Resampling

We'll be covering **resampling** in this lecture using aggregated data from Andrew Gelman's speed dating dataset.

Check speedDatingSimple.csv for the data and Speed Dating Data Key.doc for an explanation of the variables; the data has been aggregated on the level of each person being rated, so each row corresponds to a particular person's average ratings by their partners (on five different scales) as well as their self-ratings of interest in 17 different activities.

In machine learning in general, we have the problem of overfitting, where:

a problem where a functional form or algorithm performs substantially better on the data used to train it than on new data drawn from the same distribution. It occurs when the parameters used to describe the functional form end up fitting the noise or random fluctuations in the training data rather than the attributes that are common between the training data and test data.

Exercise. Given n data points and a model $a_0 + a_1x + a_2x^2 + ... + a_nx^n$ where we fit the coefficients a_i to the data, how large does n need to be before we can come up with a model that goes through every data point precisely? (Think about the linear case, where only a_0 and a_1 are nonzero.)

Our models run the risk of being *overly biased* toward the structure of the data which we have in a non-generalizable fashion. As such, we want to use *resampling* techniques, most of which involve splitting data into *train* and *test* sets. Instead of training the model on the entire dataset, we'll in general train it on a *subset* of the data and estimate the quality of the model against the data which was fed into the model.

Random number generation in R

We'll begin with a brief discussion of the random number generator (RNG) in R.

The RNG can be *seeded* with set.seed(n) for any integer n. The values that the RNG outputs will depend on its seed, and setting the seed "resets" it to the

initial state corresponding to that particular seed.

 Try using set.seed() in conjunction with runif(5) to get a sense of how this works.

This is important because we may want, *e.g.*, to generate the same random partitioning of our data consistently, in which case we would put a set.seed(1) call before our shuffling of the indices with sample().¹ Or, alternatively, you may want a sequence of reproducibly different calls of sample(), etc.

A single test/train split

- Load the speed dating dataset (using read.csv()) and restrict to a gender of your choice.
- Write a function that splits the data randomly into a train set and a test set (of equal sizes), trains a linear model on the train set to predict attractiveness from the 17 activities, and uses predict(model, newdata) to generate predictions for the test set.
 - An easy way to do this is to call sample() to shuffle the row indices
 of the data and then to use the % operator, taking the remainder
 upon division by 2, to assign each row to 0 or 1.
 - Run the function a large number of times to generate predictions for many different train/test splits.
 - For each prediction, calculate the associated R^2 (just square the cor() relation).
 - Plot the distribution of R^2 values and calculate their standard error (the standard deviation divided by sqrt(n) for n samples).
 - In general, how does the performance on the train set compare to the performance on the test set?

n-fold cross validation

In n-fold cross validation, we try to avoid the randomness of a single train/test split. The process goes as follows:

1. We randomly split the data into n different subsets.

¹In some cases, you may want to (1) save the state of the RNG for later, (2) set the seed to something specific and generate a consistent splitting of the data, and (3) change the RNG back to its saved state. This is possible using .Random.seed and is described in Cookbook for R – we won't need this for this lesson, but it's important to be aware of (as it will eventually surely come up).

- 2. For each of the *n* subsets, we train the model against *all the other subsets* and use that model to make predictions on our held-out subset.
- 3. We combine all of the predictions and calculate some measure of model quality, like R^2 or root-mean-square error.

We'll be continuing with predicting attractiveness from activities. Now, choose one of two different approaches:

- Implement a function that uses *n*-fold cross validation to generate a set of predictions for any *n*, or
- Implement 2-fold and 10-fold cross validation separately in two different functions.

Either way,

- Make a large number of predictions using both 2-fold and 10-fold cross validation.
- As before, plot the distribution of the associated *R*² values and calculate their standard errors.
- Compare the results of a single train/test split, 2-fold cross validation, and 10-fold cross validation with regard to getting an accurate measure of model quality.

Stepwise regression

We can use our methods of evaluating model quality to choose between different models.

In backward stepwise regression:

- We start with every predictor variable added to the model.
- Then, we iterate, at each step removing the variable which adds the least to the model.
- We eventually reach a stopping point based on some statistical criteria.

To that end, you'll be writing your own implementation of backward stepwise regression.

Write a function that does the following:

- It begins with the full dataset the attractiveness ratings, which we're trying to predict, as well as the 17 activity scores.
- The function should repeatedly iterate. On each iteration, it should:

- Use 10-fold cross validation to calculate an R² value, measuring the quality of how well the currently selected variables can predict attractiveness.
- Fit a linear model for attractiveness against the full set of data.
- Remove the variable associated with the coefficient which has the highest p-value. (If a linear model is stored in fit, you can access data about the coefficients by looking at summary(fit)\$coefficients.)
- The function should store the cross-validated estimates of R^2 in association with the number of features moved at each step.

Use this function to explore how cross-validated R^2 changes as you remove more and more variables from the model. Plot the results.

Using R's step()

We'll finish off by using R's built-in stepwise regression function, step(). You can more-or-less treat its functionality as a black box, but spend a minute or two skimming the official documentation. It uses the Akaike information criterion as a measure of model quality – both for determining which variable to add or remove and for determining when to stop – which you can also treat as a black box. (Just know that *a lower AIC is better*.)

- For *each of the five rating variables* (attractiveness, sincerity, intelligence, fun, ambition), use step() to run backward stepwise regression (with the direction="backward" parameter).
- Store the coefficients of the final linear model into a data frame. Interpret
 the results.
- Repeat the above for the other gender (the opposite of the one you selected at the beginning). Interpret the differences between the results.