



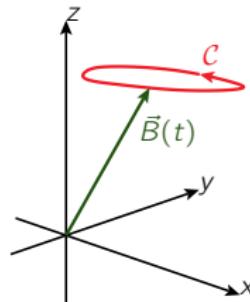
# Floquet theory for open quantum systems

Sigmund Kohler

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- I. Floquet theory for Schrödinger equation
- II. Dissipation & transport
- III. Miscellaneous topics — LZSM interference, time-dependent Liouvillians, hidden symmetries, Floquet-Gibbs states, bichromatic driving, scattering theory, adiabatic Floquet theory, ...

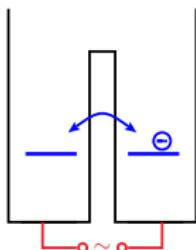
<https://sigmundkohler.github.io/download/FloquetTutorial.pdf>



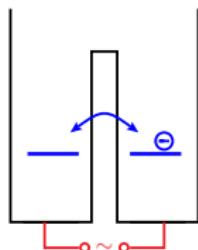
### Laser physics since the 1970s

- strong ac fields acting on atoms
- treatment beyond linear response
- **Floquet theory**
  - discrete time translation and Floquet ansatz
  - properties of Floquet states
  - relation to Berry phase
  - analytical and numerical techniques
- **Floquet-Bloch-Redfield theory**
  - Bloch-Redfield vs. Lindblad master equation
  - driving, chaos, and dissipation

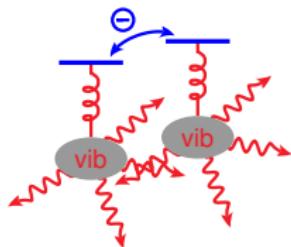
Driven double quantum dot



Driven double quantum dot

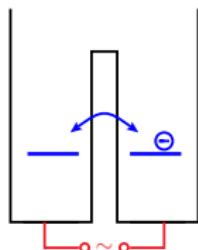


... with heat bath

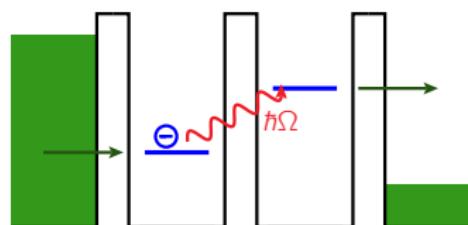


→ quantum dissipation

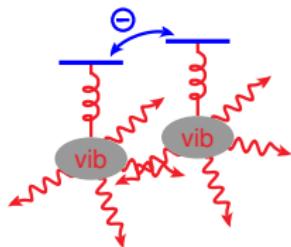
Driven double quantum dot



... electron source / drain

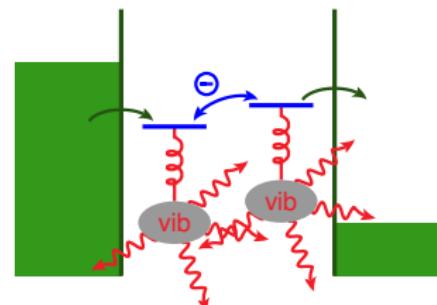


... with heat bath



→ quantum dissipation

... or both



→ quantum transport

## 1 Floquet & Schrödinger equation

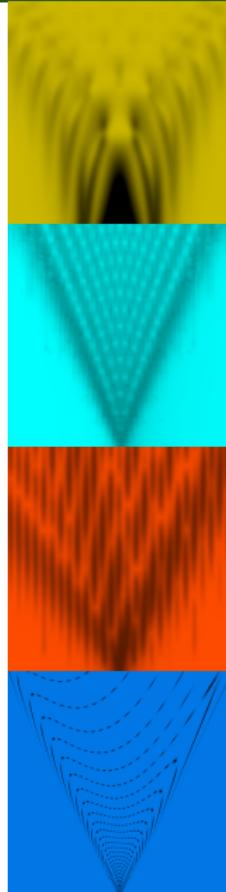
- Geometric phases
- Time-periodicity, Floquet ansatz, and all that

## 2 Quantum dissipation and transport

- System-bath model
- Floquet-Bloch-Redfield formalism

## 3 Applications & miscellaneous topics

- LZSM Interference
- Time-dependent Liouvillians
- Floquet scattering theory
- Adiabatic Floquet theory
- Hidden symmetries
- Quantum chaos and dissipation
- Floquet-Gibbs states
- Two-color Floquet theory



- Time evolution of an eigenstate:

$$|\psi(t)\rangle = e^{-iE_n t} |\phi_n\rangle$$

Notation:

$\psi$ : solution of Schrödinger equation

$\phi$ : other state vector, e.g., eigenstate

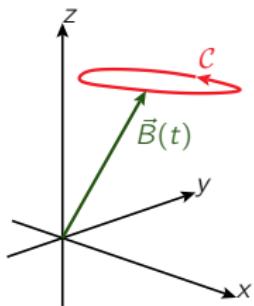
- energy  $\leftrightarrow$  phase

for (periodically) time-dependent system ?

- Spin in magnetic field

$$\vec{B}(t) = \vec{B}(t + T):$$

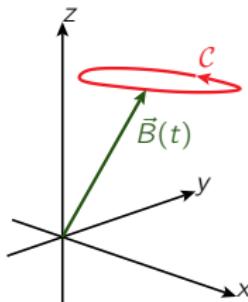
$$H(t) = \frac{1}{2} \vec{B}(t) \cdot \vec{\sigma}$$



- Spin in magnetic field

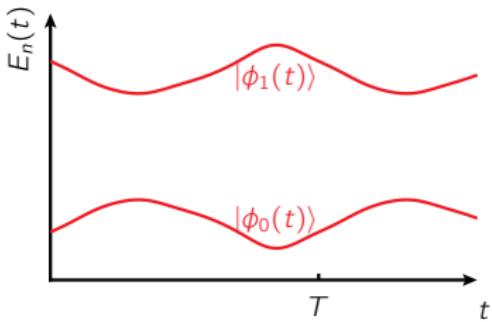
$$\vec{B}(t) = \vec{B}(t + T):$$

$$H(t) = \frac{1}{2} \vec{B}(t) \cdot \vec{\sigma}$$



- Quantum dynamics for  $\dot{B} \ll B^2$ : state follows the eigenstate adiabatically

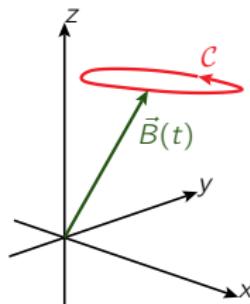
$$|\psi(t)\rangle \propto |\phi_n(t)\rangle$$



→  $|\psi(t)\rangle$  determined up to phase factor

After one period:  $|\psi(T)\rangle = e^{i\varphi} |\psi(0)\rangle$

$$\varphi = - \int_0^T dt E_n(t) + \gamma_C$$

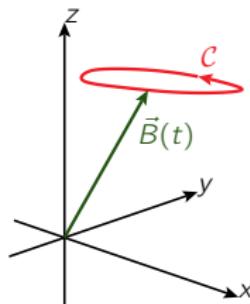


- dynamical phase  $\leftrightarrow$  mean energy
- Berry phase  $\gamma_C$ 
  - depends only on closed curve  $C$  in parameter space

M. Berry, Proc. Roy. Soc. London, Ser. A **392**, 45 (1984)

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- Assumptions:
  - 1  $\vec{B}(t)$  changes adiabatically slowly
  - 2 initial state: eigenstate  $|\phi_n(0)\rangle$

Different perspective:

State vector undergoes **periodic** time-evolution

- $|\psi(T)\rangle = e^{i\varphi}|\psi(0)\rangle$
- dynamics  $|\psi(t)\rangle$  induced by some Hamiltonian  $H(t)$

Remarks:

- no adiabatic condition
- $|\psi(t)\rangle$  need not be an eigenstate of  $H(t)$
- $H(t)$  is not unique
- only condition: **cyclic time-evolution in Hilbert space**

$$|\psi(t)\rangle = e^{if(t)}|\phi(\textcolor{red}{t})\rangle, \quad \phi(\textcolor{red}{t}) = \phi(\textcolor{red}{t} + T)$$

→ Phase acquired during cyclic evolution:  $\varphi = f(T) - f(0)$

$$\frac{df}{dt} = -\langle \phi | H(t) | \phi \rangle + \langle \phi | i \frac{d}{dt} | \phi \rangle \implies \boxed{\varphi = \gamma_{\text{dyn}} + \gamma}$$

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Dynamical phase  $\gamma_{\text{dyn}} = - \int_0^T dt \langle \phi(t) | H(t) | \phi(t) \rangle$

- depends on choice of  $H(t)$
- reflects mean energy

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Aharonov-Anandan phase (“non-adiabatic Berry phase”)

$$\gamma = \int_0^T dt \langle \phi | i \frac{d}{dt} | \phi \rangle$$

- depends only on trajectory in Hilbert space — not in parameter space!
- adiabatic limit:  $\gamma = \gamma_C$

Aharonov & Anandan, PRL 1987

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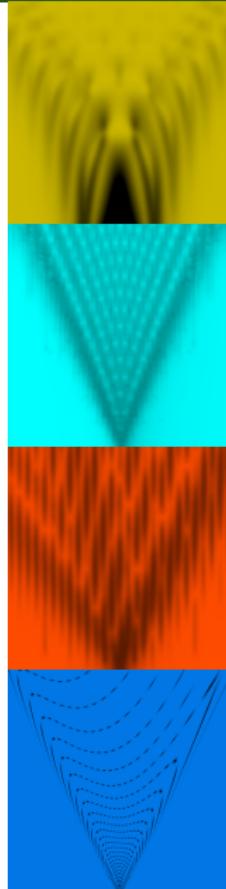
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Goal: propagator  $U(t, t')$

- Time-independent system: diagonalize Hamiltonian  $\rightarrow |\phi_n\rangle, E_n$

$$U(t, t') = U(t - t') = \sum_n e^{-iE_n(t-t')} |\phi_n\rangle\langle\phi_n|$$

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- Driven system:

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle \quad \rightarrow \text{numerical integration}$$

problem 1: time-integration not efficient for long times

problem 2: no information about structure of  $U$

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**problem 1:** time-integration not efficient for long times

**problem 2:** no information about structure of  $U$

- Solution for  $H(t) = H(t + T)$ : “Bloch theory in time”

cf.  $H(x)|\phi\rangle = \epsilon|\phi\rangle$  with  $H(x) = H(x + a)$

$\rightarrow$  Bloch waves  $\phi(x) = e^{iqx}\varphi(x)$ , where  $\varphi(x)$  is  $a$ -periodic

F. Bloch, Z. Phys. A 52, 555 (1928)

Gaston Floquet (1883):

Ann. de l'Ecole Norm. Sup. **12**, 47 (1883)

Parametric oscillator (cf. Paul trap)

$$\ddot{x} + (\omega_0^2 + \epsilon \cos \Omega t)x = 0$$

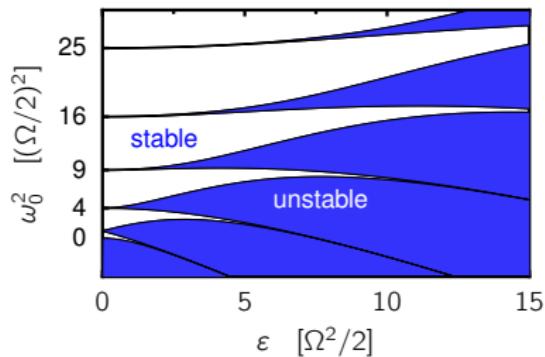
Floquet theorem:

solutions have the structure

$$x(t) = e^{\pm i\mu t} \xi(t)$$

where  $\xi(t) = \xi(t + 2\pi/\Omega)$

(undriven limit:  $\mu = \omega_0$ ,  $\xi = \text{const}$ )



$\mu$  real

→ oscillating solutions

$\mu$  imaginary

→ one solution unstable

- $H(t) = H(t + T)$ 
  - $t \rightarrow t + T$  is symmetry operation
  - solutions of Schrödinger equation with  $|\psi(t+T)\rangle = e^{i\varphi} |\psi(t)\rangle$
- Floquet ansatz

$$|\psi(t)\rangle = e^{-i\epsilon t} |\phi(t)\rangle = e^{-i\epsilon t} \sum_k e^{-ik\Omega t} |\phi_k\rangle$$

- $\epsilon$  quasienergy (cf. quasi momentum) → long-time dynamics
- $|\phi(t)\rangle = |\phi(t + T)\rangle$ , Floquet state → within driving period
- Floquet theorem:  $H(t)$  has a complete set of Floquet solutions
- Schrödinger equation  $i\partial_t |\psi\rangle = H(t) |\psi\rangle$  yields

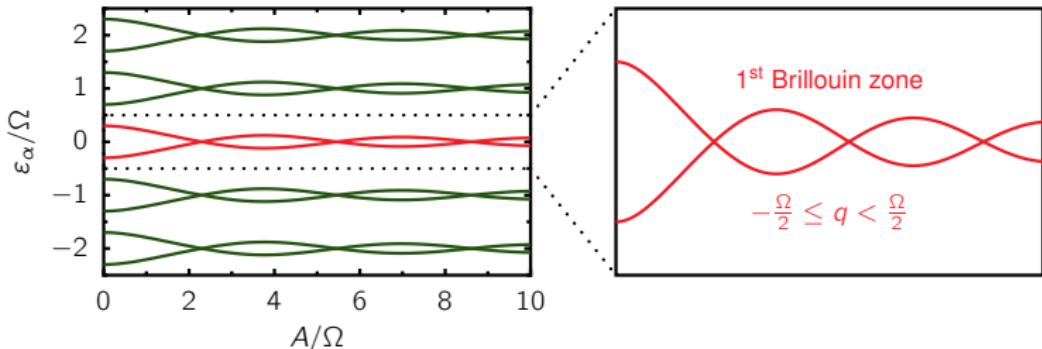
$$\left( H(t) - i \frac{d}{dt} \right) |\phi(t)\rangle = \epsilon |\phi(t)\rangle$$

$|\phi(t)\rangle$  Floquet state with quasienergy  $\epsilon$

- $e^{ik\Omega t}|\phi(t)\rangle$  Floquet state with  $\epsilon + k\Omega$
- Equivalent states: same  $|\psi(t)\rangle$

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- Equivalent states: same  $|\psi(t)\rangle$



- all Brillouin zones equivalent, choice arbitrary
- quasienergies cannot serve for ordering!

- Physical quantity / observable: mean energy

$$E = \frac{1}{T} \int_0^T dt \langle \psi(t) | H(t) | \psi(t) \rangle = \frac{1}{T} \int_0^T dt \langle \phi(t) | H(t) | \phi(t) \rangle$$

- All equivalent states have the same mean energy  
[proof: insert  $e^{ik\Omega t} |\phi(t)\rangle$ ]
- Floquet states can be ordered by their mean energy

## ■ Mean energy

$$E = \frac{1}{T} \int_0^T dt \langle \phi(t) | \{ H(t) - i\partial_t + i\partial_t \} | \phi(t) \rangle$$

where  $(H - i\partial_t)|\phi(t)\rangle = \epsilon|\phi(t)\rangle$

$$E = \epsilon + \frac{1}{T} \int_0^T dt \langle \phi(t) | i\partial_t | \phi(t) \rangle$$

- Mean energy

$$\textcolor{blue}{E} = \frac{1}{T} \int_0^T dt \langle \phi(t) | \{ H(t) - i\partial_t + \textcolor{red}{i\partial_t} \} | \phi(t) \rangle$$

where  $(H - i\partial_t)|\phi(t)\rangle = \epsilon|\phi(t)\rangle$

$$-\epsilon = \textcolor{blue}{-E} + \frac{1}{T} \int_0^T dt \langle \phi(t) | i\partial_t | \phi(t) \rangle$$

- Compare to  $\varphi = \gamma_{\text{dyn}} + \gamma$

- Mean energy

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- Compare to

$$\varphi = \gamma_{\text{dyn}} + \gamma$$

$(E - \epsilon)T$  is a geometric phase

Floquet equation as function of  $\Omega$  and  $\Omega t$ :

$$\epsilon(\Omega)\phi(\Omega t) = \left[ H(\Omega t) - i\Omega \frac{\partial}{\partial \Omega t} \right] \phi(\Omega t)$$

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Compute derivative  $(\partial/\partial\Omega)|_{\Omega t}$  and apply  $\int \frac{dt}{T} \phi^+$  to obtain

$$\frac{\partial \epsilon}{\partial \Omega} = -\frac{\gamma}{2\pi} = \frac{\epsilon - E}{\Omega} \quad \rightarrow \quad E = \epsilon - \Omega \frac{\partial \epsilon}{\partial \Omega}$$

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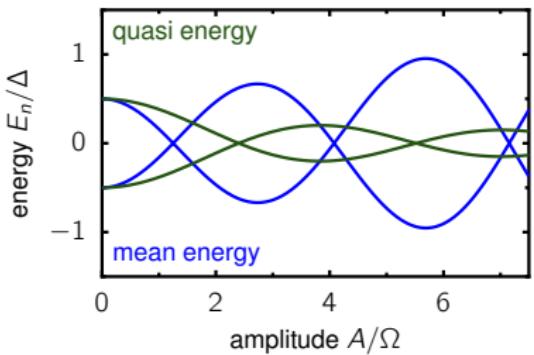
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E.g. for  $\gamma = \gamma_{\text{ad}} + \mathcal{O}(\Omega)$

$$\rightarrow \epsilon = \text{const} - (\gamma_{\text{ad}}/2\pi)\Omega + \dots \rightarrow E = E_{\text{ad}} + \mathcal{O}(\Omega^2)$$

Fainshtein, Manakov, Rapoport, J. Phys. B (1978)

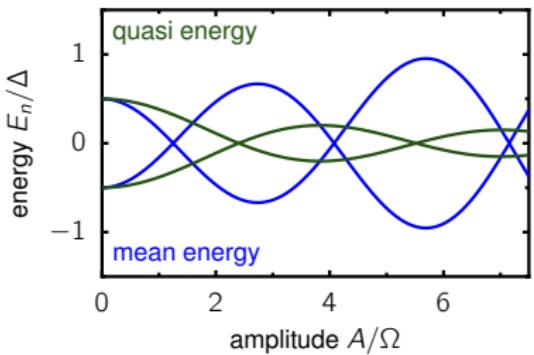
## Mean energies — two-level system



Driven **undetuned** two-level system

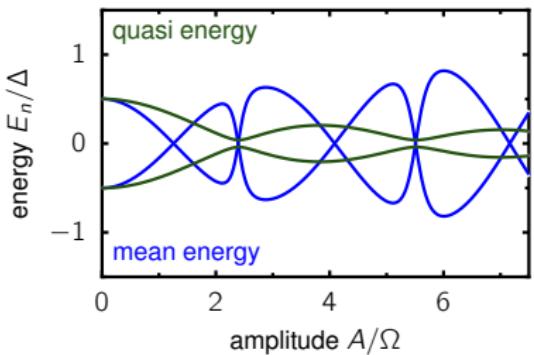
- exact crossings  
(consequence of a symmetry)

# Mean energies — two-level system



Driven **undetuned** two-level system

- exact crossings  
(consequence of a symmetry)



... with **small detuning**

- quasi energies
  - avoided crossings
- mean energies
  - exact crossings remain
  - additional crossings
  - do not follow from any eigenvalue equation

Goal: more formal treatment of  $H(t) - i\partial_t$

- $|\phi(t)\rangle \in \mathcal{R} \otimes \mathcal{T}$  composite Hilbert space / Sambe space

Shirley, PR **138**, B979 (1965), Sambe, PRA **7**, 2203 (1973)

$\mathcal{T}$ : Hilbert space of  $T$ -periodic functions with inner product

$$\langle f|g\rangle = \int_0^T f(t)^* g(t) \frac{dt}{T} = \sum_k f_k^* g_k$$

- extended Dirac notation:

- $|\phi(t)\rangle = \langle t|\phi\rangle$
- Fourier coefficient  $|\phi_k\rangle = \langle k|\phi\rangle$

e.g.:  $|\phi(t)\rangle = \langle t|\phi\rangle = \sum_k \langle t|k\rangle \langle k|\phi\rangle = \sum_k e^{-ik\Omega t} |\phi_k\rangle$

- $H - i\partial_t$  is hermitian
- Floquet states  $|\phi_\alpha\rangle$  orthonormal and complete in  $\mathcal{R} \otimes \mathcal{T}$

$$\langle\langle \phi_\alpha^{(k)} | \phi_\beta^{(k')} \rangle\rangle = \delta_{\alpha\beta} \delta_{kk'}$$

? but in  $\mathcal{R}$  ?

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? but in  $\mathcal{R}$  ?

- Consider  $\langle \phi_\alpha(\textcolor{red}{t}) | \phi_\beta(\textcolor{red}{t}) \rangle = \sum_k \lambda_k e^{-ik\Omega t}$  since  $T$ -periodic with the Fourier coefficient

$$\lambda_k = \frac{1}{T} \int_0^T dt e^{ik\Omega t} \langle \phi_\alpha(t) | \phi_\beta(t) \rangle = \langle\langle \phi_\alpha | \phi_\beta^{(k)} \rangle\rangle = \delta_{\alpha\beta} \delta_{k,0}$$

- Floquet states orthogonal at **equal** times

- propagator in terms of Floquet states

$$U(t, t') = \sum_{\alpha} |\psi_{\alpha}(t)\rangle\langle\psi_{\alpha}(t')| = \sum_{\alpha} e^{-i\epsilon_{\alpha}(t-t')} |\phi_{\alpha}(t)\rangle\langle\phi_{\alpha}(t')|$$

- long-time dynamics (depends on  $t - t'$ )
- dynamics within driving period (“micro motion”, depends on  $t$  and  $t'$ )

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- long-time dynamics (depends on  $t - t'$ )
- dynamics within driving period (“micro motion”, depends on  $t$  and  $t'$ )

- one-period propagator for kicked systems

$$H(t) = H_0 + K \sum_n \delta(t - nT)$$

$$\rightarrow U(T) = e^{-iH_0 T} e^{-iK}$$

- ✓ easy to compute
- ✓ provides quasienergies
- ✗ only long-time dynamics (stroboscopic)

Solve eigenvalue problem

$$\{H(t) - i\partial_t\}|\phi\rangle = \epsilon|\phi\rangle$$

Solve eigenvalue problem

$$\{H(t) - i\partial_t\}|\phi\rangle = \epsilon|\phi\rangle$$

Straightforward in Fourier representation (“Floquet matrix”)

$$H_0 + H_1 \cos(\Omega t) - i \frac{d}{dt} \leftrightarrow \begin{pmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & H_0 + 2\Omega & \frac{1}{2}H_1 & 0 & 0 & 0 & \dots \\ \dots & \frac{1}{2}H_1 & H_0 + \Omega & \frac{1}{2}H_1 & 0 & 0 & \dots \\ \dots & 0 & \frac{1}{2}H_1 & H_0 & \frac{1}{2}H_1 & 0 & \dots \\ \dots & 0 & 0 & \frac{1}{2}H_1 & H_0 - \Omega & \frac{1}{2}H_1 & \dots \\ \dots & 0 & 0 & 0 & \frac{1}{2}H_1 & H_0 - 2\Omega & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- 1** direct diagonalization of  $H(t) - i\partial_t$ 
  - conceptually simple → first choice
  - increasingly difficult with smaller frequency
- 2** analytical tool: **perturbation theory**  
strong driving:  $H_1 \cos(\Omega t) - i\partial_t$  as zeroth order

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- 3 numerical propagation:  $U(T, 0) \rightarrow e^{-i\epsilon T}$  and  $|\phi(0)\rangle \rightarrow |\phi(t)\rangle$
- 4 matrix-continued fractions (convenient for time-dep. Liouvillians)
- 5  $(t, t')$  formalism (very efficient propagation scheme)

Rabi-like Hamiltonian

$$H = \Omega a^\dagger a + g(a^\dagger + a)X + \dots$$

Rabi-like Hamiltonian

$$H = \Omega a^\dagger a + g(a^\dagger + a)X + \dots$$

- Fock basis  $|n_0 - k\rangle$  with  $n_0 \approx \langle a^\dagger a \rangle$
- lowest order in  $k < n_0$

$$a \longrightarrow \delta_{k,k+1}$$

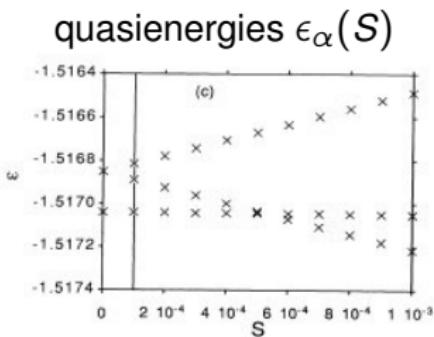
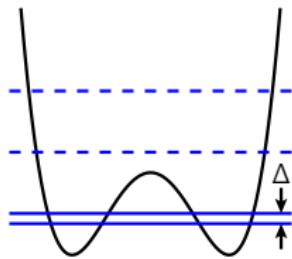
$$H \longrightarrow (n_0 - k)\Omega\delta_{k,k'} + 2g\sqrt{\langle n \rangle}\delta_{k,k' \pm 1}X + \dots$$

→ Floquet Hamiltonian with  $A = 2g\sqrt{\langle n \rangle}$

## Example I: Coherent destruction of tunneling

- 1 role of quasienergy crossings
- 2 perturbation theory (two-level approximation)

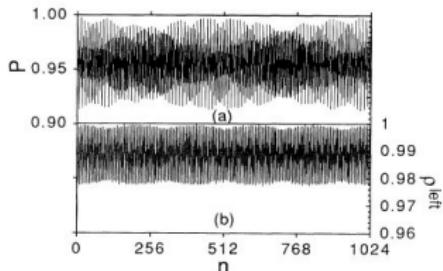
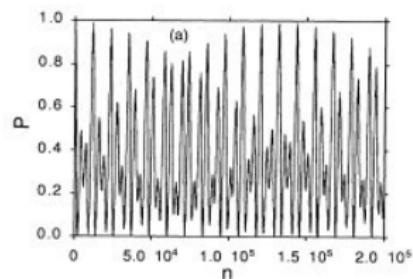
Driven double-well potential  $H(t) = H_{\text{DW}} + Sx \cos(\Omega t)$



- ? tunnel oscillations influenced by driving
- ? dynamics at quasienergy crossing

## Example I: Coherent destruction of tunneling

Occupation  $P_{\text{left}}(nT)$



far from crossing:

- tunnel oscillations

at crossing:

- particle stays in left well
- “coherent destruction of tunneling” by ac field

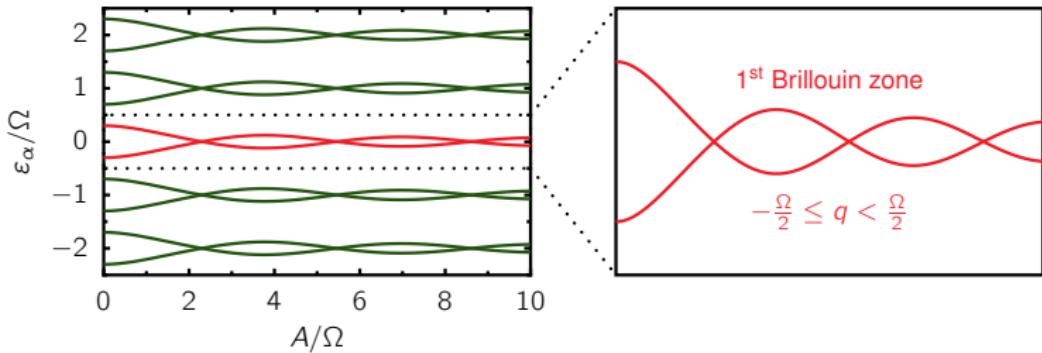
Grossmann et al., PRL 1991

Analytical understanding → two-level approximation

Driven two-level system

$$H(t) = -\frac{\Delta}{2}\sigma_x + \frac{A}{2}\cos(\Omega t)\sigma_z$$

quasienergy spectrum



Analytical approach for  $\Delta \ll \Omega$ : high-frequency limit

$$H(t) = -\frac{\Delta}{2}\sigma_x + \frac{A}{2}\cos(\Omega t)\sigma_z$$

- zeroth order propagator

$$U_{\text{dr}}(t, 0) = \exp\left(-\frac{iA}{2\Omega}\sin(\Omega t)\sigma_z\right)$$

- transformation and rotating-wave approximation (i.e. time-average)

$$H(t) \longrightarrow -\frac{\Delta}{2} U_{\text{dr}}^\dagger(t, 0)\sigma_x U_{\text{dr}}(t, 0) \longrightarrow -\frac{\Delta}{2} J_0(A/\Omega)\sigma_x$$

→ tunnel-matrix element renormalized by Bessel function  $J_0$

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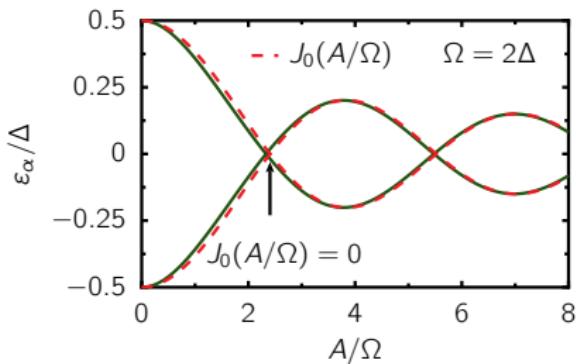
Bessel function  $J_n(x)$ :  $n$ th Fourier coefficient of  $e^{-ix\sin(\Omega t)}$

## Example I: CDT — perturbation theory

$$H_{\text{eff}} = \frac{\tilde{\Delta}}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- quasienergies

$$\epsilon_{\pm} = \pm \frac{\Delta}{2} J_0(A/\Omega)$$



## Example I: CDT — perturbation theory

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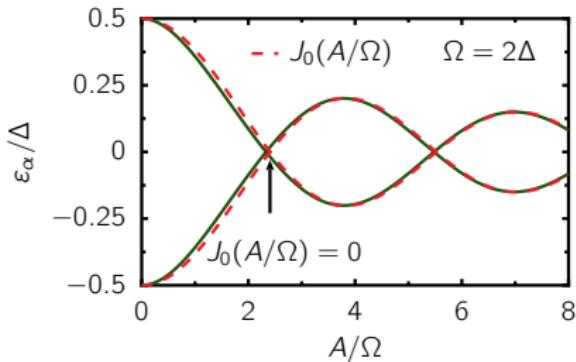
- quasienergies

$$\epsilon_{\pm} = \pm \frac{\Delta}{2} J_0(A/\Omega)$$

- mean energies  $E = \epsilon - \Omega(\partial\epsilon/\partial\Omega)$

$$E_{\pm} = \pm \frac{\Delta}{2} \left[ J_0(A/\Omega) - \frac{A}{\Omega} J_1(A/\Omega) \right]$$

no need to compute Floquet states!



Bessel function  $J_1(x) = \frac{d}{dx} J_0(x)$

$$H(t) = -\frac{\Delta}{2}\sigma_z + \frac{A}{2}\sigma_x \cos(\Omega t)$$

- close to resonance:  $\delta = \Delta - \Omega \ll \Delta$ , small amplitude:  $A \ll \Delta$

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- close to resonance:  $\delta = \Delta - \Omega \ll \Delta$ , small amplitude:  $A \ll \Delta$

$$\mathcal{H}_0 = \frac{\Omega}{2}\sigma_z - i\frac{\partial}{\partial t} \quad \mathcal{H}_1 = \frac{\delta}{2}\sigma_z + \frac{A}{2}\cos(\Omega t)\sigma_x$$

- 2nd order degenerate perturbation theory

$$\mathcal{H} \approx \frac{1}{2} \begin{pmatrix} \delta + A^2/4\Omega & A \\ A & -\delta - A^2/4\Omega \end{pmatrix} \quad \text{Rabi Hamiltonian beyond RWA}$$

- quasienergy splitting:  $\epsilon_2 - \epsilon_1 = \bar{\omega}$ , where  $\bar{\omega}^2 = \delta^2 + A^2 + \frac{A^2\delta}{8\Omega}$
- absorption maximum at

$$\Omega_{\text{res}} \approx \Delta + \frac{A^2}{16\Delta} \quad \text{Bloch-Siegert shift}$$

## Some standard references

### ■ Classic work:

- Shirley, Phys. Rev. 138, B979 (1965)
- Sambe, Phys. Rev. A 7, 2203 (1973)

### ■ Reviews:

- Grifoni, Hänggi, Phys. Rep. 304, 229 (1998)
- Hänggi, Chap.5 of “Quantum transport and dissipation” (1998)  
<http://www.physik.uni-augsburg.de/theo1/hanggi/Papers/Chapter5.pdf>
- Eckardt, Rev. Mod. Phys. 89, 011004 (2017)

- 1 Compute numerically the quasienergies of the driven TLS
- 2 Perform the corresponding perturbation theory for  $\Delta \ll \Omega$
- 3 Derive the Rapoport relation  $E = \epsilon - \Omega \frac{\partial \epsilon}{\partial \Omega}$
- 4 Compute the stability borders of the Mathieu equation  
Hint: assume integer  $\mu$  and compute the corresponding  $\omega_0^2$ ,  
why does this work?
- 5 Given  $H(t)$  and an adiabatic eigenstate  $|u(t)\rangle$  with energy  $E(t)$ .  
Write the ansatz for adiabatic following as Floquet state.  
Which gauge transformations are still allowed?

## 1 Floquet & Schrödinger equation

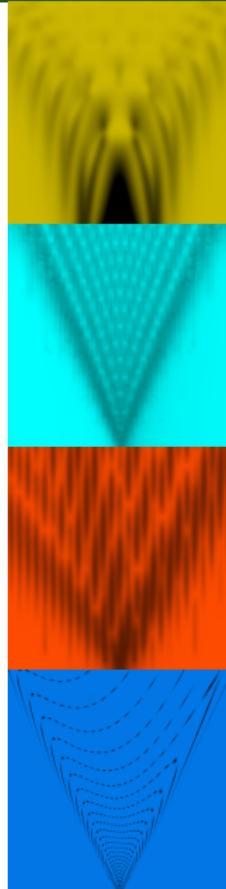
- Geometric phases
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## 2 Quantum dissipation and transport

- System-bath model
- Floquet-Bloch-Redfield formalism

## 3 Applications & miscellaneous topics

- LZSM Interference
- Time-dependent Liouvillians
- Floquet scattering theory
- Adiabatic Floquet theory
- Hidden symmetries
- Quantum chaos and dissipation
- Floquet-Gibbs states
- Two-color Floquet theory



## Heuristic approach

coupling of qubit to electromagnetic environment → spontaneous decay

$$|\psi\rangle \longrightarrow \begin{cases} \sigma_- |\psi\rangle & \text{decay with probability } \alpha \ll 1 \\ |\psi\rangle + |\delta\psi\rangle & \text{no decay, probability } 1 - \alpha \end{cases}$$

- normalization requires  $|\delta\psi\rangle = \frac{\alpha}{2}\sigma_+\sigma_-|\psi\rangle$

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- normalization requires  $|\delta\psi\rangle = \frac{\alpha}{2}\sigma_+\sigma_-|\psi\rangle$
- corresponding density operator

$$\rho \longrightarrow \rho + \frac{\alpha}{2} \left( 2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_- \right)$$

- continuous limit with coherent time-evolution → master equation

$$\frac{d}{dt}\rho = -i[H, \rho] + \frac{\gamma}{2} \left( 2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_- \right)$$

Time evolution must conserve

- hermiticity and trace of  $\rho$
- positivity (all eigenvalues of  $\rho \geq 0$ )

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$$\frac{d}{dt}\rho = -i[H, \rho] + \sum_n \gamma_n \left( 2Q_n \rho Q_n^\dagger - Q_n^\dagger Q_n \rho - \rho Q_n^\dagger Q_n \right)$$

G. Lindblad, Comm. Math. Phys. **48**, 119 (1976)

V. Gorini, J. Math. Phys. **17**, 821 (1976)

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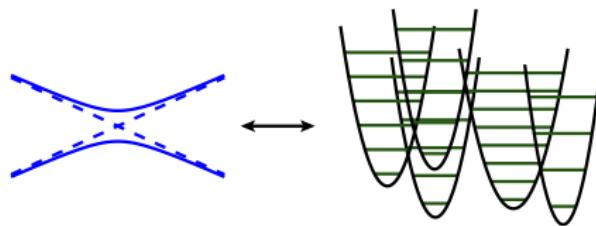
### X Critique

- request for Markovian evolution unphysical
- axiomatic, not based on physical model
- high-temperature limit typically wrong  
i.e. not the Klein-Kramers or the Smoluchowski equation

Caldeira-Leggett model

Magalinskii 1959; Caldeira, Leggett 1981

Coupling of a system to bath of harmonic oscillators



$$H = H_{\text{system}}(t) + \sum_{\nu} \gamma_{\nu} (b_{\nu}^{\dagger} + b_{\nu}) + \sum_{\nu} \omega_{\nu} b_{\nu}^{\dagger} b_{\nu}$$

- eliminate bath
- equation of motion for reduced density operator
- interpretation: bath “measures” system operator  $X$

Total density operator  $R \approx \rho \otimes \rho_{\text{bath,eq}}$

$$\dot{R} = -i[H_{\text{total}}, R]$$

2nd order perturbation theory in system-bath coupling

$$\begin{aligned} \frac{d}{dt}\rho = & -i[H_{\text{sys}}, \rho] - i \int_0^{(t-t_0) \rightarrow \infty} d\tau \mathcal{A}(\tau) [X, [\tilde{X}(-\tau), \rho(t-\tau)]_+] \\ & - \int_0^{(t-t_0) \rightarrow \infty} d\tau \mathcal{S}(\tau) [X, [\tilde{X}(-\tau), \rho(t-\tau)]] \end{aligned}$$

- Heisenberg operator  $\tilde{X}(-\tau) = U(\tau) X U^\dagger(\tau)$
- bath correlation functions  $\mathcal{A}, \mathcal{S}$
- non-Markovian
- short system-bath correlation time: **Markov approximation**

- anti-symmetric correlation function

$$\mathcal{A}(\tau) = -i\langle [\xi(\tau), \xi(0)] \rangle$$

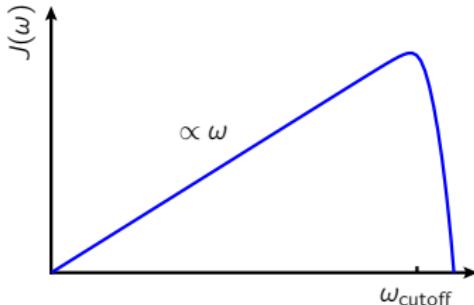
- Fourier transformed: spectral density  $\rightarrow$  continuum limit

$$\mathcal{A}(\omega) = \pi \sum_{\nu} |\gamma_{\nu}|^2 \delta(\omega - \omega_{\nu}) \rightarrow J(\omega)$$

- here: Ohmic with cutoff

$$J(\omega) = 2\pi\alpha\omega e^{-\omega/\omega_{\text{cutoff}}}$$

- dimensionless dissipation strength  $\alpha$

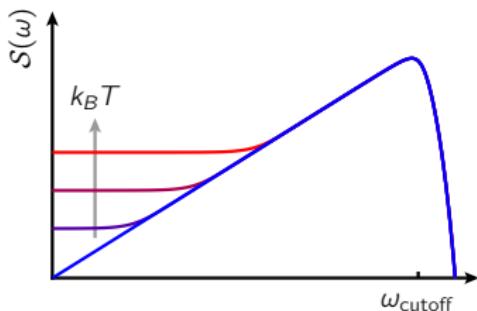


- symmetric bath correlation function

$$S(\tau) = \frac{1}{2} \langle [\xi(\tau), \xi(0)]_+ \rangle$$

$$S(\omega) = J(\omega) \coth\left(\frac{\omega}{2k_B T}\right)$$

$$= \begin{cases} 4\pi\alpha k_B T & \text{high } k_B T \\ 2\pi\alpha\omega & \text{low } k_B T \end{cases}$$

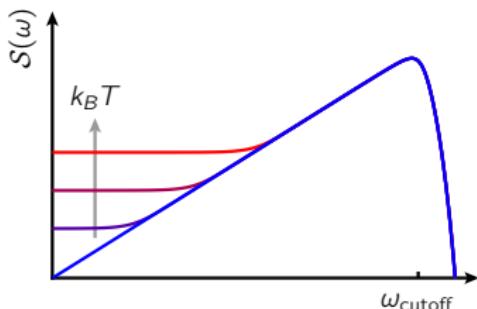


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- $S(\omega)$  evaluated at transition frequencies
- dissipation strength depends on spectrum / coherent dynamics

- Ohmic, short memory times (e.g. for  $\gamma < k_B T$ )  
→ Bloch-Redfield master equation

$$\dot{\rho} = -i[H_S, \rho] + i\gamma[X, \{[H_S, X], \rho\}] - [X, [Q, \rho]]$$

coherent dynamics    dissipation    decoherence

coherent dynamics enters via  $Q = \int_0^\infty d\tau S(\tau) \tilde{X}(-\tau)$

- Ohmic, short memory times (e.g. for  $\gamma < k_B T$ )  
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coherent dynamics    dissipation    decoherence

coherent dynamics enters via  $Q = \int_0^\infty d\tau \mathcal{S}(\tau) \tilde{X}(-\tau)$

- not of Lindblad form
  - ✗ positivity might be violated
  - ✓ happens only on unphysically small time scales
- high-temperature limit: Fokker-Planck equation

- Decomposition into energy basis and rotating-wave approximation
- rate equation for the populations (Pauli master equation)

$$\frac{d}{dt} \rho_{\alpha\alpha} = \sum_{\alpha'} \left[ w_{\alpha \leftarrow \alpha'} \rho_{\alpha'\alpha'} - w_{\alpha' \leftarrow \alpha} \rho_{\alpha\alpha} \right]$$

with the golden-rule rates

$$w_{\alpha \leftarrow \alpha'} = J(E_\alpha - E_{\alpha'}) |\langle \phi_\alpha | X | \phi_{\alpha'} \rangle|^2 n_{\text{th}}(E_\alpha - E_{\alpha'})$$

– notice:  $-n_{\text{th}}(-\omega) = n_{\text{th}}(\omega) + 1$

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– notice:  $-n_{\text{th}}(-\omega) = n_{\text{th}}(\omega) + 1$

- ✓ detailed balance:  $\frac{w_{\alpha \leftarrow \alpha'}}{w_{\alpha' \leftarrow \alpha}} = e^{-(E_\alpha - E_{\alpha'})/k_B T}$
- ✓ Lindblad form
- ✗ high-temperature limit typically wrong

full Bloch-Redfield: **golden rule for non-diagonal  $\rho_{\alpha\beta}$**

Driven system → noise term becomes time-dependent

$$\dot{\rho} = \dots - [X, [Q(t), \rho]], \quad Q(t) = \int_0^{\infty} d\tau S(\tau) \tilde{X}(t - \tau, t)$$

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Central idea:

- 1 adapted basis: Floquet states  $|\phi_\alpha(t)\rangle$  → captures coherent dynamics
- 2 master equation in Floquet basis

$$\frac{d}{dt} \rho_{\alpha\beta} = -i(\epsilon_\alpha - \epsilon_\beta) \rho_{\alpha\beta} + \sum_{\alpha'\beta'} \mathcal{L}_{\alpha\beta,\alpha'\beta'}(t) \rho_{\alpha'\beta'}$$

where  $\mathcal{L}(t) = \mathcal{L}(t + T)$

- 3 rotating-wave approximation

$$\frac{d}{dt} \rho_{\alpha\beta} = -i(\epsilon_\alpha - \epsilon_\beta) \rho_{\alpha\beta} + \sum_{\alpha'\beta'} \mathcal{L}_{\alpha\beta, \alpha'\beta'}(t) \rho_{\alpha'\beta'}$$

$$i(\epsilon_\alpha - \epsilon_\beta - k\Omega) \rho_{\alpha\beta}^{(k)} = \sum_{\alpha',\beta',k'} \mathcal{L}_{\alpha\beta, \alpha'\beta'}^{(k-k')} \rho_{\alpha'\beta'}^{(k')} \quad (\text{if } \rho \text{ is } T\text{-periodic})$$

	Liouvillian	Density operator	Remark
Full RWA	$\mathcal{L}_{\alpha\alpha, \alpha'\alpha'}^{(0)}$	populations only	efficient, Pauli type, analytic calculations
Moderate RWA	$\mathcal{L}_{\alpha\beta, \alpha\beta}^{(0)}$	transients	coherent dynamics, depends on BZ (!)
Convolution	full $\mathcal{L}$	time periodic	best Markovian long-time solution

\* for sufficiently weak dissipation

- Numerical method: compute  $\mathcal{L}$  and solve

$$\dot{\rho}_{\alpha\beta} = -i(\epsilon_{\alpha} - \epsilon_{\beta})\rho_{\alpha\beta} + \sum_{\alpha'\beta'} \bar{\mathcal{L}}_{\alpha\beta, \alpha'\beta'} \rho_{\alpha'\beta'}$$

- 1 time-independent master equation for driven system
- 2 ac driving captured by choice of basis → efficient
- 3 includes impact of bath on dissipation strength  
(very relevant for fermionic baths)
- Analytical tool: find  $H_{\text{eff}}$  and approx. for  $\overline{Q(t)}$   
→ effective time-independent Bloch-Redfield equation

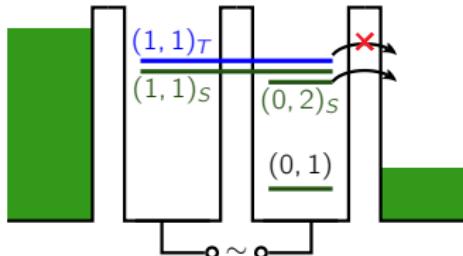
→ full RWA (Pauli master equation)

$$\frac{d}{dt} \rho_{\alpha\alpha} = \sum_{\alpha'} w_{\alpha \leftarrow \alpha'} \rho_{\alpha'\alpha'} - \sum_{\alpha} w_{\alpha' \leftarrow \alpha} \rho_{\alpha\alpha}$$

with the golden-rule rates

$$w_{\alpha \leftarrow \alpha'} = \sum_k J(\epsilon_\alpha - \epsilon_{\alpha'} + k\Omega) \left| \sum_{k'} \langle \phi_{\alpha, k+k'} | X | \phi_{\alpha', k} \rangle \right|^2 n_{\text{th}}(\epsilon_\alpha - \epsilon_{\alpha'} + k\Omega)$$

- sidebands contribute to  $w_{\alpha \leftarrow \alpha'}$   
... but NOT as independent states!
- no detailed balance between forward / backward rates



- Floquet states for central system
- evaluate rates  $w_{\alpha \leftarrow \alpha'}$
- dc current, counting statistics

	Dissipation	Transport
Environment	harmonic oscillators	electron source / drain
Coupling of mode $\nu$	$X(a_\nu^\dagger + a_\nu)$	$c^\dagger c_\nu + c_\nu^\dagger c$
Absorption / tunnel in	$n_{\text{th}}(\omega)$	$f(\epsilon - \mu)$
Emission / tunnel out	$1 + n_{\text{th}}(\omega)$	$1 - f(\epsilon - \mu)$
“Ohmic”	$J(\omega) \propto \omega$	$\Gamma(\omega) = \text{const}$

## 1 Floquet & Schrödinger equation

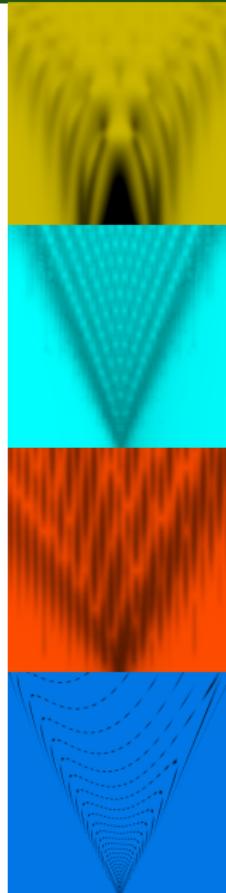
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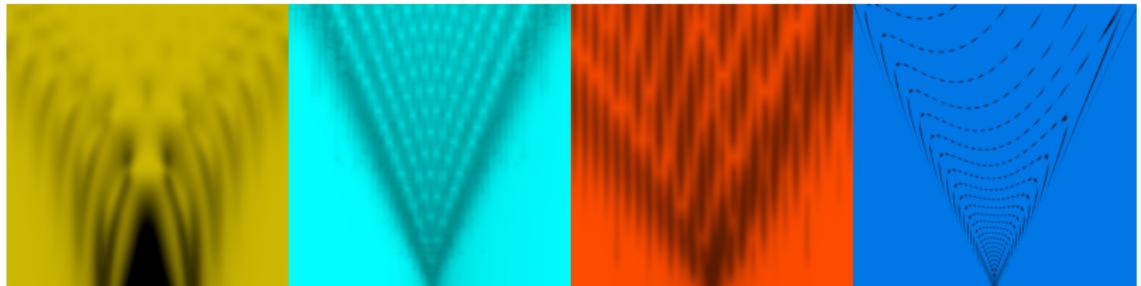
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- System-bath model
- Floquet-Bloch-Redfield formalism

## 3 Applications & miscellaneous topics

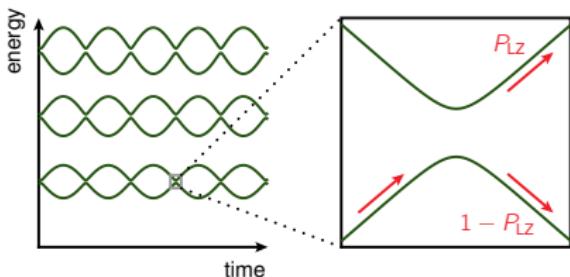
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- Time-dependent Liouvillians
- Floquet scattering theory
- Adiabatic Floquet theory
- Hidden symmetries
- Quantum chaos and dissipation
- Floquet-Gibbs states
- Two-color Floquet theory





Experiments: Stefan Ludwig (PDI Berlin)  
Jason Petta (UCLA)  
Gang Cao & Guo-Ping Guo (Hefei)

Quantum system in AC-field,  $H(t)$

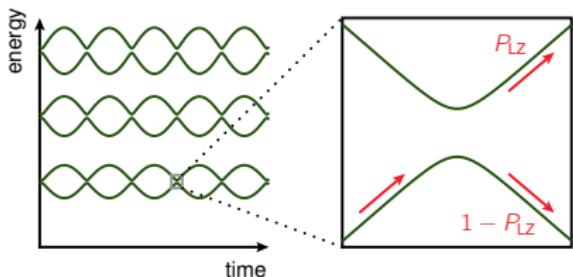


- non-adiabatic transition probability

$$P_{LZ} = e^{-\pi \Delta^2 / 2\hbar v}$$

Landau, Zener, Stückelberg,  
Majorana, 1932

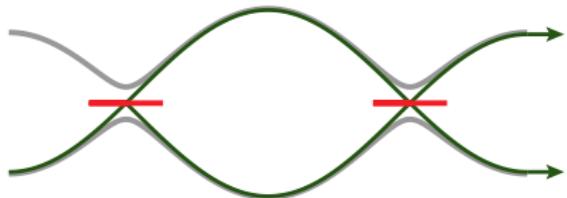
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- beam splitter, interference
- Landau-Zener-(Stückelberg-Majorana) interferometry

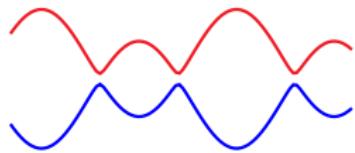
$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{g(t)}{2}\sigma_z \quad g(t) = \epsilon + A \cos(\Omega t)$$

1 (avoided) crossing requires  $A > |\epsilon|$

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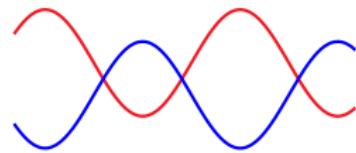
- 1 (avoided) crossing requires  $A > |\epsilon|$
- 2 relative phase between dominant paths

adiabatic:  $P_{LZ} \ll 1$



$$\varphi(T) = \int_0^T dt |g(t)|$$

diabatic:  $1 - P_{LZ} \ll 1$



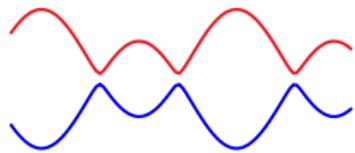
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→ fringes for  $\varphi(T) = 2\pi k$

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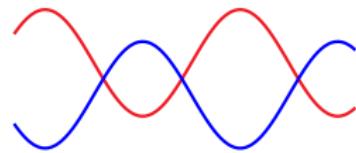
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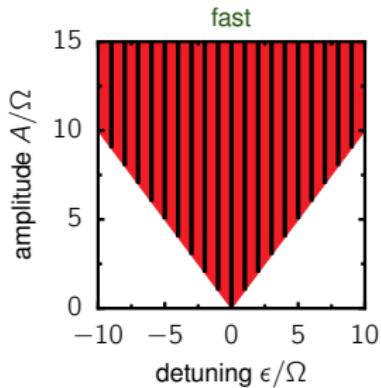
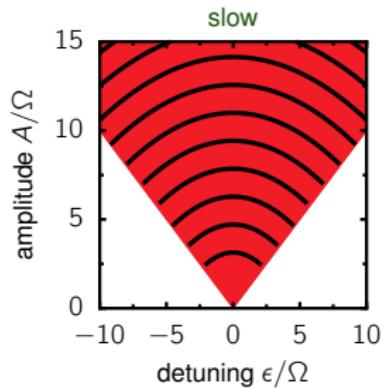
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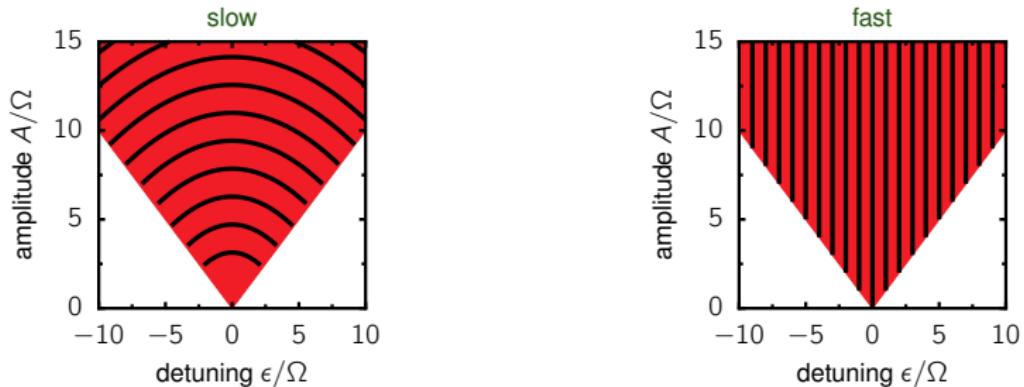
$$\varphi(T) = \int_0^T dt g(t) = \epsilon T$$

$\epsilon = k\Omega$  “ $k$ -photon resonance”

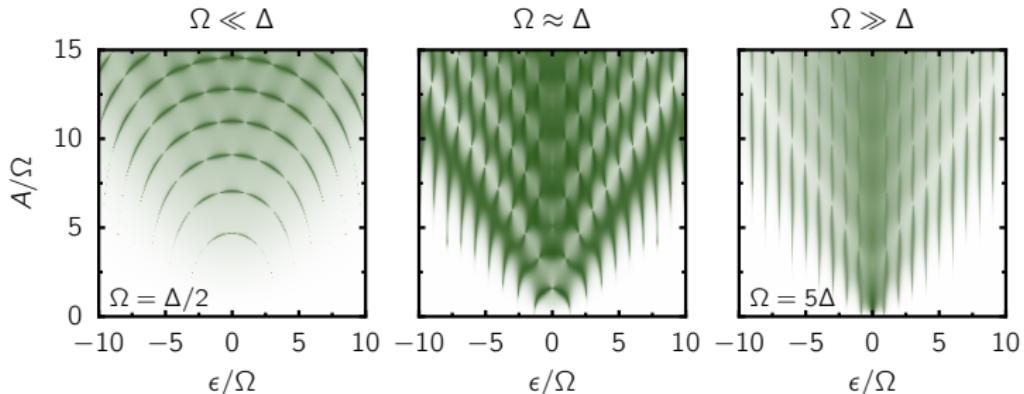
# Patterns for two-level systems



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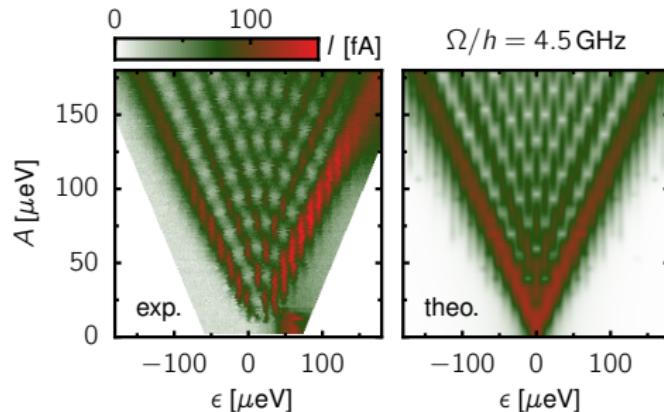
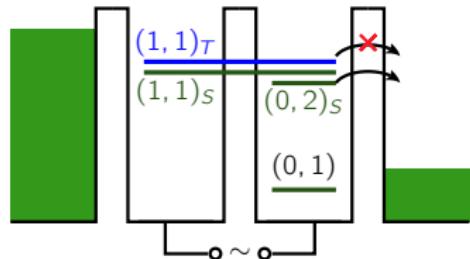
E.g. excitation probabilities of a TLS:



# Measurement I: Transport

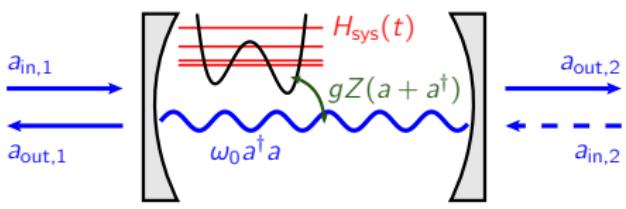
DC current:

$$I(t) = e_0 \Gamma_R \langle n_R(t) \rangle \rightarrow \overline{\langle n_R(t) \rangle}^T$$



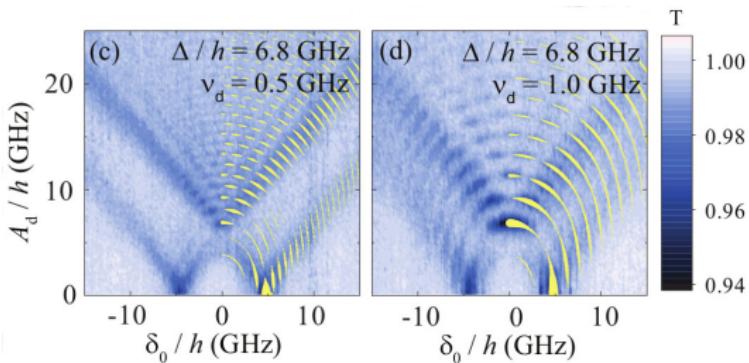
Forster et al., PRL 2014

## Measurement II: Cavity transmission



Dispersive frame:  
effective cavity frequency

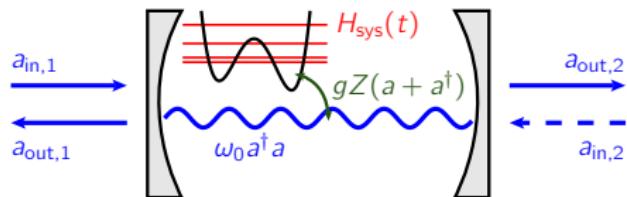
$$\omega_0 \longrightarrow \omega_0 + \frac{g^2}{\epsilon_{qb} - \omega_0} \sigma_z$$



Theory for readout of  
driven system?

Koski *et al.*, PRL 2018

## Qubit-cavity Hamiltonian



$$H = H_{\text{sys}}(t) + gZ(a^\dagger + a) + \omega_0 a^\dagger a$$

- Backaction: cavity  $\rightarrow$  qubit  $\rightarrow$  cavity
- Cavity equation (input/output formalism)

$$\frac{d}{dt}a = -i\omega_0 a - \frac{\kappa}{2}a - \sum_{\nu=1,2} \sqrt{\kappa_\nu} a_{\text{in},\nu} - igZ$$

- (non equilibrium) Kubo formula  $Z(t) = g \int dt' \chi(t - t') a(t')$   
with the response function (may depend on the initial state!)

$$\chi(t) = -i \langle [Z(t), Z] \rangle \theta(t - t')$$

$$\rightarrow -i\omega a = -i(\omega_0 + g^2 \chi(\omega)) a - \frac{\kappa}{2} a - \sum_{\nu=1,2} \sqrt{\kappa_\nu} a_{\text{in},\nu}$$

- measured quantity: (non-equilibrium) susceptibility

Response of periodically driven system

$$\chi(t, t') = -i \langle [Z(t), Z(t')] \rangle_{\text{non-eq}} = \chi(t+T, t'+T)$$

such that

$$\chi(t, t - \tau) = \sum_k e^{-ik\Omega t} \int d\omega e^{-i\omega\tau} \chi^{(k)}(\omega)$$

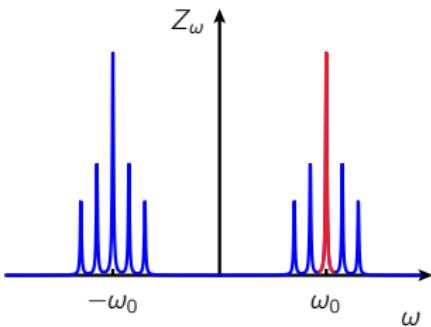
Response of periodically driven system

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such that

$$\chi(t, t - \tau) = \sum_k e^{-ik\Omega t} \int d\omega e^{-i\omega\tau} \chi^{(k)}(\omega)$$

- resonant cavity driving,  $\omega = \omega_0$
- response  $Z(t)$  acquires sidebands
- good cavity limit,  $\kappa \ll \omega_0, \Omega$



Relevant component:

$$\chi^{(0)}(\omega_0) = \sum_{\beta,\alpha,k} \frac{(p_\alpha - p_\beta)|Z_{\beta\alpha,k}|^2}{\epsilon_\alpha - \epsilon_\beta + \omega_0 + k\Omega + i\gamma/2}$$

- Floquet theory → quasi-energies  $\epsilon_\alpha$
- Floquet-Bloch-Redfield → populations  $p_\alpha$

Relevant component:

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- Floquet theory → quasi-energies  $\epsilon_\alpha$
- Floquet-Bloch-Redfield → populations  $p_\alpha$

Resonance conditions

cavity response:

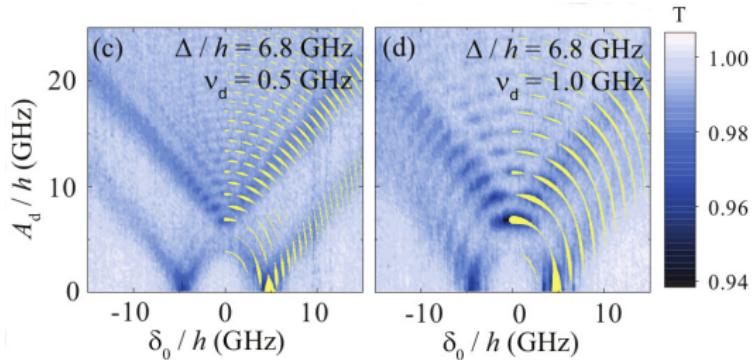
$$\Delta\epsilon = \omega_0 + k\Omega$$

cf. population:

$$\Delta\epsilon = k\Omega$$

e.g. Ivakhnenko *et al.*, Phys.Rep. 2023

→ Agree only for low-frequency oscillator!

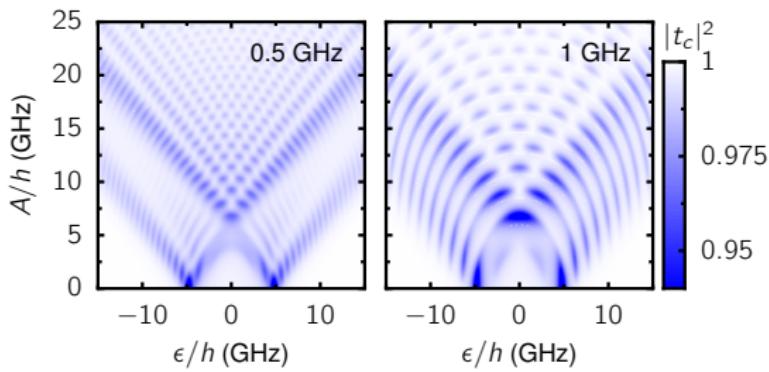


Double QD in GaAs

→ two-level system

Koski *et al.*, PRL 2018

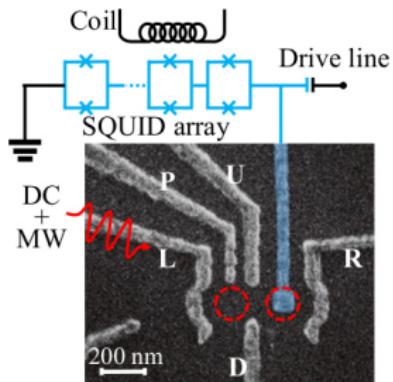
(yellow: heuristic theory)



Present theory

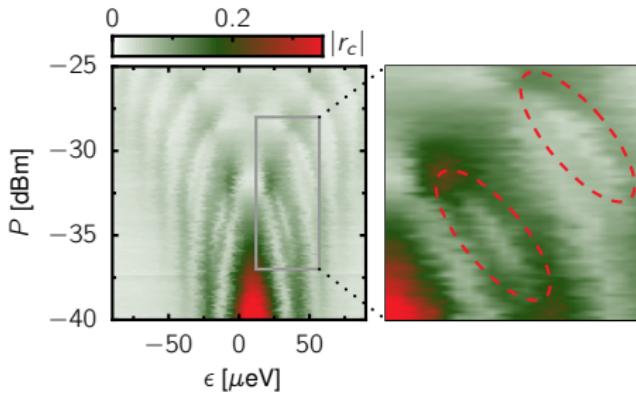
SK, PRA 2018

# Readout of Floquet state population



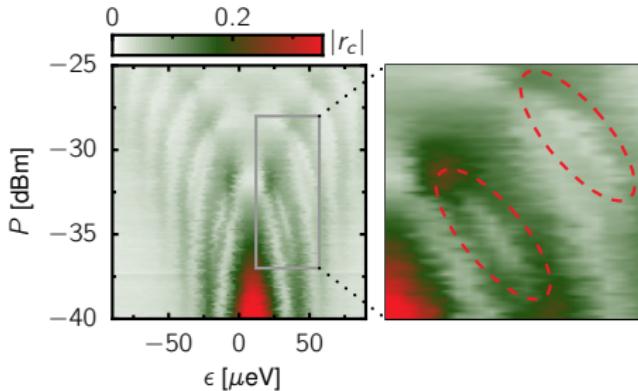
Chen *et al.*, Phys. Rev. B **103**, 205428 (2021)

## Motivation: Holes in interference fringes



Experiment (Cao & Guo, Hefei)

- holes in LZSM pattern
- GaAs DQD → two-level sys.



Experiment (Cao & Guo, Hefei)

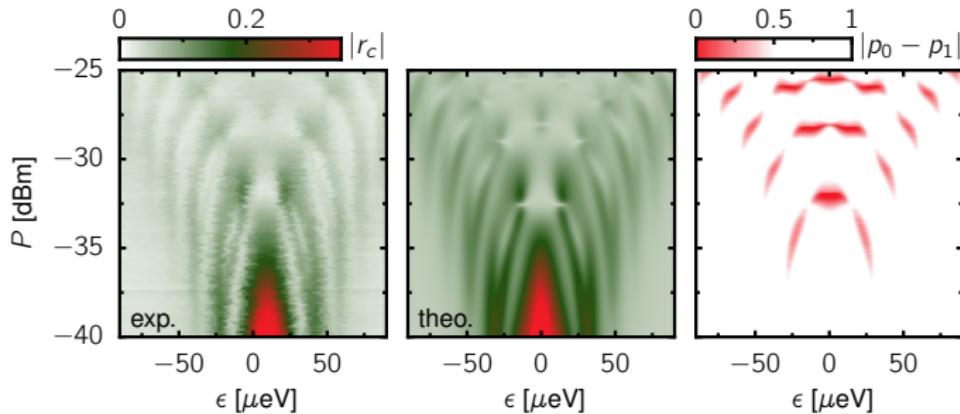
- holes in LZSM pattern
- GaAs DQD → two-level sys.

Recap: susceptibility (two-level system)

$$\chi^{(0)}(\omega_0) = (p_0 - p_1) \sum_k \frac{|Z_{10,k}|^2}{\epsilon_1 - \epsilon_0 + \omega_0 + k\Omega + i\gamma/2}$$

response determined by

- resonance condition for cavity signal
- Floquet state population



- competing resonance conditions
- holes in fringes when  $p_0 \approx p_1 \approx 1/2$
- cavity response provides information about Floquet state population

## Periodically time-dependent Liouvillians

$$\frac{d}{dt}P = L(t)P$$

Master equation of type

$$\frac{d}{dt}P = L(t)P$$

- Floquet-Bloch-Redfield beyond moderate RWA
- time-dependent system with Lindblad dissipator

$$\dot{\rho} = -i[H(t), \rho] + \gamma(2a^\dagger \rho a - a^\dagger a \rho - \rho a^\dagger a)$$

- very weak dissipation
- transport problem with large bias

- long-time solution  $T$ -periodic
- Floquet ansatz with “quasienergy” zero

$$P(t) = \sum_k e^{-ik\Omega t} p_k$$

$$\frac{d}{dt}P = L(t)P \text{ with}$$

$$L(t) = L_0 + 2L_1 \cos(\Omega t)$$

→ kernel of tridiagonal Floquet matrix

$$L_0 + 2L_1 \cos(\Omega t) - \partial_t \leftrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & L_0 + 2i\Omega & L_1 & 0 & 0 & 0 & \cdots \\ \cdots & L_1 & L_0 + i\Omega & L_1 & 0 & 0 & \cdots \\ \cdots & 0 & L_1 & L_0 & L_1 & 0 & \cdots \\ \cdots & 0 & 0 & L_1 & L_0 - i\Omega & L_1 & \cdots \\ \cdots & 0 & 0 & 0 & L_1 & L_0 - 2i\Omega & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- ansatz  $P(t) = \sum_k e^{-ik\Omega t} p_k$  yields

$$\textcolor{blue}{L}_1 p_{k-1} + (\textcolor{green}{L}_0 + \textcolor{red}{ik}\Omega) p_k + \textcolor{blue}{L}_1 p_{k+1} = 0$$

- idea: truncate and iterate  $p_{k-1} = -\textcolor{blue}{L}_1^{-1} \{(L_0 - ik\Omega)p_k + L_1 p_{k+1}\}$

- ansatz  $P(t) = \sum_k e^{-ik\Omega t} p_k$  yields

$$\textcolor{blue}{L}_1 p_{k-1} + (\textcolor{teal}{L}_0 + \textcolor{red}{ik}\Omega) p_k + \textcolor{blue}{L}_1 p_{k+1} = 0$$

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✗ fails,  $L_1$  generally singular

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$$L_1 p_{k-1} + (L_0 + ik\Omega) p_k + L_1 p_{k+1} = 0$$

- idea: truncate and iterate  $p_{k-1} = -L_1^{-1} \{ (L_0 - ik\Omega) p_k + L_1 p_{k+1} \}$   
✗ fails,  $L_1$  generally singular
- solution: ansatz  $p_k = S_k L_1 p_{k\pm 1}$  ( $k \gtrless 0$ ) leads to

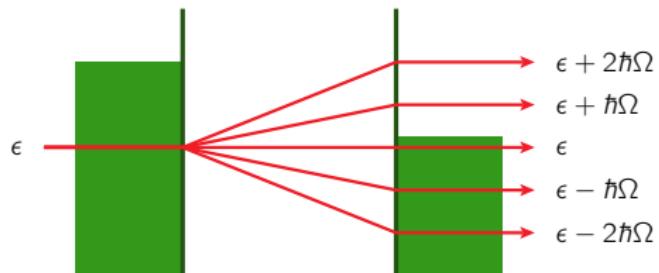
$$S_k = -(L_0 + ik\Omega + L_1 S_{k\pm 1} L_1)^{-1} \quad \longrightarrow \quad S_{\pm 1} \quad (1)$$

$$0 = (L_1 S_{-1} L_1 + L_0 + L_1 S_1 L_1) p_0 \quad (2)$$

- truncate at  $\pm k_0$ , iterate (1), and solve (2)  
 → time-averaged  $P(t) = p_0 \rightarrow$  time-averaged expectation values

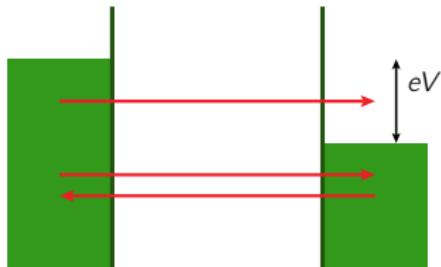
Risken, "The Fokker-Planck Equation"  
 Appendix of Forster *et al.*, PRB 2015

## Floquet scattering theory

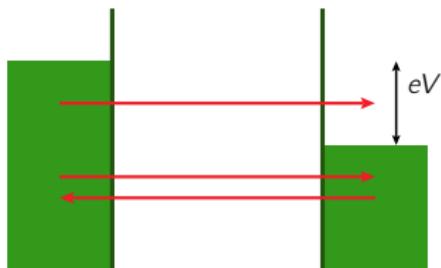


SK, Lehmann, Hänggi, Phys. Rep. **406**, 379 (2005)

- Landauer (1957): “conductance is transmission”



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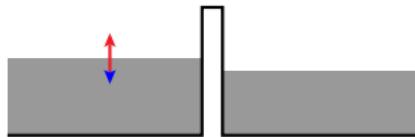
- current  $I = \frac{e}{h} \int dE T(E) [f(E + eV) - f(E)]$
- transmission of an electron with energy  $E$

$$T(E) = \Gamma_L \Gamma_R |\langle 1 | G(E) | N \rangle|^2$$

Fischer, Lee, PRB 1981; Meir, Wingreen, PRL 1992

- ac bias voltage:

$$V_0 \longrightarrow V_0 + V_{\text{ac}} \cos(\Omega t)$$

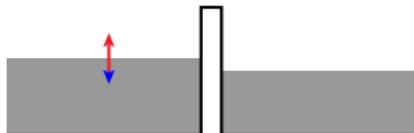


- time-dependent energy shift by  $eV_{\text{ac}} \cos(\Omega t)$

$$e^{-iEt} \longrightarrow \exp \left( -iEt - i \frac{eV_{\text{ac}}}{\Omega} \sin(\Omega t) \right)$$

- ac bias voltage:

$$V_0 \longrightarrow V_0 + V_{\text{ac}} \cos(\Omega t)$$



- time-dependent energy shift by  $eV_{\text{ac}} \cos(\Omega t)$

$$e^{-iEt} \longrightarrow \exp \left( -iEt - i \frac{eV_{\text{ac}}}{\Omega} \sin(\Omega t) \right) = \sum_k J_k(eV_{\text{ac}}/\Omega) e^{-i(E+k\Omega)t}$$

- sidebands occupied with probability  $J_k^2(\dots)$
- energy  $k\Omega$  corresponds to additional DC bias voltage  $k\Omega/e$

$$I(V_0, V_{\text{ac}}) = \sum_k J_k^2 \left( \frac{eV_{\text{ac}}}{\Omega} \right) I_0(V_0 + k\Omega/e)$$

DC conductivity determines the current !

- Approach rather heuristic
- Rigorous derivation?
- When is Tien-Gordon theory applicable?

Transport and driving:

Green's function and Landauer formula for time-dependent situation

## Transport and driving:

Green's function and Landauer formula for time-dependent situation

- Floquet equation

with self-energy  $\Sigma = |1\rangle \frac{i\Gamma_L}{2} \langle 1| + |N\rangle \frac{i\Gamma_R}{2} \langle N|$

$$\left( H(t) + \Sigma - i \frac{d}{dt} \right) |\varphi_\alpha(t)\rangle = (\epsilon_\alpha - i\gamma_\alpha) |\varphi_\alpha(t)\rangle$$

- propagator in the presence of the contacts

$$G(\textcolor{red}{t}, t - \tau) = \sum_{\mathbf{k}} e^{i\mathbf{k}\Omega t} \int d\epsilon e^{-i\epsilon\tau} \underbrace{\sum_{\alpha, k'} \frac{|\varphi_{\alpha, \mathbf{k}+k'}\rangle \langle \varphi_{\alpha, k'}|}{\epsilon - (\epsilon_\alpha + k'\Omega - i\gamma_\alpha)}}_{G^{(\mathbf{k})}(\epsilon)}$$

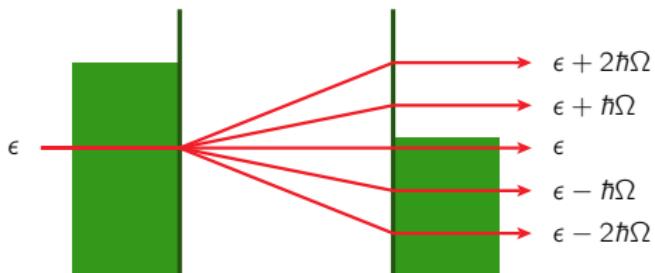
propagation under absorption/emission of  $|\mathbf{k}|$  photons

- dc current

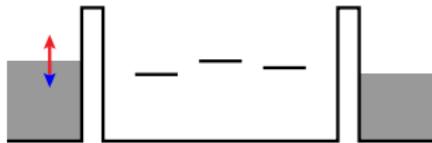
$$I = \frac{e}{h} \sum_k \int d\epsilon \left\{ T_{LR}^{(k)}(\epsilon) f(\epsilon - \mu_L) - T_{RL}^{(k)}(\epsilon) f(\epsilon - \mu_R) \right\}$$

- transmission under absorption of  $k$  photons

$$T_{LR}^{(k)}(\epsilon) = \Gamma_L \Gamma_R |\langle 1 | G^{(k)}(\epsilon) | N \rangle|^2 \neq T_{RL}^{(\pm k)}(\epsilon \pm k\Omega)$$

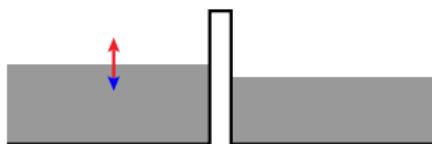


# When is Tien-Gordon theory applicable ?

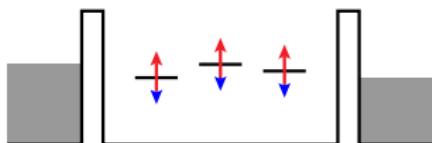


applicable for

- AC bias voltage

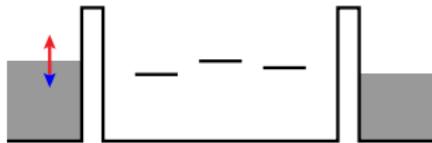


- tunnel barriers  
(studied by Tien & Gordon)



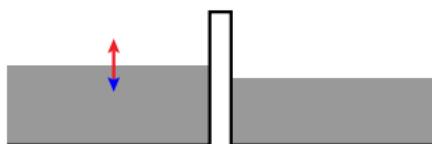
- uniform AC gate voltage

# When is Tien-Gordon theory applicable ?

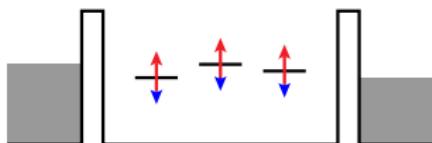


applicable for

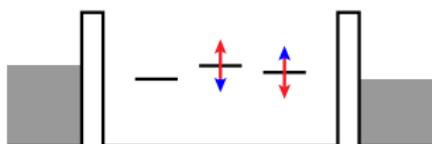
- AC bias voltage



- tunnel barriers  
(studied by Tien & Gordon)



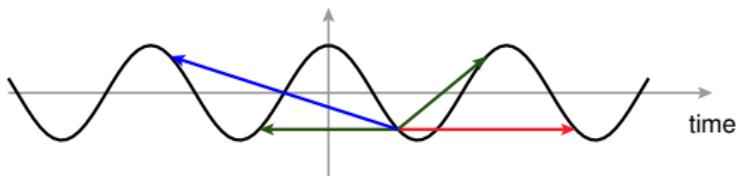
- uniform AC gate voltage



but not for

- non-uniform gating
- dipole force

$$H_{\text{dipole}} \propto x \cos(\Omega t)$$



- 1 time periodicity  $t \rightarrow t + T$  → Floquet theory applicable
- 2 time reversal  $t \rightarrow -t$  → Floquet states real
- 3 generalized parity  $(x, t) \rightarrow (-x, t + T/2)$   
e.g. symmetric potential with dipole driving  
→ Floquet states even/odd
- 4 time-reversal parity  $(x, t - T/4) \rightarrow (-x, T/4 - t)$   
– combination of the other three

Consequences for scattering probabilities ?

Symmetry-related processes have the same probability

time-reversal

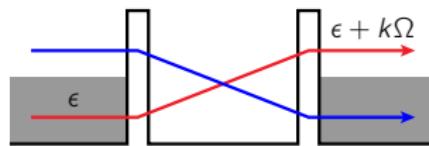
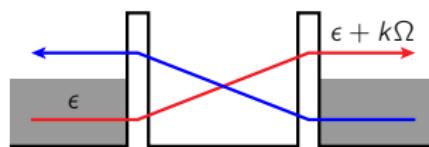
$$t \rightarrow -t$$

generalized parity

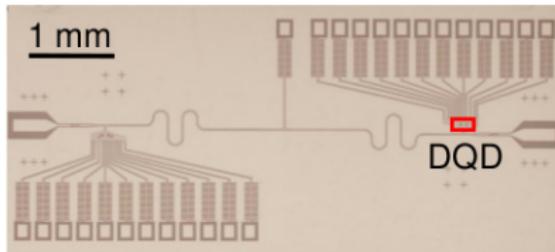
$$(x, t \rightarrow -x, t + \frac{T}{2})$$

time-reversal parity

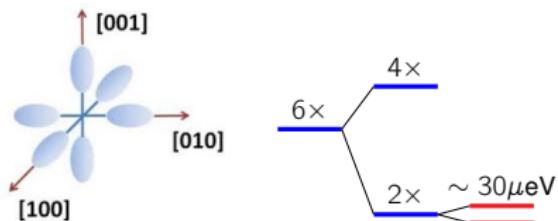
$$(x, t \rightarrow -x, -t)$$



Dispersive readout of LZSM patterns in Si/SiGe DQDs  
(under low-frequency driving)

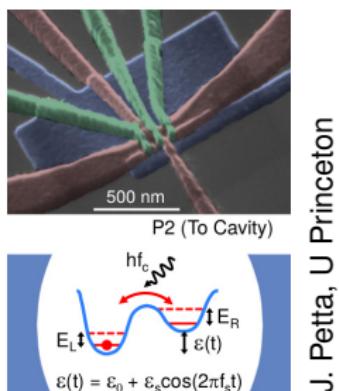


Mi, SK, Petta, Phys. Rev. B **98**, 161404(R) (2018)



## Level structure

- strain, confinement
- valley splitting
- **valley qubit**



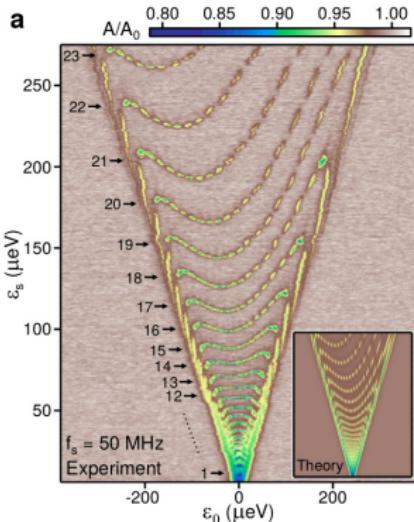
J. Petta, U Princeton

## Si double quantum dots

- position, spin, valley
- coupling to cavity
- “dispersive readout”

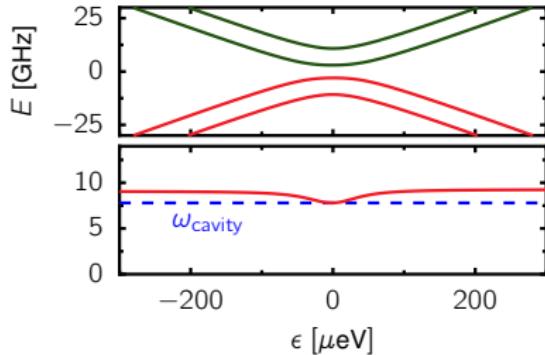
# Motivation: LZSM with silicon double quantum dots

ICMM



Experiment (J.R. Petta, Princeton)

- very slow driving: 50–100 MHz  
→ non-eq. population tiny
- heuristic resonance condition:  
 $\bar{E}_1 - \bar{E}_0 = \omega_{\text{cavity}} + k\Omega$   
 (mean energies!)



Computational problem:

- amplitude: up to 300 GHz
- Floquet theory requires  
 $\gtrsim 10^4$  sidebands

- adiabatic eigenstates  $H(t)|u(t)\rangle = (\bar{E} + \delta E(t))|u(t)\rangle$

$$\rightarrow |\psi(t)\rangle = e^{-i\bar{E}t} e^{i\varphi(t)} |u(t)\rangle$$

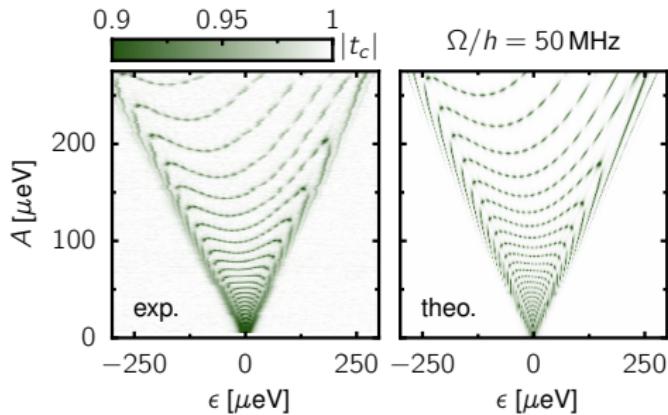
- adiabatic Floquet states

$$|\phi(t)\rangle = e^{i\varphi(t)} |u(t)\rangle \quad \text{with} \quad \varphi(t) = - \int_0^t dt' \delta E(t') \quad T \text{ periodic!}$$

$$\text{with quasienergies } \epsilon = \bar{E} = \frac{1}{T} \int_0^T dt E(t)$$

[here: time-reversal symmetry  $\rightarrow \langle u | \partial_t | u \rangle = 0$ , i.e., no Berry phase]

# Silicon double quantum dots



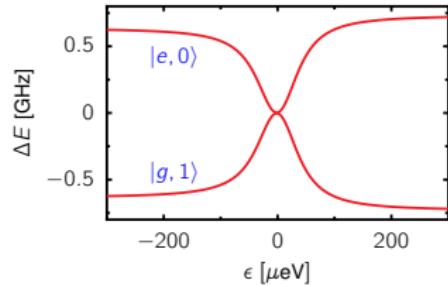
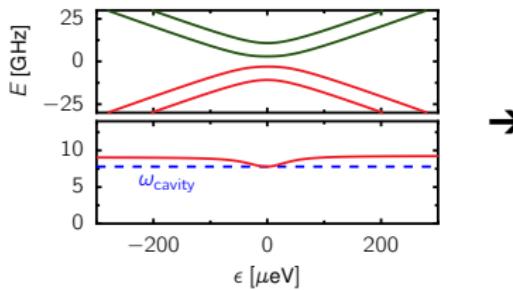
Dressed-state interpretation

- LZSM between states

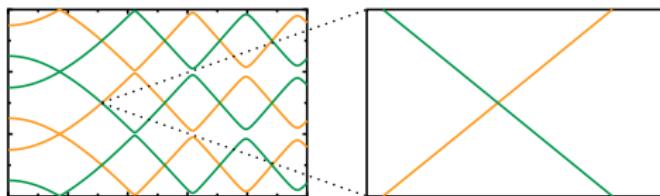
$$|g, 1\rangle, \quad |e, 0\rangle$$

→ cavity-assisted LZSM

Mi, SK, Petta, PRB 2018

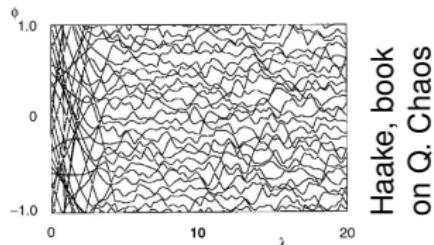


## Hidden Symmetries in Floquet Systems

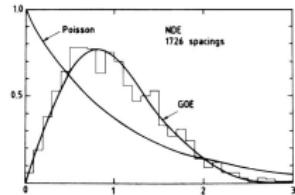


in collaboration with J. Casado-Pascual, U Sevilla

- Spectra of classically chaotic systems



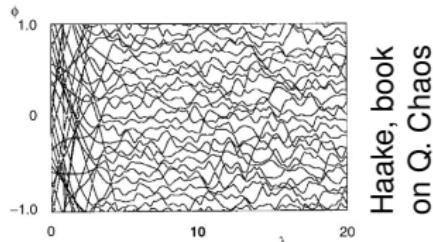
Haake, book  
on Q. Chaos



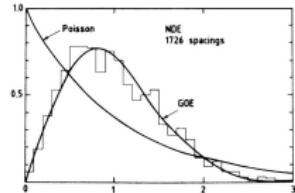
Bohigas 1983

→ level repulsion → anti crossings

- Spectra of classically chaotic systems



Haake, book  
on Q. Chaos



Bohigas 1983

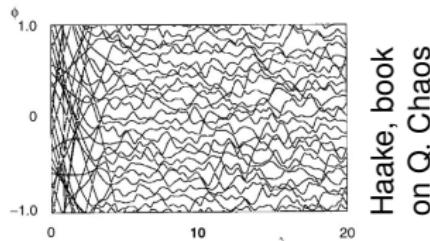
- level repulsion → anti crossings

- symmetry: Hamiltonian block diagonal (here: even/odd)

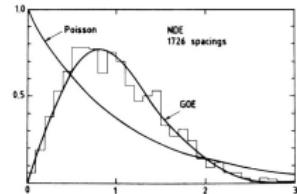
$$H = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}$$

- allows exact level crossings

- Spectra of classically chaotic systems



Haake, book  
on Q. Chaos



Bohigas 1983

→ level repulsion → anti crossings

- symmetry: Hamiltonian block diagonal (here: even/odd)

$$H = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}$$

→ allows exact level crossings

→ Exact level crossings indicate symmetries / integrals of motion

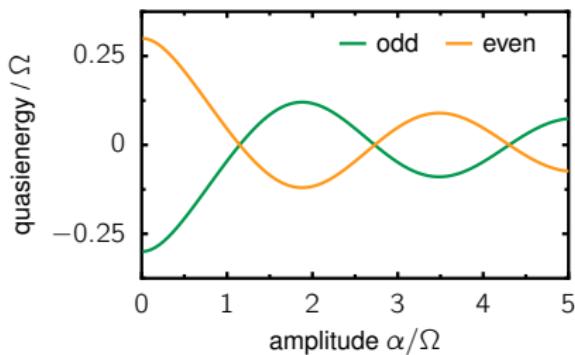
## TLS Hamiltonian

$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t)$$

generalized parity  $G = \sigma_x e^{(T/2)\partial_t}$

- even / odd states
- exact crossings

Peres, PRL 1991

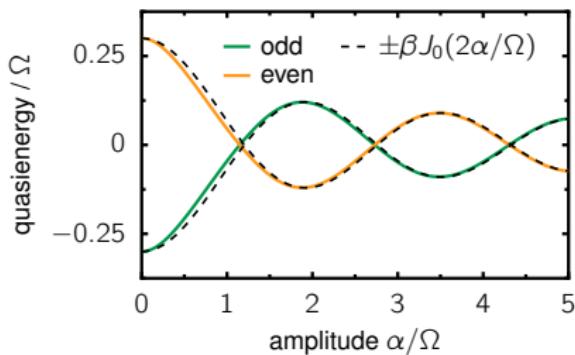


TLS Hamiltonian

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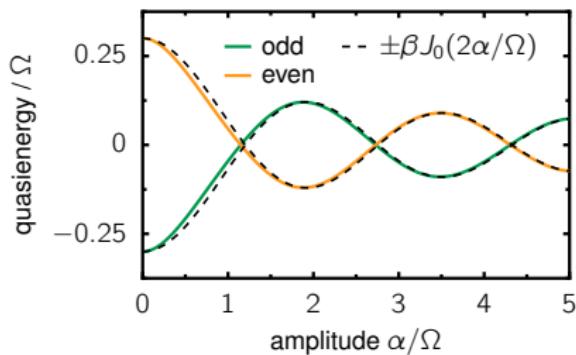
High-frequency approximation

$$q_{\pm} = \pm\beta J_0(2\alpha/\Omega)$$

TLS Hamiltonian

$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t) + \frac{n\Omega}{2}\sigma_z$$

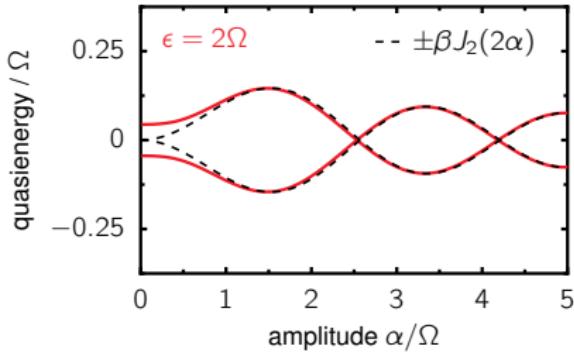
generalized parity  $G = \sigma_x e^{(T/2)\partial_t}$



High-frequency approximation

$$q_{\pm} = \pm\beta J_0(2\alpha/\Omega) \longrightarrow \pm\beta J_n(2\alpha/\Omega)$$

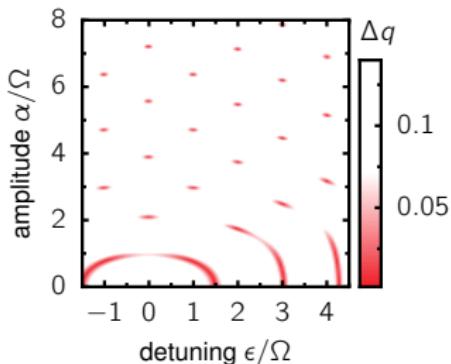
- suggests that crossings are exact



$$H(t) = \beta\sigma_x + \alpha\sigma_z \cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$

Quasienergy splittings tiny at isolated points

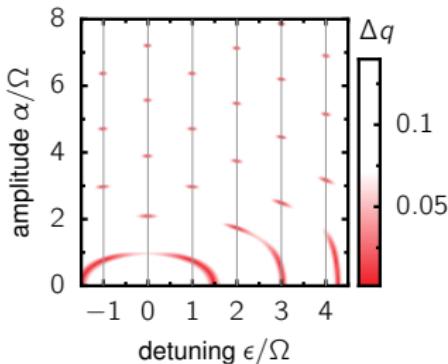
(intermediate frequency,  
beyond Bessel function approx.)



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Quasienergy splittings tiny at isolated points

(intermediate frequency,  
beyond Bessel function approx.)



- crossings exact for integer detuning ?
- underlying symmetry / integral of motion ?

- time-independent: integral of motion  $[H, J] = 0$
- for time-dependent system:  $[H(t) - i\partial_t, J(t)] = 0$

Lewis & Riesenfeld, J. Math. Phys. 1969

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- ansatz for time non-local LR invariant:  $J(t) = Q(t)e^{(T/2)\partial_t}$   
[motivation: for  $\epsilon = 0 \rightarrow Q = \sigma_x$ ]

$$i\partial_t Q = \left[ \frac{\epsilon}{2}\sigma_z + \beta\sigma_x, Q \right] + \{ \alpha\sigma_z, Q \} \cos(\Omega t)$$

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$$i\partial_t Q = \left[ \frac{\epsilon}{2}\sigma_z + \beta\sigma_x, Q \right] + \{ \alpha\sigma_z, Q \} \cos(\Omega t)$$

- trivial (and useless) solution: projector on Floquet state

$$J = |\phi_\ell(t)\rangle\langle\phi_\ell(t)| \rightarrow Q = |\phi_\ell(t)\rangle\langle\phi_\ell(t+T/2)|$$

depends on state  $\ell \rightarrow$  cannot be used to classify state

- 1 Fourier ansatz  $Q(t) = \sum_k e^{-ikt} Q_k$
  - 2 matrix recurrence relation
  - 3 scalar recurrence
  - 4  $\epsilon/\Omega = n = \text{integer}$ : recurrence terminates at  $k_{\max} = n$
  - 5  $J^2 = 1$ , i.e.  $\mathbb{Z}_2$  symmetry
- eigenvalues  $j = \pm 1$ , “even / odd” Floquet states

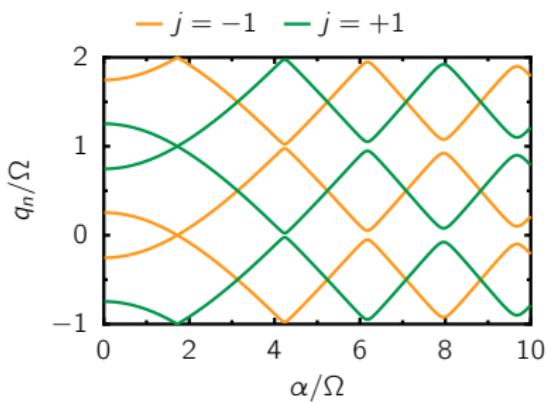
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Such  $J = Qe^{(T/2)\partial_t}$  exists for any integer value of  $\epsilon/\Omega$

## Solution ( $n = 1$ )

For  $\epsilon = \Omega$

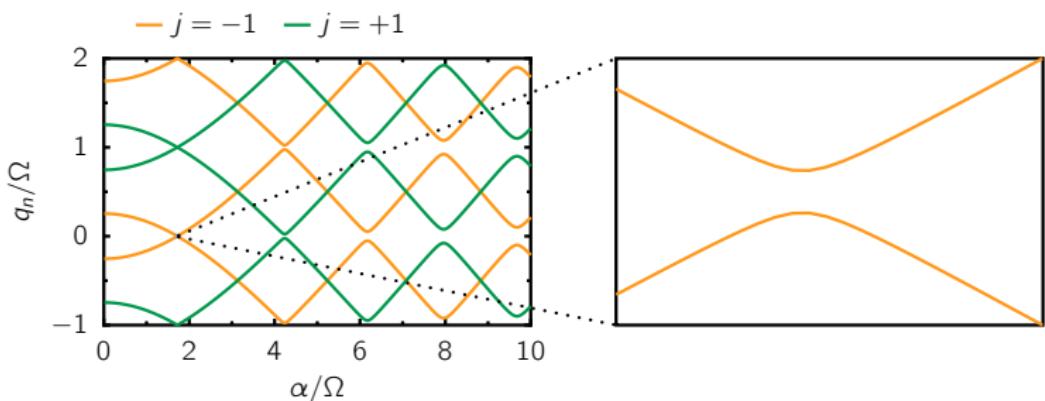
$$Q(t) = \frac{1}{\sqrt{\alpha^2 + \beta^2}} \begin{pmatrix} \beta & \alpha e^{-i\Omega t} \\ -\alpha e^{i\Omega t} & \beta \end{pmatrix} \quad \text{with} \quad J^2 = QQ^\dagger = 1$$



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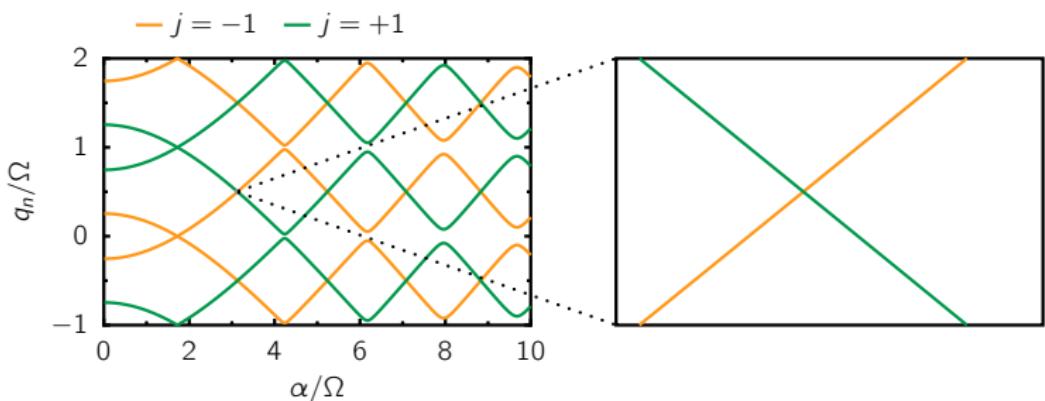
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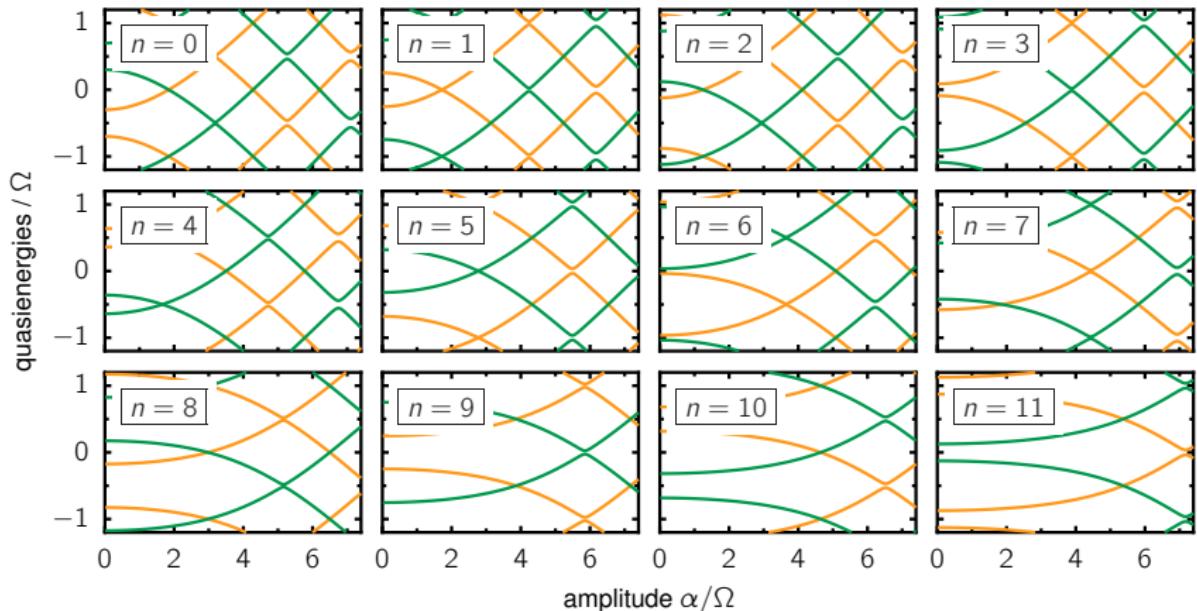
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→ accidental degeneracy due to hidden symmetry

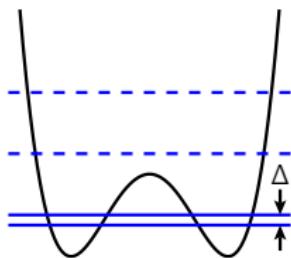
## Solutions for $\epsilon/\Omega = n > 1$

- numerical iteration:  $J$  with eigenvalues  $j = +1$  and  $j = -1$



## Example: Driven double-well potential

- 1 long-time solution of a “non-trivial” problem → populations
- 2 semi-classical limit → capability of the formalism



$$H(x, p, t) = H_{\text{DW}}(x, p) + Fx \cos(\Omega t)$$

Symmetries:

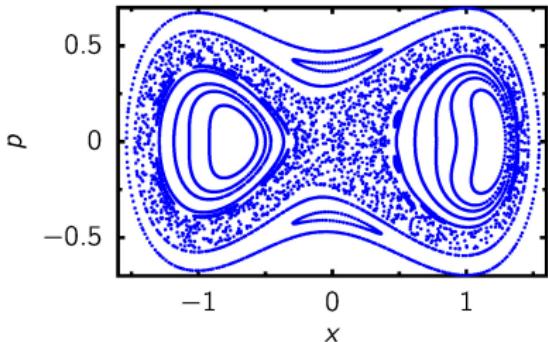
**TR:**  $(x, p, t) \rightarrow (x, -p, -t)$

**GP:**  $(x, p, t) \rightarrow (-x, -p, t + T/2)$

SK, PhD thesis, 1999

SK, Utermann, Dittrich, Hänggi, PRE 1998

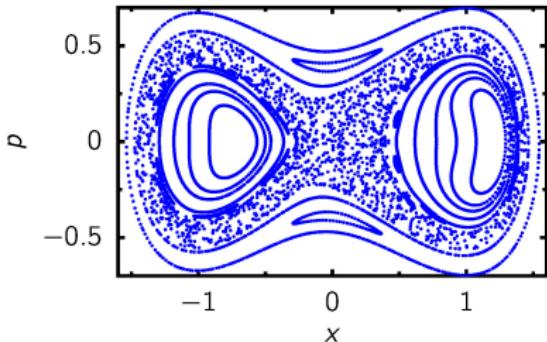
## Classical phase space



Stroboscopic map  $[x(nT), p(nT)]$

- regular vs. chaotic
- chaos augments with amplitude
- symmetry  $p \rightarrow -p$   
(consequence of TR)

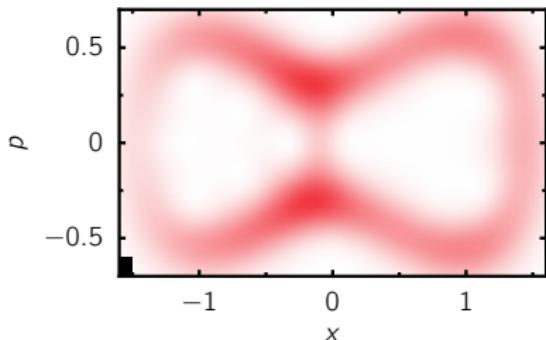
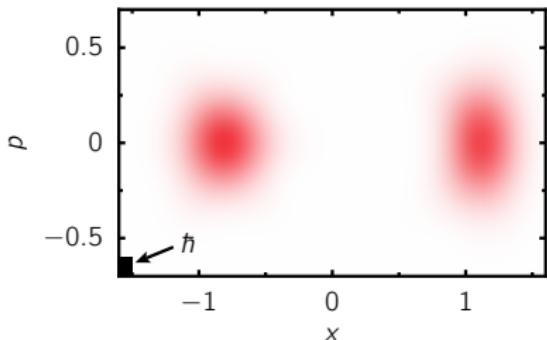
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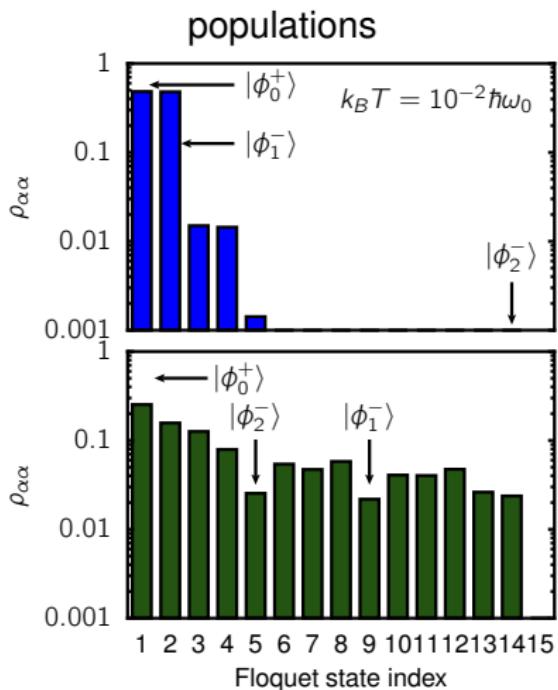
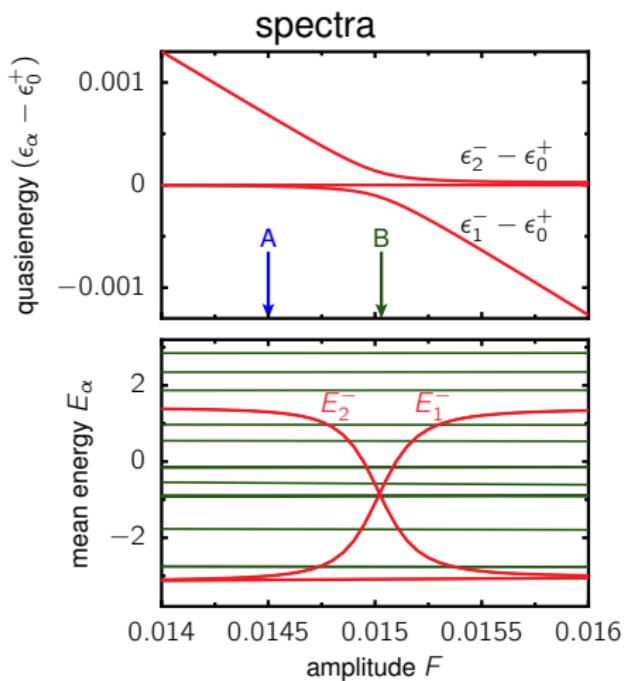
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(consequence of TR)

## Husimi functions of Floquet states (at times $nT$ )



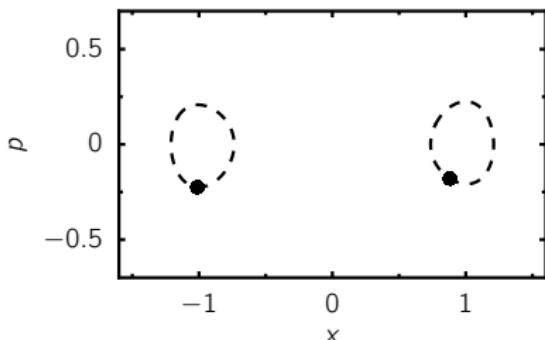
# Population of Floquet states



- far from crossing: occupation according to  $E_\alpha$
- at crossing: no general rule

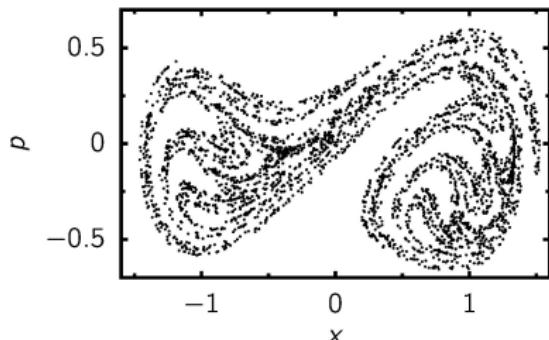
large dissipation:

→ fixed points, limits cycles



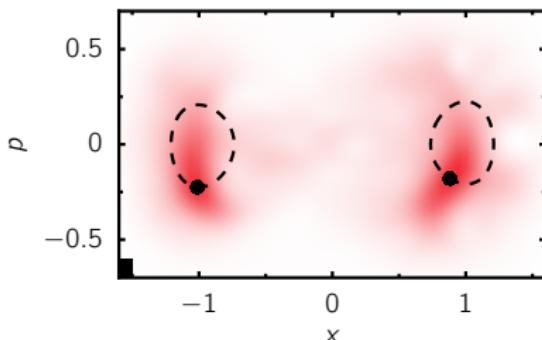
weak dissipation:

→ strange attractor

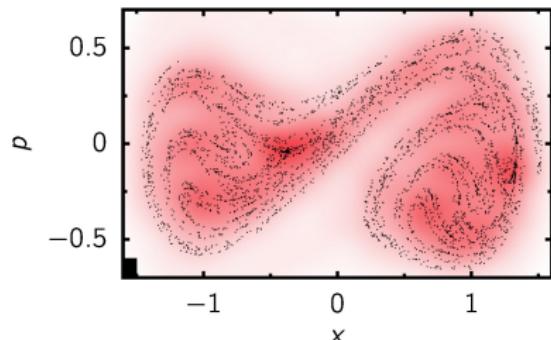


- dissipative term  $-\gamma \dot{x}$  breaks time reversal  
→ phase lag due to dissipation → no longer symmetric in  $p$

large dissipation:  
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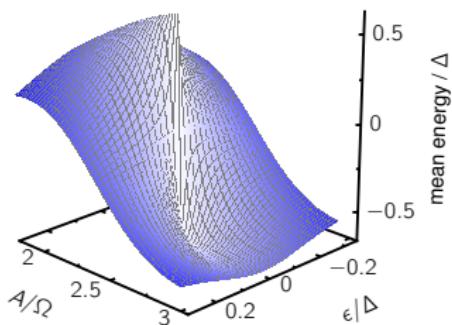
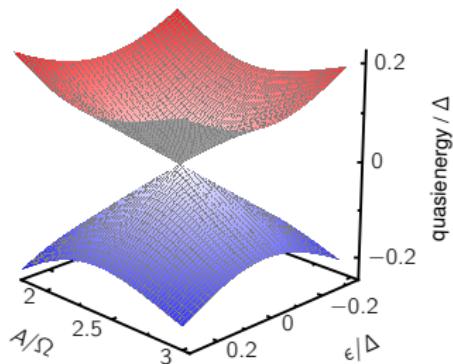


weak dissipation:  
 → strange attractor



- dissipative term  $-\gamma \dot{x}$  breaks time reversal  
 → phase lag due to dissipation → no longer symmetric in  $p$
- ✓ Floquet-Bloch-Redfield capable of phase lag
- ✗ RWA:  $\rho = \sum_{\alpha} p_{\alpha} |\phi_{\alpha}\rangle\langle\phi_{\alpha}|$  would preserve symmetry

## Dissipation at Conical Intersections



SK, Phys. Rev. A **110**, 052218 (2024)

SK, J. Chem. Phys. **162**, 154101 (2025)

Stationary state of driven dissipative quantum system

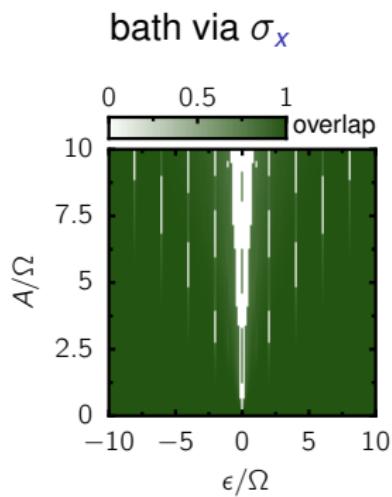
mean energy:  $p_\alpha \propto e^{-E_\alpha/kT}$

quasi energy:  $p_\alpha \propto e^{-q_\alpha/kT}$

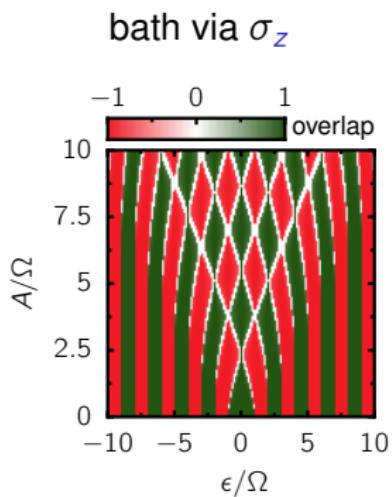
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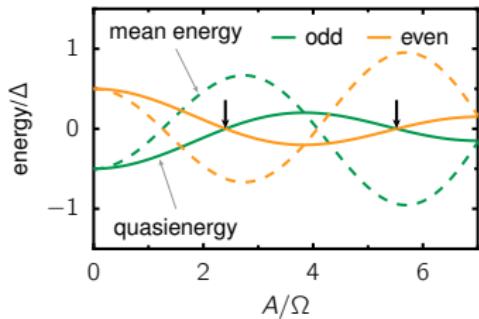


mean-energy state



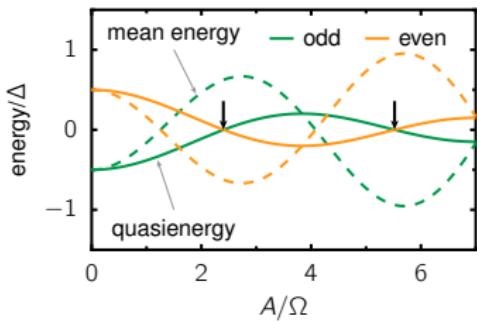
(anti-) Floquet-Gibbs state

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{A}{2}\sigma_z \cos(\Omega t)$$

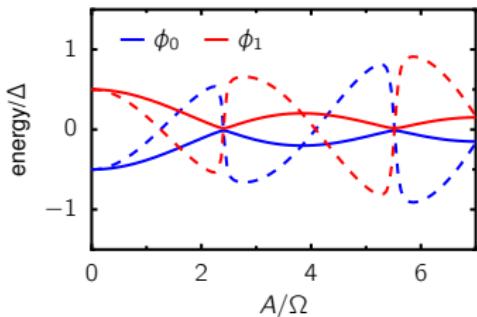


- spatio-temporal symmetry  $G$ :  
 $\sigma_x$  &  $(t \rightarrow t + T/2)$
- even / odd states

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{A}{2}\sigma_z \cos(\Omega t) + \frac{\epsilon}{2}\sigma_z$$

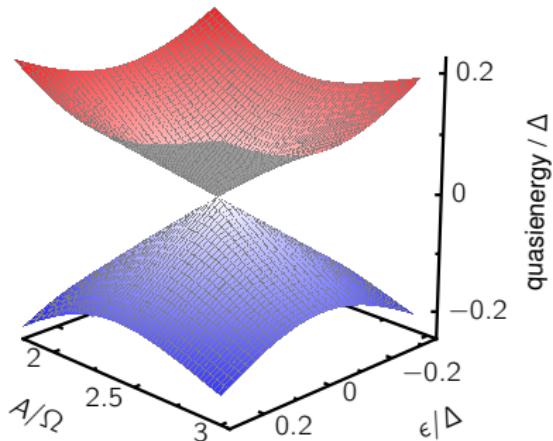


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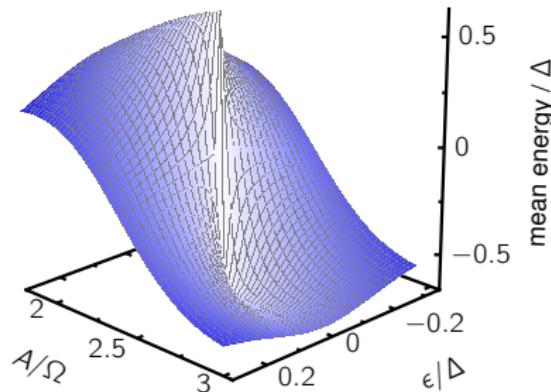


- with detuning
- quasienergies: anti-crossing
  - mean energies: exact crossing

quasienergy



mean energy



On circle around tip:

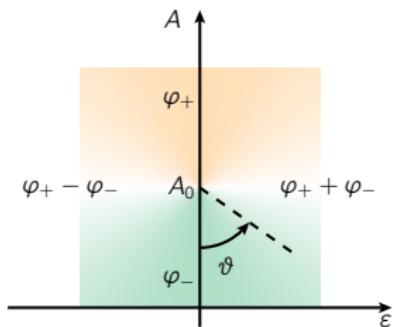
- constant quasienergy
- maximal variation of mean energy

Two-state approximation for  $\phi_0$

- basis:  $\varphi_- / \varphi_+$  at tip
- $\vartheta = 0 \rightarrow \vartheta = \pi$ :

$$\phi_0 \rightarrow \phi_1$$

$$p_0 \rightarrow p_1 = 1 - p_0$$

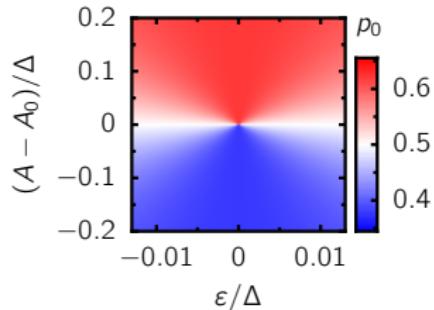
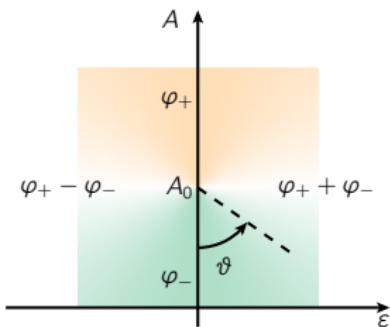


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Population for  $\sigma_x$ -coupling

- angles with  $p_0 = p_1 = 1/2$ 
  - full mixture, max. entropy
- behaves like mean energy of  $\phi_0$

cf. discontinuity along  $A$ -axis

Engelhardt et al., PRL 2019

Golden-rule rate with  $S_{01}(t) = \langle \phi_0(t) | S | \phi_1(t) \rangle \rightarrow$  sidebands  $S_k$

Generic

- Ohmic  $J(\omega) = \frac{\pi}{2} \alpha \omega$
- $k = 0$  suppressed

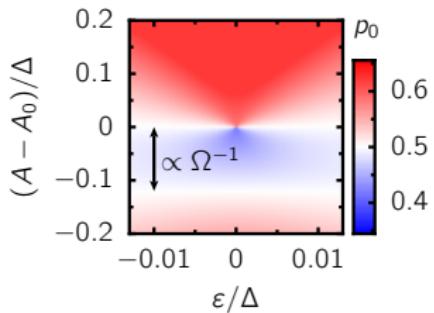
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Generic

- Ohmic  $J(\omega) = \frac{\pi}{2} \alpha \omega$
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Exception:  $S = \sigma_z$

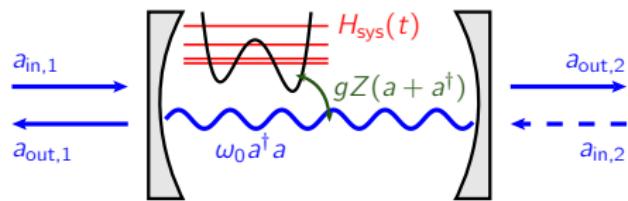
- time-independent for large  $\Omega$
- $k \neq 0$  suppressed



- Crossover to Floquet-Gibbs:  
two lines with  $p_0 = 1/2$  merge  
with increasing  $\Omega$
- Measurable signature of  
 $\sigma_z$ -coupling

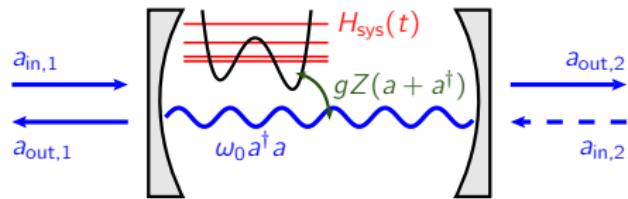
# How to measure the population?

Dispersive readout



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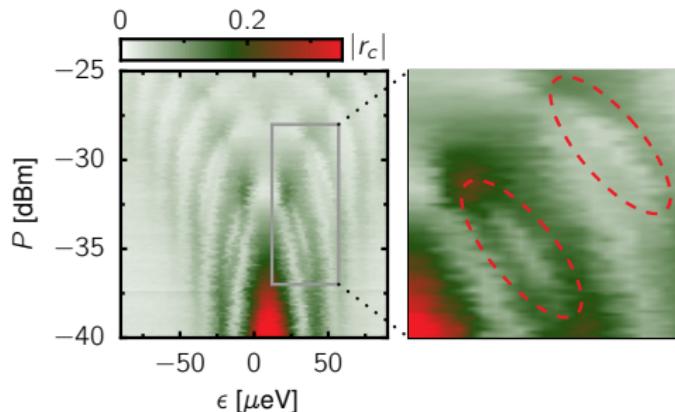
Dispersive readout



Signal: cavity transmission / reflection

$$\chi^{(0)}(\omega_0) = (p_0 - p_1) \sum_k \frac{|Z_{10,k}|^2}{\epsilon_1 - \epsilon_0 + \omega_0 + k\Omega + i\gamma/2} + (0 \leftrightarrow 1)$$

## Dispersive readout



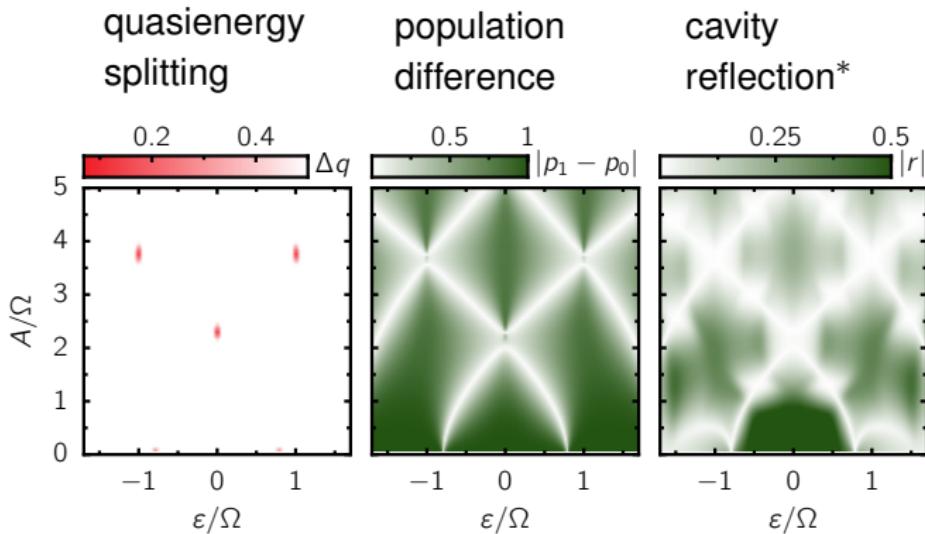
Signal: cavity transmission / reflection

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→ vanishes when  $p_0 \approx p_1 \approx 1/2$

Chen et al., PRB 2021

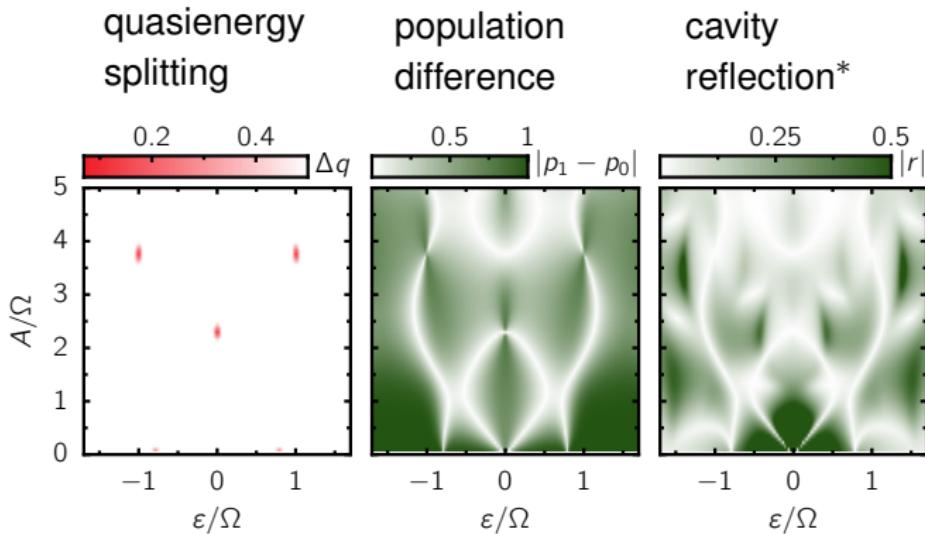
Bath coupling via  $\sigma_z$



→ experimental signature of bath coupling operator

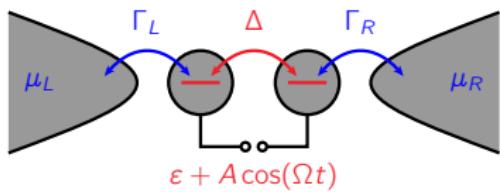
\* for DQD and cavity of Chen *et al.*, PRB 2021,  $\Omega/2\pi = 10$  GHz

Bath coupling via  $\sigma_x$



→ experimental signature of bath coupling operator

\* for DQD and cavity of Chen *et al.*, PRB 2021,  $\Omega/2\pi = 10$  GHz



- zero bias
  - no interaction
  - no spin
- master eq. for populations  $P_i$

$$\frac{d}{dt} \begin{pmatrix} P_L \\ P_R \end{pmatrix} = \begin{pmatrix} -\Gamma_L & \Gamma_R \\ \Gamma_L & -\Gamma_R \end{pmatrix} \begin{pmatrix} P_L \\ P_R \end{pmatrix}$$

(numerics: beyond RWA)

Special case  $\Gamma_L = \Gamma_R$ :

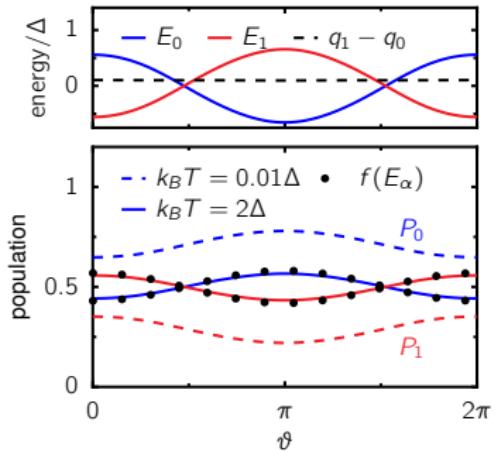
$$P = \sum_k f(q + k\Omega - \mu) \langle \phi_k | \phi_k \rangle$$

Special case  $\Gamma_L = \Gamma_R$ :

$$P = \sum_k f(q + k\Omega - \mu) \langle \phi_k | \phi_k \rangle$$

use  $f(x) \approx \frac{1}{2} + \frac{x}{2k_B T}$

$$P \approx f(E - \mu)$$



→ mean-energy state

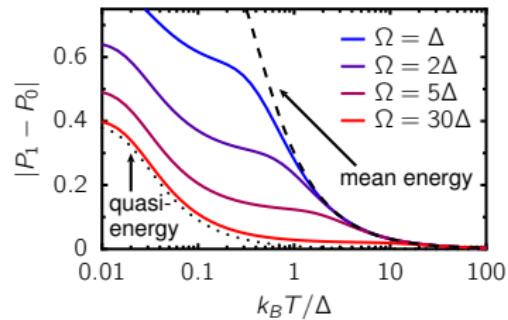
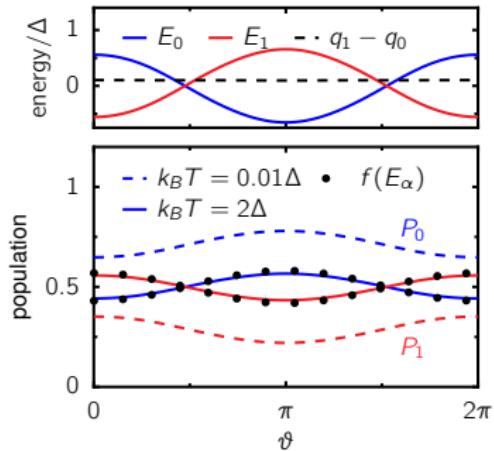
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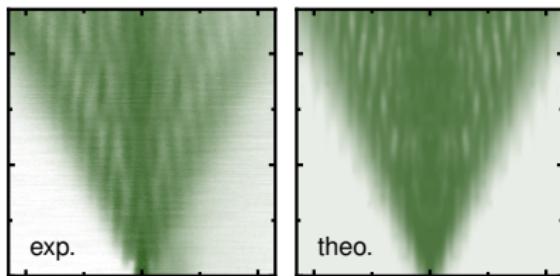
## Mean energy state

- dissipation:  
(almost) any bath coupling
- intermediate / high  
temperature  $kT \gtrsim \hbar\Omega$
- open TLS:  $\Gamma_L \approx \Gamma_R$

## Floquet-Gibbs state

- huge driving frequency,  
 $\hbar\Omega \gtrsim 25\Delta$
- coupling to environment  
via  $\sigma_z$  or  $\Gamma_L \approx \Gamma_R$

## LZSM Interference for Bichromatic Driving



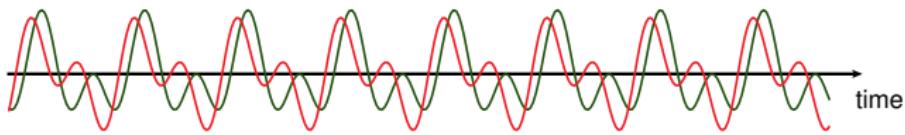
Forster, Mühlbacher, Blattmann, Schuh, Wegscheider, Ludwig & SK

PRB **92**, 245422 (2015)

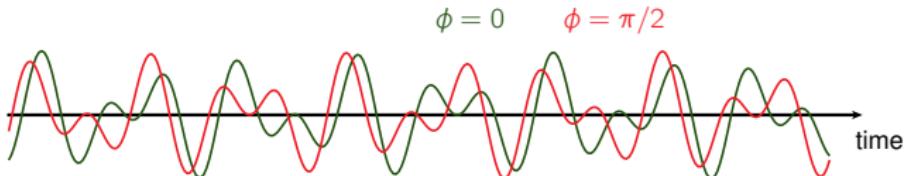
- $g(t) = \cos(\Omega t) + \eta \cos(\Omega' t + \phi)$

$\Omega', \Omega$  commensurable vs. incommensurable

→  $g(t)$  periodic vs. quasi-periodic



$$\frac{\Omega'}{\Omega} = 2$$

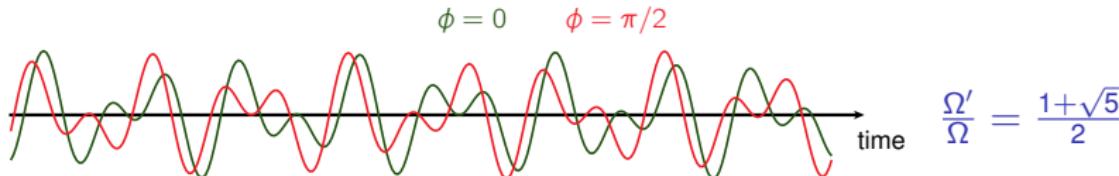
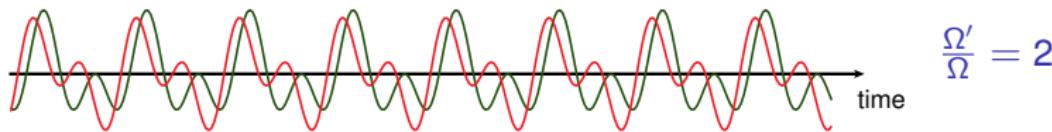


$$\frac{\Omega'}{\Omega} = \frac{1+\sqrt{5}}{2}$$

■  $g(t) = \cos(\Omega t) + \eta \cos(\Omega' t + \phi)$

$\Omega'$ ,  $\Omega$  commensurable vs. incommensurable

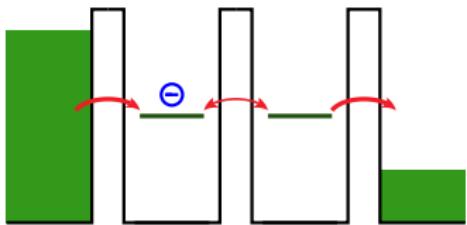
→  $g(t)$  periodic vs. quasi-periodic



■ quasi-periodic:

- dynamical phases (quasi) random

- ? regular LZSM pattern, symmetries



E.g. Lindblad approach:

- source-to-dot tunneling

$$|\psi\rangle \longrightarrow c_1^\dagger |\psi\rangle$$

$$\rightarrow \dot{\rho} = \dots + \Gamma (c_1^\dagger \rho c_1 - \frac{1}{2} c_1 c_1^\dagger \rho - \frac{1}{2} \rho c_1 c_1^\dagger)$$

$$\frac{d}{dt} P = L(t)P \quad \text{where} \quad L(t) = -i[H_{\text{DQD}}(t), \cdot] + L_{\text{in}} + L_{\text{out}}$$

Quantum master equation

$$\frac{d}{dt}\rho = L(t)\rho \quad \text{with} \quad L(t) = L_0 + L_1 \cos(n\Omega t) + L'_1 \cos(n'\Omega t)$$

## Quantum master equation

$$\frac{d}{dt}\rho = L(t)\rho \quad \text{with} \quad L(t) = L_0 + L_1 \cos(n\Omega t) + L'_1 \cos(n'\Omega t)$$

- long-time solution periodic, “Floquet solution with eigenvalue 0”

$$\rho(t) = \rho(t + 2\pi/\Omega) = \sum_k e^{-ik\Omega t} \rho_k$$

- homogeneous set of equations for  $\rho_k$
- $\rho_0$ , time-averaged expectation values, e.g. dc current

- $\frac{d}{dt}\rho = L(t)\rho$  with

$$L(t) = L_0 + L_1 \cos(\Omega t) + L'_1 \cos(\omega t)$$

- auxiliary angular coordinate  $\omega t \rightarrow \theta$

$$\frac{d}{dt}\mathcal{P} = \mathcal{L}(t, \theta)\mathcal{P}$$

$$\mathcal{L}(t, \theta) = L_0 + L_1 \cos(\Omega t) + L'_1 \cos(\theta) - \omega \frac{\partial}{\partial \theta}$$

- $2\pi/\Omega$ -periodic time-dependence
- solve by usual Floquet tools — here: matrix-continued fractions

- $\frac{d}{dt}\rho = L(t)\rho$  with

$$L(t) = L_0 + L_1 \cos(\Omega t) + L'_1 \cos(\omega t)$$

- auxiliary angular coordinate  $\omega t \rightarrow \theta$

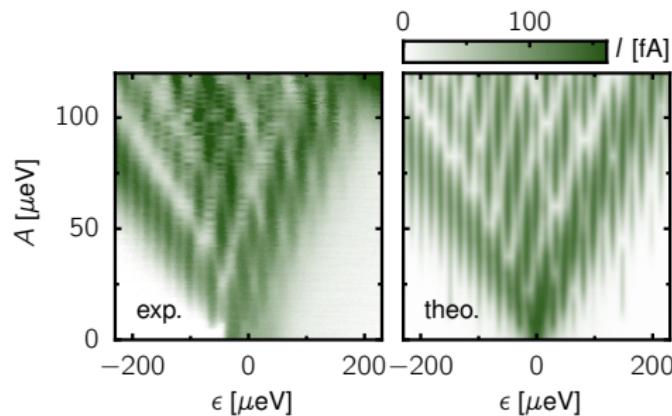
$$\frac{d}{dt}\mathcal{P} = \mathcal{L}(t, \theta)\mathcal{P}$$

$$\mathcal{L}(t, \theta) = L_0 + L_1 \cos(\Omega t) + L'_1 \cos(\theta) - \omega \frac{\partial}{\partial \theta}$$

- $2\pi/\Omega$ -periodic time-dependence
- solve by usual Floquet tools — here: matrix-continued fractions
- connection  $\rho(t) = \mathcal{P}(t, \theta)|_{\theta=\omega t}$

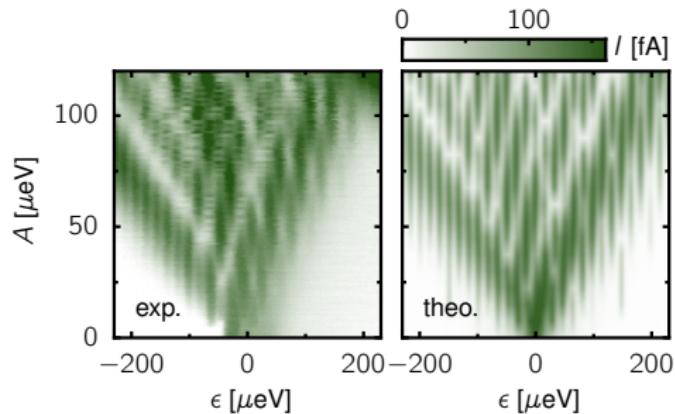
cf.  $t-t'$  formalism, see Peskin & Moiseyev, J.Chem.Phys. 1993

# Theory vs. experiment (commensurable)



- $g(t) = \sin(\Omega t) + \eta \sin(2\Omega t + \phi)$   
[4.8 GHz,  $\eta = 1.25$ ]
- pattern asymmetric ...

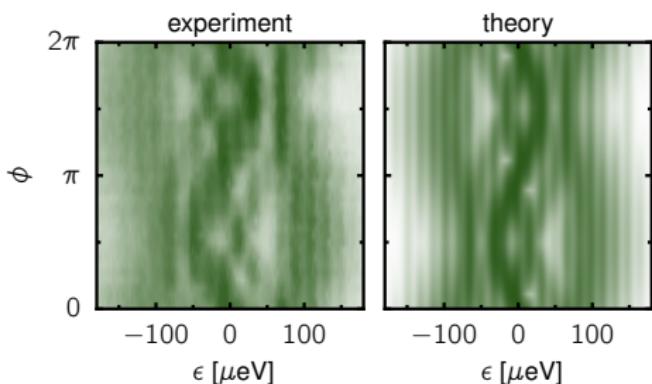
# Theory vs. experiment (commensurable)



- $g(t) = \sin(\Omega t) + \eta \sin(2\Omega t + \phi)$

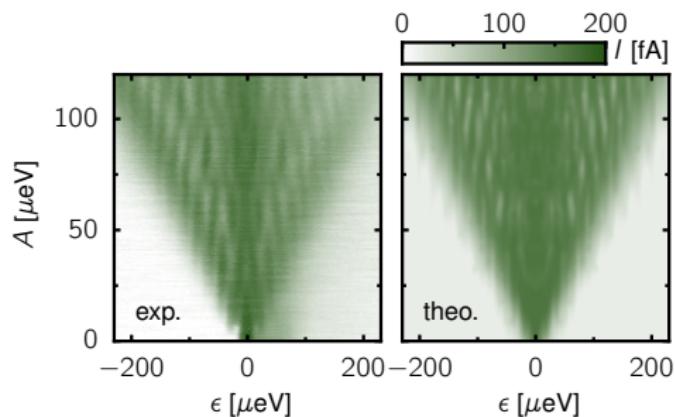
[4.8 GHz,  $\eta = 1.25$ ]

- pattern asymmetric ...



- ... and  $\phi$ -dependent

→ phase calibration



■ golden ratio  $\frac{\Omega_2}{\Omega_1} = \frac{1 + \sqrt{5}}{2}$

→ pattern symmetric  
and  $\phi$ -independent

Forster *et al.*, PRB 2015

## 1 Floquet & Schrödinger equation

- Geometric phases
- Time-periodicity, Floquet ansatz, and all that

## 2 Quantum dissipation and transport

- System-bath model
- Floquet-Bloch-Redfield formalism

## 3 Applications & miscellaneous topics

- LZSM Interference
- Time-dependent Liouvillians
- Floquet scattering theory
- Adiabatic Floquet theory
- Hidden symmetries
- Quantum chaos and dissipation
- Floquet-Gibbs states
- Two-color Floquet theory

