Digital Filter Design: Linear Phase vs Minimum Phase FIR
BIRMINGHAM CITY UNIVERSITY
BSc (Hons) Sound Engineering and Production

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Abstract

Through the design of three graphic EQs with different phase responses, the report explores filter design methodologies with specific attention drawn to phase and its imparted effect. In this report, linear phase filters are shown to perfectly reconstruct the input signal due to their symmetric impulse response, but it is also shown that this symmetric impulse response imposes an unnatural pre-ring upon the output signal. Minimum phase filters are shown to have frequency dependent phase and extended post ring.

The report uses objective analysis and listening tests to investigate whether these phase related issues are audible, and to what extent. Transient and square wave forms are found to expose the filter's phase relationship to the listener. Pre-ring is shown to play an integral part in audibly perceptible differences between phase responses. Listeners were shown to prefer samples in linear phase, however, the reason for this is inconclusive and the area invites further study.

The graphic EQ was tested at different EQ settings in order to establish where in the spectrum phase related issues were most evident, however, this was inconclusive, so further research in to this area is recommended.

Acknowledgements

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Table of Contents

List of diagrams, tables and plates

Introduction

Problem definition

The inner ear is known to react to acoustic stimuli in a variety of ways. Studies from Glasberg and Moore's research into time varying sounds (2002) to Harvey Fletcher and Wilden A. Munson's equal loudness contours (1933) have revealed the psycho-acoustic impact of both the magnitude and frequency response of a system. Although records have been made of the awareness of frequency dependent phase, it's consequences and perceived significance has had less research. As of yet, no standards are in place for a specified phase response in high quality audio systems and few manufacturers offer phase specifics. The implications of this study lead to speaker, headphone and microphone design.

Scope

Multiple filter designs are required to fulfil the projects requirements; therefore, the aid of open source code and libraries may be required to complete the research within the time frame. There were initial concerns that real-time processing would be beyond the scope of this project, however, the Gantt chart which was used to predict timings indicated that there would be time.

Rationale

This study is of significant importance to the high-end digital audio industry. It is generally assumed that linear phase FIR filters perform best due to their waveform-preserving properties, however, there is an argument that the symmetrical impulse response of a linear phase filter causes a 'pre-ringing' effect. Minimal phase filters may subtly distort the waveform, due to phase offset between frequencies, however, they do not suffer from this 'pre-ring'. Compromises must be made in the process of designing and selecting digital filters; this paper should help engineers to make better informed decisions.

Aims and Objectives

Project Aim

To investigate LTI FIR digital filter designs and with a series of tests, analyse and assess the perceived notion of group delay.

Project Objectives

- Design digital implementations of linear phase, minimum phase and maximum phase FIR digital filters.
- Using industry specific techniques implement linear phase, minimum phase and maximum phase graphic EQs.
- Compose objective testing methods to analyse the filters.
- Formulate a series of listening tests; differentiate and critique the digital filter responses and collect and assess findings.
- Examine the categorisation of digital filters and find appropriate applications for their implementations.
- Debate, criticise and appraise relevant research material, building on previous studies.

Review of Existing Knowledge

A Brief History of EQ

The earliest equalisation came by the invention of the Harmonic Telegraph by Alexander Graham Bell, which enabled multiple messages to be sent simultaneously when the signals occupied different frequency bands. (loc.gov, 2002)

One of the earliest examples of variable equalisation is John Volkman's external equalizer design. Used to improve sound reproduction for motion pictures, it featured cut and boost setting for bass and treble. In 1949 Gramophones were fitted with switchable tone controls and in 1952 Baxandall introduced controls using potentiometers. (McGrath, D. 1994)

Throughout the 1950s the industry recognised the importance of equalisation in the production and playback of records and standards were set by The Recording Industry Association of America and National Association of Broadcasters. (Bohn, D., 1990)

In 1962 the Texan physics professor C P Boner developed new theories on room acoustics. Art Davis, Jim Noble and Don Davis went on to develop the industry's first passive 1/3-octave variable notch filter for Altec Lansing in 1967. Through the next decade Equalisation continued to progress in various ways, most notably parametric and graphic EQs were developed

Yamaha introduced the DEQ7 in 1987, the first Digital Variable Equalizer. The DEQ7 had 30 different built-in configurations and combined both graphic and parametric technology. (ibid)

Linear and Non-Linear Systems

Linear systems adhere to two conditions, homogeneity and superposition.
Homogeneity states that if an input is scaled by a constant, the output shall be proportionally scaled. Superposition says if the input is composed of the summation of two inputs, the output shall also be composed of the two inputs and behave the same as if each signal were fed through the filter separately and added at the output. An example of a non-linear system would be any case where both of these conditions did not hold true. (Smith, J. O. 2007)

Time Invariant Systems and Time Variant Systems

A shift of N samples to the input of a time invariant system results in a shift of N samples in the output, preserving the waveform as expected (ibid). This paper concerns itself with Linear Time Invariant (LTI) digital filters. However, in most real-time, analog application, a component's stability will drift in respect to time, e.g. the resistance of the windings in a motor will vary from when the motor first starts to after it's increased usage due to its temperature rising. This is an example of a Time Variant System (TVS). (McGraw-Hill Professional, 2017)

Phase Delay and Group Delay of a Linear Time Invariant (LTI) Digital Filter

Phase Delay can be defined as 'the time delay in seconds experienced by each sinusoidal component' input to a digital system (ibid)

In contrast, the group delay of a system is the 'delay experienced by a narrow-band ``group'' of sinusoidal components'. This can be a useful specification when analysing filters whose 'phase delay' alters over the spectrum (ibid).

'High Frequency Phase Response Specifics – Useful or Misleading?' by Deane Jensen addresses problems associated with how the industry viewed phase response (1988). She discusses two factors pertaining to the phase of a signal: frequency dependent and frequency independent phase, also two required specifics: 'Deviation from linear phase' and 'Group delay'.

These are calculated with the following formula:

$$Deviation\ from\ Linear\ Phase = -(|Td| - |fID|) \times f \times 360$$

(Eq. 1) Deviation from linear phase is in degrees, |Td| is the absolute time delay in seconds, |fID| is the absolute frequency independent delay in seconds and f is frequency in hertz.

$$Group\ Delay = (|P1| - |P2|)/(f2 - f1) \times 360$$

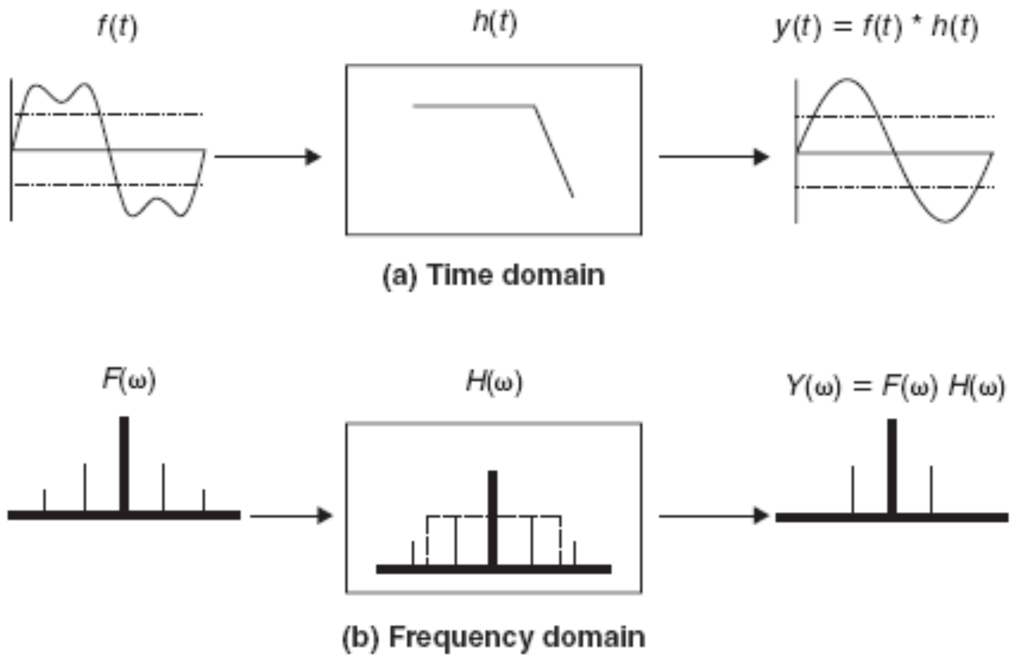
(Eq. 2) Group Delay is in Seconds, f1/f2 is in Hertz, |P1| is the Absolute Phase at f1 specified in degrees, |P2| is the Absolute Phase at f2 specified in degrees, f1 and 2 are equally less and more than the specified frequency by a very small amount, defining the group.

Filter Structure

A filter can be depicted as a 'black box' which takes a time-domain signal x(n), applies a change to the signal's spectral content and results in an output y(n). This 'black box' allows certain frequencies to pass while attenuating other frequencies. (Smith, J. O., 2007)

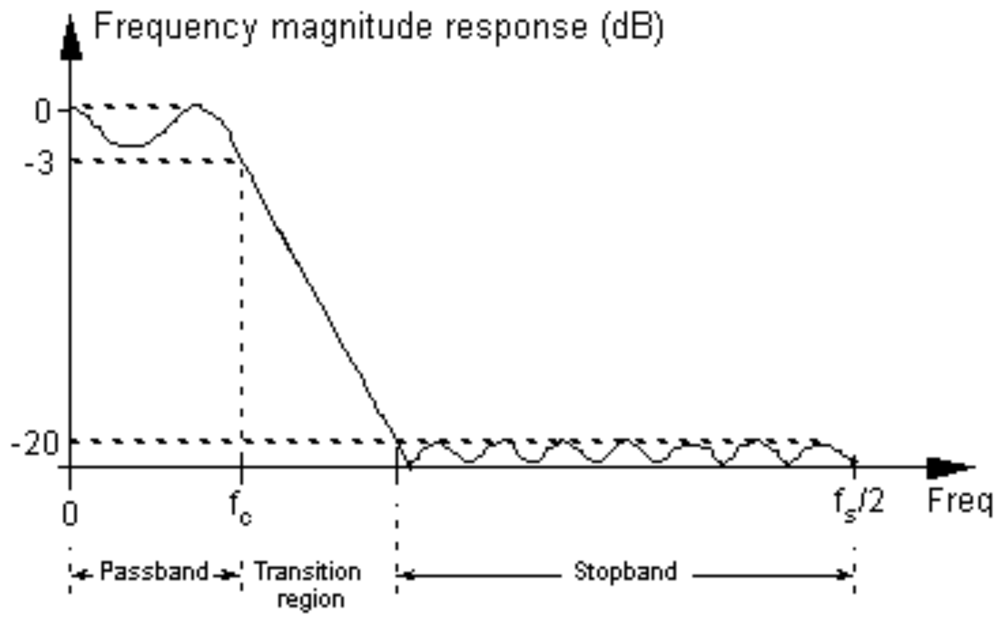
(fig. 1) Black Box representation of a filter.
(Lyons, 2011)

Different filter types allow the attenuation of different areas of the spectrum. 'Low pass'/'high cut' filters attenuate frequencies above a set cut off frequency. 'High pass'/'low cut' filters attenuate frequencies below the cut-off and 'band pass' filters 'pass one frequency band' and attenuate 'frequencies above and below that band' (Lyons, 2011)



(fig. 2.) Shows a block diagram and a mathematical representation of the input signal (consisting of two frequencies), filtered by the transfer function, resulting in a single sinusoid. Where f(t) is the input signal, h(t) is the filters impulse response and y(t) is the output signal in the time domain. F(ω), H(ω) and Y(ω) are representations in the frequency domain.
(Broesch, J. D. 2008)

Three important attributes for filter design are the filter's order, the pass band/stop band ripple and the transition width. The amount of play between these parameters can be expressed by a triangle; the total sum of angles is fixed, therefore only two parameters can be changed and the third is conditional. (Losada, A. R., 2008)



(fig 3.) A Lowpass filter, showing passband and stopband ripple and the transition region.
(dspguru.com, n.d.)

Delay, Phase and Filtering

A time delay of one signal in regard to another by less than 40ms is perceived as a single sound. This was discovered by Dr. Helmut Haas in 1949 and is commonly referred to as the Hass effect; it relates to how we perceive the location of a sound in an acoustic environment: the first sound to reach our ear defines the sound's source regardless of delayed reflections. The combination of these delayed signals produces a comb filtering effect, resulting in constructive and deconstructive interference at different sinusoidal frequencies. It can also give a sense of spaciousness and depth which may or may not be preferable.

Eric Tarr of Hack Audio describes how parallel processing and the delay of sinusoidal frequencies are integral to equalization. Delay is commonly expressed in programming in samples and in mathematics by radians. Important to filtering, the summation of a signal with a delayed version by n samples will result in a different amount of phase shift (radians) for each sinusoidal frequency. Frequencies where their instantaneous phase match will have constructive interference; their amplitudes will increase, whereas frequencies where their instantaneous phase are polarized will have destructive interference and their amplitudes will cancel. (Tarr, E., 2018).

'Traditional linear digital filters typically come in two flavors: finite impulse response (FIR) filters and infinite impulse response (IIR) filters' (Lyons, 2011)

FIR (Finite Impulse Response)

FIR filters are also known as feedforward filters due to the computation relying purely on their input. When given a finite amount of non-zero inputs, a FIR filter has a finite amount of non-zero outputs. This makes them inherently stable (ibid.). FIR filters can be easily made linear phase. Their implementation can be done in a single looped instruction and they are suitable for multi-rate application, therefore both decimation (reducing the sampling rate) and interpolation (increasing the sampling rate) can be used to reduce computational expense. They can also be implemented with integer mathematics which can be useful when DSP hardware requires fixed point scaling, commonly used in devices such as Texas Instruments MSP430, FPGA, or ASIC (iowahills.com, n.d). Improved technology has allowed long FIR filters to be implemented with low latency for live use. Low latency fast convolution techniques are used enabling FIR filters to mimic the response of IIR filters whilst being computationally less expensive (McGrath. D.S., 1993)

The processing of a FIR filter is performed by the Multiply-Accumulate (MAC). A number of delays to the signal ('taps'), often denoted by N, are multiplied by coefficients and summed at their output (dspguru.com, n.d). The most basic example of this approach is shown in the moving average filter.

In 'Understanding Signal Processing' (2011), Lyons uses the analogy of finding the average number of cars passing over a bridge in five-minute periods. If we know how many cars pass each minute, we simply add together the last five minute intervals and divide by five, or in the case of MAC, multiply by 1/5th. The order of addition and multiplication is interchangeable, giving us the equation

$$y(n) = \frac{1}{5}x(n) + \frac{1}{5}x(n-1) + \frac{1}{5}x(n-2) + \frac{1}{5}x(n-3) + \frac{1}{5}x(n-4) + \frac{1}{5}x(n-5)$$

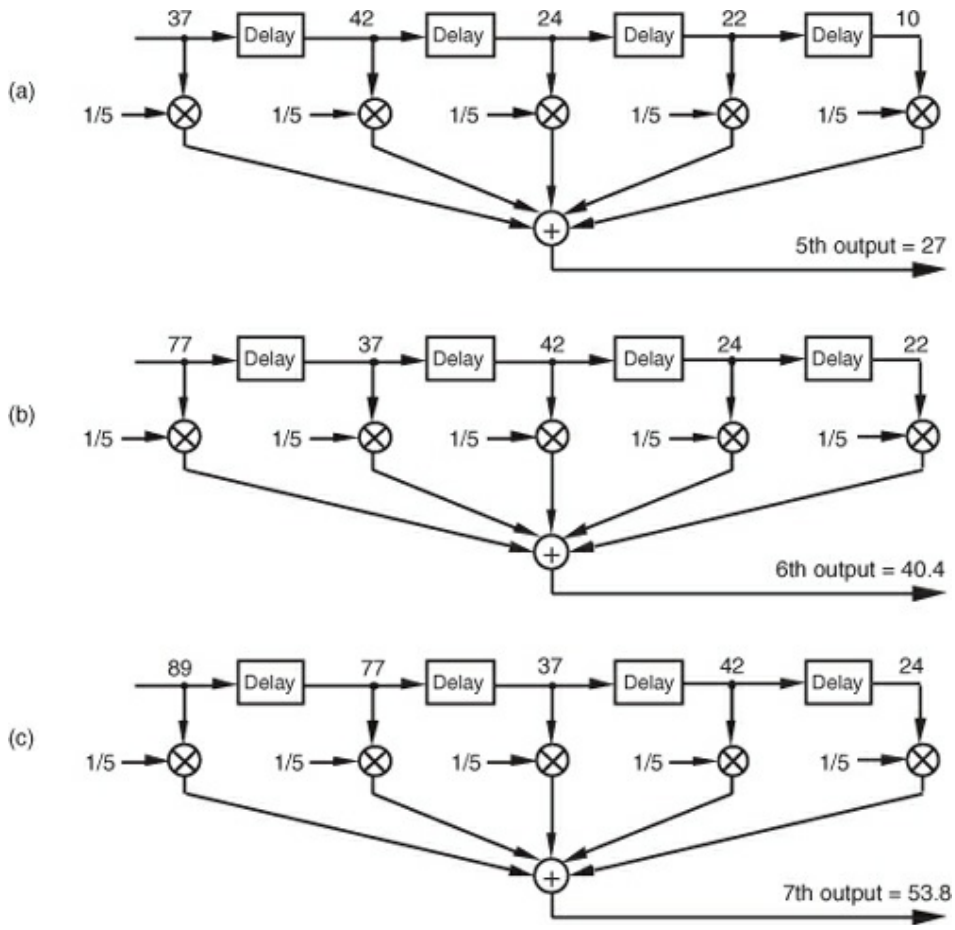
(Eq. 3) Where y(n) is the average number of cars per five minutes, x(n) is the number of cars per minute.

In a more general, concise form:

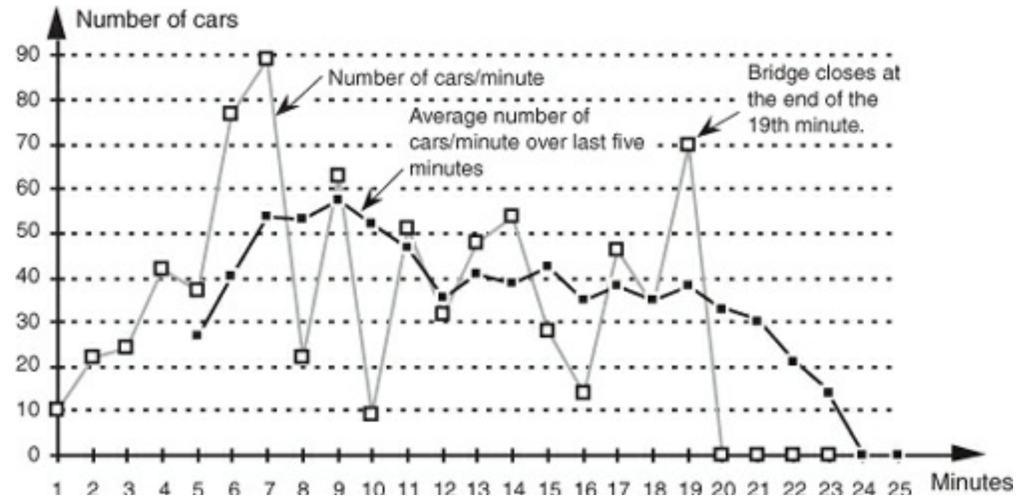
$$y(n) = \sum_{k=1}^{4} \frac{x(n-k)1}{5}$$

(Eq. 4) Where k is the number of delays to the signal x(n)

The filter structure is graphically depicted in the block diagram.



(fig. 4) Block diagram describing a moving average filter, a) Output at fifth minute. b) Output at sixth minute. c) Output at seventh minute. (Lyons, 2011)



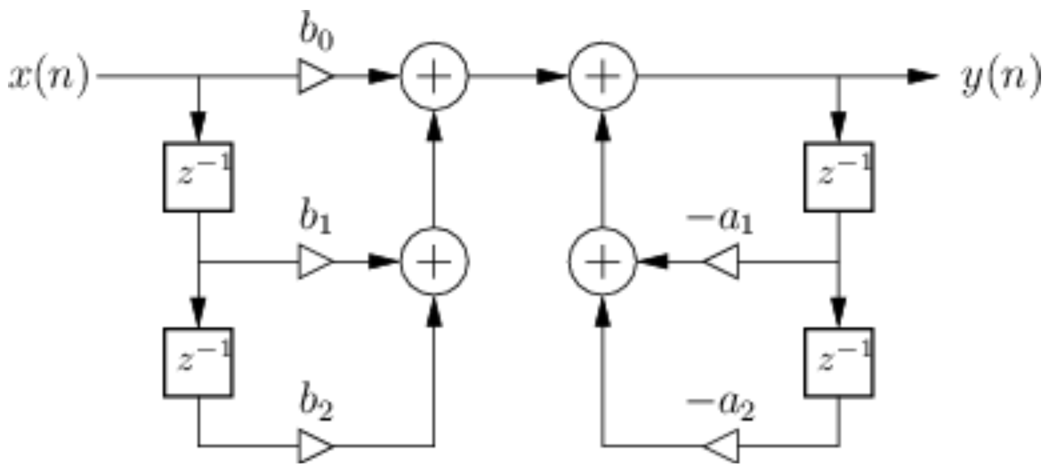
(fig.5) Plot of cars per minute and the response of the moving average filter. (Lyons, 2011)

The key factors regarding the filter's response are the number of taps and the value of the coefficients.

IIR (Infinite Impulse response)

In contrast to the FIR structure, an IIR filter relies both on the input and the output of the system, this allows it to achieve a required filtering characteristic with fewer computations; hence it uses less computational memory. (dspguru.com, n.d.) The biquadratic filter is commonly used by engineers for its computationally efficient benefits and simplicity of use; the feed-forward and feedback gains can be altered to create all types of filter.

An example of this is shown in direct form in the block diagram of figure 5. The z-n expression can be considered as simply a delay; the properties of the z transform shall be discussed in more depth at a later stage in this report.



(fig. 6) Block diagram of a second order Biquadratic IIR filter. (Lyons, 2011)

This can be shown in the difference equation:

$$
\begin{aligned}
y(n) \;=\; & b_0 x(n) + b_1 x(n-1) + \cdots + b_M x(n-M) \\
& - a_1 y(n-1) - \cdots - a_N y(n-N) \\
=\; & \sum_{i=0}^{M} b_i x(n-i) - \sum_{j=1}^{N} a_j y(n-j)
\end{aligned}
$$

(Eq. 5) Where 'b' are the feed-forward coefficients and 'a' are the feed-back coefficients (Smith, J. O., 2011)

The transfer function for the biquadratic, second order recursive filter is derived directly from analogue filter designs by the use of bilinear transform (Milivojević, Z., 2009)

A simple example of an IIR structured filter for analysis is the Leaky Integrator this is a first order IIR and is defined by:

$$y[n] = \lambda y[n-1] + (1-\lambda)x[n]$$

(Eq. 6) Where λ is N -1/N, y[n] is the output signal and x[n] is the input signal. (Coursera.org, n.d)

When the order of the filter N is large, λ is very close to 1. This is used to scale both the feed forward and feedback paths. Where y[n] keeps a fraction of the accumulated value so far (λ*y[n-1]) and forgets or leaks a fraction (1-λ) from the inputs accumulator.

Much literature is available on the subject of IIR filters however this project is concerned solely with FIR filters.

Impulse Response

Referring back to Lyon's moving average filter in figure 3, the impulse response of the system, also known as the filter kernel, is simply the coefficients scaling the input signal by $1/5^{th}$ at each tap. This can be seen as a signal in its own right; there are five of them therefore the impulse response is five samples long. The reason it's called an impulse response is due to the fact that if a unit impulse (a single 1 followed by a stream zeros) was passed through the system, the output would contain five samples equating exactly to these $1/5^{th}$ coefficients. Therefore, in the case of a 'black box' filter we can derive its impulse response in exactly this manner; from this the transfer function can be obtained. The impulse response is commonly referred to in literature by h(n) in the time domain and h(ω) in the frequency domain. Where n is the index for the coefficients and ω is the angular frequency in radians. In this report however it shall be referred to by h(k) to avoid confusion when set alongside the standard input and output variables x(n) and y(n). In this example all the k variables are set to $1/5^{th}$ however altering the values of k changes the response, this is important to consider when implementing a desired filter.

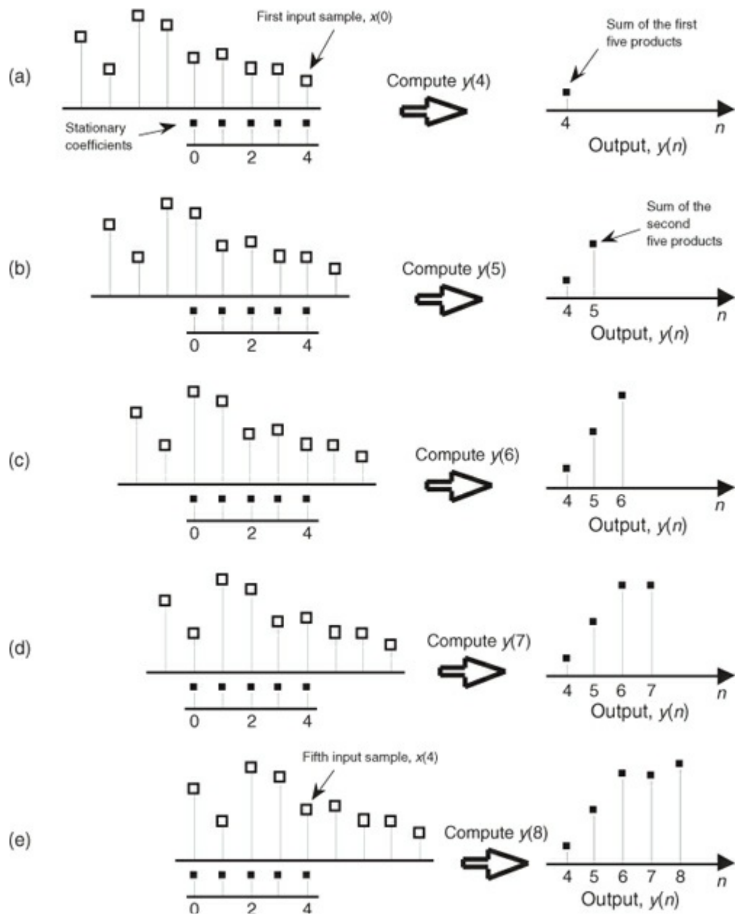
Convolution

Convolution has already been defined in this report in both the FIR and IIR transfer functions. The transfer function for the FIR filter is precisely this; convolution of the input signal with the impulse response. Using the newly presented notation for the impulse response, h(k), it can be shown in the time domain as:

$$y(n) = \sum_{k=0}^{N-1} x(n-k)h(k)$$

(Eq. 7) Where y(n) is the output, x(n) is the input and h(k) is the impulse response. (Lyons, 2011)

A graphic illustration is presented in Lyon's book (2011) of the aforementioned moving average filter (figure 6), where the black dots on the left indicate discrete measurements of the impulse response, the white dots are the input signal and the black dots on the right are the filter's output. While the filters kernel remains stationary, the input signal is passed over the coefficients making a Multiply Accumulate at each progression, resulting in a sequential output.



(Fig. 7) Graphic illustration of convolution for the moving average filter. (Lyons, 2011)

The Convolution theorem states that when x(n) is convolved with h(n) in the time domain, the product of X(f) and H(f) is shown in the frequency domain and vice versa. This can be shown in the expression.

$$h(k) * x(k) \underset{\text{IDFT}}{\overset{\text{DFT}}{\rightleftarrows}} H(m) \cdot X(m).$$

(Eq. 8) Where h(k) and x(k) is the impulse response and input signal in the time domain and H(m) and X(m) is the impulse response and the input signal in the frequency domain. * denotes convolution.
(Lyons, 2011)

DFT

The DFT (Discrete Fourier Transform) is the process by which the frequency and harmonic content of a time domain signal can be determined (Lyons, 2010). The mathematical process of the DFT is derived from the continuous Fourier transform, which was hailed by Lord Kelvin as 'One of the most beautiful results of modern analysis' (Bracewell, 2000). The continuous Fourier transform is shown as:

$$\mathrm{X}(f) = \int_{-\infty}^{+\infty} x(t) e^{-i\omega t}\, dt$$

(Eq. 9) Where x(t) is a continuous time domain signal and X(f) is a continuous frequency domain signal.
(Lyons, 2010)

Where integration is used in continuous time, a summation is used in discrete time. The DFT is therefore:

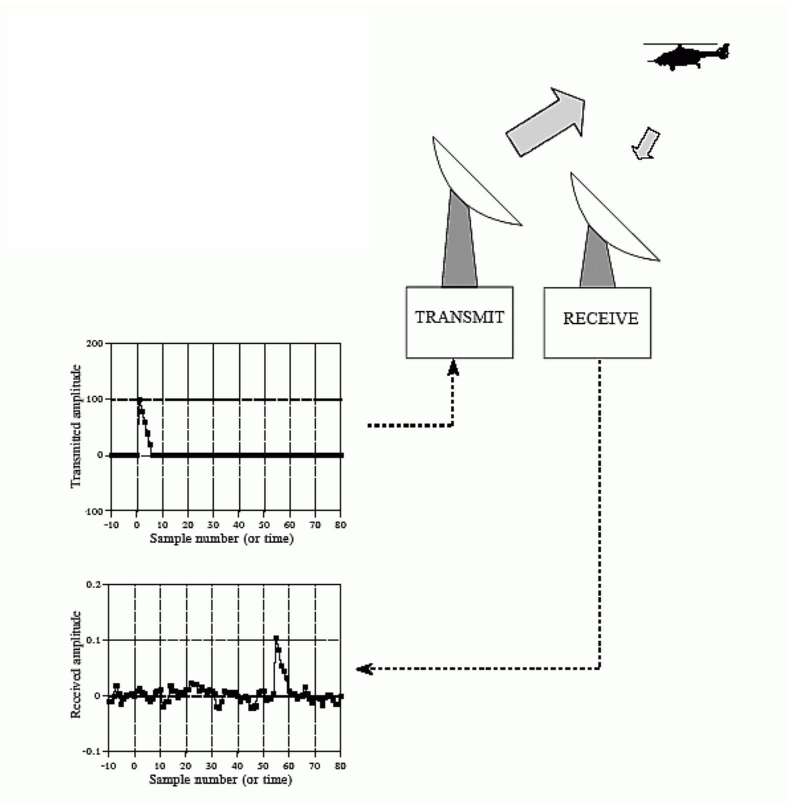
$$X(m) = \sum_{n=0}^{N-1} x(n) e^{-i\omega nm/N}$$

(Eq. 10) Where x(n) is a discrete time domain signal and X(m) is a discrete complex frequency domain signal.
(Lyons, 2010)

In depth study of the DFT is beyond the scope of this project however key concepts are discussed which relate both to this and the project.

Correlation

Like convolution, correlation is a mathematical procedure on two input signals to produce a third signal, however, in practice they represent very different DSP procedures. The output signal is described as the ‘cross-correlation’ of the two input signals, if a signal is correlated with its self the resulting signal is the ‘auto-correlation’. S, W Smith describes correlation in his book ‘The Scientist and Engineer’s Guide to Digital Signal Processing’ (2011), giving an example of its common uses in radar systems. In the graphic of figure 7, a short pulse of radio wave activity is transmitted in the direction of the helicopter, the waves are then reflected back off its surface to the receiving antennae. The received signal is a shifted and scaled version of the original, accompanied by noise and static picked up from other radio technology and electronic interference. The speed of radio waves is known and therefore the distance of the helicopter can be computed by finding the time shift between the two signals. Finding whether the pulse is present in the received signal or at what position in time the pulse is located within the random noise can be a problem; correlation is the answer.



(fig. 8) Graphic illustration of a radar system in which correlation could be used to evaluate the distance of a helicopter.
(Smith, S, W., 2011)

Correlation is presented as:

$$r(k) = \sum_{n=0}^{N} x(n)y(n-k)$$

(Eq. 11) Where r(k) is the correlated signal.

(Smith, S, W., 2011)

It is a measure of how similar two signals are. If two signals are the same and their phase is aligned, when both signals are positive their values will multiply to give a positive and when both signals are negative their values will also multiply to give a positive. These values are then summed, and the output will be positive. In the case when the two signals are 180 degrees out of phase, the positive values will multiply with negative values to give negative and vice versa, therefore the correlation will be negative. It is in the case where the two signals are 90 degrees out of phase where the signal is 'orthogonal', the output will be a DC signal where the two signals have exactly cancelled each other out, it is said to be 'de-correlated' (Practicalcryptography.com, n.d.). The latter case has much relevance to quadrature signals which shall be discussed further. Correlation is at the heart of the DFT, where a time domain signal x(n) is correlated with another signal $e^{-i\omega t}$, and an important part of the Z transform.

Complex Exponentials, Quadrature signals and the complex plane

Quadrature signals (also known as complex exponentials) allow the additional measurement of a sinusoid's phase. They are two dimensional signals, specified in a single complex number (Lyons, 2010).

Real numbers reside wholly on the one-dimensional number line, whereas complex numbers can be plotted on a two-dimensional plane, also known as the complex plane.

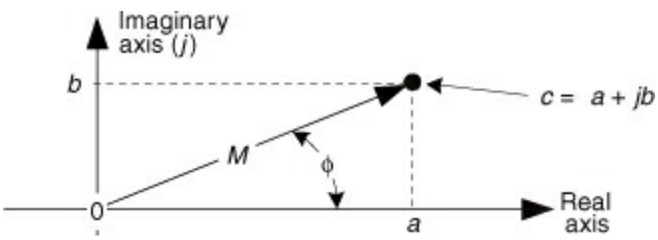
Euler's formula states

$$e^{i\theta} = cos\theta + j\ sin\theta$$

(Eq. 12) e is an irrational number and is the natural logarithm's base; j indicates an imaginary number and θ is the angular frequency.
(Lyons, 2010)

e raised to an imaginary number represents a complex number whose components are functions of sine and cosine and can be plotted as Cartesian coordinates on the complex plane.

In its rectangular form these numbers can be expressed as a + jb. (Lyons, 2010)



(fig. 9) Cartesian coordinates where a is real and b is imaginary.
(Lyons, 2010)

Whilst a and b are functions of cosine and of sine, and functions of time, c is the tip of a phaser on the complex plane spinning CCW about its centre.

The magnitude and phase of the complex exponential can be taken with the following equations.

$$m = \sqrt{a^2 + b^2}$$

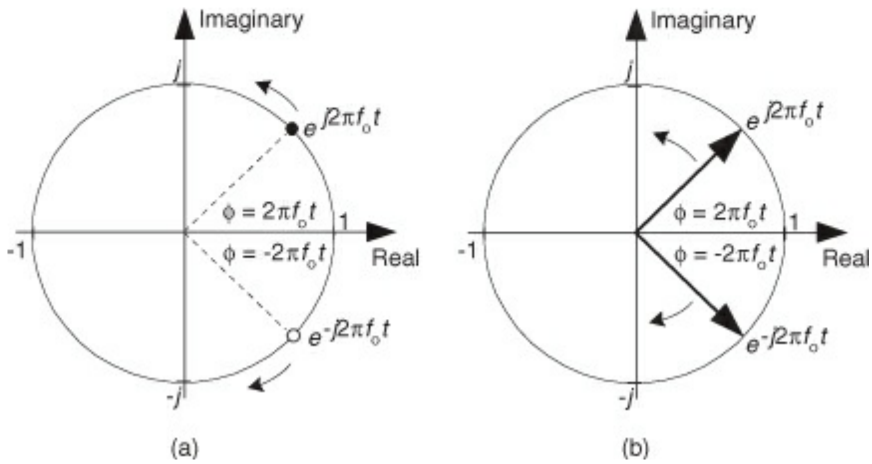
$$\theta = \tan^{-1}({}^{b}/_{a})$$

(Eq. 13) Where m is the magnitude of the complex exponent and θ is its phase in radians.
(Lyons, 2010)

It can therefore be said that the following equation turns this phaser about CCW in motion.

$$e^{-i\theta} = cos\theta - j\ sin\theta$$

(Eq. 14) Where $e^{-i\theta}$ is a complex exponential.
(Lyons, 2010)



(Fig. 10) A pair of conjugate complex exponentials a) expressed as coordinates on the complex plane.
b) expressed as phasors.
(Lyons, 2010)

The terms *in phase* and *quadrature phase* are used to describe a signal, this refers to another part of Euler's identity:

$$\cos(2\pi fot) = \frac{e^{j2\pi fot} + e^{-j2p\pi fot}}{2} = \frac{e^{j2\pi fot}}{2} + \frac{e^{-j2\pi fot}}{2}$$

(Eq. 15) Where $e^{j2\pi fot}$ is a complex exponential rotating CCW and $e^{-j2p\pi fot}$ rotates CW.

This states that every real cosine signal comprises of a conjugate pair of complex exponentials. The correlation of these two exponentials results with the real cosine parts, 'in phase', constructively summing and the imaginary sine parts decorrelated, in 'quadrature phase'. Lyons describes this identity as the 'Rosetta Stone of quadrature signal processing', as it allows us to translate back and forth between real sinusoids and complex exponentials.

Looking back at the calculation for the DFT, the input signal is cross-correlated with a complex exponential. The magnitude of the output from the DFT is how correlated the complex exponent is with our signal and the phase is how much the input signal leads or lags this complex exponential.

Z Transform

The Z transform is a generalization of the DFT and the discrete version of the Laplace transform. it allows the analysis of sinusoidal frequencies not present in the DFT and introduces new concepts of stability and causality. The unilateral z transform is presented as:

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

(Eq.16) where X is the output frequency response x is the input and z is a complex exponential
(Fehlhaber, W. 2019)

In the DFT (Eq. 9); the complex exponential which the input signal is correlated, has a magnitude of one and sits neatly on the unit circle.

$$|e^{j\omega}| = 1$$

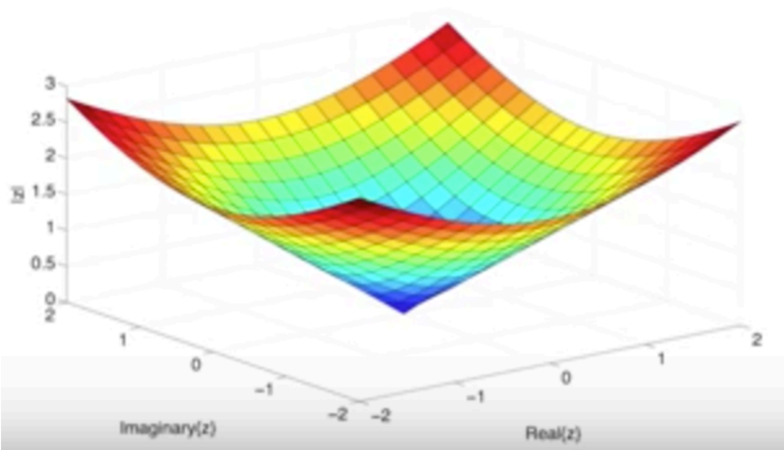
(Eq. 17) Where $|e^{j\omega}|$ is the absolute of the complex exponential from the DFT
(Fehlhaber, W. 2019)

For the stability of a system the DFT must be absolutely summable and therefore finite.

$$X(e^{j\omega}) = \sum_{n=0}^{N} x(n) * 1$$

(Eq. 18)
(Stables, R. 2019)

Hence the following depiction holds true.



(Fig. 11) Z plane of the DFT when z = $e^{j\omega}$ (allsignalprocessing.com, n.d)

Z is also a complex exponential and when shown in its polar form the magnitude of z can be notated with the variable r.

$$z = \mathrm{r}\, e^{j\omega}$$

(Eq. 19) (Fehlhaber, W. 2019)

The Z transform can therefore be rewritten:

$$\mathrm{X(z)} = \sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-j\omega n}$$

(Eq. 20) (Fehlhaber, W. 2019)

ROC

The Region of Convergence (ROC) is a range of values for r where X(z) approaches 0. In the previous example r converges when r < ∞ This is proven by the following equation.

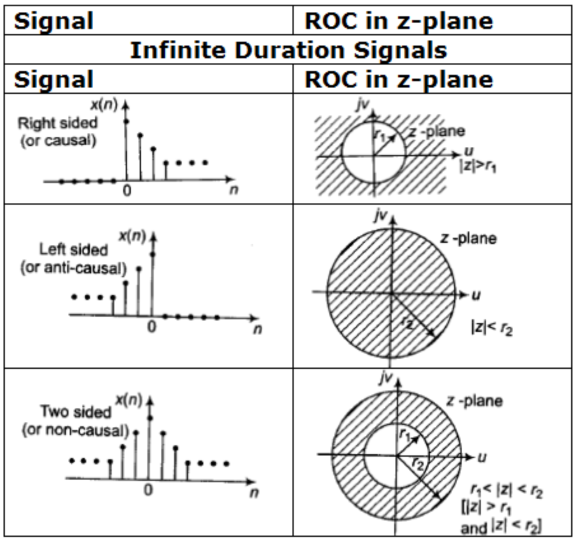
$$\sum_{n=-\infty}^{\infty} r^{-n} = \sum_{n=-\infty}^{\infty} (1/r)^{-n}$$

(Eq.21) where r is the magnitude of z.
(Fehlhaber, W. 2019)

Changing the value of X(z) to a reciprocal ie. X(z) = 1/z-0.5 means that when z = 0.5 the magnitude will be infinite. When X(z) = z-0.5 and z = 0.5 the magnitude will be zero. These are known as poles and zeros. At points when the transfer functions denominator evaluates to 0 there is a 'pole' and when the numerator evaluates to 0 there is a 'zero' (Stables, Ryan., 2018).

FIR filters have a single undetermined pole at 0 + 0j and therefore are inherently stable (Fehlhaber, W. 2019), however, knowing the ROC and the position of the poles of a recursive filter is essential to their design. The ROC must cover the unit circle for the DFT to exist and for the filter to be stable.

Knowing the region of convergence, the causality of a system can also be found. When the ROC includes the unit circle and extends out, the system relies only on past and present samples and is said to be causal and stable. An example of this is real-time filtering implementations. When the ROC is located on the unit circle and within, the system is anti-causal, it relies on present and future samples, e.g. when future samples are stored within a computer's memory. A two-sided non-causal version also exists; this is defined between the furthest and closest pole and it relies on past, present and future samples.



(Fig 12) Shows the ROC for causal, anti causal and non-causal filters.
(allsignalprocessing.com, n.d)

As stated, the z transform relates mostly to the design of IIR filters as it can be used to analyse the magnitude response, the stability and the causality of a filter. The following example shows its use to determine the magnitude response of a FIR filter.

As previously stated, a filter can be characterised by its impulse response (h(k)), a mathematical approach to obtaining the magnitude response of the filter would be to measure the impulse response against various values of z and plot the results.

Take the transfer function

$$y(n) = 0.5x(n) + 0.5\,x(n-1)$$

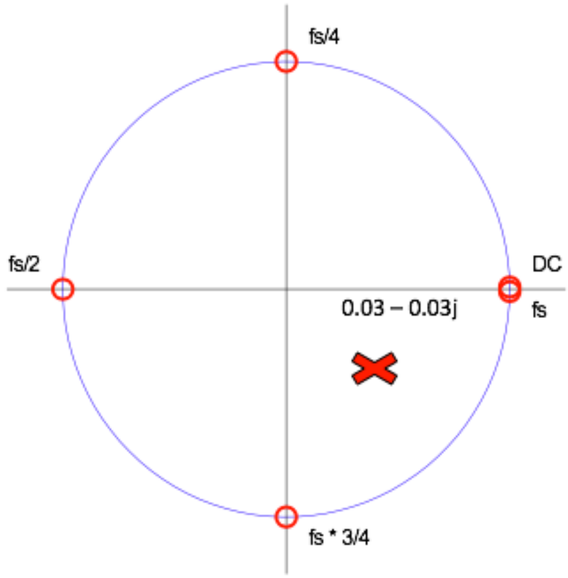
(Eq. 22)

The z transform of the impulse response is therefore:

$$h(z) = 0.5z^{0} + 0.5z^{-1}$$

(Eq.23)

Pick a coordinate on the Z plane to evaluate:



(Fig. 13) The z plane and the chosen coordinate to evaluate z for.
(Fcorthay, n.d)

Converting to the polar form:

$$0.03 + 0.03j \equiv 0.4243\, e^{-\frac{j\pi}{4}}$$

(Eq. 24)

Replacing the values of n gives:

$$0.4243^{0} e^{-\frac{j\pi}{4}0} = 1$$

$$0.4243^{-1} e^{-\frac{j\pi}{4}(1)} \;= 1.66 - 1.66j$$

(Eq. 25)

Replacing the values of z into the transfer function:

$$h(z) = \; 0.5(1) + 0.5(1.66 - 1.66j)$$

$$\therefore 0.03 + 0.03j \; = \; 1.33 - 0.83j$$

(Eq. 26)

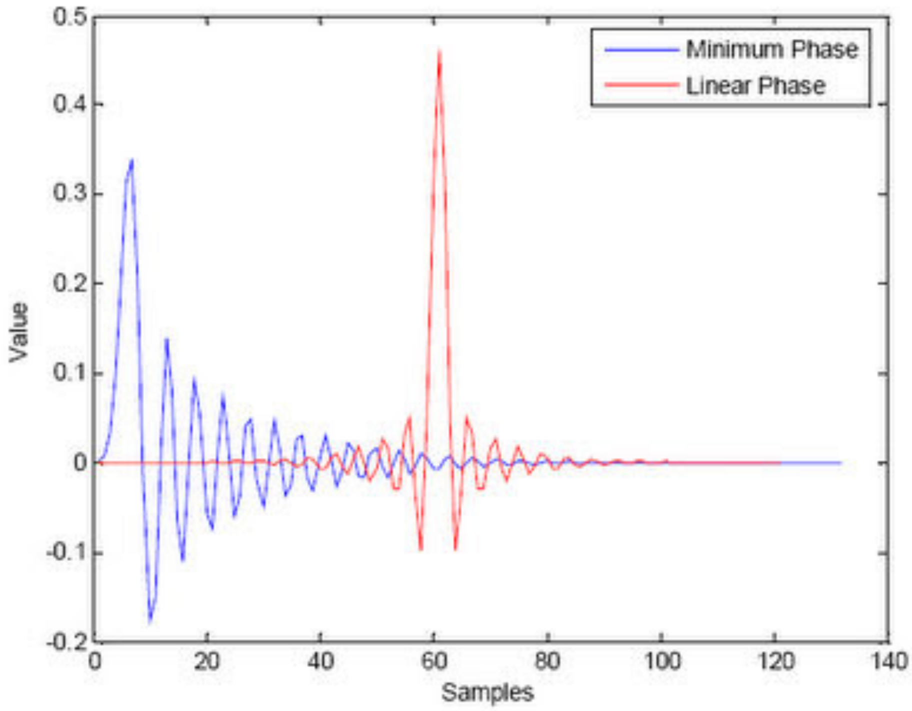
The magnitude and phase at this value for z can be taken (eq. 12):

$$\sqrt{a^2 + b^2} \;=\; \sqrt{1.33^2 + 0.83^2} = 1.5677$$
$$\tan^{-1}(b/a) \;= \tan^{-1}\left(\frac{0.83}{1.33}\right) = 0.5579$$

(Eq. 27)

Evaluating more values of Z and plotting the results against the corresponding angular frequency of z on the unit circle gives the frequency and phase response of the filter (Fehlhaber, W. 2019).

Linear Phase and Minimum Phase filters



(Fig. 14) The Impulse response of a minimum phase and linear phase filter.
(ARDUINO FOR HI-FI, 2009)

Linear Phase Filters

A linear phase filter has a symmetrical impulse response (see fig. 14). It is described as linear phase as its phase is a linear function of its frequency, thus:

$$X(k) = G(k)e^{j\emptyset(k)}$$

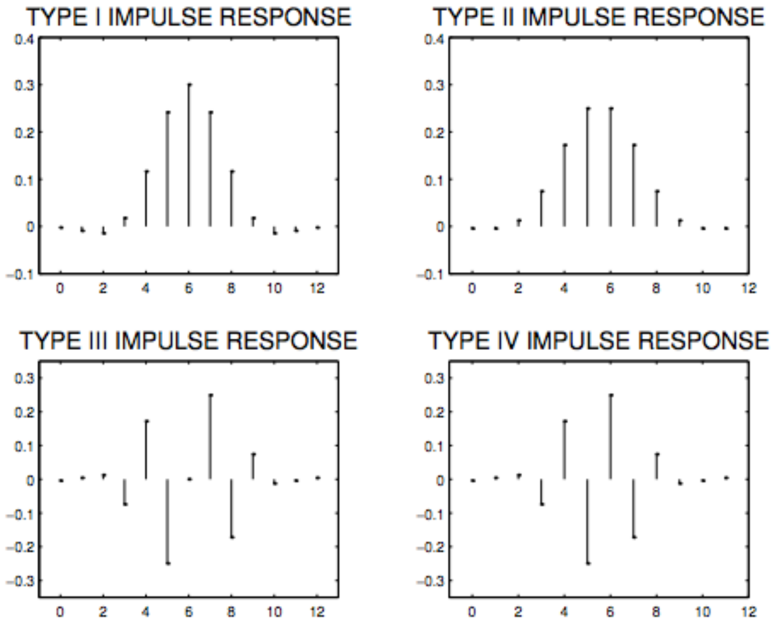
(Eq. 28) Where X(k) is a complex value of frequency G(k) is the gain and $e^{j\emptyset(k)}$ is a complex exponential representing the phase of the signal.
(Smith, J. O., 2007)

A linear phase FIR (Finite Impulse Response) filter has both a phase delay and group delay of:

$N - 1/2$

Where N is the order of the filter and 'Phase Delay' and 'Group Delay' is measured in seconds.

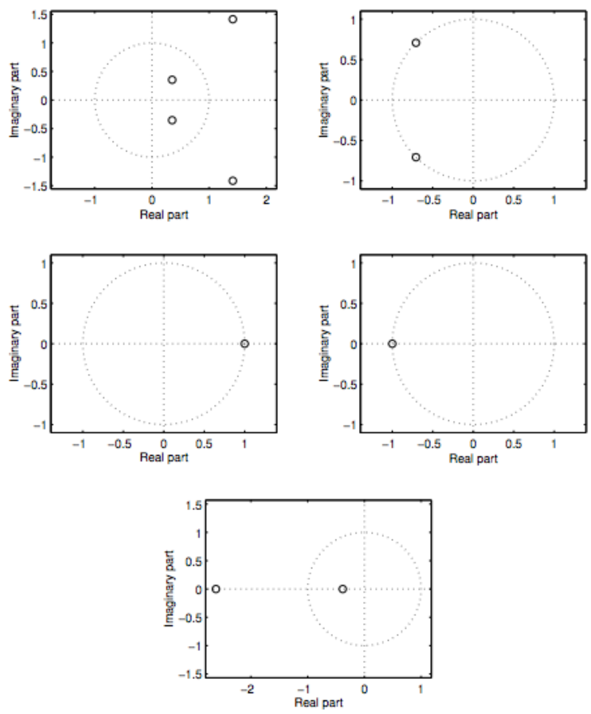
This corresponds to exactly half the total order of the filter at all frequencies. 'Delaying all frequency components by the same amount preserves the wave-shape as much as possible for a given amplitude response.' (Smith, J. O., 2007). It is therefore deemed desirable for its transparent quality (Keith Wood, Crave DSP, 2019). Four types of linear phase FIR filter are shown, types one and three, have magnitudes that are respectively symmetrical and asymmetrical about zero and have an odd number of taps. Similarly types two and four display the same magnitudes but have an even number of taps.



(fig. 15) Four types of linear phase impulse responses
(Selesnick, I., n, d)

An impulse response with an even number of coefficients can be described as zero phase as It is a special case when an impulse response is symmetric about time zero. It cannot rely on past samples therefore it is an example of an anti-causal system. (Smith, J. O., 2007).

When analyzing a FIR linear phase filter in the z domain, four conditions are required of the positioning of the roots (zeros) of z.



(fig.16) Four conditions for the locations of zeros in the z domain, (a) – (e) read left to right.
(Selesnick, I., n, d)

Typically, the roots of a linear-phase FIR filter exist in groups of four; conjugate pairs on the top and bottom of the unit circle and similarly mirrored outside the unit circle (a). Zeros on the unit circle exist simply as conjugate pairs (b). Roots at 1 (0) and −1(π) exist alone (c, d).
Zeros on the real line come in pairs inside and outside the unit circle (e).
(Selesnick, I., n, d)

Pre-ring

'Pre-ringing is a backwards echo. It sounds like a strange sucking sound'
(Keith Wood, Crave DSP, 2019)

One side affect of Linear and maximum phase filtering is an unnatural ringing which occurs prior to the transient. The level of this pre-ringing is determined by the shape of the EQ curve. The only true way to achieve linear phase is to apply EQ twice, once forward in time and once backwards in time to cancel out any phase changes, therefore pre-ringing is perceived as 'a backwards echo' and is only possible with digital audio. The fixed group delay and latency present in linear phase filtering is the result of this forward and backwards filtering. (Keith Wood, Crave DSP, 2019)

The article 'The difference between minimum-phase and linear-phase EQ on transient signals such as snare drum' by David Mellor (2018) compares the responses of both linear phase and minimum phase filters using 'FabFilter's Pro-Q 2'. By applying extreme settings of the respective EQs to a snare sample and providing the output audio for analysis he shows objective analysis and clearly audible results, confirming Woods claims that linear phase filtering can cause a '*strange sucking sound'* on transient material.

A strategy for negating this pre-ring is shown on the web site for audio enthusiasts 'Troll audio' (2019). Here, the effect of pre-ringing in ADC and DAC (Analog to Digital/Digital to Analog Converters) is discussed. It is shown that the frequency of the pre-ring of an anti-alias filter is identical to the cut-off frequency specified by the sinc function that creates it. As the magnitude of the output is a direct result of multiplication of the input signal with a filters impulse response, it therefore relates that for the pre-ring to occur, the input signal must contain frequency content at the filters cut-off frequency. It is concluded that pre-ring in a DAC can be completely obliterated by leaving 'a small margin above the highest frequency in a recording'

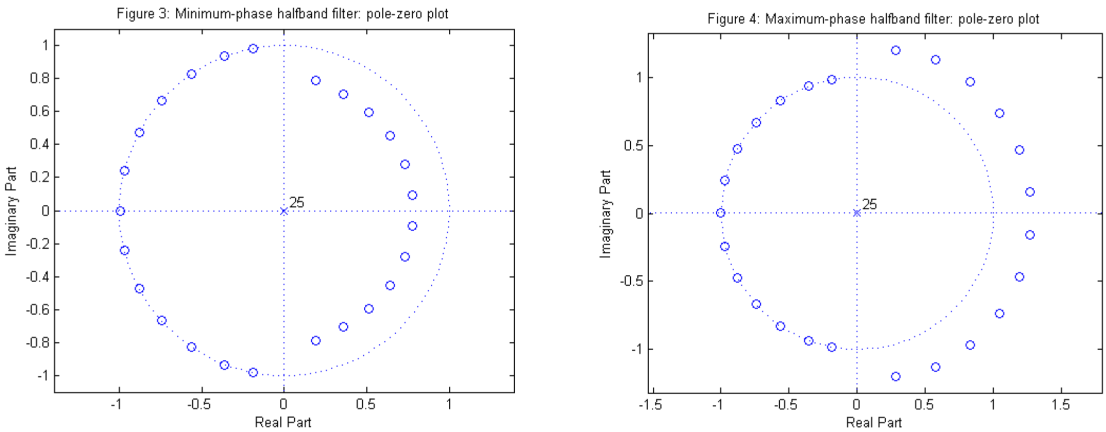
'If the goal is to avoid ringing from DAC filters, it can be achieved entirely in the studio'
(Troll Audio, 2019).

This passes control of the pre-ringing to the producer and arguably makes the decision to include more or less ringing, by controlling the EQ curve, a creative one. This is far from a scientific solution.

Further research leads to a study by Peter Craven, 'Anti-alias Filters and System Transient Response at High Sample Rates' (Craven, P., 2004), his 'Apodizing filter' places a slow roll-off minimum phase low pass filter just before the Nyquist frequency in order to eliminate pre-ring that may have been incurred in ADC. However, this trades the pre-ring with a slight loss of high frequency content.

Minimum and Maximum Phase Filters

All analog components contribute some phase distortion due to band limiting, it is found that high frequency roll off contributes to a lesser extent to phase irregularities than low frequency roll off therefore it is a necessity of engineers to reduce this frequency dependent lag. A minimum phase system is a system with the least possible lag, therefore the frequency response and the phase response of a minimum phase filter are closely tied. It is desirable for analog filters to be minimum phase; when the frequency response of a minimum phase system is flattened by minimum phase equalization the phase response is flattened, closer to linear phase. (Lipshitz, S. P., et al. 1982). A convenience of both minimum phase and maximum phase filters is that their phase response can be considered arbitrary to their design, every minimum phase filter can be considered stable as all the roots are within or on the unit circle, whereas the roots of a maximum phase filter exist outside or on the unit circle. (Smith, J. O., 2007).



(fig. 17) Example of a minimum and maximum phase FIR filter in the z domain.
(Milić, L., 2009)

This can be proven by the following equations:

A first order FIR filters z transform in it's simplest form is shown:

$$h(z) = 1 + h(1)z^{-1}$$

(Eq. 29) Where h(z) is the z transform of the impulse response h(k) and h(0) is made 1 for simplicity. There is a zero located at z = -h(1) as the polynomial factorizes. Therefore when |h(1)| < 1 the filter is minimum phase; when |h(1)|>1 the filter is maximum phase (Smith, J. O., 2007).

An analogy is described by Ron Nicholson (Nicholson, R,. 2004) to describe the relationship the positions of poles and zeros have on phase. He imagines a crank handle attached to every pole and zero that points to the location you are standing at the edge of the unit circle. As you walk around the unit circle these handles follow, and counters count by 2π for every full rotation. When the poles and zeros are within the unit circle the handles will turn a full 2π per rotation of the unit circle, whereas if the roots are outside the unit circle or far away from the unit circle their handles will wiggle back and forth and never complete a full rotation. 'Unless you have a time machine', the poles must remain in the unit circle to keep turning (this explains the causality of a filter). The sum of multiple rotations of these roots within the unit circle equals -2π times the number of poles and +2π times the number of zeros. If there are the same number of poles and zeros within the unit circle the sum will cancel and be near zero this explains the properties of a stable minimum phase filter. If some of the zeros were outside the unit circle the sum would continue to diverge, 'this ever-rising curve is unlikely to be called "minimum"' (Nicholson, R,. 2004).

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Critical Review

The book ‘Introduction to Digital Filters with Audio Applications’ (Smith, J. O., 2011) reflects upon whether linear phase or minimum phase filters are ideal for audio. A listening experiment is provided in Matlab code where unit impulses are pushed through respective versions of lowpass filter with a cut off at 2k Hz. The expected result is a click or compact thud; the minimum phase filter performs closest to this expectation however pre-ringing is clearly heard on the linear phase filter, making a chirp like effect. Although elliptic IIR filters are used in this chapter because of the extended ring time in their impulse response, this inspired further research and informed the report at an early stage.

Using broadband clicks passed through filters with the same order but different cut-off frequencies, previous experimental research by Preiss and Bloom (1983) suggests that the ear is more sensitive to group delay at mid frequencies (4khz) than at high frequencies (15khz). Tests will therefore be limited to mid-range frequencies to get more reliable results and short broadband signals shall be considered for use.

‘On the Perception of Phase Distortion’ (Suzuki, H., et al., 1980) concludes that the effect of phase distortion is perceptually much less noticeable than the objective waveform suggests, this is important to note for the projects own objective tests. The effect of changing the phase response of the test signal is noted as a ‘highly individual’ experience due to wide ranging results and when subjects are presented with popular music in the test not a single person notices a degradation in sound quality. The purpose of this project is to judge the perceptual qualities of minimum phase and linear phase filters; there will be only a small amount of phase shift and the effect may be subtle, so the selection of test material is an important consideration.

‘On the Audibility of Midrange Phase Distortion in Audio Systems’, (Lipshitz, S.P,. et al., 1982) suggests phase distortion is more noticeable when listening through headphones than from speakers therefore headphones shall be used in this project.

‘All About Audio Equalization: Solutions and Frontiers’ (Välimäki, V. Reiss, J. D., 2016) offers a deep understanding of Digital Signal Processing (DSP) techniques for the design of digital filters and Matlab source code is available online to correspond with the paper. This will provide both theoretical support and insight into how to implement filters in Matlab.

Methodology

Two distinct methodologies are proposed, one pertaining to system design, the other to subjective testing. The system design methodology includes objective testing and is depicted in Figure 1. This is a modified version of the waterfall method, that avoids common issues associated with the linearity of that method; there are opportunities to revise phases and make use of new information.

The subjective testing methodology (Fig 2) provides qualitative and quantitative analysis of the filters. An ABX test will be used to select appropriate test material and a MUSHRA styled test will be conducted for subjective analysis. The WAET testing software shall be used.

Introduction to the methodology

The project requires a successful implementation of linear phase, minimum phase and maximum phase filters, as individual low pass filters and as banks of band pass filters for 10 band, octave graphic EQs. Appropriate input signals need to be selected for testing the filters. Objective tests are required to plot the magnitude and phase response of their output and listening tests are needed to obtain psycho-acoustic data.

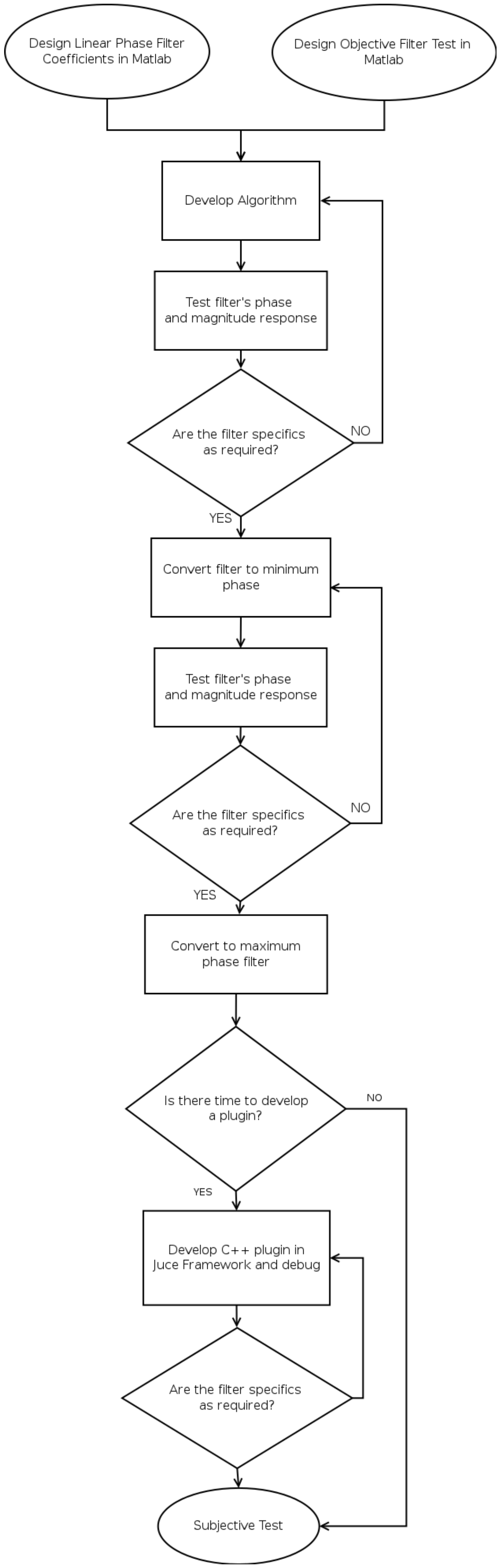


Figure 18 System Design Methodology

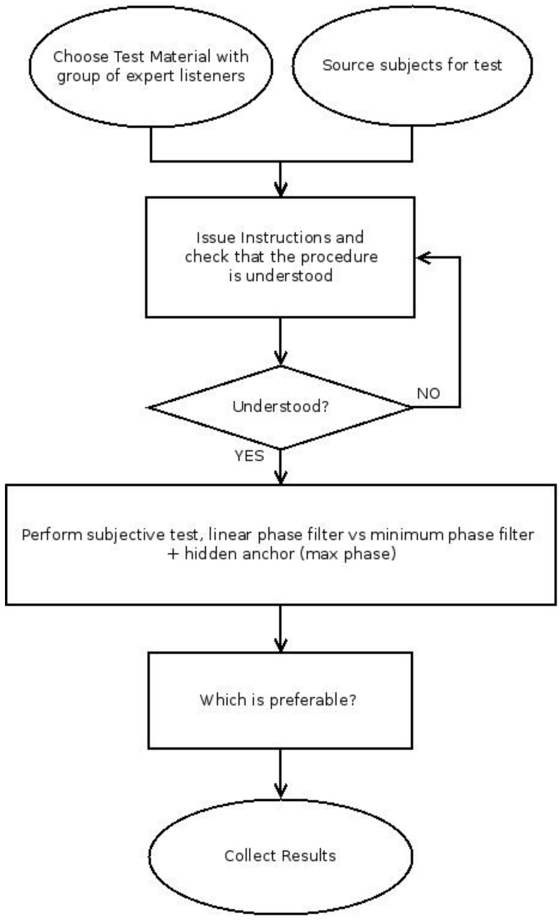


Figure 19 Subjective Test Methodology

Secondary research

Literature Search Methodology

Initial research has revealed a plethora of information on digital filter design. Smith's 'Introduction to Digital Filters with Audio Applications' is well regarded and offers a deep understanding of DSP theory (Smith, J. O., 2007). Reiss and McPherson's 'Audio Effects: Theory, Implementation and Application' is also an important work and shall prove useful in this project (Reiss, J. D., McPherson, A. P., 2014). The BCU library has a wealth of reliable digital and physical information in the form of books, articles, journals and papers. The search engine 'Copac' enables searching of over 100 UK and Irish academic libraries; material may then be acquired through BCU's Inter-library loan service. Websites such as Google Scholar, Microsoft Academic and ResearchGate offer further online resources. Reading the abstracts of papers can be a quick way to determine if they are suitable for study, however, the reliability of articles should be considered; has the article been peer-reviewed by experts in the field? Using the CRAAP test (Currency, Relevance, Authority, Accuracy, Purpose), web material can be given a score of 1-10 in each of these categories; this helps to identify key texts. Further, citation programs such as Mendeley, Endnote and Zotero offer ways of collating these resources.

Keywords: 'Digital filter design', 'Group delay', 'Impulse response', 'Minimum phase', 'Linear phase', 'Phase distortion', 'Pre-ring', 'Convolution'.

The project objectives have been reviewed and methods have been identified from secondary research (see critical review). Along with the work of Reiss and McPherson (2014), system design methods shall be taken from 'All About Audio Equalization: Solutions and Frontiers' (Välimäki, V., Reiss, J. D, 2016) and further theoretical advice is taken from 'The Scientist and Engineer's Guide to Digital Signal Processing' (Smith, S. W., 2011). The MUSHRA subjective test (Mason, A., 2002) acts as the main source for the testing methodology. Subjective testing methods are also identified from the work of Preiss and Bloom in the papers 'Perceptual Identification and Discrimination of Phase Distortions' (1983) and 'Perception of Phase Distortion in Anti-Alias Filters' (1984). The papers 'On the Perception of Phase Distortion' (Suzuki, H. et al., 1980) and 'On the Audibility of Midrange Phase Distortion in Audio Systems' (Lipshitz, S. P, et al., 1982,) offer further methodological advice.

Primary Research

Test Material

MUSHRA advises that test material is chosen by a group of expert listeners in order to find the most revealing material for the filters. BCU sound engineering students and musicians between the ages 19 - 40 shall be asked to perform this task prior to the main testing procedure, as hearing is known to decline with age. (Cruickshanks, K.J., et al. 1998). The trial of test material through minimum phase and linear phase filters shall be conducted as an ABX test.

The test subjects are played two signals, A and B. In the case of this project, A is a signal passed through a linear phase filter and B is the same signal passed through a minimum phase filter. A final signal, X, is played which is a random selection of either A or B, the subject is asked to identify which of these two signals X is. A range of different applied audio shall be tested multiple times and the material that is noted with the highest approval rating shall be considered the most appropriate for use in the main subjective test.

Broadband clicks are used in a test by Preiss and Bloom (1984) and it is concluded that phase distortion is most noticeable in the mid-range frequencies, MUSHRA advises that synthesized sounds should not be used in subjective testing therefore a suitable acoustic source might be a drum kit, various drums and truncated cymbal sounds shall be tested.

The paper 'On the Perception of Phase Distortion' (ibid) showed that phase distortion is much less noticeable in listening tests than the plotted waveform suggests, and that pop music hindered the test subject's ability to hear the effect of phase distortion. This infers that the comparison of filters may be perceived as being subtle and that single hits may be more revealing of a filter's phase response than predictable rhythmic patterns. Single hits are proposed for the afore mentioned trials.

'On the Audibility of Midrange Phase Distortion in Audio Systems' (ibid), showed that phase distortion is most noticeable with the use of headphones. The tests shall take place in a studio environment using the same pair of high-quality closed back headphones. This will ensure reduced background noise from external sources and strict lab conditions.

Primary Research Testing Procedure

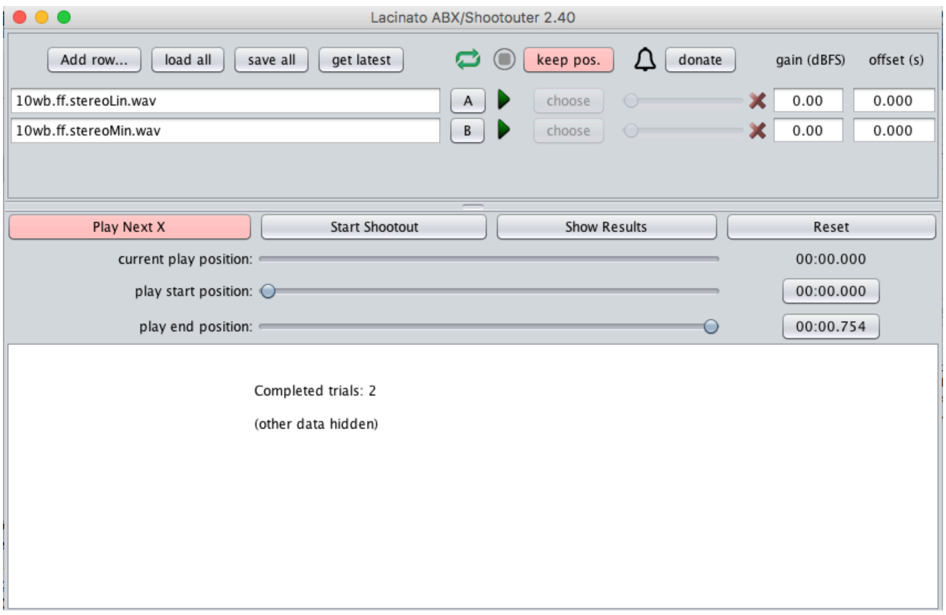
Test Equipment

Equipment	Brand	Model
Laptop	Macbook Pro	2.4 GHz Intel Core i5
Software	Lacinato	n/a
Audio Interface	RME	Fireface 800
Headphones	Beyerdynamic	T1

Table 1. Test equipment used.

Methodology

A group of four listeners where selected to find material which most highlighted the filters architecture, all were between the ages of 26 – 34 and all from musical backgrounds. Based on evidence from secondary research, a pool of 42 samples were chosen for testing, these included studio and anechoic recordings and synthesized sounds, both sustained and truncated sounds were used. The samples where passed through both linear phase and minimum phase lowpass filters with identical magnitude responses and a cut off frequency of 1kHz, it was thought that a midrange cutoff frequency would allow all samples to be audible. The ABX testing software Lacinato was used (see fig. 13). This allowed both states of the signal, linear phase (A) and minimum phase (B), to be played back by the user then a randomly selected state (X) was played. The test subject was to choose which of A or B X was. The samples could be looped and a 'keep position' function made continuous playback possible when changing between samples. There was a minimum of 15 trials for each pair of filtered samples to ensure enough data for the results to be accurate and subjects were given regular breaks between trials to minimize ear fatigue. Listening levels were made controllable for the user.



(fig. 20) Preliminary testing software Lacinato.

Results

Accuracy and Confidence ratings were procured from the primary research testing procedure (see appendix), confidence ratings of over 95% were deemed successful candidates for the main subjective test. Ten samples fulfill this criteria (see table 2).

Sound File.	Number Correct. Accuracy (%)	Confidence Value (%)
square100.wav	15 (100.0%)	0.999969482421875 (99.9969482421875%)
saw100.wav	15 (100.0%)	0.999969482421875 (99.9969482421875%)
Kick_1-018.wav	12 (80.0%)	0.982421875 (98.2421875%)
Kick_1-020.wav	13 (86.666664%)	0.996307373046875 (99.6307373046875%)
Snare_3-014.wav	16 (72.72727%)	0.9737606048583984 (97.37606048583984%)
castanet1.ff.stereo.wav	13 (86.666664%)	0.996307373046875 (99.6307373046875%)
Trumpet.novib.ff.Db4.stereo.wav	15 (100.0%)	0.999969482421875 (99.9969482421875%)
AltoSax.NoVib.ff.Db4.stereo.wav	14 (87.5%)	0.9979095458984375 (99.79095458984375%)
BassTrombone.ff.Db2.stereo.wav	14 (93.333336%)	0.99951171875 (99.951171875%)
Viola.arco.ff.sulC.D4.stereo	13 (72.22222%)	0.951873779296875 (95.1873779296875%)
trumpet anechoic.wav	n/a	n/a

Table 2. Samples that obtained a 95% confidence rating, samples highlighted in red were emitted from the main subjective test.

Discussion

As suggested in previous studies, (Lipshitz, S, P. et al 1982) generated square and sawtooth waves resulted in 100% correct comparison. Although these were emitted from the main testing procedure, this raised questions regarding the appropriate use of these filters for equalization in electronic music production. Further sets of samples were taken by generating preset patches from Native instrument's soft synth 'Massive' using predominantly square and sawtooth oscillators (see Appendix). Results were found to be inconclusive (none of the samples met the afore mentioned criteria), those analysed that came close to the threshold for success were monotonous tones with little time varying parameters. It was thought that sinusoidal LFOs (Low Frequency Oscillation) and slow modulations of the signal were cause for distraction.

The trumpet and the viola sample were found to suggest conclusive differentiation between filter types. A brass instruments waveform is close to a square wave, as a bowed instruments waveform is close to sawtooth wave however complications in their actualized wave shape lie in the time variant nature of the acoustic waveform. It is suggested that it is this time variant property that renders a listener incapable of differentiating between different frequency dependent phase relationships. For this reason a musical passage for trumpet is included in the main subjective testing procedure. The viola sample was however omitted from the main subjective test as it's accuracy levels were less definite, the test was also too long so exclusion of the viola sample also helped to reduced ear fatigue, string samples could be an area for further research.

Methods

System Design

The coefficients for a specified order of linear phase low pass filter are conceived with the use of truncated sinc functions in Matlab. Windowing techniques are then required to smooth the impulse response. Various types of window function are available, (Hann, Hamming, Blackman, Kaiser etc.) offering different resolutions and spectral leakage of the DFT (Smith, S. W., 2011). The Kaiser window is proposed for the minimum phase, linear phase and maximum phase filters. This offers a narrow transition band and low stop band attenuation. High pass filters can be implemented by 'spectral inversion' or 'spectral reversal' of the low pass filter (Smith, J. O., 2007). Both techniques were tried, and spectral reversal was shown to provide the narrowest transition band, so was thought to be the most appropriate for the implementation of a graphic EQ. It was first thought that band pass filters for a graphic EQ could be established by using a technique known as 'complex mixing' (Smith, J. O., 2007), however, this was seen to be flawed and is not an established industry technique. Research shows that band pass filters can be derived through the convolution of low and high pass filters (Välimäki, V., Reiss, J. D., 2016). Objective tests shall then be performed in Matlab; once the filters are shown to be linear phase the filters can be made minimum phase using an open source matlab function (1997-1998, McCaslin, S).

System Design Realisation

Although the system design was followed and found to be successful in designing the intended linear phase octave filter bank, the narrow bandwidth of low octave filters resulted in filter orders of several thousand (Välimäki, V., Reiss, J. D., 2016), computationally this was very expensive so alternative techniques were considered. Common ways to counter the computational expense involve fast convolution and multi-rate designs. A multi-rate design was adopted; this used decimation filters which down-sampled the individual filter kernels to a factor no less than their Nyquist frequency and summed their outputs back at the original input signal's sample rate. This greatly sped up the calculation for the filters however, the order of the filters remained the same.

Much deliberation was made to reduce the order of the filters to below the permissible threshold for conversion, and in some instances, this meant having a higher stopband attenuation and a larger transition width than expected still some spectral error was shown, but these were thought to be permissible.

An updated version of the open source code was also required to solve spectral errors found when calculating the minimum phase coefficients for the high order filters. The updated code (Zhu, X.) required Matlab 'System Objects', therefore, functions from the 'Signal Processing Toolbox' were used to generate coefficients, automating the impulse responses design procedure (see table 3). Once the minimum phase filter's magnitude response was shown to be an exact replica of the linear phase version and their phase responses were shown to correspond respectively, a maximum phase filter was derived through time reversal of the minimum phase kernel (Smith, J. O., 2007).

Other unresolved issues during the design process were undesired peaks and notch filtering at the crossover regions between bands. The filter bandwidths were adjusted in an attempt to flatten the frequency response, however, the limitations imposed by the need for a reduced filter order meant narrower bandwidths in the low octaves and steeper transition widths throughout could not be achieved. Therefore, the summation of magnitudes at these points inferred spectral errors.

A solution to reduce the order of the filters can be found by using multistage techniques. This involves cascading the filtering procedure to multiple lower order stages, resulting in fewer Multiply accumulates. This is a topic for further research, however, as the main concern of this paper lies in the phase response of the filters rather than the magnitude response.

Matlab Function	Arguments	Purpose	Use
fdesign.decimator()	M,‘bandpass’,'Fst1,Fp1,Fp2,Fst2,Ast1,Ap,Ast2' M = the factor of decimation ‘bandpass’ is the type of filter. 'Fst1,Fp1,Fp2,Fst2,Ast1,Ap,Ast2' specifies the stop band and pass band frequencies and the amount of attenuation in the stop band and pass band.	Returns the specifics for a decimation filter	Specification object passed as argument for design() to create impulse response.
design()	D,METHOD,'Systemobject',true D = specifications object. METHOD = design method used, a Kaiser Windowing method was used. (‘kaiserwin’).	Applies the design method to a filter specification object	Impulse response derived from the filter specification object.
minphase_1()	Filter Specification Object	Convert the linear phase filter to minimum phase.	‘’
flip()	Filter coefficients	Apply time reversal to the impulse response.	Convert minimum phase filter to maximum phase.
filter()	b, a, x b = numerators of the impulse response. a = denominator of the impulse response. x = input signal.	Apply filtering to input signal	‘’
audioread();	filename	Read audio samples for filtering	‘’
fft()	X, n X = input time domain signal n = the window size for the fft, default is the length of x.	Covert a real time domain signal to a complex frequency domain signal.	To analyse the output of the filters.
abs()	X = input complex frequency signal.	To find the magnitude of the input complex frequency	‘’
angle()	X = input complex frequency signal.	To find the phase of the input complex frequency	‘’

Table 3. Prominent Matlab functions used to generate filters.

Subjective Test

Methods devised by the MUSHRA audio subjective test proposed by the BBC are to be used. (Mason, A., 2002)

'MUS' in MUSHRA stands for multi stimulus. Multiple versions of the test signal are available to the subjects of the experiment, processed and unprocessed.

- 'HRA' in MUSHRA stands for hidden reference and anchors. The subjects grade the signals under test along with the original unprocessed version of the signal and a low passed version of the original signal. This ensures the full range of the grading scale is used. As the project's test involves the comparison of filter types, considerations will need to be made on how to adapt the testing procedure.
- The grading scale used in the MUSHRA test is from 100 to 0 with descriptive terms over intervals of 20. 100 – 80 is described as 'excellent', 80 – 60 as 'good', 60 – 40 as 'fair', 40 – 20 as 'poor' and 20 – 0 as 'bad'. Subjects can specify an exact value, so the scale is continuous from 100 – 0. This will enable close scrutiny of the filters which may only subtly differ.
- Purely artificial signals are not recommended, however an artificial signal such as the broadband clicks used by Preiss and Bloom might inspire similar acoustic sounds, such as snare drums.
- It is suggested that a group of expert listeners are given the task of selecting the test material, a range of material should be processed, and a set should be chosen that is most revealing of the systems that are under test.
- It is stressed that different characteristics from listening environments could affect the quality of results.
- MUSHRA advises that the subjects of the experiment should be expert listeners. This implies that they are experienced in listening for qualities in audio.
- Instructions given to the test subjects should be simple and it should be noted that the wording can significantly affect the performance of the individual.
- Recording of the test session can be helpful in solving issues found in the results for example tests could be filmed.
- The MUSHRA method for analysing results is to calculate the mean grades and 95% confidence interval of these mean values.

A consent form was prepared and given to test subjects which informed them of the testing procedure and any risks involved.

Methods from the MUSHRA subjective listening test were used in the methodology, however, some aspects were found to be too reliant on the unprocessed signal and therefore an adapted version of MUSHRA was proposed using the WAET (Web Audio Evaluation Tool) interface. Tests were held both locally, with high quality headphones (Beyerdynamic T1), and online, in which case the type and quality of the headphones were at the participants regard. MUSHRA advises that simply worded instructions are given to the subjects prior to testing to make sure there is no confusion in carrying out their task. A suitable statement was prepared, and a short survey was carried out prior to the testing. Subjects were asked their name and age. 20 subjects between the ages of 19 and 40 were sourced; 10 were students studying Sound Engineering and Production at BCU and 10 were participants with a musical background. They were also asked to rate their experience in audio analysis on a scale of 1 - 10; any subject rating below 5 was excluded from the test.

A linear phase filtered signal was used as a reference track. This was compared with minimum phase and maximum phase filtered versions of the same signal and a second version of the linear phase filtered signal was included for comparison as a hidden reference. The subjects were asked which of the signals most closely resembled the reference track. This was graded on a scale, 0 being 'most different' to 100 being 'most similar'. Although the maximum phase filter was initially intended as the hidden anchor, the phase distortion was found to be subtler than expected, so it was used as a further reference.

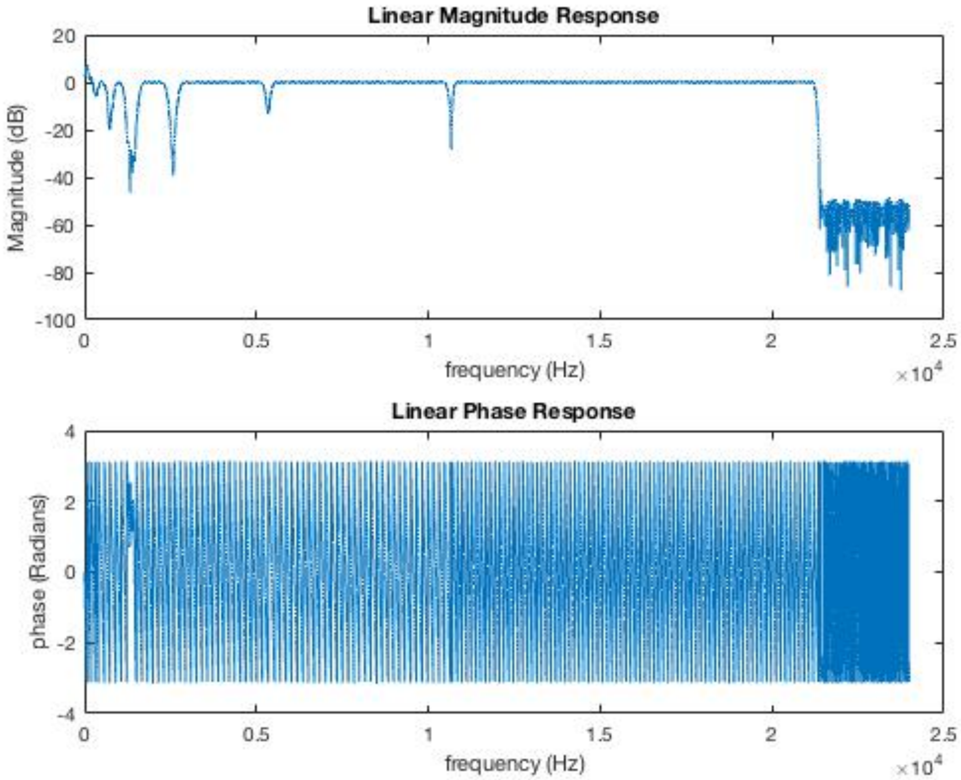
Four EQ settings were tested, a setting with mainly low frequency content, a setting with mainly high frequency content, a mid heavy EQ and a flat response. Seven samples were brought forward for analysis from the primary research testing procedure; when passed through the four EQ settings this made a total of 28 comparisons.

Both ABX and APE (Audio Perceptual Evaluation) tests were considered for the main subjective test however the afore mentioned MUSHRA was found the most suitable. Multiple filters could be compared at once allowing for a quick turn-around; lessening ear fatigue for the test subjects. Another benefit of MUSHRA was that multiple sliders allowed the subjects to grade samples closely if there was no discernible difference, widening the range of analytical grading.

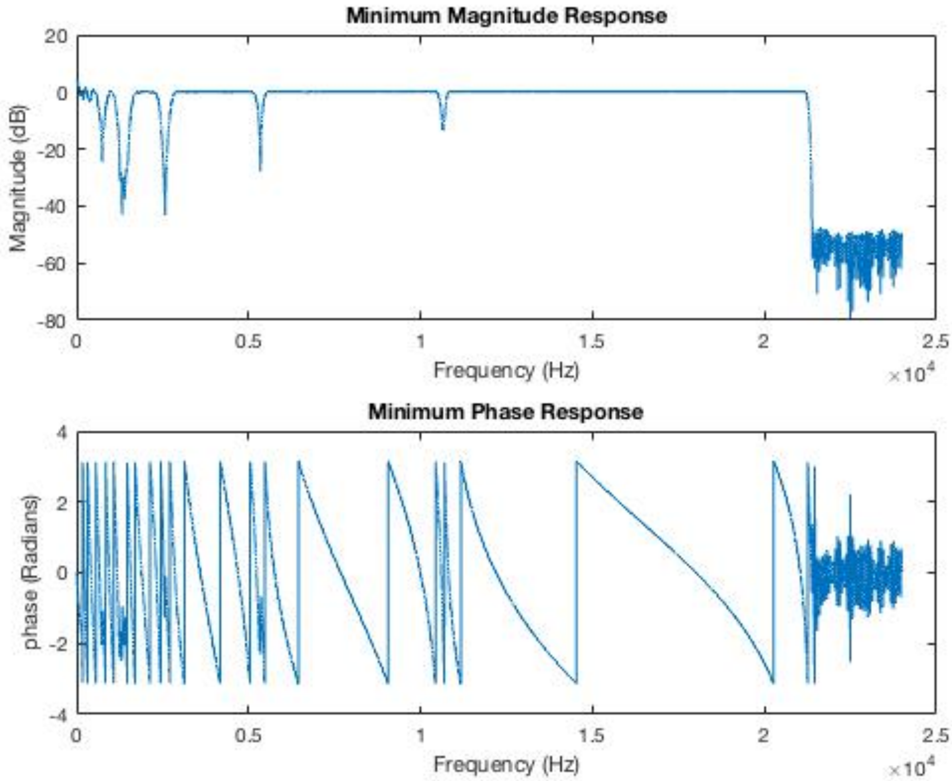
Results and Findings

Objective Analysis

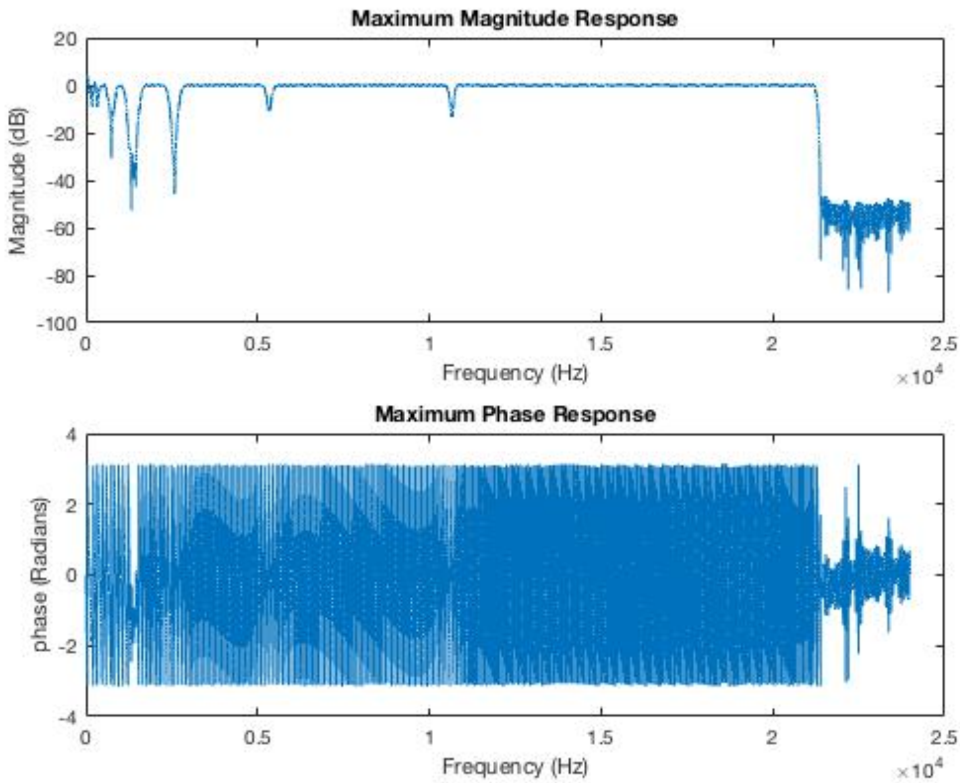
A unit impulse was passed through all three designs of Graphic EQ, the FFT was taken and both the magnitude and phase responses were plotted.



(fig. 21) Magnitude response and phase response of the Linear phase Graphic EQ. Notice the fluctuations from π to -π radians are constant throughout the passband's spectrum.



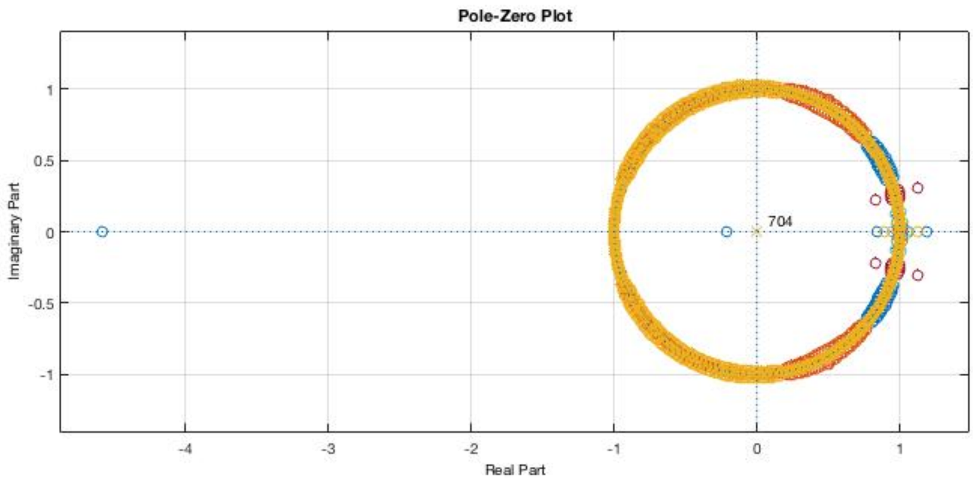
(fig. 22) Magnitude and phase response of the minimum phase graphic EQ. The transition from π to -π is noticeably less frequent than the linear phase response.



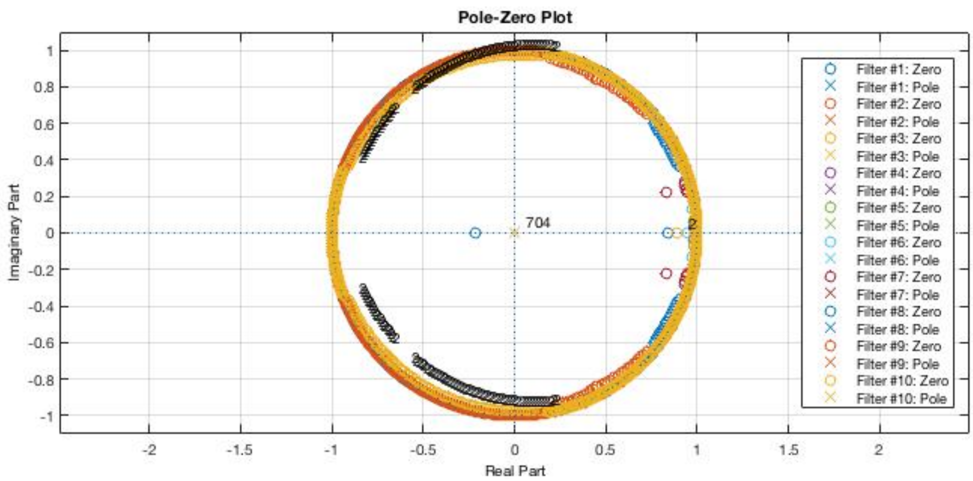
(fig. 23) The magnitude and phase response of the maximum phase graphic EQ. Transitions from π to- π are even more frequent than linear phase.

The magnitude responses are near identical (slight alterations are thought to be due to spectral leakage and should be seen as negligible) whereas the phase responses are shown to be significantly different.

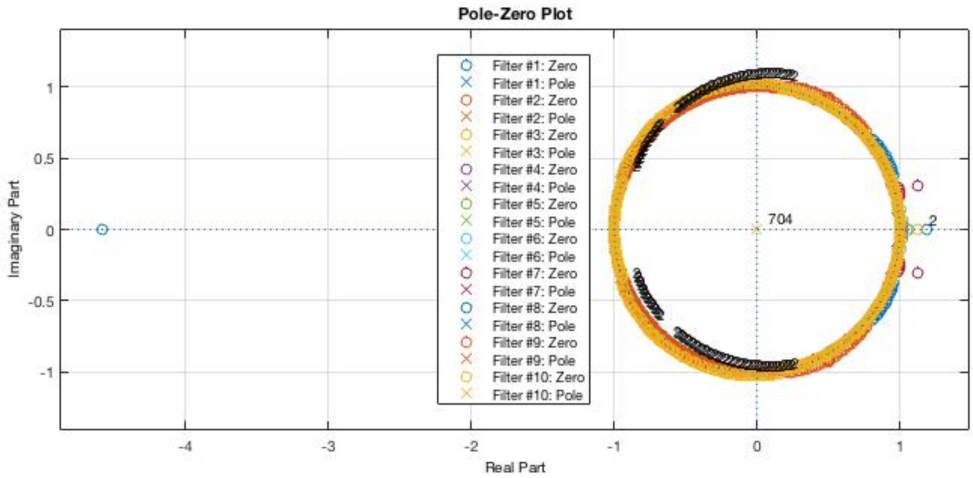
The unit circle is examined and the positions of the roots confirm the phase responses of the filters.



(fig. 24) The unit circle of the linear phase graphic EQ. Notice the zeroes meet the afore mentioned criteria for linear phase and the central poles meet the criteria for a FIR filter.

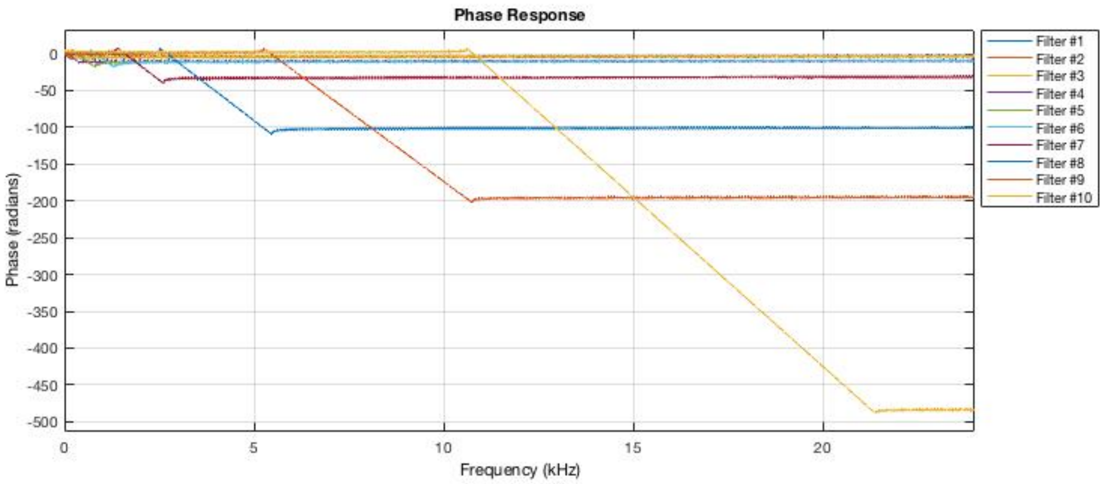


(fig. 25) The unit circle of the minimum phase graphic EQ. Notice the zeroes are all in or on the unit circle.

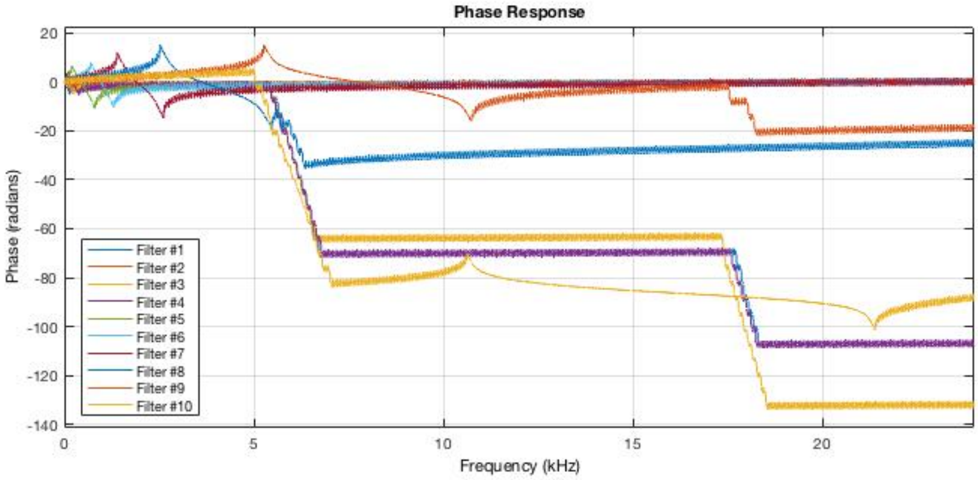


(fig. 26) The unit circle of the maximum phase graphic EQ. Notice the zeros are all outside or on the unit circle.

Further objective analysis was done using Matlab's filter visualization tool (fvtool). A clearer description of the phase is plotted showing the full amount radians past, not limited to 2π radians.

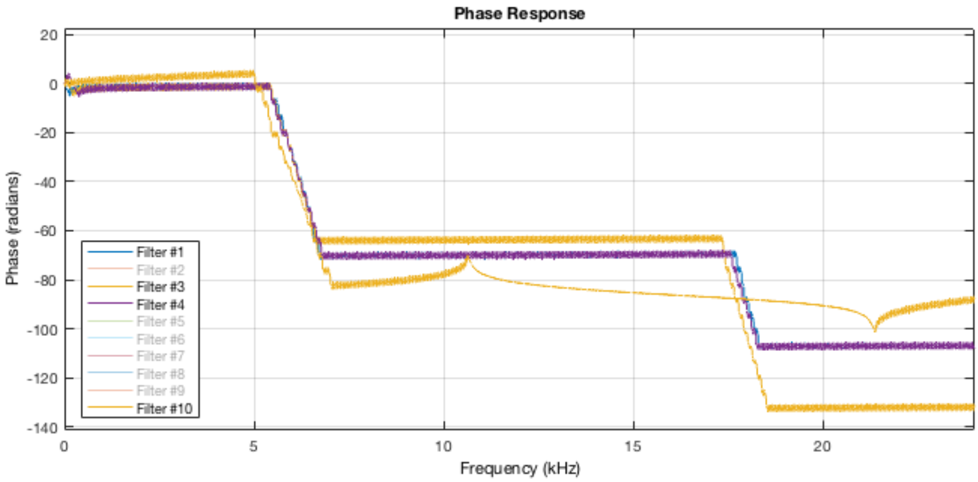


(fig. 27) The phase response of the linear phase graphic EQ as viewed in fvtool. Notice the linear function of phase and frequency in the pass band of each filter with a phase delay of more than 450 radians for filter 10.

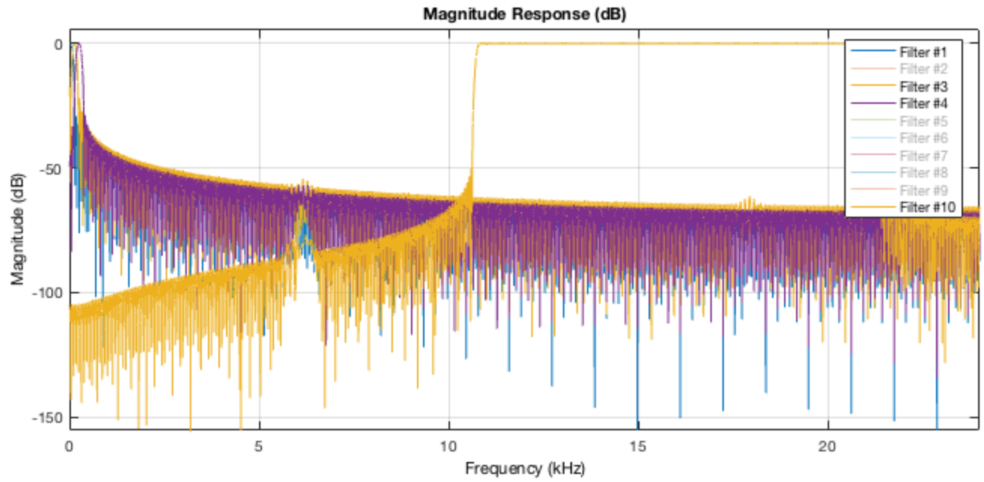


(fig. 28) The phase response of the minimum phase graphic EQ as viewed in fvtool.

Notice the constrained relationship between phase and frequency in the pass bands of each filter compared with the linear phase filter. The shallow curve suggests high frequency content is delayed more than low frequency content and the symmetry about this curve shows that central frequencies of the bands are delayed by roughly the same amount. As these are the frequencies the users of the EQ are interested in, it follows that the more bands made available to the user the less frequency dependent delay will occur causing less transient smearing.

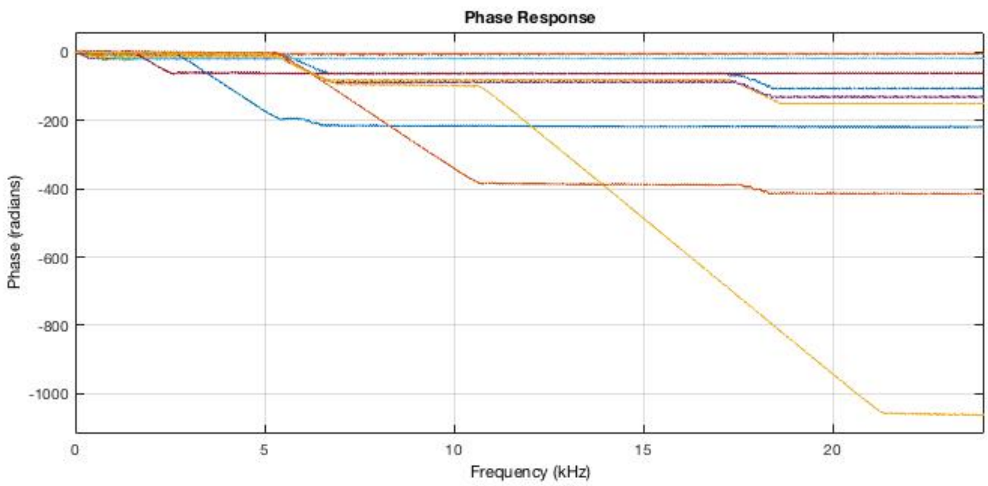


(fig. 29) A closer look at filters 1, 3, 4 and 10 show unexpected phase shifts at 6 and 18kHz. This is thought to have been caused by factorizing errors when converting the high order filter to minimum phase. The phase shift has no effect on the passbands of filters 1,3 and 4, however, filter 10s passband is delayed by around 80 radians.



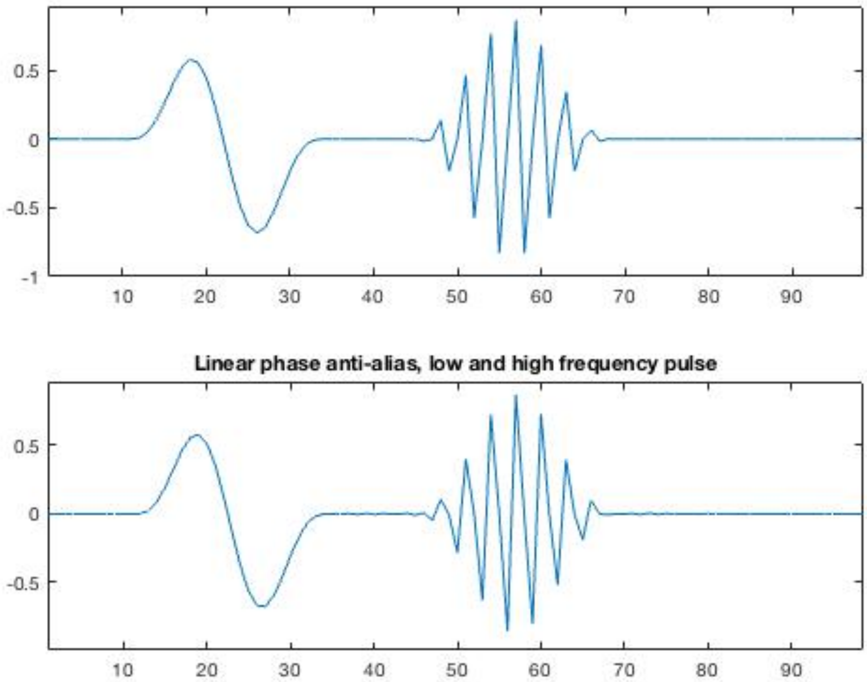
(fig. 30) Minor spectral flux is seen at the same points in the magnitude response.

Care was taken to lower the order of the filters to reduce the extent of this issue in the magnitude response, whilst in the stopband these small notches in the spectrum were not seen as a significant issue. Further analysis of the phase response has revealed complications for filter 10; although this band is not truly minimum phase, it does show the same constrained behavior in its passband and still has the least lag in comparison to the other filter designs. This has been considered when analyzing the subjective results.

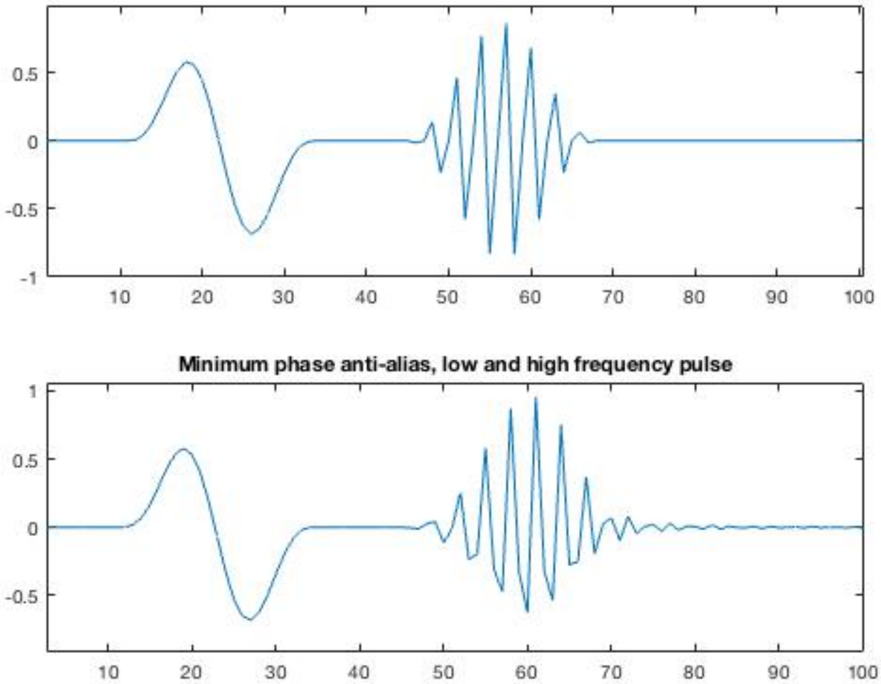


(fig. 31) The phase response of the maximum phase graphic EQ as viewed in fvtool. Notice the similarly staggered response for filters 1, 3, 4, and 10 and over 1000 radians of phase delay in the passband of filter 10.

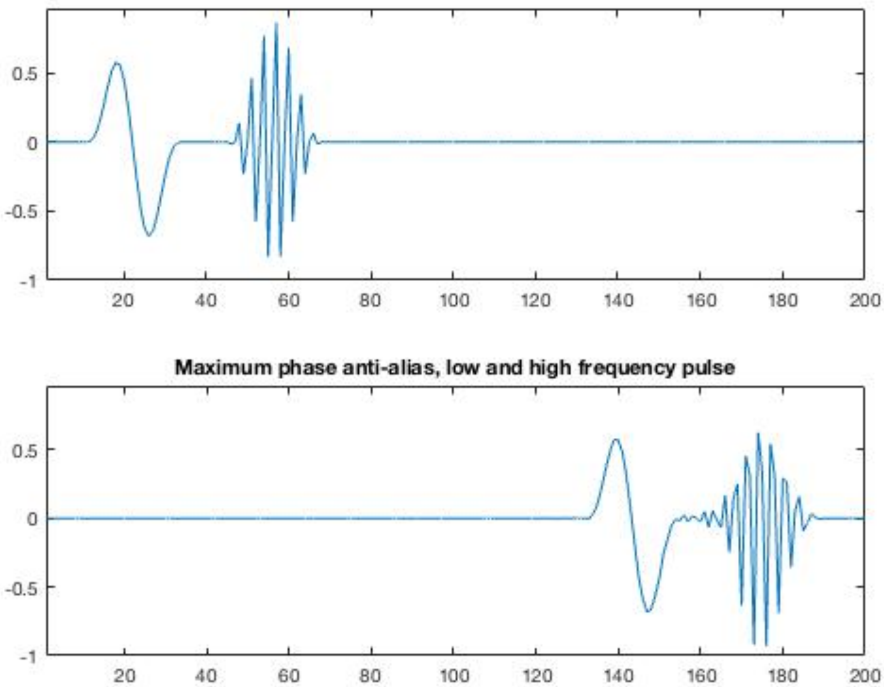
By applying a Hanning window to 2kHz and 16kHz sine waves, low and high frequency pulses were generated and passed through linear, minimum and maximum phase versions of anti-alias-like lowpass filters, with cut offs at 20kHz. The output waveform gives a clear impression of how the filters react over an extended bandwidth.



(fig. 32) The two frequencies were passed through the linear phase anti alias-like filters with a flat frequency response up to 20kHz. The considerable fixed delay has been compensated for and it shows a close resemblance to the input waveform.

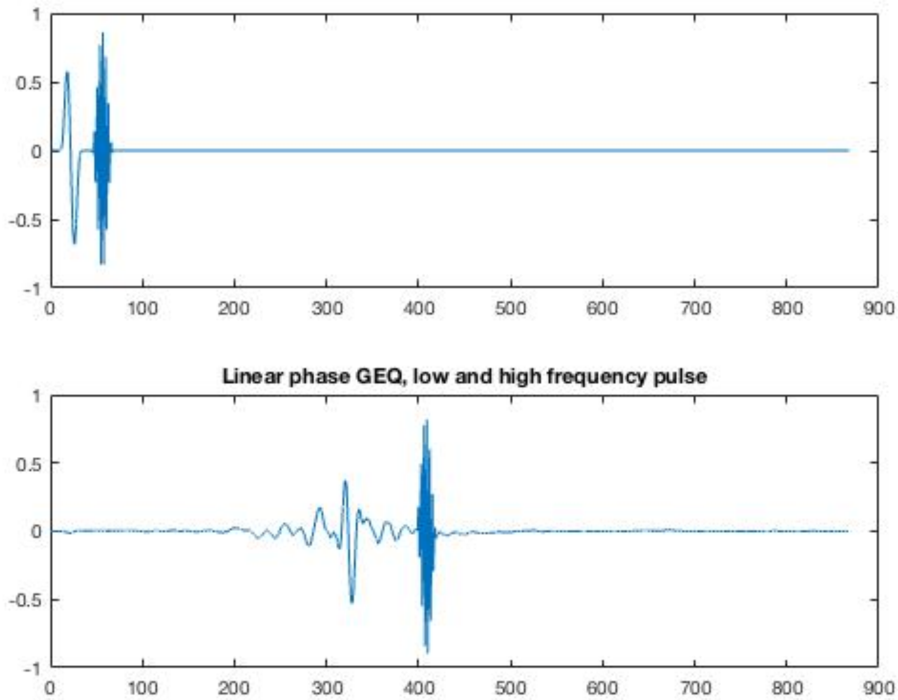


(fig. 33) When passed through the minimum phase anti alias filter there is minor asymmetric distortion, lag to the high frequency pulse and extended post ringing.

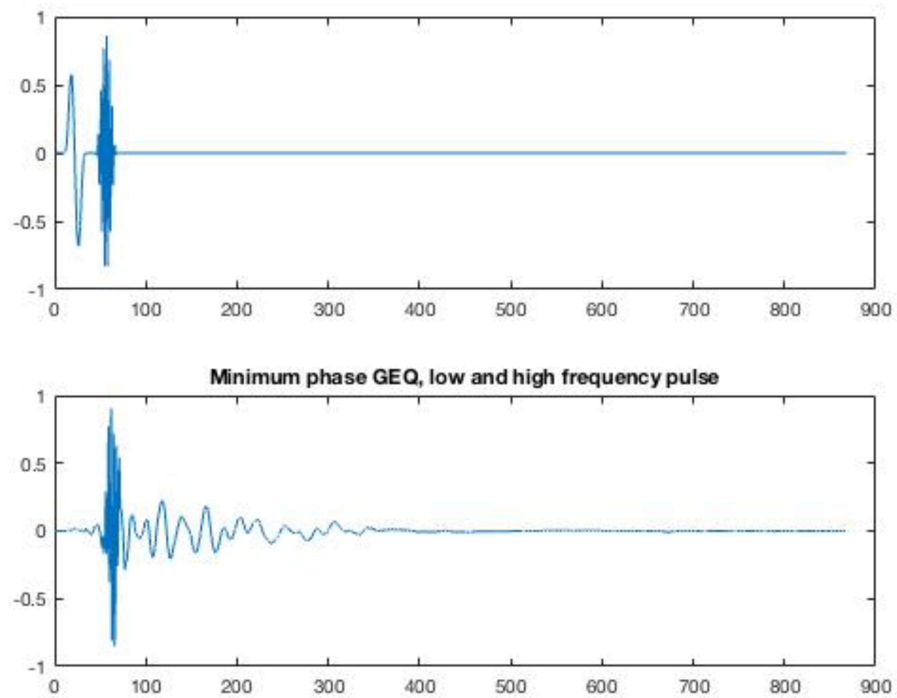


(fig. 34) Having accounted for the fixed linear delay, the maximum phase anti alias filter shows considerable deviation from linear phase (over 130 samples), therefore, it shows the most lag. There is also extended pre-ring and asymmetric distortion to the high frequency waveform.

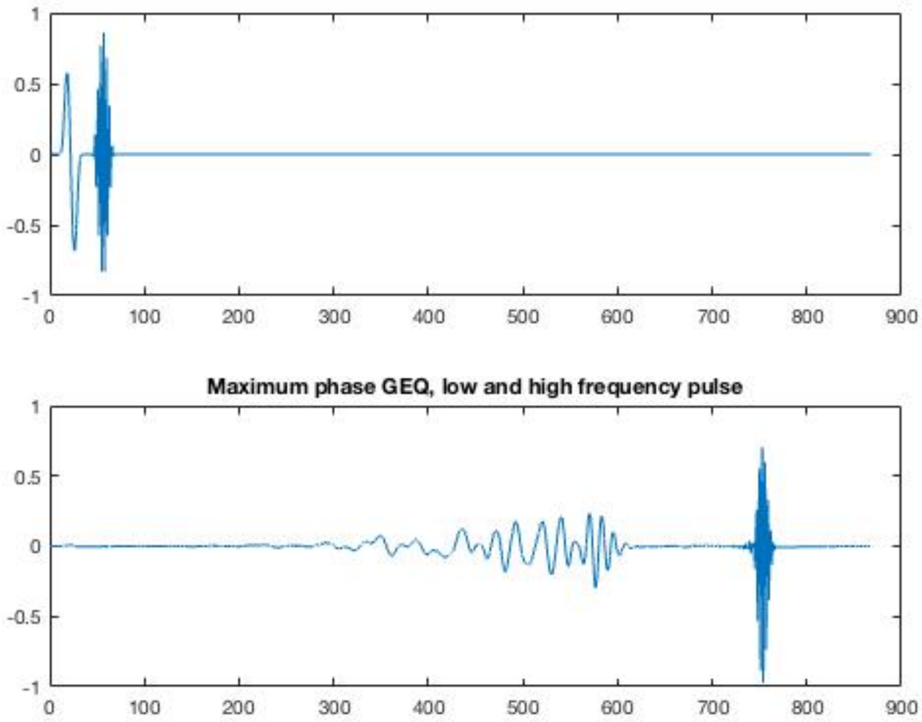
The low and high frequency pulses were then passed through the designed graphic EQs



(fig. 35) Displayed with the considerable fixed delay, the linear phase graphic EQ preserves the input waveform. Both the low frequency and high frequency pulse are clearly visible although there is considerable pre and post ring to the low frequency pulse.

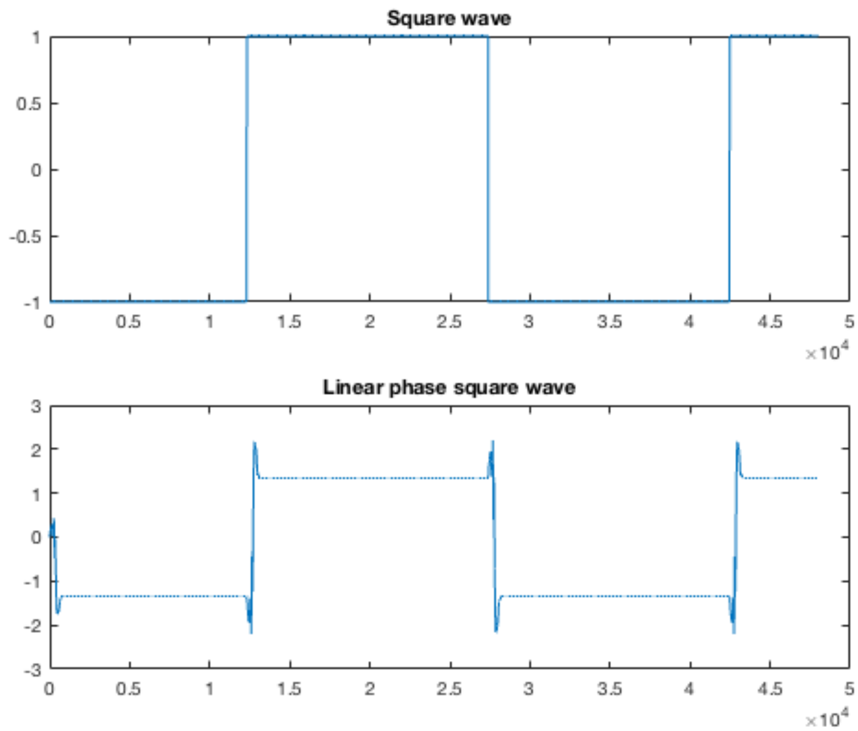


(fig. 36) The minimum phase graphic EQ shows least lag but little resemblance to the input waveform; the low frequency pulse appears to have been merged with the high frequency pulse and there is extended post ring.

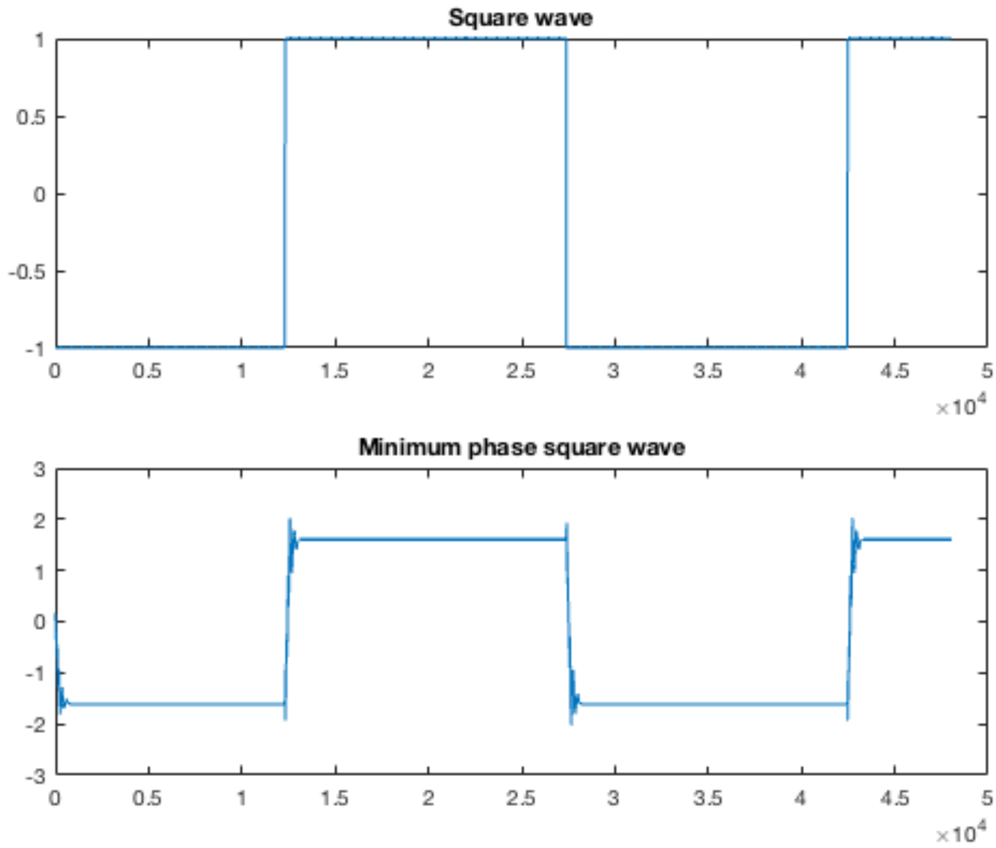


(fig 37) The maximum phase graphic EQ shows extended pre-ring to the low frequency pulse and extended delay between waveforms, suggesting high frequency content is delayed more than low frequency content.

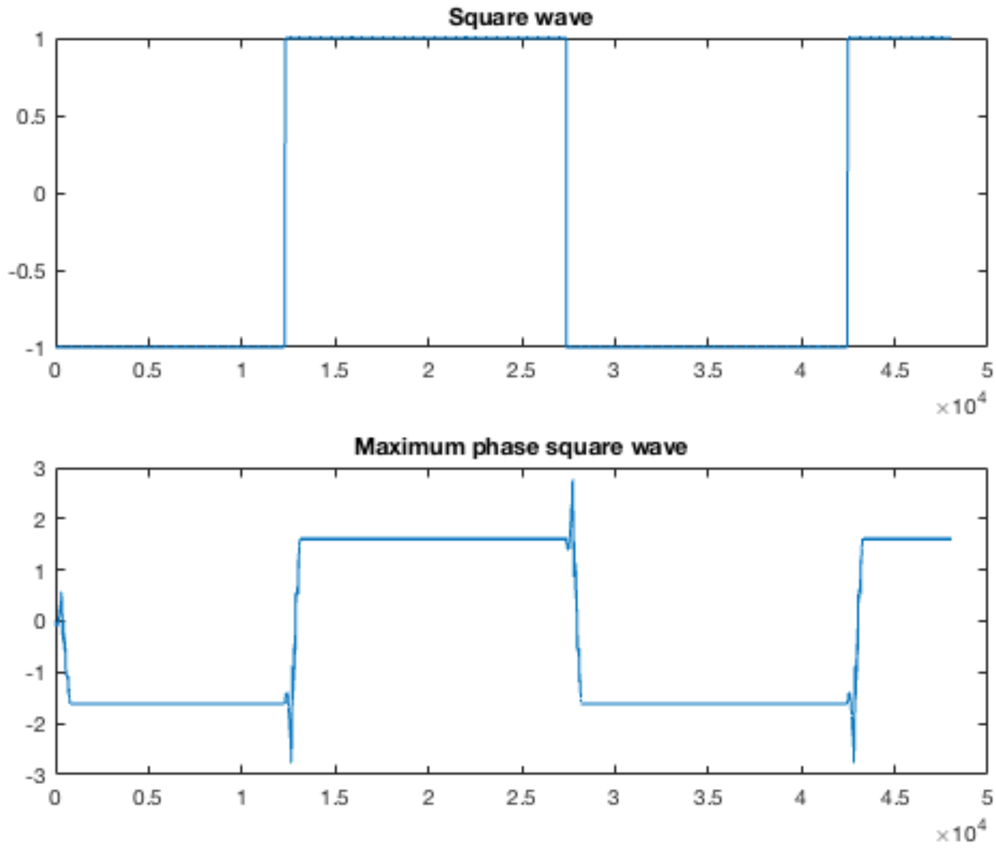
It can further be seen that when square waves are passed through the systems, ringing occurs at the waveforms transitions.



(fig. 38) A square wave passed through the linear phase graphic EQ, notice the pre and post ring.



(fig. 39) A square wave passed through the minimum phase graphic EQ - observe the post ring. The rise before transitions is due to the Gibbs phenomena, this is suggested as an area for further research as to why ringing occurs.



(fig. 40) The maximum phase graphic EQ shows only pre-ring at transitions.

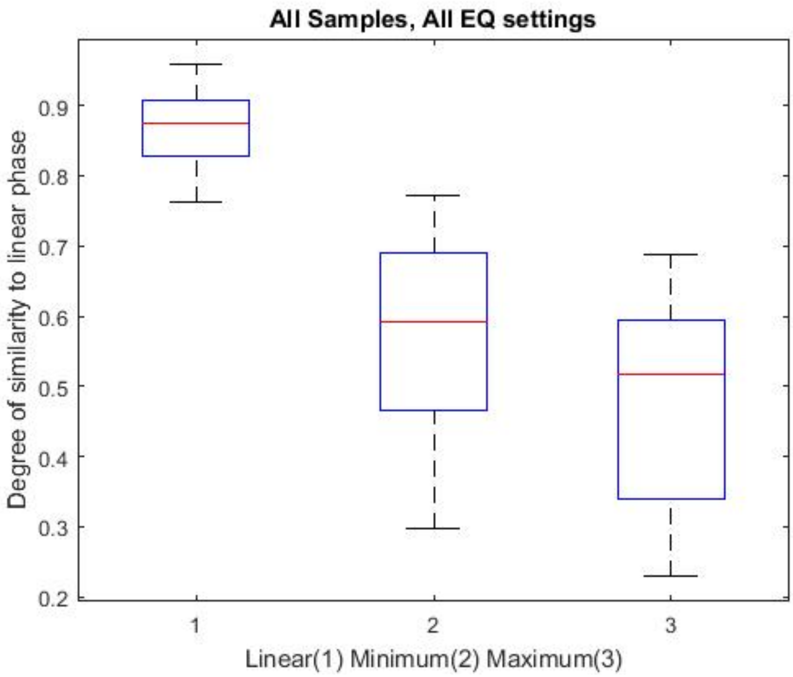
Subjective Test Results

The listening test was performed on 20 subjects and serves to extend and confirm findings from the primary research procedure. Primary research suggested there was significant differentiation between linear phase and minimum phase lowpass filters, specifically on horn and percussion samples. These samples were therefore used in the main test. The audible difference is thought to be due to brass instruments having mostly square waveforms and the transient nature of percussion.

Tests were performed on all three designs of graphic EQ. The linear phase graphic EQ was used as the reference against which a minimum phase, a maximum phase and an additional linear phase filter were compared. 7 signals were passed through 4 different EQ settings making a total of 28 samples to be contrasted. The 4 EQ setting meant results could be drawn from specific frequency responses.

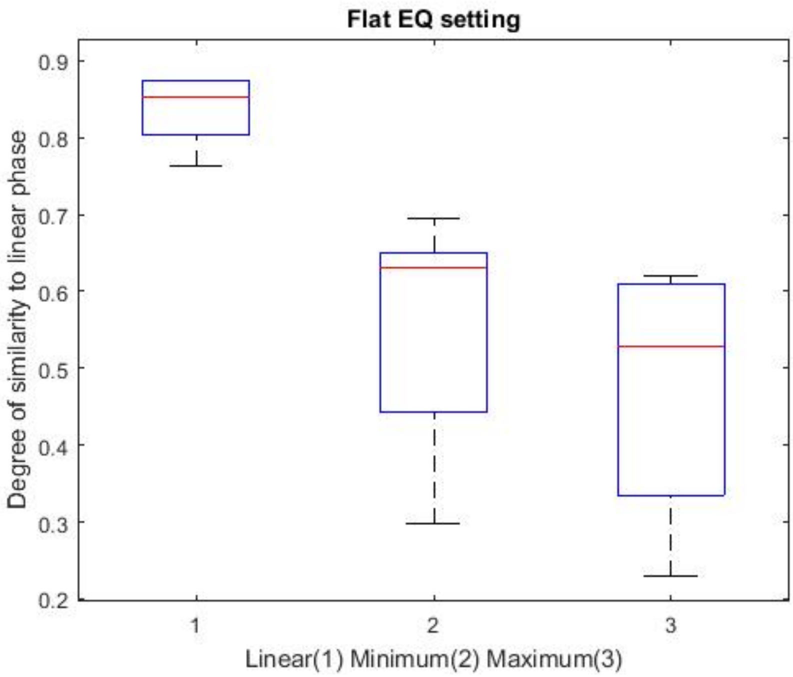
Box and whisker diagrams were generated for each sample by the Web Audio Evaluation Tool and further plots were made using Matlab's boxplot() function. The range of findings is represented between the outer whiskers, the 25 to 75 percentiles of findings is defined within the box and the red line shows the average. Outliers are defined by circles.

Conclusions were drawn from the data by averaging the results for each sample to see how closely the hidden reference was perceived. For all 28 averages taken, the hidden reference (ie. the linear phase filter) was found to score highest in similarity.

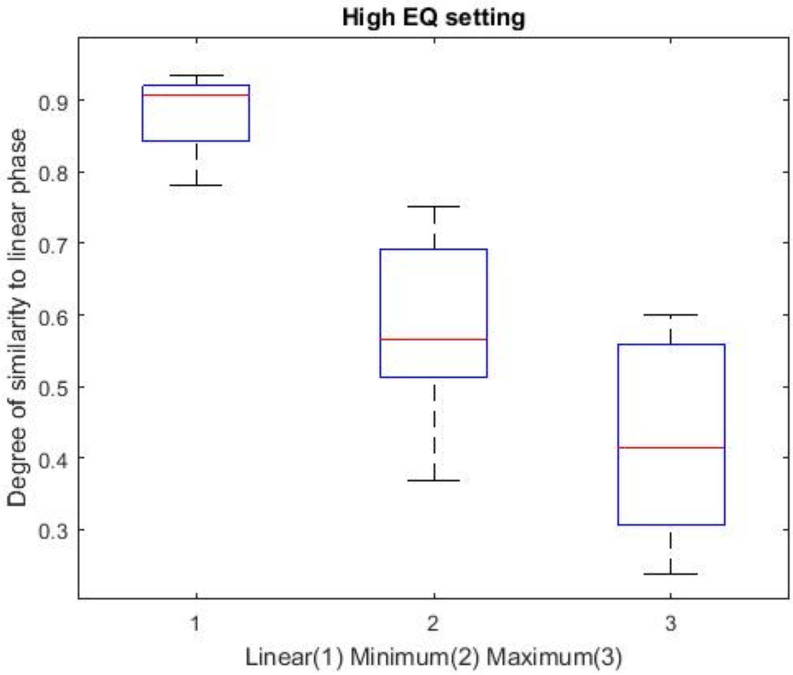


(fig. 41) Taking an average of all the samples at all EQ settings shows the linear phase filter is perceived correctly by a significant degree.

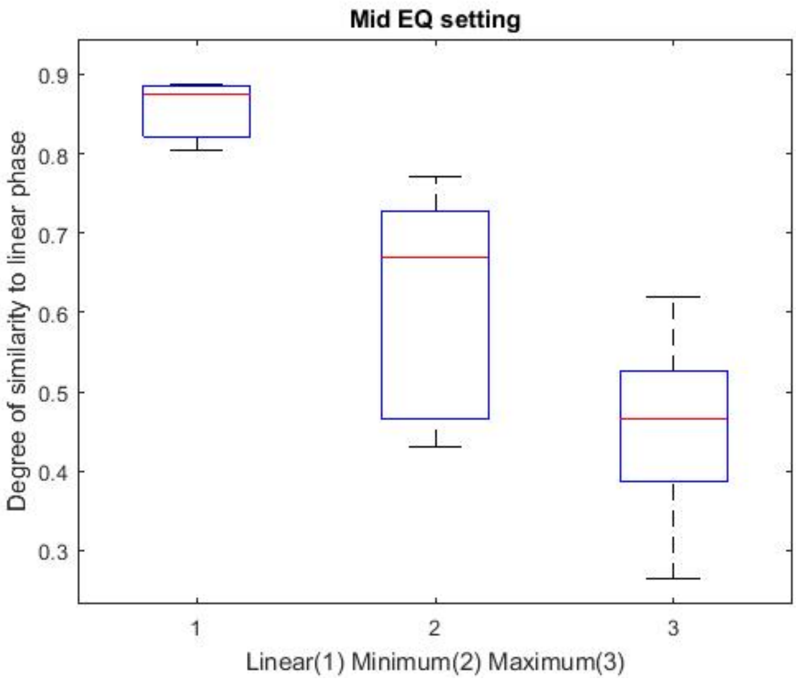
Further analysis groups the findings by their EQ settings. In all averaged cases the linear phase filter was correctly perceived, and in all cases bar the low frequency EQ curve the minimum phase filter scored second in similarity.



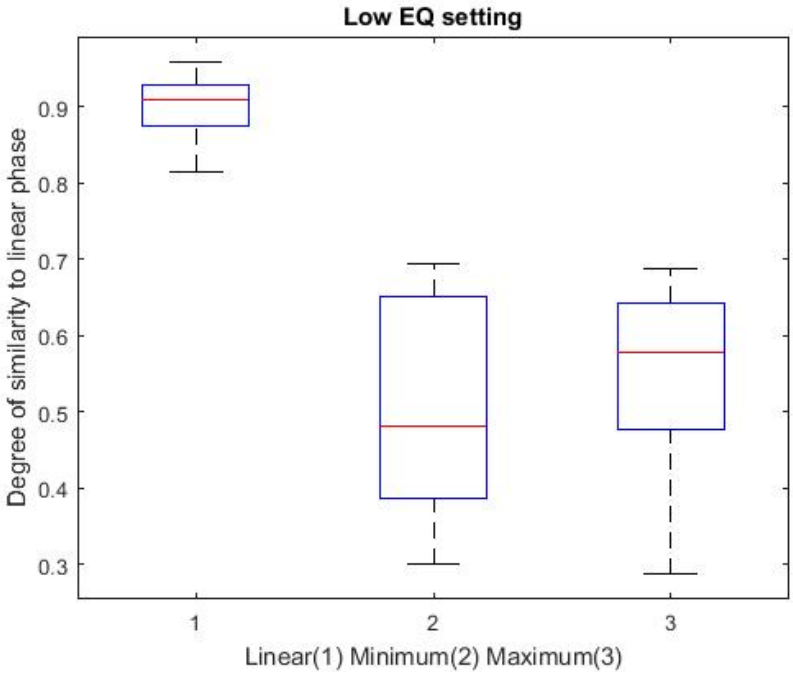
(fig. 42) Results from the flat EQ setting show that test subjects scored the minimum and maximum phase filters over a wide range meaning there was indecision as to which was most similar to the reference.



(fig. 43) The high EQ setting shows tighter boxes meaning closer agreement between subjects.

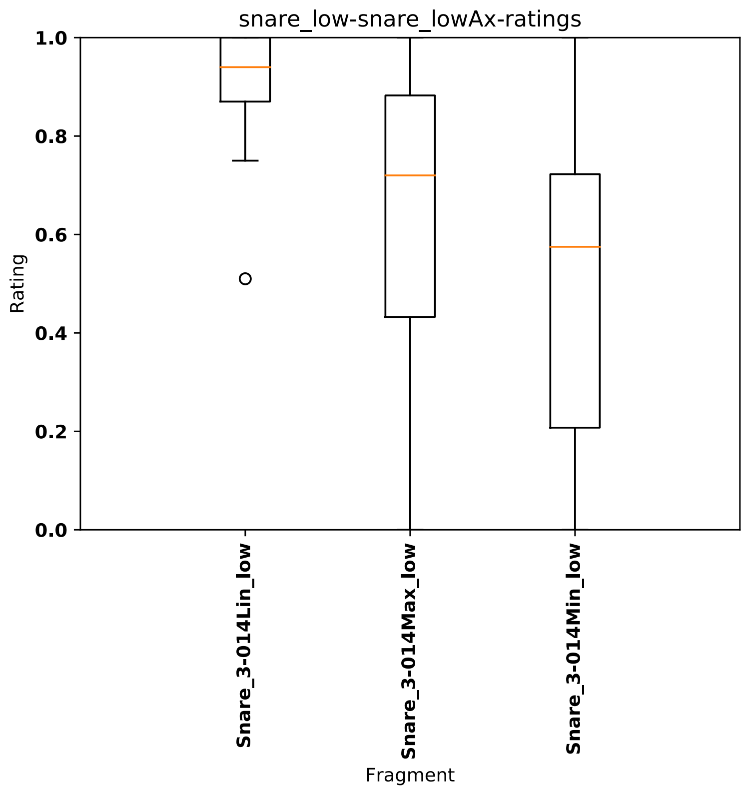


(fig. 44) A comparatively tall box for the minimum phase filter again shows indecision between test subjects.



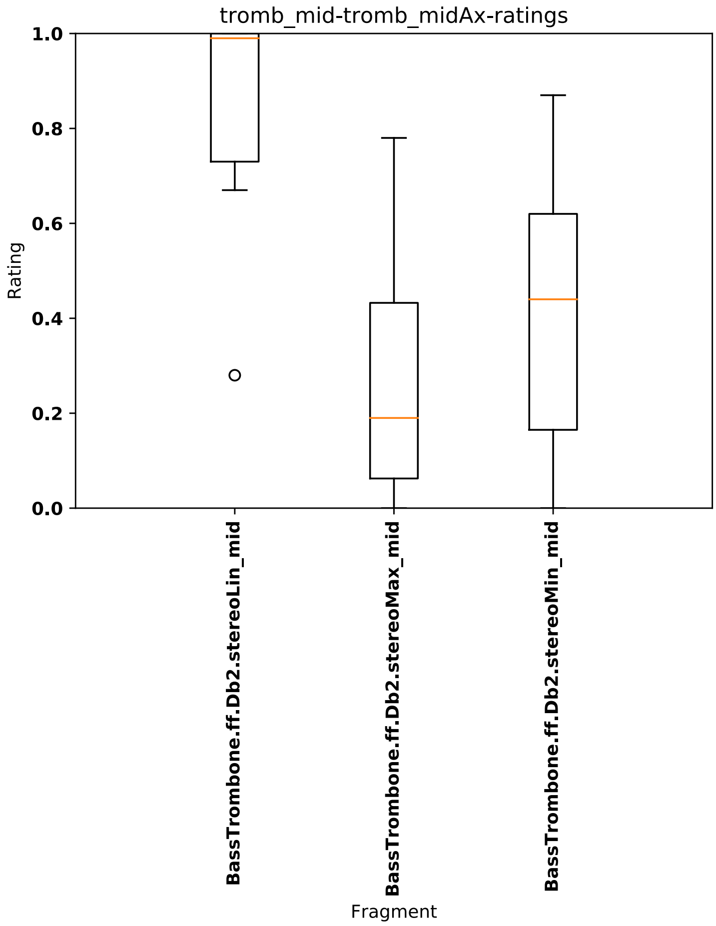
(fig. 45) The minimum phase filter scores less than the maximum phase filter on average, however, the overlapping minimum and maximum phase boxes and similar ranges displayed by the whiskers shows disagreement between test subjects.

Following on from these findings, more was ascertained by looking at the boxplots for individual samples. The low frequency filtered snare sample shows an instance where the maximum phase filtered signal was perceived as closer to the linear phase reference.



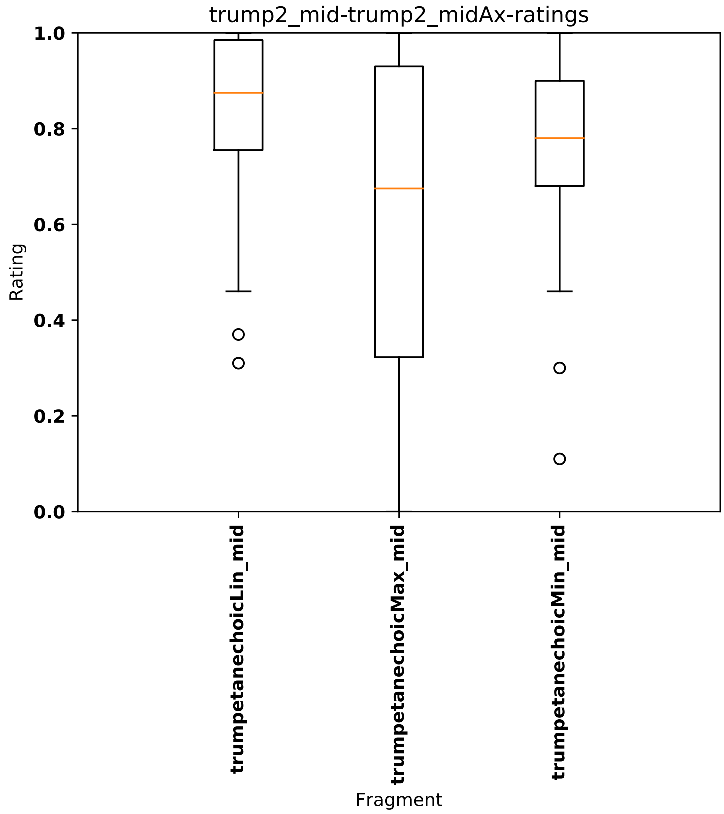
(fig 46) On average the low frequency maximum phase filtered snare sample was perceived as close to linear phase.

Brass instruments scored high in their confidence ratings in the AB test and concurrently in the main listening test.



(fig. 47) The mid frequency filtered trombone sample showed the highest degree of separation between axes, this concurs with the primary research where it scored highest in the AB test confidence ratings.

An additional melodic passage for trumpet was included for analysis.



(fig. 48) The boxplot shows the results for the melodic passage for trumpet, on average the linear filter was perceived correctly however tall boxes show disagreement among test subjects and outliers show rebel results, suggesting subjects were guessing.

The test subjects were then asked which signal they preferred and the results were tallied. There was little variance between the different EQ settings and in all cases the linear phase filter was found most preferable with a significant average of 81% for all samples with a flat EQ. No relevant information can be drawn from the selection of minimum or maximum phase filters as they appear randomly and are chosen infrequently however both the held tones of trumpet 1 and the trombone were favored linear phase in 100% of cases.

Listener	kick	snare	castanet	trump I	alto sax	trombone	trump II	
1	L	L	L	L	L	L	L	
2	L	L	L	L	L	L	L	
3	L	L	L	L	m	L	L	
4	M	m	L	L	L	L	M	
5	L	L	L	L	M	L	L	
6	L	L	L	L	L	L	L	
7	L	L	M	L	L	L	m	

8	L	L	L	L	L	L	L	
9	m	L	L	L	L	L	L	
10	L	m	m	L	m	L	m	
11	L	L	L	L	M	L	L	
12	M	m	L	L	M	L	L	
13	m	m	L	L	L	L	L	
14	L	m	L	L	L	L	m	
15	m	L	m	L	L	L	M	
16	L	L	m	L	L	L	L	
17	m	L	L	L	L	L	m	
18	L	L	L	L	m	L	L	
19	L	L	L	L	L	L	L	
20	L	L	L	L	L	L	L	Average
LINEAR	15	15	16	20	14	20	14	16.2857143
	75%	75%	80%	100%	70%	100%	70%	81%

Table 4. Showing which phase response was preferable for the flat EQ setting, the linear phase filter is shown to be significantly favoured.

Discussion

Initial primary research found that lowpass versions of linear and minimum phase filters show audible differences between their phase types, specifically on transient and square waveforms. A similar procedure was enacted on the graphic EQ's in the main listening test. Different frequency responses of the signals resulted in little variance of the test subject's responses. Perhaps steeper EQ curves could be used to find points at which phase distortion is unnoticed by these transient and square waveforms.

Objective analysis performed on the graphic EQs confirms their phase types and when square waves and short pulses are passed through their systems ringing qualities are shown to be present in the output waveforms. It is shown that there is more ringing to the graphic EQ designs than to a single anti-alias like lowpass filter however in the case of minimum phase anti alias like filters high frequency pulses are shown to be delayed in contrast to low frequency signals. It follows that Linear phase filters may be more suitable to the role of anti-alias filtering.

Secondary research suggests that ringing occurs at filter cutoff frequencies *(Troll Audio, 2019)*, the increased ringing in the graphic EQs is thought to be due to the multiple cut-offs imposed by the use of band pass filters. The phase response of the minimum phase filter shows that the frequency dependent delay is limited by the filter's pass band width; this implies that the more filters used the less frequency dependent delay is present, this is however countered by the previous argument for increased ringing therefore it is extended post-ringing that distorts the output waveform rather than transient smearing. This should be considered an area for further research.

Listening tests show that phase irregularities in the graphic EQ's can be clearly deciphered and that linear phase is preferable. Is this because linear phase is more transparent and devoid of transient smearing or because the test subjects had previously heard the reference being linear phase? Further studies could repeat this experiment with minimum and maximum phase filters as the reference.

The different EQ settings did not show a lot of variance in the findings although, the test does show in some instances that maximum phase filters are perceived as closer to linear phase filters at low frequencies this could arguably be due to linear and maximum phase filters sharing an unnatural pre-ring at low frequencies.

Having created the linear phase, minimum phase and maximum phase Graphic EQs in Matlab and performed the subjective tests, the filter coefficients were hard coded into the Juce framework to create linear phase and minimum phase versions of realtime plugins. Various functions were called from the DSP module (see table 2).

The coefficients were stored as arrays of floats in the header file and zero padding was applied so that their orders remained the same. The coefficients were read to multiple instances of the AudioSampleBuffer class within a circular buffer. These impulse response delay lines could then be copied to the Convoluton class and parallel processing was made available by storing multiple copies of the input buffer in AudioBlock obects. Convolution was then applied to the parallel channels and their output AudioBlocks summed.

Juce::dsp class/function	Arguments	Purpose	Use
Convolution::prepare ( )	const **ProcessSpec** & reference to processSpec struct containing SampleRate, block size, number of channels.	Specifics for convolution defined before audio processing	″
Convolution::reset()		Called before audio processing	″
Convolution::copyAndLoadImpulseResponseFromBuffer()	Buffer, bufferSampleRate, wantsStereo, wantsTrimming, wantsNormalisation, size	Load coefficients to Convolution class for processing	″
AudioBlock()	Buffer	Contains pointer to buffer for use in process()	″
ProcessContextReplacing<float>()	AudioBlock	Contains context information for process()	″
Convolution::process()	Context = ProcessContextReplacing <float>(dsp::AudioBlock)	Applies convolution to the specified AudioBlock	Apply Filtering

Table 5. Prominent Juce DSP Classes/functions used to generate AU plugins.

Conclusion

Digital implementations of linear phase, minimum phase and maximum phase FIR filters have been constructed and tested. Graphic EQs have been designed both in Matlab and in the Juce framework, using industry specific approaches. These include techniques that reduce the computational expense and the filter order. Objective testing methods have been composed which appraise how well the filters reconstruct their input waveforms and which justify how the filters phase responses may affect their output. Listening tests have proven that there are clear audible differences between the phase responses given the appropriate input signal. Instruments with a mostly square waveform and transient signals are noted as showing the most audible differences. The psycho-acoustic test suggests that linear phase filtering is preferable, however this may be due to the linear phase filter being used as the reference in the testing procedure, as previously discussed. This report has also suggested that linear phase anti alias filters might be more suitable than minimum phase anti-alias filters due to evidence found from objective analysis.

It can be concluded that given specific audio material, linear phase, minimum phase and maximum phase filters can be deciphered with a high degree of accuracy, both by waveform analysis and by listening tests via headphones. This is in keeping with secondary research, however, the finding that maximum phase filters are perceived as closer to linear phase filters at low frequencies and that minimum phase filters frequency dependent delay is limited by the filter's bandwidth suggests that low frequency pre-ring plays an integral role in this differentiation.

Recommendations

Both primary and secondary research (Lipshitz, S.P., Pocock, M., Vanderkooy, J., 1982) suggests that phase distortion can be perceived with triangular waveforms. The primary research procedure found string samples that could be deciphered with over 95% confidence, so this would be an area for further study. Future listening tests should produce more qualitative findings, in order to signify what the perceived differences between samples are. Given greater scope to the study, further research would be conducted on the Gibbs phenomena, which may lead to the reasons ringing occurs in filters. Having created real-time implementations of the graphic EQs, further studies could involve test subjects interacting with the filters. Multi-stage filtering is thought to be an effective way of reducing the filter's order, and this should be investigated further for a more accurate magnitude response.

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Appendices

Sound File	Number Correct. (Accuracy)	Confidence
BassTrombone.ff.Db2.stereo.wav	14 (93.333336%)	0.99951171875 (99.951171875%)
castanet1.ff.stereo.wav	13 (86.666664%)	0.996307373046875 (99.6307373046875%)
Kick_1-020.wav	13 (86.666664%)	0.996307373046875 (99.6307373046875%)
AltoSax.NoVib.ff.Db4.stereo.wav	14 (87.5%)	0.9979095458984375 (99.79095458984375%)
Snare_3-014.wav	16 (72.72727%)	0.9737606048583984 (97.37606048583984%)
square100.wav	15 (100.0%)	0.999969482421875 (99.9969482421875%)
saw100.wav	15 (100.0%)	0.999969482421875 (99.9969482421875%)
clap2.wav	5 (33.333336%)	0.059234619140625 (5.9234619140625%)
10wb.ff.stereo	7 (46.666668%)	0.303619384765625 (30.3619384765625%)
13crash.mallet.roll.stereo.wav	10 (66.66667%)	0.84912109375 (84.912109375%)
16chinese.chokecrash.stereo.wav	10 (66.66667%	0.84912109375 (84.912109375%)
19chinese.mallet.roll.stereo.wav	11 (73.333336%)	0.940765380859375 (94.0765380859375%)
21ride.bow.stereo.wav	8 (53.333336%)	0.5 (50.0%)
40tamtam.ff.stereo.wav	8 (53.333336%)	0.5 (50.0%)
AltoSax.NoVib.ff.Db4.stereo.wav	14 (93.333336%)	0.99951171875 (99.951171875%)
Bass.arco.ff.sulC.D1.stereo.wav	8 (53.333336%)	0.5 (50.0%)
Bass.pizz.ff.sulC.D1.stereo.wav	9 (60.000004%)	0.696380615234375 (69.6380615234375%)
BassFlute.ff.D3.stereo.wav	8 (53.333336%)	0.5 (50.0%)
bongo3.wav	11 (73.333336%)	0.940765380859375 (94.0765380859375%)
Cello.arco.ff.sulC.Db3.stereo.wav	7 (46.666668%)	0.303619384765625 (30.3619384765625%)
Cello.pizz.ff.sulC.D3.stereo.wav	7 (46.666668%)	0.303619384765625 (30.3619384765625%)
saw100.wav	16 (100.0%)	0.9999847412109375 (99.99847412109375%)
timbalehit10.wav	7 (43.75%)	0.2272491455078125 (22.72491455078125%)

Trumpet.novib.ff.Db4.stereo.wav	15 (100.0%)	0.999969482421875 (99.9969482421875%)
woodtambshake1.wav	9 (60.000004%)	0.696380615234375 (69.6380615234375%)
Kick_1-018.wav	12 (80.0%)	0.982421875 (98.2421875%)
Kick_1-020.wav	7 (43.75%)	0.2272491455078125 (22.72491455078125%)
Snare_3-014.wav	8 (53.333336%)	0.5 (50.0%)
hihat.bell.ff.stereo.wav	7 (46.666668%)	0.303619384765625 (30.3619384765625%)
hihat.shoulder.ff.stereo.wav	6 (40.0%)	0.15087890625 (15.087890625%)
99243__soundbytez__ratchet-large.wav	8 (53.333336%)	0.5 (50.0%)
05-Revolt-190403_1543-2.wav	7 (46.666668%)	0.303619384765625 (30.3619384765625%)
02-3 Octave Buzzer-190403_1534-2.wav	8 (53.333336%)	0.5 (50.0%)
03-Agressor-190403_1535-2.wav	6 (40.0%)	0.15087890625 (15.087890625%)
387186__alexiero-1__ai-snare-20.wav	9 (60.000004%)	0.696380615234375 (69.6380615234375%)
02-Hyper-190403_1839.wav	12 (75.0%)	0.9615936279296875 (96.15936279296875%)
04-Propeller Saw-190403_1843.wav	6 (37.5%)	0.1050567626953125 (10.50567626953125%)
05-Buzz Lightyear-190403_1845.wav	4 (26.666668%)	0.017578125 (1.7578125%)
Violin.arco.ff.sulG.C4.stereo.wav	5 (33.333336%)	0.059234619140625 (5.9234619140625%)
Violin.pizz.ff.sulG.D5.stereo.wav	7 (46.666668%)	0.303619384765625 (30.3619384765625%)
Viola.arco.ff.sulC.D3.stereo.wav	9 (60.000004%)	0.696380615234375 (69.6380615234375%)
Viola.arco.ff.sulC.D4.stereo.wav	13 (72.22222%)	0.951873779296875 (95.1873779296875%)

A. The results of the primary research methodology.

listener								
1	kick Hi	snare Hi	castanet Hi	trump I Hi	alto sax Hi	trombone Hi	trump II Hi	
2	L	L	L	L	L	L	L	
3	L	L	L	L	L	L	L	
4	L	L	L	L	L	L	L	
5	L	L	L	L	m	L	L	
6	L	m	L	L	L	L	M	
7	L	L	L	L	M	L	L	
8	L	L	L	L	L	L	L	

9	L	L	m	L	L	L	m	
10	L	L	L	L	L	L	L	
11	L	L	L	L	L	L	L	
12	L	m	m	L	m	L	m	
13	L	L	L	L	M	L	L	
14	L	m	L	L	M	L	L	
15	L	m	L	L	L	L	L	
16	L	m	L	L	L	L	m	
17	L	L	m	L	L	L	L	
18	L	L	m	L	L	L	L	
19	L	L	L	L	L	L	m	
20	L	L	L	L	m	L	L	
LINEAR	L	L	L	L	L	L	L	
	20	15	16	20	14	20	15	17.1428571
	100%	75%	80%	100%	70%	100%	75%	86%
listener	kick Low	snare Low	castanet Low	trump I Low	alto sax Low	trombone Low	trump IILow	
1	L	L	L	L	L	L	L	
2	L	L	L	L	L	L	m	
3	L	L	L	L	L	L	L	
4	m	L	L	L	L	L	L	
5	L	L	L	L	L	L	L	
6	L	L	L	L	L	L	L	
7	L	m	L	L	L	L	L	
8	L	L	L	L	L	L	L	
9	L	L	L	L	L	L	L	
10	m	m	L	m	L	m	m	
11	L	L	L	L	L	L	L	
12	m	L	L	L	L	L	m	
13	m	L	L	L	L	L	m	
14	m	L	L	L	L	L	L	
15	L	m	L	L	L	L	L	
16	L	m	L	L	L	L	L	
17	L	L	L	L	L	L	L	
18	L	L	L	L	L	L	M	
19	L	L	L	L	L	L	L	
20	L	L	L	L	L	L	L	
LINEAR	15	16	20	19	20	19	16	17.8571429
	75%	80%	100%	95%	100%	95%	80%	89%
listener	kick Mid	snare Mid	castanet Mid	trump I Mid	alto sax Mid	trombone Mid	trump II Mid	

1	L	L	L	L	L	L	L	
2	L	L	L	L	L	L	m	
3	L	L	m	L	L	L	M	
4	L	L	L	L	M	L	L	
5	L	L	M	L	L	L	L	
6	L	L	L	L	L	L	L	
7	m	m	L	L	m	L	m	
8	L	L	L	L	L	L	L	
9	L	L	L	L	L	L	M	
10	L	L	m	L	m	L	L	
11	L	L	M	L	L	L	L	
12	L	L	M	L	L	L	L	
13	L	L	L	L	L	L	m	
14	L	L	L	L	m	L	L	
15	m	m	L	L	M	L	L	
16	L	L	L	L	L	L	L	
17	L	L	L	L	m	L	L	
18	L	L	m	L	L	L	L	
19	L	L	L	L	L	L	m	
20	L	L	L	L	L	L	L	
LINEAR	18	19	14	20	14	20	14	17
	90%	95%	70%	100%	70%	100%	70%	85%

B Further results showing the preferred phase type for High, Low and Mid frequency Graphic EQ settings