Impact of Imperfect CSI on scheme performances

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REMARQUES PHILIPPE/ JULIEN/FRANCOIS SUR PRESENTATION PPT 26/11/2020:

- Modele de l'erreur: cela peut être Eve qui rajoute du bruit si on considère que l'erreur est une erreur du type bruit thermique et non une erreur dûe à la réciprocité du schéma TDD;
- Bob pourrait matched filter le signal reçu et diminuer l'effet du mauvais precoding
- fig p7 presentation: axe y in dB
- Si Alice connait la variance de l'erreur: precodage plus robuste possible?
- Lien entre α et σ
- p8: σ a mettre en dB comme ça on peut avoir un lien direct avec le SNR
- une des figures: c'est pas en % l'axe y
- Interprétation des formules: déevloppement première ordre pour $\sigma \to 1$ et voir si ça permet de faire des simplifications (asymptotes)
- courbe $\sigma_{\max}$: quand $\Delta \to 0$, normalement l'erreur max devrait tendre vers 1 (asymptote verticale attendue normalement).

$$\tilde{\mathbf{H}}_B = \sqrt{1-\sigma}\mathbf{H}_B + \sqrt{\sigma}\Delta\mathbf{H}_B \tag{1}$$

- $\mathbf{H}_B = \mathbf{H}_{B,x} + j\mathbf{H}_{B,y} \sim \mathcal{CN}(0,1) \sim \mathcal{N}(0,\frac{1}{2}) + j\mathcal{N}(0,\frac{1}{2})$
- $\Delta\mathbf{H}_B = \Delta\mathbf{H}_{B,x} + j\Delta\mathbf{H}_{B,y} \sim \mathcal{CN}(0,1) \sim \mathcal{N}(0,\frac{1}{2}) + j\mathcal{N}(0,\frac{1}{2})$

- $h_{B,i} \perp\!\!\!\perp h_{B,j}, \forall i \neq j$

- $\Delta h_{B,i} \perp\!\!\!\perp \Delta h_{B,j}, \forall i \neq j$

- $\Delta h_{B,i} \perp\!\!\!\perp h_{B,j}, \forall i, j$

$$\begin{aligned}
\mathbf{y}_B^H &= \sqrt{\alpha}\mathbf{S}^H\mathbf{H}_B\tilde{\mathbf{H}}_B^*\mathbf{S}\mathbf{x} + \mathbf{S}^H\mathbf{v}_B + \mathbf{S}^H\mathbf{H}_B\mathbf{w} \\
&= \sqrt{\alpha}\mathbf{S}^H\mathbf{H}_B\left[\sqrt{1-\sigma}\mathbf{H}_B^* + \sqrt{\sigma}\Delta\mathbf{H}_B^*\right]\mathbf{S}\mathbf{x} + \mathbf{S}^H\mathbf{v}_B + \mathbf{S}^H\mathbf{H}_B\mathbf{w} \\
&= \sqrt{\alpha(1-\sigma)}\mathbf{S}^H\left|\mathbf{H}_B\right|^2\mathbf{S}\mathbf{x} + \sqrt{\alpha\sigma}\mathbf{S}^H\mathbf{H}_B\Delta\mathbf{H}_B^*\mathbf{x} + \mathbf{S}^H\mathbf{v}_B + \mathbf{S}^H\mathbf{H}_B\mathbf{w}
\end{aligned} \tag{2}$$

with:

$$\mathbf{S}^H\mathbf{H}_B\mathbf{w} \neq 0 \tag{3}$$

since AN designed to be in the null space of $\tilde{\mathbf{H}}_B^*$

$$\mathbb{E}\left[\|\text{data}\|^2\right] = \frac{\alpha\left[(U+1)(1-\sigma)+\sigma\right]}{U} \tag{4}$$

$$\mathbb{E}\left[\|\text{noise}\|^2\right] = \sigma_B^2 \tag{5}$$

$$\mathbb{E}\left[\|\text{AN}\|^2\right] = \frac{(1-\alpha)\sigma}{U} \tag{6}$$

$$\mathbb{E}\left[\gamma_{B,n}\right] = \frac{\alpha\left[(U+1)(1-\sigma)+\sigma\right]}{U\sigma_B^2 + (1-\alpha)\sigma} \tag{7}$$

MF DECODER:

$$R_s^{MF} \approx \log_2\left(1 + \frac{\alpha\left[(U+1)(1-\sigma)+\sigma\right]}{U\sigma_B^2 + (1-\alpha)\sigma}\right) - \log_2\left(1 + \frac{\alpha\frac{U+3}{U}}{\sigma_{\text{V,E}}^2 + \frac{1-\alpha}{U+1}}\right) \tag{8}$$

$$\sigma_{\max} = \frac{\alpha(U+1) - U\sigma_B^2\gamma_{E,n}}{(1-\alpha)\gamma_{E,n} + \alpha U} \tag{9}$$

$$\delta_{B,\infty} = 10\log_{10}\left[\frac{\alpha(U+2^\Delta) + U(2^\Delta - 1)}{\alpha^2(2^\Delta B\sigma - U(U+1)(1-\sigma)) + \alpha(2^\Delta U\sigma - \sigma 2^\Delta B + (U+1)(1-\sigma)U - \sigma U) + \sigma U(1-2^\Delta)}\right] \tag{10}$$

$$B = U^2 + 3U + 3 \tag{11}$$

$$\sigma_{\max,\infty} = \frac{U(U+1)}{2^{\Delta}B + NoEveNoise_U(U+1)} \tag{12}$$

Hypothesis

- Q subcarriers, back off rate $= U$, $N = Q/U$ symbols sent per OFDM block
- $\mathbf{H}_B = \mathbf{H}_{B,x} + j\mathbf{H}_{B,y} \sim \mathcal{CN}(0,1) \sim \mathcal{N}(0,\frac{1}{2}) + j\mathcal{N}(0,\frac{1}{2})$
- $\mathbf{H}_E = \mathbf{H}_{E,x} + j\mathbf{H}_{E,y} \sim \mathcal{CN}(0,1) \sim \mathcal{N}(0,\frac{1}{2}) + j\mathcal{N}(0,\frac{1}{2})$
- $h_{B,i} \perp\!\!\!\perp h_{B,j}, \forall i \neq j$
- $h_{E,i} \perp\!\!\!\perp h_{E,j}, \forall i \neq j$
- $h_{B,i} \perp\!\!\!\perp h_{E,j}, \forall i, j$

AN derivation

We want to compute the mean energy per symbol received at Eve for the articial noise (AN) component when she performs a matched filtering. The AN term at Eve is given by:

$$\begin{aligned}
\mathbf{v} &= \mathbf{S}^H \mathbf{H}_B |\mathbf{H}_E|^2 \mathbf{w} && (13)\\
&= \mathbf{A} |\mathbf{H}_E|^2 \mathbf{V}_2 \mathbf{w}' && (14)\\
&= \mathbf{U} \begin{pmatrix} \mathbf{\Sigma} & \mathbf{0}_{N-Q\times N} \end{pmatrix} \begin{pmatrix} \mathbf{V}_1^H \\ \mathbf{V}_2^H \end{pmatrix} |\mathbf{H}_E|^2 \mathbf{V}_2 \mathbf{w}' && (15)\\
&= \mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H |\mathbf{H}_E|^2 \mathbf{V}_2 \mathbf{w}' && (16)
\end{aligned}$$

where:

- $\mathbf{U}$ is a $N \times N$ unitary matrix, i.e., $\mathbf{U}^H\mathbf{U} = \mathbf{I}_N$, its columns form an orthonormal basis of $\mathcal{C}^N$ and are the left singular vectors of each singular value of $\mathbf{A}$;
- $\mathbf{\Sigma}$ is a $N \times N$ diagonal matrice containing the singular values of $\mathbf{A}$ in the descending order, i.e., $\sigma_i = \mathbf{\Sigma}_{i,i}$;
- $\mathbf{V}_1$ is a $Q \times N$ complex matrix that contains the right singular vectors associated to the non-zero singular values;

- $\mathbf{V}_2$ is a $Q \times Q - N$ complex matrix that contains the right singular vectors associated to the zeroes singular values, i.e., that span the right null-space of $\mathbf{A}$;
- $\mathbf{V} = (\mathbf{V}_1 \ \mathbf{V}_2)$ is a $Q \times Q$ unitary matrix, i.e., $\mathbf{V}^H \mathbf{V} = \mathbf{I}_Q$, its columns form an orthonormal basis of $\mathcal{C}^Q$ and are the right singular vectors of each singular value of $\mathbf{A}$;
- $\mathbf{w'}$ is a $Q - N \times 1$ complex normal random variable such that $\mathbf{w'} \sim \mathcal{CN}(0, 1)$

Let us now look at the covariance matrix

$$\mathbb{E}\left(\mathbf{v}\mathbf{v}^H\right) = \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H |\mathbf{H}_E|^2 \mathbf{V}_2 \mathbf{w}' \left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H |\mathbf{H}_E|^2 \mathbf{V}_2 \mathbf{w}'\right)^H\right) \tag{17}$$

$$= \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H |\mathbf{H}_E|^2 \mathbf{V}_2 \mathbf{w}' \mathbf{w}'^H \mathbf{V}_2^H |\mathbf{H}_E|^2 \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{18}$$

Note that $\mathbf{w}'$ is independent of other random variable and has a unit covariance matrix. We can thus put the expectation inside to get

$$\mathbb{E}\left(\mathbf{v}\mathbf{v}^H\right) = \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H |\mathbf{H}_E|^2 \mathbf{V}_2 \mathbf{V}_2^H |\mathbf{H}_E|^2 \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{19}$$

We rewrite $|\mathbf{H}_E|^2 = \sum_{q=1}^{Q} |H_{E,q}|^2 \mathbf{e}_q \mathbf{e}_q^T$ where $\mathbf{e}_q$ is an all zero vector except a 1 at row q to isolate the independent random variable H_E

$$\mathbb{E}\left(\mathbf{v}\mathbf{v}^H\right) = \sum_{q=1}^{Q} \sum_{q'=1}^{Q} \mathbb{E}(|H_{E,q}|^2 |H_{E,q'}|^2) \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_2 \mathbf{V}_2^H \mathbf{e}_{q'} \mathbf{e}_{q'}^T \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{20}$$

$$= \sum_{q=1}^{Q} \mathbb{E}(|H_{E,q}|^4) \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_2 \mathbf{V}_2^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{21}$$

$$+ \sum_{q=1}^{Q} \sum_{q' \neq q}^{Q} \mathbb{E}(|H_{E,q}|^2 |H_{E,q'}|^2) \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_2 \mathbf{V}_2^H \mathbf{e}_{q'} \mathbf{e}_{q'}^T \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{22}$$

$$= 2 \sum_{q=1}^{Q} \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_2 \mathbf{V}_2^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{23}$$

$$+ \sum_{q=1}^{Q} \sum_{q' \neq q}^{Q} \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_2 \mathbf{V}_2^H \mathbf{e}_{q'} \mathbf{e}_{q'}^T \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{24}$$

$$= \sum_{q=1}^{Q} \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_2 \mathbf{V}_2^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{25}$$

$$+ \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \sum_{q=1}^{Q} \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_2 \mathbf{V}_2^H \sum_{q'=1}^{Q} \mathbf{e}_{q'} \mathbf{e}_{q'}^T \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{26}$$

$$= \sum_{q=1}^{Q} \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_2 \mathbf{V}_2^H \mathbf{e}_q \mathbf{e}_q^T \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) + \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \mathbf{V}_2 \mathbf{V}_2^H \mathbf{V}_1 \mathbf{\Sigma}^H \mathbf{U}^H\right) \tag{27}$$

Using the fact that $\mathbf{V}_2^H\mathbf{V}_1 = \mathbf{0}$, the second term cancels and

$$\mathbb{E}\left(\mathbf{v}\mathbf{v}^H\right) = \mathbb{E}\left(\mathbf{U}\mathbf{\Sigma}\mathbf{V}_1^H \sum_{q=1}^{Q}\left(\mathbf{e}_q\mathbf{e}_q^T\mathbf{V}_2\mathbf{V}_2^H\mathbf{e}_q\mathbf{e}_q^T\right)\mathbf{V}_1\mathbf{\Sigma}^H\mathbf{U}^H\right) \tag{28}$$

Since all elements of $\mathbf{v}$ have same variance, we can compute it as

$$\frac{1}{N}\mathbb{E}\left(\|\mathbf{v}\|^2\right) = \frac{1}{N}\mathbb{E}\ \mathrm{tr}\left(\mathbf{v}\mathbf{v}^H\right) \tag{29}$$

$$= \frac{1}{N}\mathbb{E}\ \mathrm{tr}\left(\mathbf{\Sigma}^2\mathbf{V}_1^H \sum_{q=1}^{Q}\left(\mathbf{e}_q\mathbf{e}_q^T\mathbf{V}_2\mathbf{V}_2^H\mathbf{e}_q\mathbf{e}_q^T\right)\mathbf{V}_1\right) \tag{30}$$

Let us rewrite $\mathbf{V}_1 = \sum_l \mathbf{e}_l\mathbf{v}_{1,l}^H$ where $\mathbf{v}_{1,l}^H$ is the l-th row of $\mathbf{V}_1$ (of dimension $N \times 1$) with only one nonzero element.

$$\frac{1}{N}\mathbb{E}\left(\|\mathbf{v}\|^2\right) = \frac{1}{N}\sum_{q=1}^{Q}\sum_{l}\sum_{l'}\mathbb{E}\ \mathrm{tr}\left(\mathbf{\Sigma}^2\mathbf{v}_{1,l}\mathbf{e}_{l'}^T\mathbf{e}_q\mathbf{e}_q^T\mathbf{V}_2\mathbf{V}_2^H\mathbf{e}_q\mathbf{e}_q^T\mathbf{e}_l\mathbf{v}_{1,l}^H\right) \tag{31}$$

$$= \frac{1}{N}\sum_{q=1}^{Q}\sum_{l}\sum_{l'}\delta_{l'-q}\delta_{l-q}\mathbb{E}\ \mathrm{tr}\left(\mathbf{\Sigma}^2\mathbf{v}_{1,l}\mathbf{e}_q^T\mathbf{V}_2\mathbf{V}_2^H\mathbf{e}_q\mathbf{v}_{1,l}^H\right) \tag{32}$$

$$= \frac{1}{N}\sum_{q=1}^{Q}\mathbb{E}\ \mathrm{tr}\left(\mathbf{\Sigma}^2\mathbf{v}_{1,q}\mathbf{e}_q^T\mathbf{V}_2\mathbf{V}_2^H\mathbf{e}_q\mathbf{v}_{1,q}^H\right) \tag{33}$$

Let us rewrite $\mathbf{V}_2 = \sum_l \mathbf{e}_l\mathbf{v}_{2,l}^H$ where $\mathbf{v}_{2,l}^H$ is the l-th row of $\mathbf{V}_2$ (of dimension $Q - N \times 1$) with $U - 1$ nonzero elements

$$\frac{1}{N}\mathbb{E}\left(\|\mathbf{v}\|^2\right) = \frac{1}{N}\sum_{q=1}^{Q}\sum_{l}\sum_{l'}\mathbb{E}\ \mathrm{tr}\left(\mathbf{\Sigma}^2\mathbf{v}_{1,q}\mathbf{e}_q^T\mathbf{e}_l\mathbf{v}_{2,l}^H\mathbf{v}_{2,l'}\mathbf{e}_{l'}^T\mathbf{e}_q\mathbf{v}_{1,q}^H\right) \tag{34}$$

$$= \frac{1}{N}\sum_{q=1}^{Q}\mathbb{E}\ \mathrm{tr}\left(\mathbf{\Sigma}^2\mathbf{v}_{1,q}\mathbf{v}_{2,q}^H\mathbf{v}_{2,q}\mathbf{v}_{1,q}^H\right) \tag{35}$$

$$= \frac{1}{N}\sum_{q=1}^{Q}\mathbb{E}\left(\|\mathbf{v}_{2,q}\|^2\mathbf{v}_{1,q}^H\mathbf{\Sigma}^2\mathbf{v}_{1,q}\right) \tag{36}$$

where $\mathbf{v}_{1,q}^H\mathbf{\Sigma}^2\mathbf{v}_{1,q} := \|\mathbf{v}_{1,q}\|^2\sigma_n^2$ is a scalar. Therefore, we obtain:

$$\frac{1}{N}\mathbb{E}\left(\|\mathbf{v}\|^2\right) = \frac{1}{N}\sum_{q=1}^{Q}\mathbb{E}\left(\|\mathbf{v}_{2,q}\|^2\|\mathbf{v}_{1,q}\|^2\sigma_n^2\right) \tag{37}$$

Since $\mathbf{V}$ forms an orthonormal basis, i.e., $\mathbf{V}^H\mathbf{V} = \mathbf{I}_Q$, we have $\|\mathbf{v}_{1,q}\|^2 + \|\mathbf{v}_{2,q}\|^2 = 1$. We then have:

$$\frac{1}{N}\mathbb{E}\left(\|\mathbf{v}\|^2\right) = \frac{1}{N}\sum_{q=1}^{Q}\mathbb{E}\left[\left(\|\mathbf{v}_{1,q}\|^2 - \|\mathbf{v}_{1,q}\|^4\right)\sigma_n^2\right] \tag{38}$$

To determine eq.38, we need to know the transformations performed by the singular value decomposition on the input matrix $\mathbf{A}$ to obtain $\mathbf{v}_{1,q}$ and σ_n^2, i.e., we have to find an analytic expression of $\mathbf{v}_{1,q}$ and σ_n^2. We know that:

$$\mathbf{A} = \mathbf{S}^H \mathbf{H}_B = \begin{bmatrix} z_1 & 0 & \dots & 0 & z_2 & 0 & \dots & 0 & \dots & z_U & 0 & \dots & 0 \\ 0 & z_{U+1} & \dots & 0 & 0 & z_{U+2} & \dots & 0 & \dots & 0 & z_{2U} & \dots & 0 \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots & \dots & \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & z_{(N-1)U+1} & 0 & 0 & \dots & z_{(N-1)U+2} & \dots & 0 & 0 & \dots & z_Q \end{bmatrix} \tag{39}$$

where $\mathbf{A} \in \mathcal{C}^{N \times Q}$ and $z_i = z_{i,x} + jz_{i,y} \sim \mathcal{CN}(0, \frac{1}{U}) \sim \mathcal{N}(0, \frac{1}{2U}) + j\mathcal{N}(0, \frac{1}{2U})$. After singular value decomposition, we obtain:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_N \end{bmatrix} \tag{40}$$

where $\sigma_n = \sqrt{\sum_{i=1}^{U} \left| z_{(n-1)U+i} \right|^2}$, $n = 1...N$

$$\mathbf{V}_1 = \begin{bmatrix} v_1 & 0 & \dots & 0 \\ 0 & v_{U+1} & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & v_{(U-1)N+1} \\ v_2 & 0 & \dots & 0 \\ 0 & v_{U+2} & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & v_{(U-1)N+2} \\ \vdots & \vdots & & \vdots \\ v_U & 0 & \dots & 0 \\ 0 & v_{2U} & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & v_Q \end{bmatrix} \tag{41}$$

where $v_i = \frac{z_i^*}{\sigma_k}$, $i = 1..Q$, $k = 1...N$ represents the column of $\mathbf{V}_1$ where v_i belongs.

From that, we obtain:

$$\mathbb{E}\left[\sigma_n^2\right] = \mathbb{E}\left[\sum_{i=1}^{U}\left|z_{(n-1)U+i}\right|^2\right] \tag{42}$$

$$= U\mathbb{E}\left[\left|z_{(n-1)U+i}\right|^2\right] \tag{43}$$

$$= U\frac{1}{U} \tag{44}$$

$$= 1 \tag{45}$$

Without loss of generality, we compute $\mathbb{E}\left[\|v_1\|^2\right]$ and $\mathbb{E}\left[\|v_1\|^4\right]$ since all components of $\mathbf{V}_1$ are identically distributed:

$$\mathbb{E}\left[\|v_1\|^2\right] = \mathbb{E}\left[\left|\frac{z_1^*}{\sigma_1}\right|^2\right] \tag{46}$$

$$= \mathbb{E}\left[\frac{|z_1|^2}{\sigma_1^2}\right] \tag{47}$$

$$= \mathbb{E}\left[\frac{|z_1|^2}{\sum_{i=1}^{U}|z_i|^2}\right] \tag{48}$$

$$= \mathbb{E}\left[\frac{|z_1|^2}{U\,|z_1|^2}\right] \tag{49}$$

$$= \frac{1}{U} \tag{50}$$

For the moment of order 4, we note that $\mathbb{E}\left[|z_i|^4\right] = \frac{2}{U^2}$, cfr *"Momentum of complex normal* *random variables"* pdf.

$$\mathbb{E}\left[\|v_1\|^4\right] = \mathbb{E}\left[\left|\frac{z_1^*}{\sigma_1}\right|^4\right] \tag{51}$$

$$= \mathbb{E}\left[\frac{|z_1|^4}{\sigma_1^4}\right] \tag{52}$$

$$= \mathbb{E}\left[\frac{|z_1|^4}{\left(\sum_{i=1}^U |z_i|^2\right)^2}\right] \tag{53}$$

$$= \mathbb{E}\left[\frac{|z_1|^4}{\sum_{i=1}^U |z_i|^4 + 2\sum_{i=1}^U \sum_{j<i} |z_i|^2 |z_j|^2}\right] \tag{54}$$

$$= \mathbb{E}\left[\frac{|z_1|^4}{U |z_1|^4 + 2\frac{(U-1)U}{2} |z_i|^2 |z_j|^2}\right] \tag{55}$$

$$= \frac{\frac{2}{U^2}}{U\frac{2}{U^2} + 2\frac{(U-1)U}{2}\frac{1}{U}\frac{1}{U}} \tag{56}$$

$$= \frac{\frac{2}{U^2}}{\frac{U+1}{U}} \tag{57}$$

$$= \frac{2}{U(U+1)} \tag{58}$$

The double sum on the denominator of eq.54 contains $\frac{(U-1)U}{2}$ double products.

Finally, we can compute eq.38 as:

$$\frac{1}{N}\mathbb{E}\left(\|\mathbf{v}\|^2\right) = \frac{1}{N}\sum_{q=1}^{Q}\left[\left(\frac{1}{U} - \frac{2}{U(U+1)}\right)1\right] \tag{59}$$

$$= \frac{1}{N}Q\frac{U-1}{U(U+1)} \tag{60}$$

$$= \frac{U-1}{U+1} \tag{61}$$

which is the mean energy per symbol of the AN component when Eve implements a matched filtering. It is exactly what we observe in the simulations.

$$\mathbb{E}\left[\gamma_{E,n}\right] = \frac{\alpha(U+1)(U+3)}{U\left[(U+1)\sigma_E^2 + (1-\alpha)\right]} \tag{62}$$

$$C_s = \log_2\left(1 + \frac{\alpha(U+1)}{U\sigma_B^2}\right) - \log_2\left(1 + \frac{\alpha(U+1)(U+3)}{U\left[(U+1)\sigma_E^2 + (1-\alpha)\right]}\right) \tag{63}$$