parity $G = \sigma_x e^{(T/2)\partial_t}$

➔ even / odd states

➔ exact crossings   Peres, PRL 1991



Driven two-level system

TLS Hamiltonian

$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t)$$

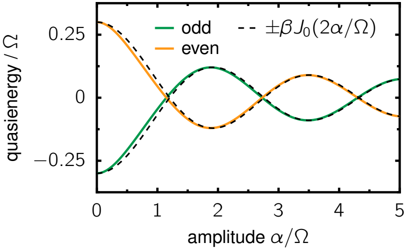
generalized parity $G = \sigma_x e^{(T/2)\partial_t}$

➔ even / odd states

➔ exact crossings   Peres, PRL 1991

High-frequency approximation

$$q_\pm = \pm\beta J_0(2\alpha/\Omega)$$



Driven two-level system

TLS Hamiltonian

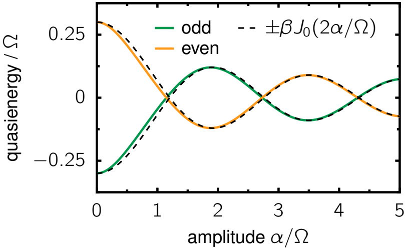
$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t) + \frac{n\Omega}{2}\sigma_z$$

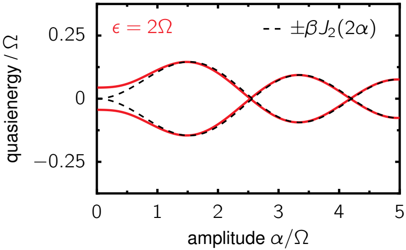
~~generalized parity $G = \sigma_x e^{(T/2)\partial_t}$~~

High-frequency approximation

$$q_\pm = \pm\beta J_0(2\alpha/\Omega) \longrightarrow \pm\beta J_n(2\alpha/\Omega)$$

- suggests that crossings are exact

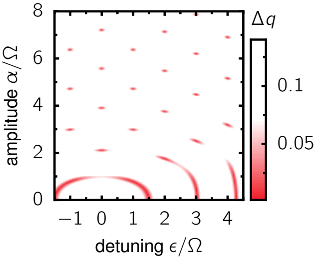




$$H(t) = \beta\sigma_x + \alpha\sigma_z\cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$

Quasienergy splittings tiny at isolated points

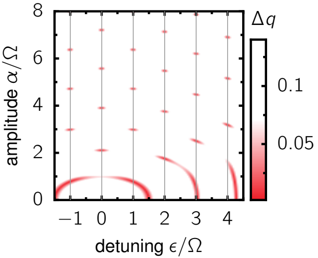
(intermediate frequency,
beyond Bessel function approx.)



$$H(t) = \beta\sigma_x + \alpha\sigma_z\cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$

Quasienergy splittings tiny at isolated points

(intermediate frequency,
beyond Bessel function approx.)



- crossings exact for integer detuning ?
- underlying symmetry / integral of motion ?

- time-independent: integral of motion $[H, J] = 0$
- for time-dependent system: $[H(t) - i\partial_t, J(t)] = 0$

Lewis & Riesenfeld, J. Math. Phys. 1969

- time-independent: integral of motion $[H, J] = 0$
- for time-dependent system: $[H(t) - i\partial_t, J(t)] = 0$

Lewis & Riesenfeld, J. Math. Phys. 1969

- ansatz for time non-local LR invariant: $J(t) = Q(t)e^{(T/2)\partial_t}$

  [motivation: for $\epsilon = 0$ ➔ $Q = \sigma_x$]

$$i\partial_t Q = \left[\frac{\epsilon}{2}\sigma_z + \beta\sigma_x, Q\right] + \left\{\alpha\sigma_z, Q\right\}\cos(\Omega t)$$

Lewis-Riesenfeld invariant

- time-independent: integral of motion $[H, J] = 0$
- for time-dependent system: $[H(t) - i\partial_t, J(t)] = 0$

Lewis & Riesenfeld, J. Math. Phys. 1969

- ansatz for time non-local LR invariant: $J(t) = Q(t) e^{(T/2)\partial_t}$

[motivation: for $\epsilon = 0$ ➔ $Q = \sigma_x$]

$$i\partial_t Q = \left[\frac{\epsilon}{2}\sigma_z + \beta\sigma_x, Q\right] + \left\{\alpha\sigma_z, Q\right\}\cos(\Omega t)$$

- trivial (and useless) solution: projector on Floquet state

$$J = |\phi_\ell(t)\rangle\langle\phi_\ell(t)| \quad \rightarrow \quad Q = |\phi_\ell(t)\rangle\langle\phi_\ell(t+T/2)|$$

depends on state ℓ ➔ cannot be used to classify state

1. Fourier ansatz $Q(t) = \sum_k e^{-ikt} Q_k$
2. matrix recurrence relation
3. scalar recurrence
4. $\epsilon/\Omega = n =$ integer: recurrence terminates at $k_{\max} = n$
5. $J^2 = 1$, i.e. $\mathbb{Z}_2$ symmetry

→ eigenvalues $j = \pm 1$, “even / odd” Floquet states

1. Fourier ansatz $Q(t) = \sum_k e^{-ikt} Q_k$
2. matrix recurrence relation
3. scalar recurrence
4. $\epsilon/\Omega = n =$ integer: recurrence terminates at $k_{\max} = n$
5. $J^2 = 1$, i.e. $\mathbb{Z}_2$ symmetry

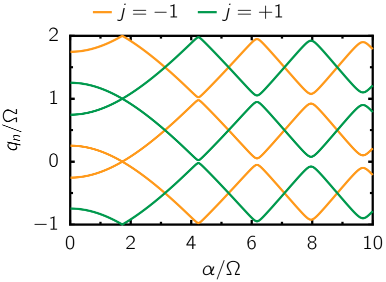
→ eigenvalues $j = \pm 1$, "even / odd" Floquet states

$\boxed{\text{Such } J = Q e^{(T/2)\partial_t} \text{ exists for any integer value of } \epsilon/\Omega}$

Solution ($n = 1$)

For $\epsilon = \Omega$

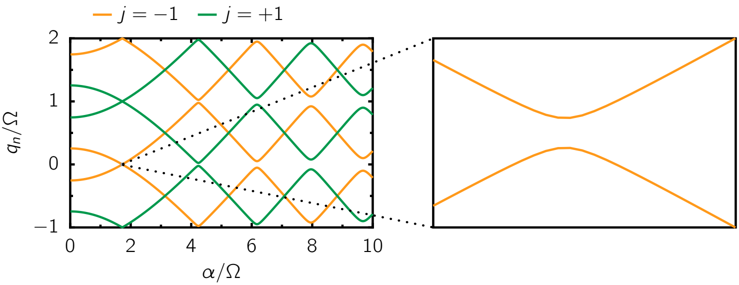
$$Q(t) = \frac{1}{\sqrt{\alpha^2+\beta^2}} \begin{pmatrix} \beta & \alpha e^{-i\Omega t} \\ -\alpha e^{i\Omega t} & \beta \end{pmatrix} \quad \text{with} \quad J^2 = QQ^\dagger = 1$$



Solution ($n = 1$)

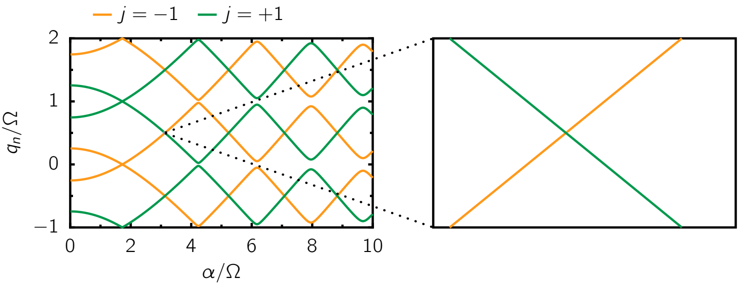
For $\epsilon = \Omega$

$$Q(t) = \frac{1}{\sqrt{\alpha^2+\beta^2}} \begin{pmatrix} \beta & \alpha e^{-i\Omega t} \\ -\alpha e^{i\Omega t} & \beta \end{pmatrix} \quad \text{with} \quad J^2 = QQ^\dagger = 1$$



For $\epsilon = \Omega$

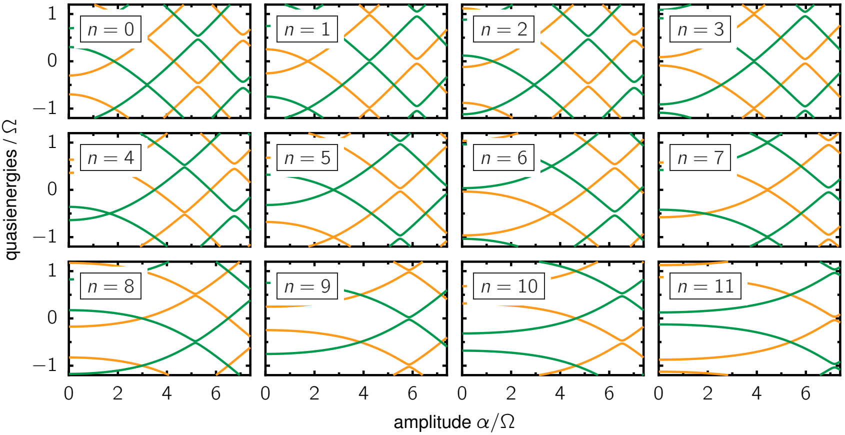
$$Q(t) = \frac{1}{\sqrt{\alpha^2+\beta^2}} \begin{pmatrix} \beta & \alpha e^{-i\Omega t} \\ -\alpha e^{i\Omega t} & \beta \end{pmatrix} \quad \text{with} \quad J^2 = QQ^\dagger = 1$$



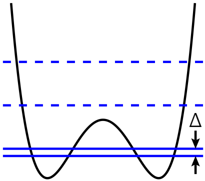
➔ accidential degeneracy due to hidden symmetry

Solutions for $\epsilon/\Omega = n > 1$

- numerical iteration: J with eigenvalues $j = +1$ and $j = -1$



1. long-time solution of a “non-trivial” problem ➔ populations
2. semi-classical limit ➔ capability of the formalism



$$H(x,p,t) = H_{\mathrm{DW}}(x,p) + Fx\cos(\Omega t)$$

Symmetries:

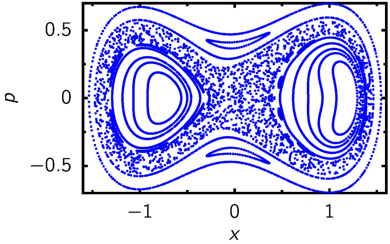
TR: $(x,p,t) \to (x,-p,-t)$

GP: $(x,p,t) \to (-x,-p,t+T/2)$

SK, PhD thesis, 1999

SK, Utermann, Dittrich, Hänggi, PRE 1998

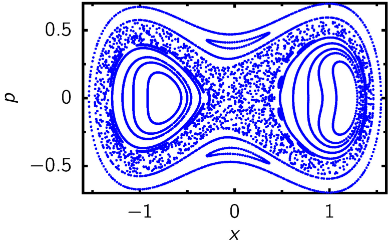
Classical phase space



Stroboscopic map $[x(nT), p(nT)]$

- regular vs. chaotic
- chaos augments with amplitude
- symmetry $p \to -p$
  (consequence of TR)

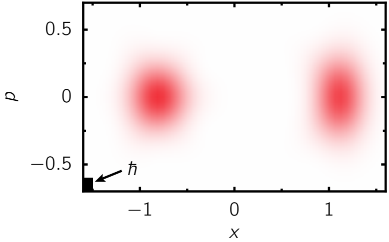
Classical phase space

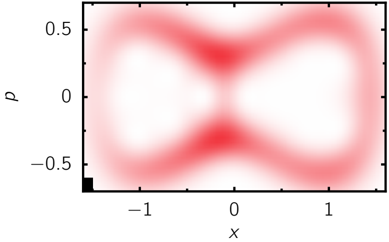


Stroboscopic map $[x(nT), p(nT)]$

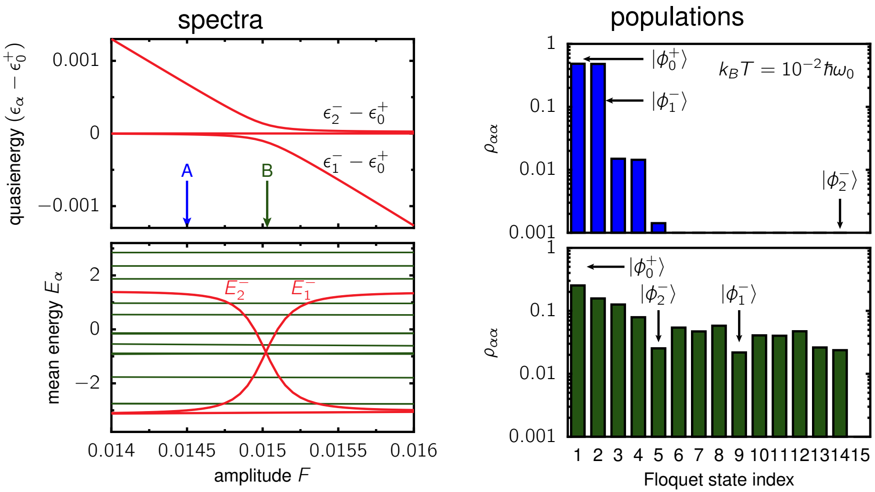
- regular vs. chaotic
- chaos augments with amplitude
- symmetry $p \to -p$ (consequence of TR)

Husimi functions of Floquet states (at times nT)





Population of Floquet states

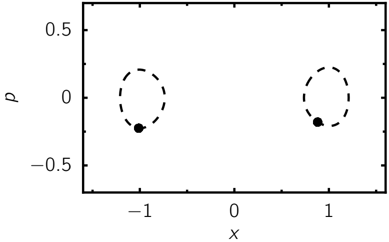


- far from crossing: occupation according to E_α
- at crossing: no general rule

Phase-space structure (dissipative)

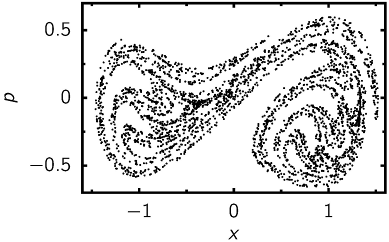
large dissipation:

➔ fixed points, limits cycles



weak dissipation:

➔ strange attractor

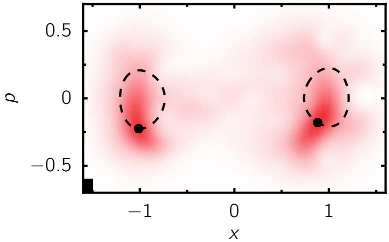


- dissipative term $-\gamma\dot{x}$ breaks time reversal

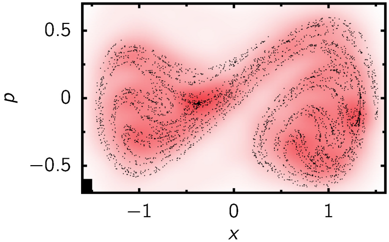
  ➔ phase lag due to dissipation ➔ no longer symmetric in p

Phase-space structure (dissipative)

large dissipation:
➔ fixed points, limits cycles



weak dissipation:
➔ strange attractor



- dissipative term $-\gamma\dot{x}$ breaks time reversal

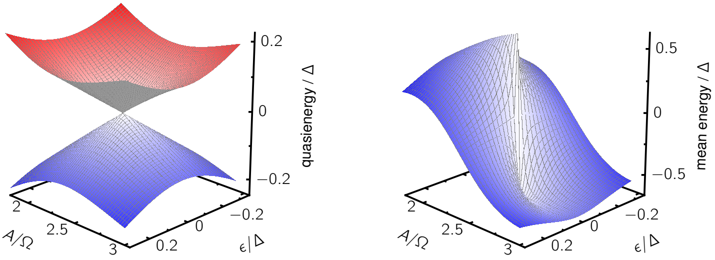
  ➔ phase lag due to dissipation ➔ no longer symmetric in p

✓ Floquet-Bloch-Redfield capable of phase lag

✗ RWA: $\rho = \sum_\alpha p_\alpha |\phi_\alpha\rangle\langle\phi_\alpha|$ would preserve symmetry

for ratchet potential: Denisov, SK, Hänggi, EPL 2009

Dissipation at Conical Intersections



SK, Phys. Rev. A **110**, 052218 (2024)

SK, J. Chem. Phys. **162**, 154101 (2025)

Mean energy state *vs.* Floquet-Gibbs state

Stationary state of driven dissipative quantum system

mean energy: $p_\alpha \propto e^{-E_\alpha/kT}$

quasi energy: $p_\alpha \propto e^{-q_\alpha/kT}$

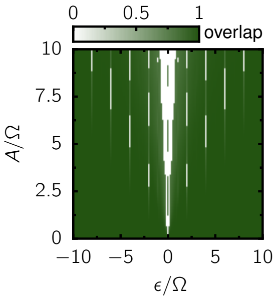
Mean energy state *vs.* Floquet-Gibbs state

Stationary state of driven dissipative quantum system

mean energy: $p_\alpha \propto e^{-E_\alpha/kT}$

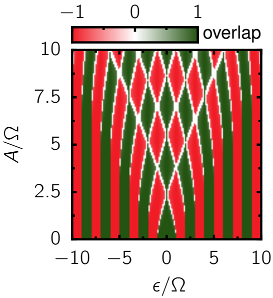
quasi energy: $p_\alpha \propto e^{-q_\alpha/kT}$

bath via σ_x



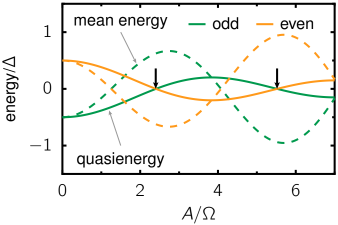
mean-energy state

bath via σ_z



(anti-) Floquet-Gibbs state

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{A}{2}\sigma_z \cos(\Omega t)$$



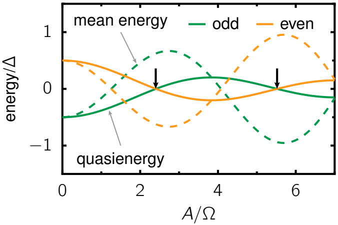
- spatio-temporal symmetry G:

$$\sigma_x \ \& \ (t \rightarrow t + T/2)$$

➔ even / odd states

Conical intersection

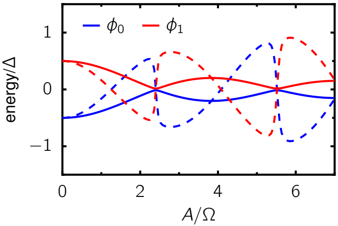
$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{A}{2}\sigma_z \cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$



- spatio-temporal symmetry G:

  $$\sigma_x \;\&\; (t \to t + T/2)$$

➔ even / odd states



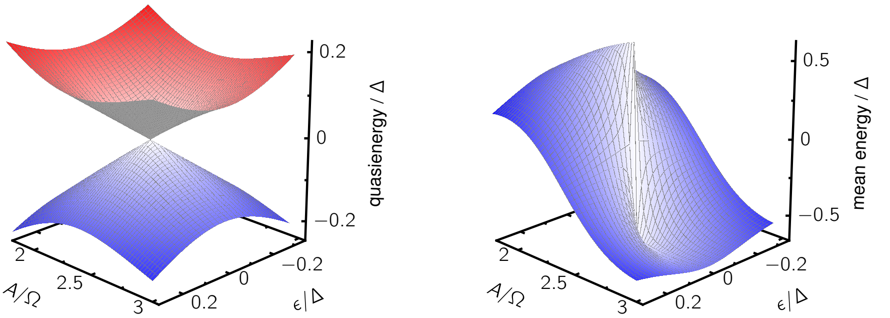
with detuning

- quasienergies: anti-crossing
- mean energies: exact crossing

Floquet “ground state”

quasienergy

mean energy



On circle around tip:

- constant quasienergy
- maximal variation of mean energy

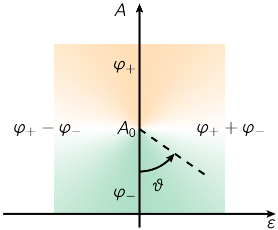
Floquet "ground state"

Two-state approximation for ϕ_0

- basis: φ_- / φ_+ at tip
- $\vartheta = 0 \longrightarrow \vartheta = \pi$:

$$\phi_0 \longrightarrow \phi_1$$

$$p_0 \longrightarrow p_1 = 1 - p_0$$



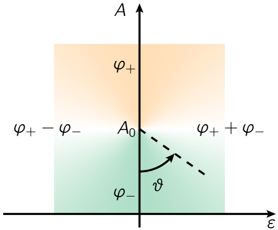
Floquet "ground state"

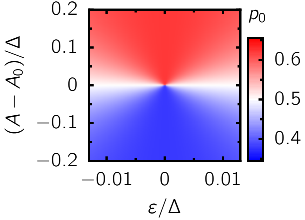
Two-state approximation for ϕ_0

- basis: φ_- / φ_+ at tip
- $\vartheta = 0 \longrightarrow \vartheta = \pi$:

  $\phi_0 \longrightarrow \phi_1$

  $p_0 \longrightarrow p_1 = 1 - p_0$





Population for σ_x-coupling

- angles with $p_0 = p_1 = 1/2$
  - full mixture, max. entropy
- behaves like mean energy of ϕ_0

cf. discontinuity along A-axis

Engelhardt *et al.*, PRL 2019

Transition to Floquet-Gibbs state

Golden-rule rate with $S_{01}(t) = \langle \phi_0(t)|S|\phi_1(t)\rangle$ ➔ sidebands S_k

Generic

- Ohmic $J(\omega) = \frac{\pi}{2}\alpha\omega$

➔ $k = 0$ suppressed

Transition to Floquet-Gibbs state

Golden-rule rate with $S_{01}(t) = \langle\phi_0(t)|S|\phi_1(t)\rangle$ ➔ sidebands S_k

Generic

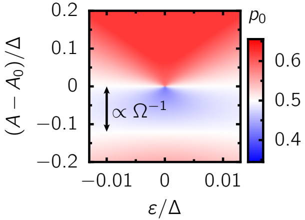
- Ohmic $J(\omega) = \frac{\pi}{2}\alpha\omega$

➔ $k = 0$ suppressed

Exception: $S = \sigma_z$

- time-independent for large Ω

➔ $k \neq 0$ suppressed

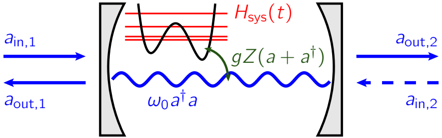


- Crossover to Floquet-Gibbs: two lines with $p_0 = 1/2$ merge with increasing Ω

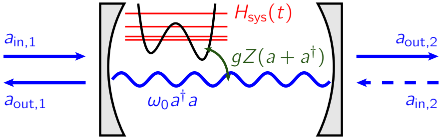
➔ Measurable signature of σ_z-coupling

SK, PRA 2024

Dispersive readout



Dispersive readout

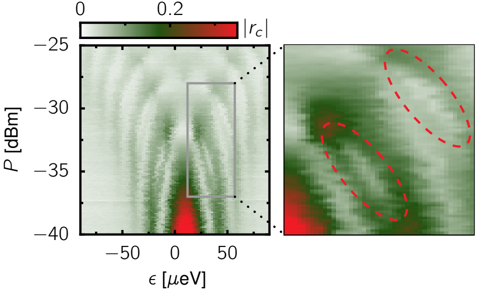


Signal: cavity transmission / reflection

$$\chi^{(0)}(\omega_0) = (p_0 - p_1) \sum_k \frac{|Z_{10,k}|^2}{\epsilon_1 - \epsilon_0 + \omega_0 + k\Omega + i\gamma/2} + (0 \leftrightarrow 1)$$

How to measure the population?

Dispersive readout



Signal: cavity transmission / reflection

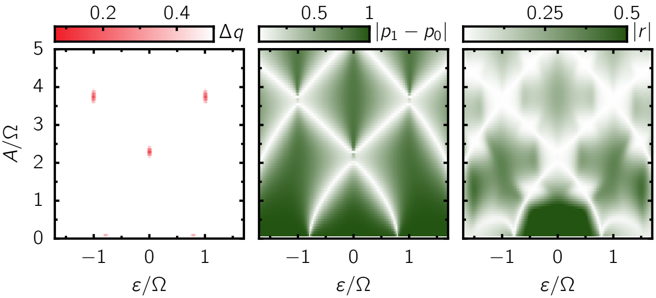
$$\chi^{(0)}(\omega_0) = (p_0 - p_1) \sum_k \frac{|Z_{10,k}|^2}{\epsilon_1 - \epsilon_0 + \omega_0 + k\Omega + i\gamma/2} + (0 \leftrightarrow 1)$$

➔ vanishes when $p_0 \approx p_1 \approx 1/2$

Chen *et al.*, PRB 2021

Bath coupling via σ_z

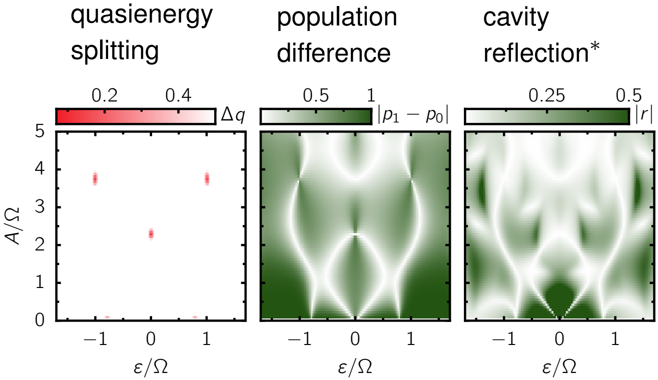
quasienergy splitting | population difference | cavity reflection*



➔ experimental signature of bath coupling operator

* for DQD and cavity of Chen *et al.*, PRB 2021, $\Omega/2\pi = 10\,\mathrm{GHz}$

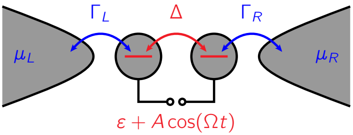
Bath coupling via σ_x



➔ experimental signature of bath coupling operator

* for DQD and cavity of Chen *et al.*, PRB 2021, $\Omega/2\pi = 10\,\mathrm{GHz}$

Open two-level system



- zero bias
- no interaction
- no spin

➔ master eq. for populations P_i

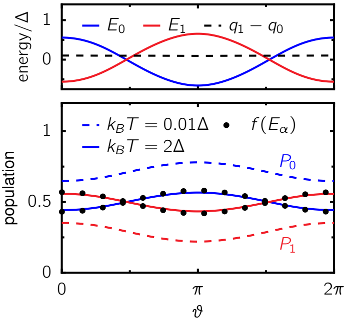
$$\frac{d}{dt}\begin{pmatrix} P_L \\ P_R \end{pmatrix} = \begin{pmatrix} -\Gamma_L & \Gamma_R \\ \Gamma_L & -\Gamma_R \end{pmatrix} \begin{pmatrix} P_L \\ P_R \end{pmatrix}$$

(numerics: beyond RWA)

Special case $\Gamma_L = \Gamma_R$:

$$P = \sum_k f(q + k\Omega - \mu)\langle \phi_k | \phi_k \rangle$$

Open two-level system



Special case $\Gamma_L = \Gamma_R$:

$$P = \sum_k f(q + k\Omega - \mu)\langle \phi_k | \phi_k \rangle$$

use $f(x) \approx \frac{1}{2} + \frac{x}{2k_B T}$

$$P \approx f(E - \mu)$$

➔ mean-energy state

Open two-level system

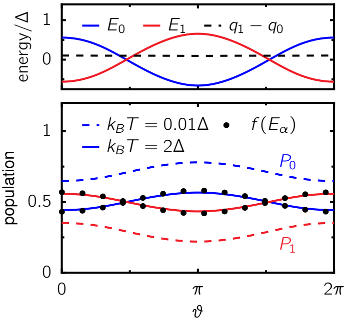
Special case $\Gamma_L = \Gamma_R$:

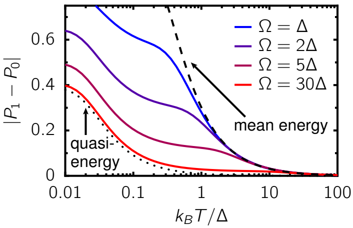
$$P = \sum_k f(q + k\Omega - \mu)\langle\phi_k|\phi_k\rangle$$

use $f(x) \approx \frac{1}{2} + \frac{x}{2k_B T}$

$$P \approx f(E - \mu)$$

➔ mean-energy state





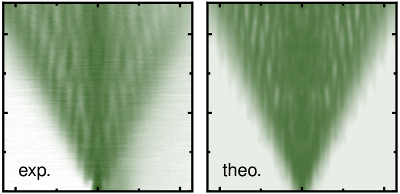
Mean energy state

- dissipation:
  (almost) any bath coupling
- intermediate / high temperature $kT \gtrsim \hbar\Omega$
- open TLS: $\Gamma_L \approx \Gamma_R$

Floquet-Gibbs state

- huge driving frequency, $\hbar\Omega \gtrsim 25\Delta$
- coupling to environment via σ_z or $\Gamma_L \approx \Gamma_R$

LZSM Interference for Bichromatic Driving



Forster, Mühlbacher, Blattmann, Schuh, Wegscheider, Ludwig & SK

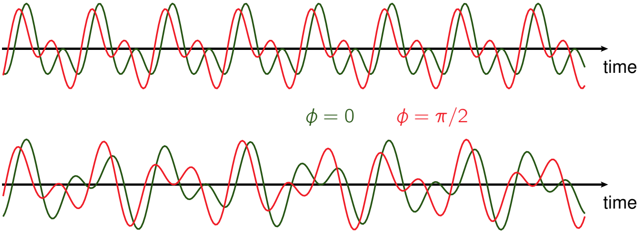
PRB **92**, 245422 (2015)

Bichromatic driving

- $g(t) = \cos(\Omega t) + \eta \cos(\Omega' t + \phi)$

  Ω', Ω commensurable *vs.* incommensurable

  ➔ $g(t)$ periodic *vs.* quasi-periodic



$\frac{\Omega'}{\Omega} = 2$

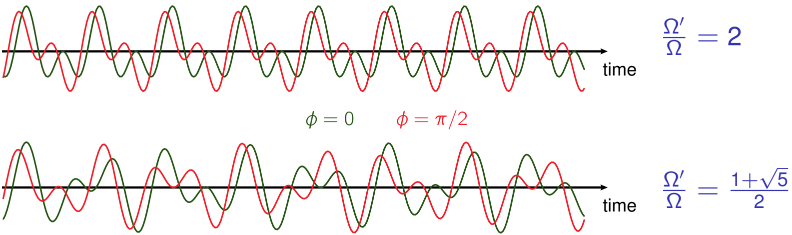
$\frac{\Omega'}{\Omega} = \frac{1+\sqrt{5}}{2}$

Bichromatic driving

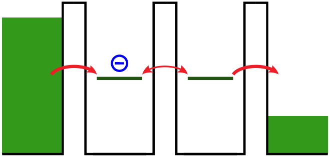
- $g(t) = \cos(\Omega t) + \eta\cos(\Omega' t + \phi)$

  Ω', Ω commensurate *vs.* incommensurable

  ➔ $g(t)$ periodic *vs.* quasi-periodic



- quasi-periodic:
  - dynamical phases (quasi) random
  - ? regular LZSM pattern, symmetries



E.g. Lindblad approach:

- source-to-dot tunneling

$$|\psi\rangle \longrightarrow c_1^\dagger|\psi\rangle$$

➔ $\dot\rho = \ldots + \Gamma\left(c_1^\dagger \rho c_1 - \frac{1}{2} c_1 c_1^\dagger \rho - \frac{1}{2}\rho c_1 c_1^\dagger\right)$

$$\frac{d}{dt}P = L(t)P \qquad \text{where} \qquad L(t) = -i\left[H_{\mathrm{DQD}}(t), \cdot\right] + L_{\mathrm{in}} + L_{\mathrm{out}}$$

Quantum master equation

$$\frac{d}{dt}\rho = L(t)\rho \quad \text{with} \quad L(t) = L_0 + L_1 \cos(n\Omega t) + L_1' \cos(n'\Omega t)$$

Two-color Floquet theory (commensurable)

Quantum master equation

$$\frac{d}{dt}\rho = L(t)\rho \quad \text{with} \quad L(t) = L_0 + L_1 \cos(n\Omega t) + L_1' \cos(n'\Omega t)$$

- long-time solution periodic, “Floquet solution with eigenvalue 0”

$$\rho(t) = \rho(t + 2\pi/\Omega) = \sum_k e^{-ik\Omega t}\rho_k$$

➔ homogeneous set of equations for ρ_k

➔ ρ_0, time-averaged expectation values, e.g. dc current

Two-color Floquet theory (incommensurable)

- $\frac{d}{dt}\rho = L(t)\rho$ with

$$L(t) = L_0 + L_1 \cos(\Omega t) + L_1' \cos(\omega t)$$

- auxiliary angular coordinate $\omega t \longrightarrow \theta$

$$\frac{d}{dt}\mathcal{P} = \mathcal{L}(t, \theta)\mathcal{P}$$

$$\mathcal{L}(t, \theta) = L_0 + L_1 \cos(\Omega t) + L_1' \cos(\theta) - \omega \frac{\partial}{\partial \theta}$$

➔ $2\pi/\Omega$-periodic time-dependence

➔ solve by usual Floquet tools — here: matrix-continued fractions

Two-color Floquet theory (incommensurable)

- $\frac{d}{dt}\rho = L(t)\rho$ with

$$L(t) = L_0 + L_1 \cos(\Omega t) + L_1' \cos(\omega t)$$

- auxiliary angular coordinate $\omega t \longrightarrow \theta$

$$\frac{d}{dt}\mathcal{P} = \mathcal{L}(t,\theta)\mathcal{P}$$

$$\mathcal{L}(t,\theta) = L_0 + L_1 \cos(\Omega t) + L_1' \cos(\theta) - \omega\frac{\partial}{\partial\theta}$$

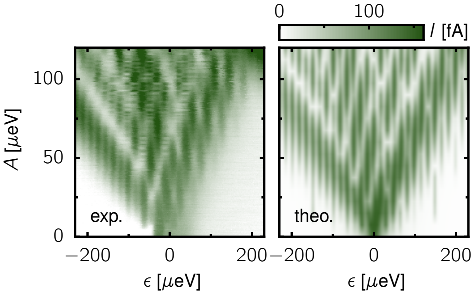
➔ $2\pi/\Omega$-periodic time-dependence

➔ solve by usual Floquet tools — here: matrix-continued fractions

- connection $\rho(t) = \mathcal{P}(t,\theta)\big|_{\theta=\omega t}$

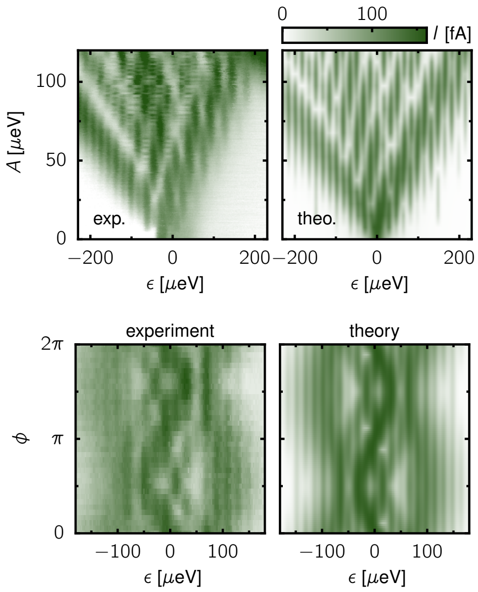
cf. t-t' formalism, see Peskin & Moiseyev, J.Chem.Phys. 1993

Theory *vs.* experiment (commensurable)



- $g(t) = \sin(\Omega t)$
  $+ \eta \sin(2\Omega t + \phi)$
  $[4.8\,\mathrm{GHz}, \eta = 1.25]$
- pattern asymmetric . . .

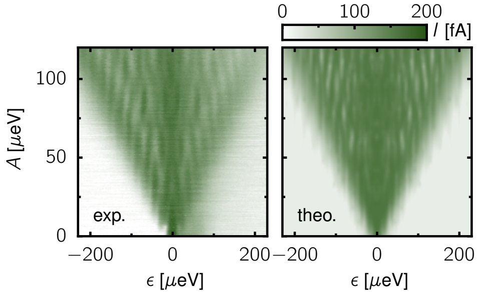
Theory *vs.* experiment (commensurable)



- $g(t) = \sin(\Omega t)$
  $+ \eta \sin(2\Omega t + \phi)$
  $[4.8\,\mathrm{GHz}, \eta = 1.25]$
- pattern asymmetric ...

- ... and ϕ-dependent

➔ phase calibration



- golden ratio $\frac{\Omega_2}{\Omega_1} = \frac{1+\sqrt{5}}{2}$

➔ pattern symmetric and ϕ-independent

Forster *et al.*, PRB 2015

Summary

1. **Floquet & Schrödinger equation**
   - Geometric phases
   - Time-periodicity, Floquet ansatz, and all that
2. **Quantum dissipation and transport**
   - System-bath model
   - Floquet-Bloch-Redfield formalism
3. **Applications & miscellaneous topics**
   - LZSM Interference
   - Time-dependent Liouvillians
   - Floquet scattering theory
   - Adiabatic Floquet theory
   - Hidden symmetries
   - Quantum chaos and dissipation
   - Floquet-Gibbs states
   - Two-color Floquet theory