Floquet State Population at Conical Intersections of Quasienergies

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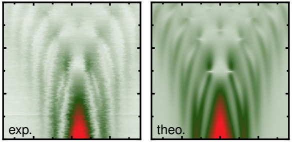
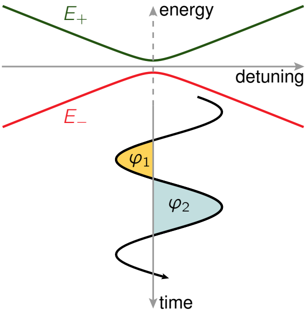
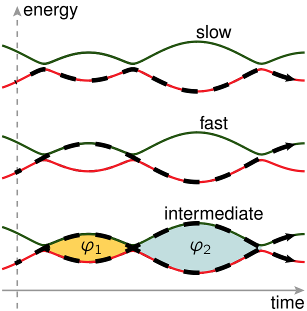


CMD31, Braga, September 2024

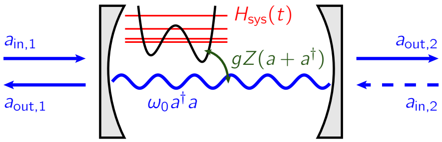
Landau-Zener-Stückelberg-Majorana interference





➔ Interference pattern as function of
detuning ϵ, amplitude A, (and driving frequency Ω)

Measurement: Cavity transmission



Dispersive frame:

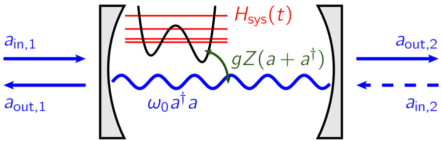
- cavity frequency shift

$$\omega_0 \longrightarrow \omega_0 + \frac{g^2}{\epsilon_{qb} - \omega_0} \sigma_z$$

➔ qubit excitation

(under certain conditions)

Measurement: Cavity transmission



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$$\omega_0 \longrightarrow \omega_0 + \frac{g^2}{\epsilon_{\mathrm{qb}} - \omega_0}\sigma_z$$

➔ qubit excitation
(under certain conditions)

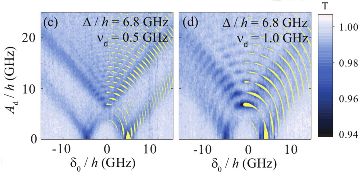
Non-equilibrium linear response:

- cavity ➔ qubit ➔ cavity
- response function

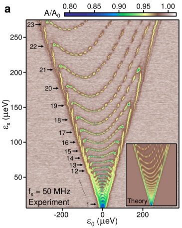
$$\chi(t) = -i\langle [Z(t), Z] \rangle \theta(t)$$

➔ $\omega_0 \longrightarrow \omega_0 + g^2 \chi(\omega_0)$

Experiments



Koski *et al.*, PRL 2018



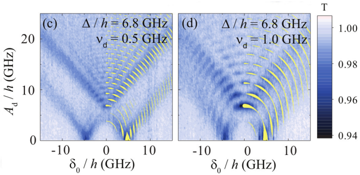
Mi, Petta, SK, PRB 2018

Experiments with DQDs in GaAs and Si are beyond . . .

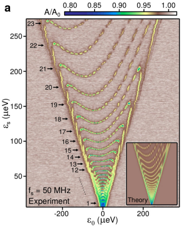
- low-frequency cavity
- two-level system
- dispersive limit

➔ cavity signal $\neq$ excitation probability

Experiments



Koski *et al.*, PRL 2018



Mi, Petta, SK, PRB 2018

Experiments with DQDs in GaAs and Si are beyond . . .

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➔ cavity signal $\neq$ excitation probability

? (Floquet) theory for measurement

Susceptibility of driven system

Response function

$$\chi(t, t') = -i\big\langle [Z(t), Z(t')] \big\rangle_{\text{non-eq}} \theta(t - t') = \chi(t+T, t'+T)$$

such that

$$\chi(t, t-\tau) = \sum_k e^{-ik\Omega t} \int d\omega\, e^{-i\omega\tau} \chi^{(k)}(\omega)$$

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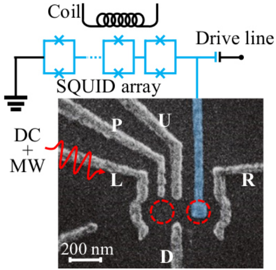
Relevant component:

$$\chi^{(0)}(\omega_0) = \sum_{\beta,\alpha,k} \frac{(p_\alpha - p_\beta)|Z_{\beta\alpha,k}|^2}{\epsilon_\alpha - \epsilon_\beta + \omega_0 + k\Omega + i\gamma/2}$$

- Floquet theory ➔ quasi-energies ϵ_α
- Floquet-Bloch-Redfield ➔ populations p_α

SK, PRL 2017 & PRA 2018

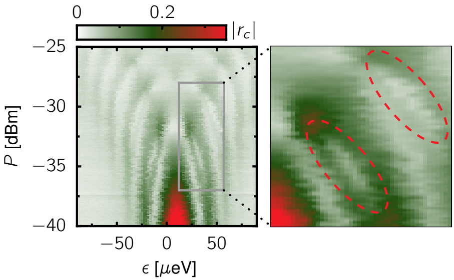
Readout of Floquet State Population



Chen *et al.*, Phys. Rev. B **103**, 205428 (2021)

SK, arXiv:2405.12093

Motivation: Holes in interference fringes



Experiment (Cao & Guo, Hefei)

- holes in LZSM pattern
- GaAs DQD ➔ two-level sys.

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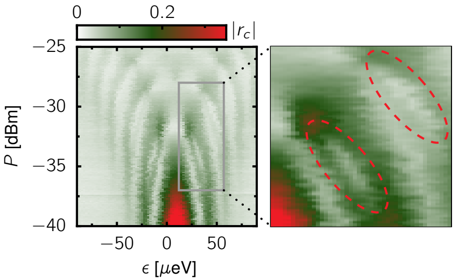


Experiment (Cao & Guo, Hefei)

- holes in LZSM pattern
- GaAs DQD ➔ two-level sys.

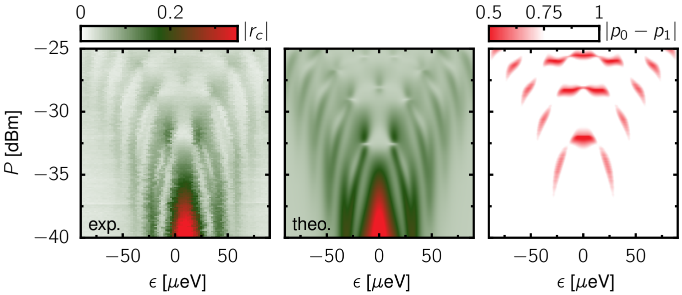
Susceptibility (two-level system)

$$\chi^{(0)}(\omega_0) = (p_0 - p_1) \sum_k \frac{|Z_{10,k}|^2}{\epsilon_1 - \epsilon_0 + \omega_0 + k\Omega + i\gamma/2}$$

Response determined by

- Floquet state population $\quad \Delta\epsilon = k\Omega$
- resonance condition $\quad \Delta\epsilon + \omega_0 = k\Omega$

Experiment *vs.* theory



- holes in fringes when $p_0 \approx p_1 \approx 1/2$
- competing resonance conditions verified

➔ cavity response provides information about Floquet state population

Floquet-Gibbs state *vs.* mean energy state

Steady state of driven dissipative quantum system

quasi energy: $p_\alpha \propto e^{-\epsilon_\alpha / kT}$ mean energy: $p_\alpha \propto e^{-E_\alpha / kT}$

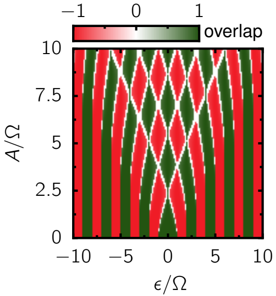
Floquet-Gibbs state *vs.* mean energy state

Steady state of driven dissipative quantum system

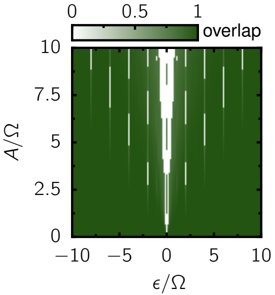
quasi energy: $p_\alpha \propto e^{-\epsilon_\alpha/kT}$

mean energy: $p_\alpha \propto e^{-E_\alpha/kT}$

bath via σ_z



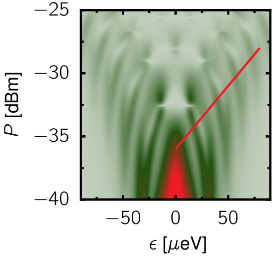
bath via σ_x

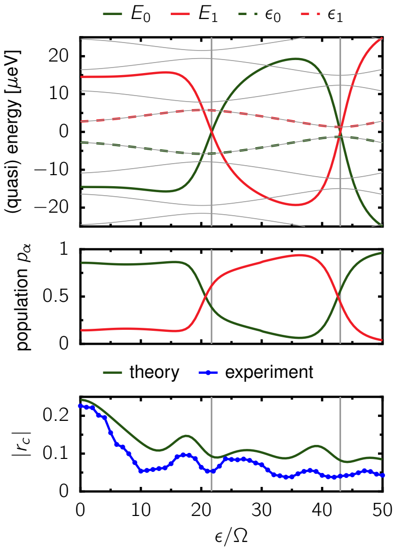


Floquet-Gibbs state
vs. anti Floquet-Gibbs

The present case!

Floquet state population





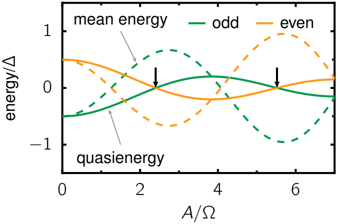
Measurement consistent with

- bath coupling via σ_x
- p_α determined by E_α

  i.e. mean-energy state

Conical intersection

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{A}{2}\sigma_z \cos(\Omega t)$$



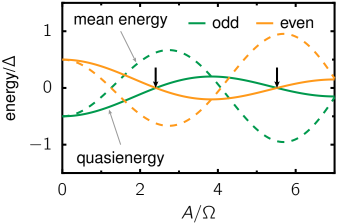
- spatio-temporal symmetry G:

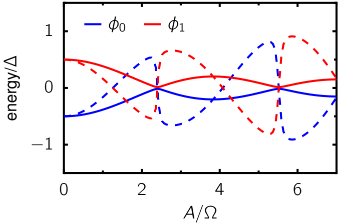
  σ_x & $(t \rightarrow t + T/2)$

➔ even / odd states

Conical intersection

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{A}{2}\sigma_z \cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$





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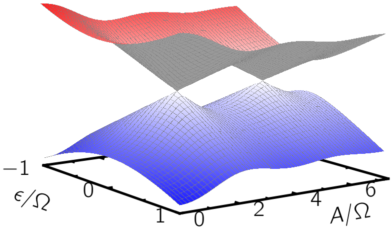


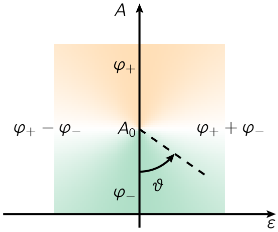
Floquet "ground state"

Two-state approximation for ϕ_0

- basis: φ_- / φ_+ at tip
- $\vartheta = 0 \longrightarrow \vartheta = \pi$:

$$\phi_0 \longrightarrow \phi_1$$

$$p_0 \longrightarrow p_1 = 1 - p_0$$



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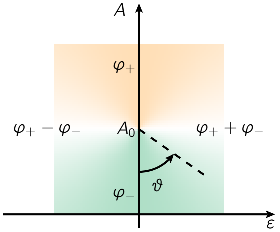

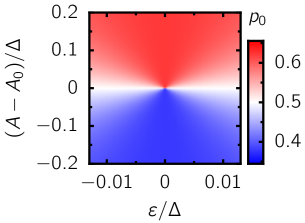

Population for σ_x-coupling

- angles with $p_0 = p_1 = 1/2$
  - full mixture, max. entropy
  - cavity signal vanishes
- behaves like mean energy of ϕ_0

cf. discontinuity along A-axis

Engelhardt *et al.*, PRL 2019

Transition to Floquet-Gibbs state

Golden-rule rate with $S_{01}(t) = \langle \phi_0(t)|S|\phi_1(t)\rangle$ ➔ sidebands S_k

Generic

- Ohmic $J(\omega) = \frac{\pi}{2}\alpha\omega$

➔ $k = 0$ suppressed

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Generic

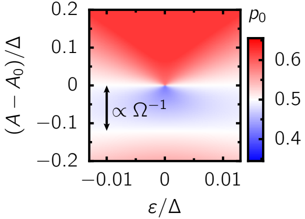
- Ohmic $J(\omega) = \frac{\pi}{2}\alpha\omega$

➔ $k = 0$ suppressed

Exception: $S = \sigma_z$

- time-independent for large Ω

➔ $k \neq 0$ suppressed



- Crossover to Floquet-Gibbs: two lines with $p_0 = 1/2$ merge with increasing Ω

➔ Measurable signature of σ_z-coupling

SK, arXiv:2405.12093

Floquet State Population at Conical Intersections

- Dispersive readout
  - Theory for ac-driven systems
  - Non-equilibrium susceptibility
- Floquet state population
  - Holes in LZSM pattern
  - Floquet-state population
- Conical intersections of quasi-energies
  - Signature of qubit-bath coupling

Thanks to J. R. Petta (UCLA) and G. Cao (Hefei)