

Regularized Linear Regression

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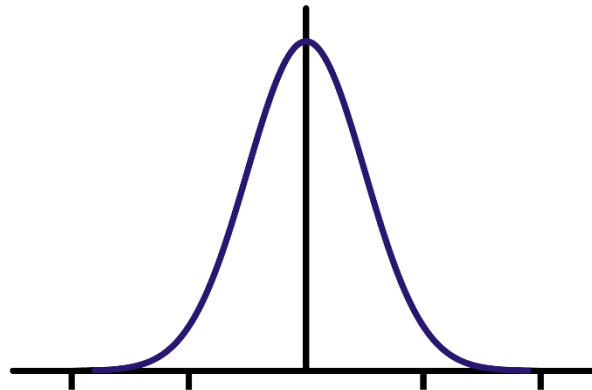
Signal Data Science

Motivation for regularization

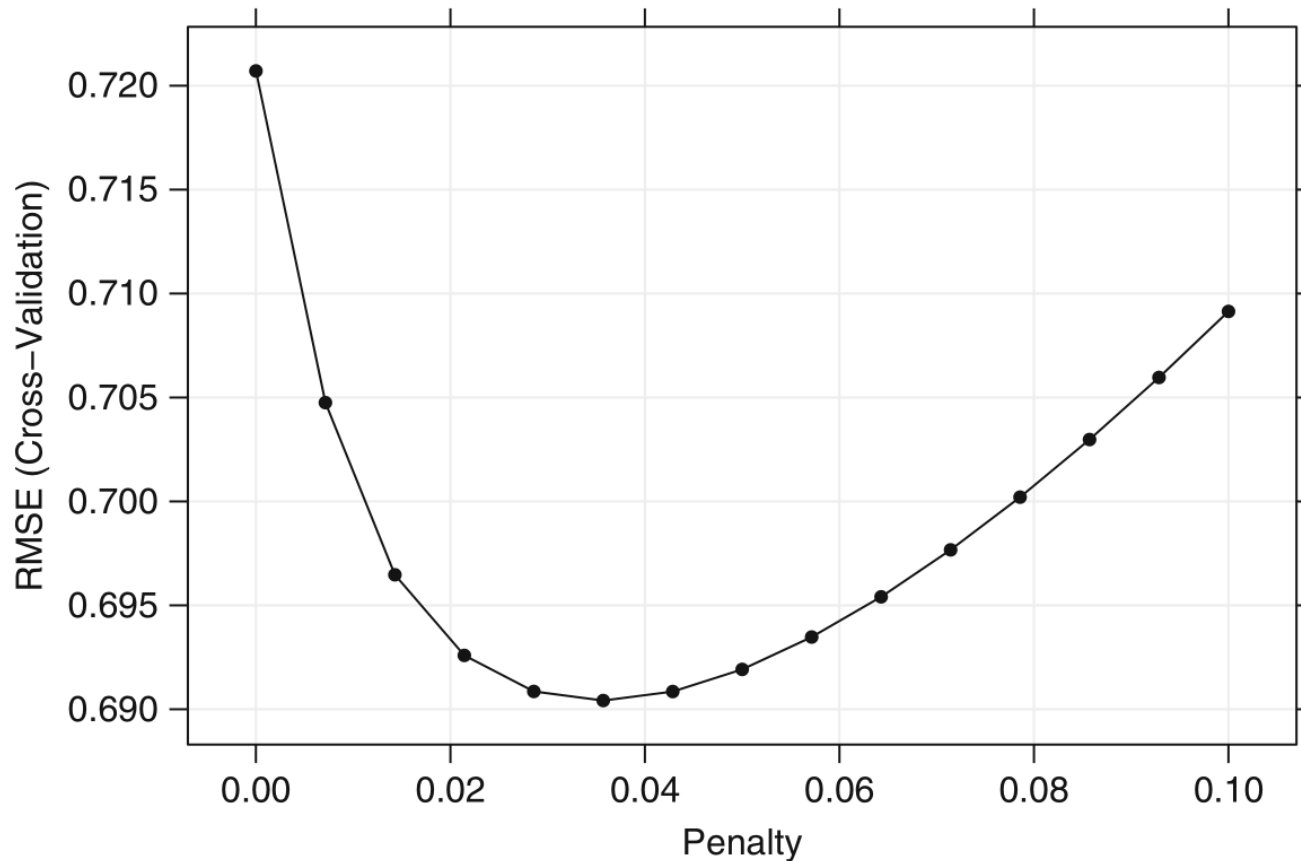
- $\text{MSE} = (\text{intrinsic error})^2 + (\text{bias})^2 + \text{variance}$
- Ordinary least squares is unbiased
 - Isn't the model with the lowest MSE in general
- Introducing some bias can reduce variance
- Two major problems result in inflated coefficients
 - Collinearity among predictors
 - Overfitting

Two equivalent formulations

- Penalize large coefficients
 - Instead of minimizing sum of squared errors (SSE), minimize $SSE + \lambda * \sum(|\text{coefficients}|)$
 - Or minimize $SSE + \lambda * \sum(|\text{coefficients}|^2)$
- Impose Bayesian prior on coefficients
 - Use a Gaussian or Laplacian prior for coefficients being closer to 0



Penalty leads to lower RMSE

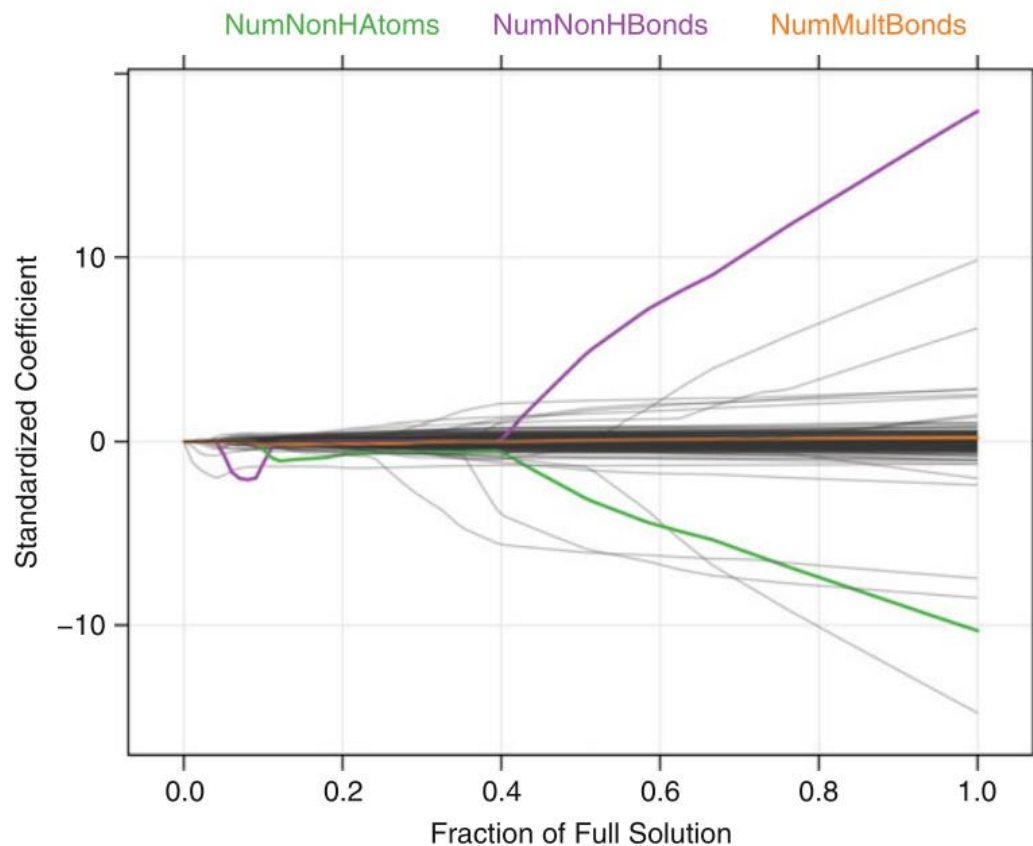


(Applied Predictive Modeling, p. 125)

Lasso and ridge regression

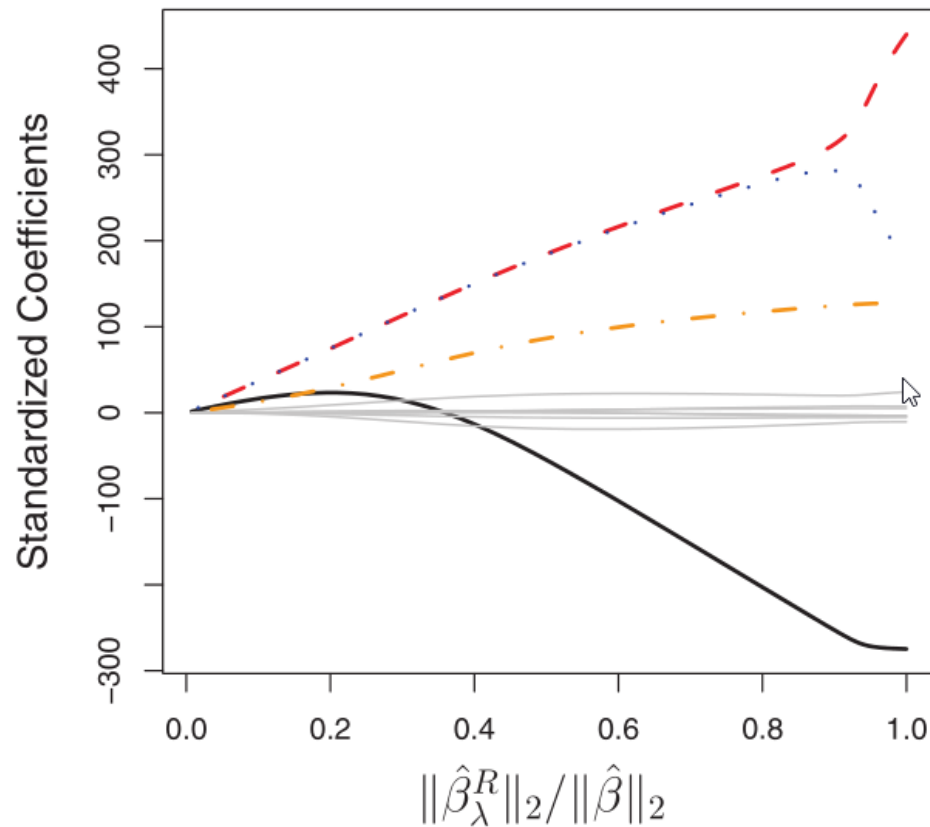
- Minimizing $SSE + \lambda * \text{sum}(|\text{coefficients}|)$
 - Called “lasso” or “ L^1 penalization”
 - Tends to shrink some coefficients to 0 and leave others
 - Easy to interpret
- Minimizing $SSE + \lambda * \text{sum}(|\text{coefficients}|^2)$
 - Called “ridge regression” or “ L^2 penalization”
 - Tends to shrink coefficients uniformly
- “ L^p penalization” comes from notion of p -norm
 - $|x|_p = (|x_1|^p + |x_2|^p + \dots + |x_n|^p)^{1/p}$

L^1 coefficient shrinkage



(Applied Predictive Learning, p. 126)

L^2 coefficient shrinkage

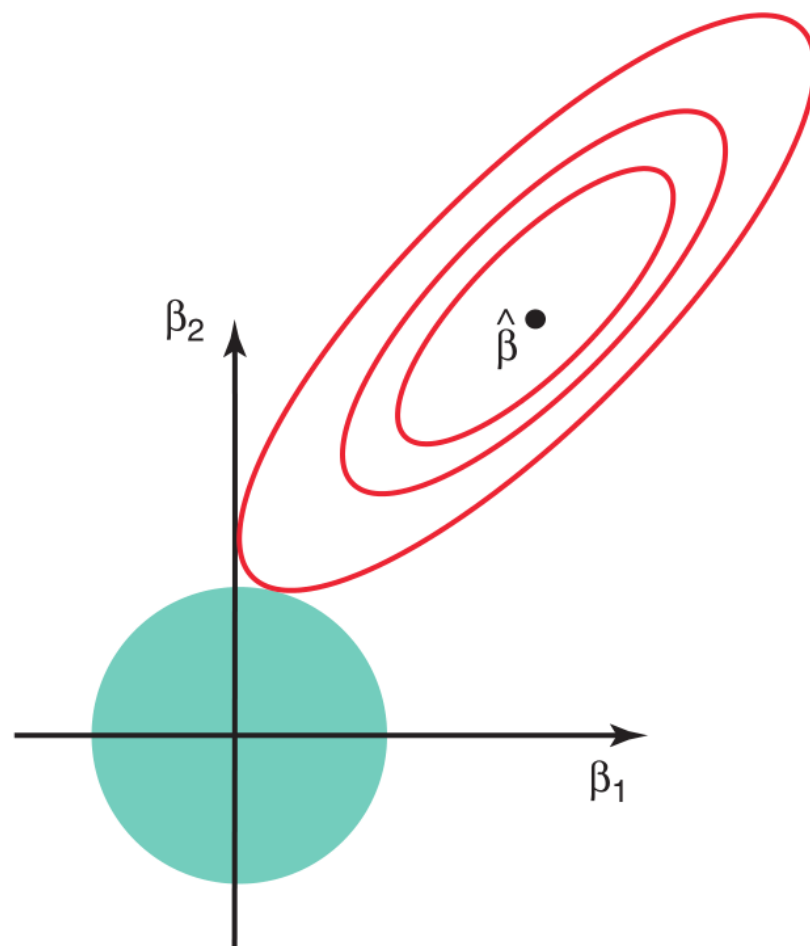
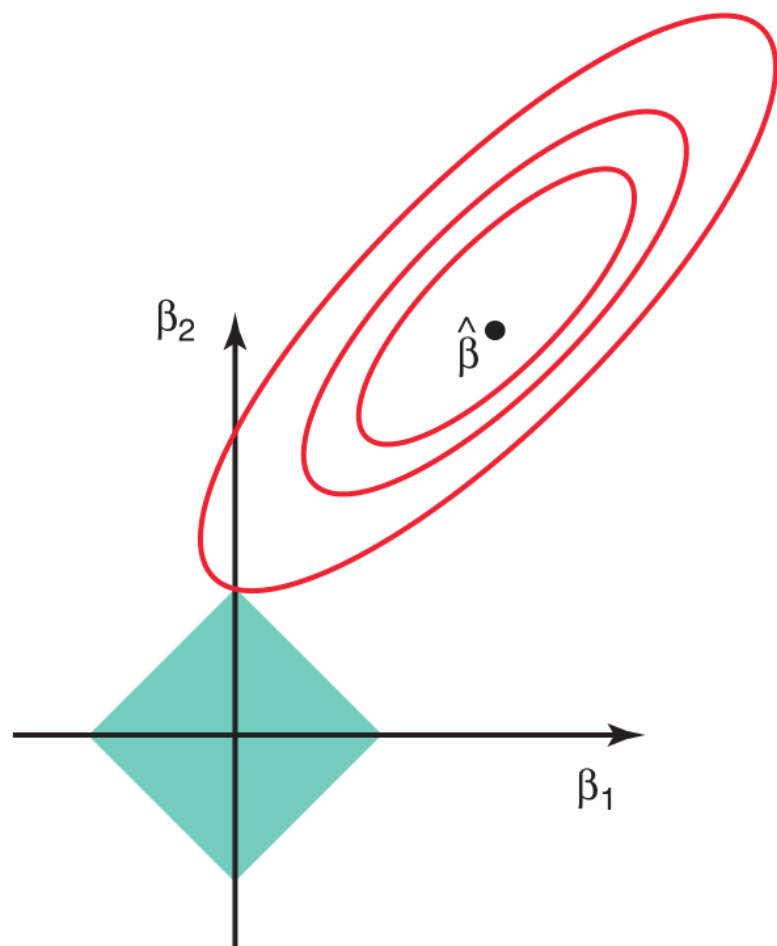


(Introduction to Statistical Learning, p. 216)

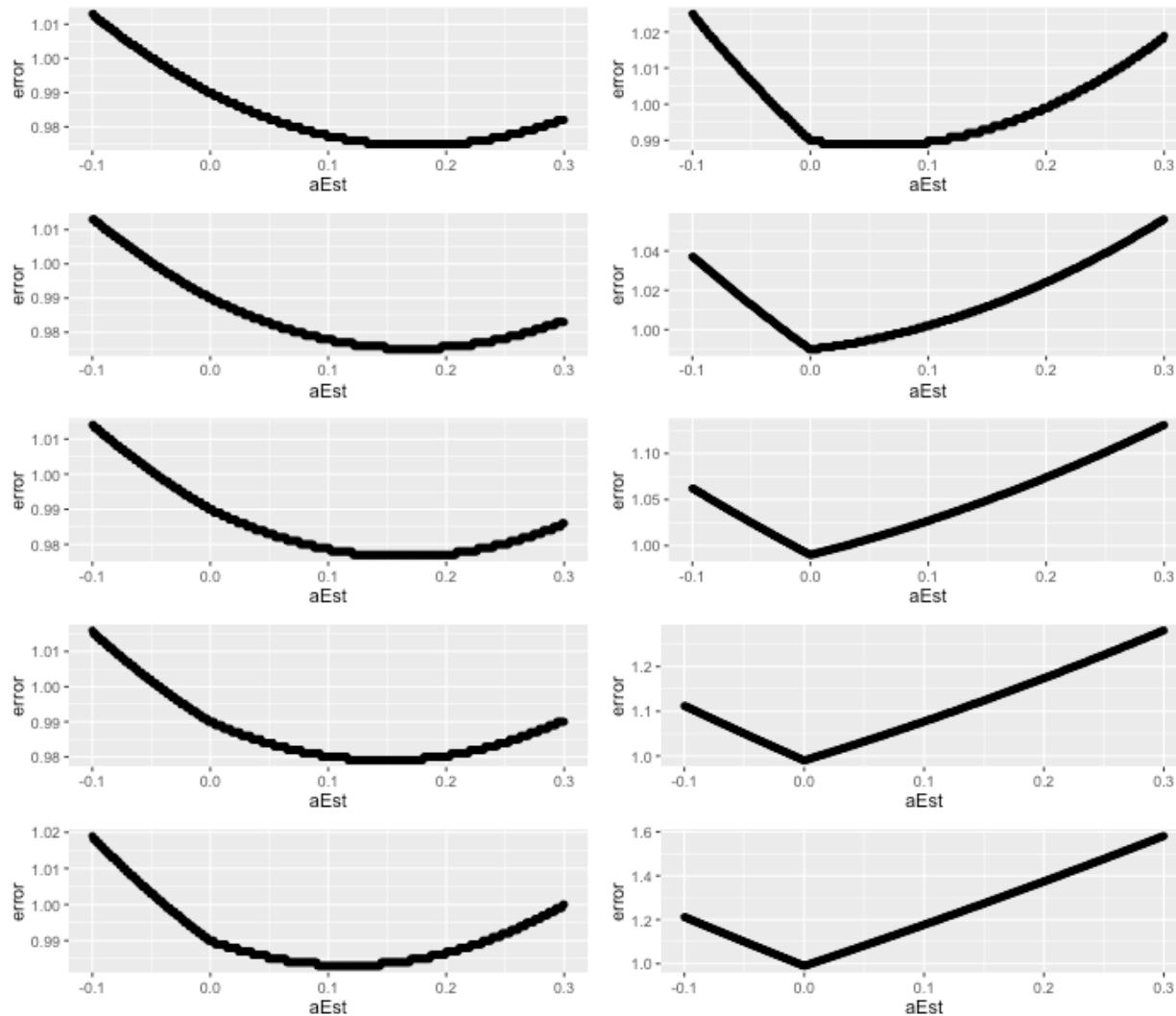
Duality of optimization

- Two equivalent mathematical formulations
 - Minimizing $SSE + \lambda * \text{sum}(|\text{coefficients}|)$
 - Minimizing SSE *subject to* $\text{sum}(|\text{coefficients}|) \leq s_1(\lambda)$
- Same for ridge regression
 - Minimizing $SSE + \lambda * \text{sum}(|\text{coefficients}|^2)$
 - Minimizing SSE *subject to* $\text{sum}(|\text{coefficients}|^2) \leq s_2(\lambda)$

Visual intuition



L^1 regularization



L^2 regularization

