Validity of the Kronecker Model for MIMO Correlated Channels

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Abstract– **The goal of this paper is to investigate the validity of the Kronecker model for MIMO correlated channels. First, mathematical and equivalent propagation conditions of validity are detailed. In a second part, we address several areas where the Kronecker model is not valid. We also review the one-ring model and experimental results dealing with the validation of the Kronecker model. Based on these scarce results, the preliminary conclusion is that the Kronecker might not be valid for arrays larger than 2×2 when antenna correlations are high across both receive and transmit arrays. The paradox is that the Kronecker model has been developed and is being widely used to account for correlation. This is in strong contrast with the popular belief that the Kronecker model is valid in many indoor and outdoor cases.**

I. Introduction

A well-known result of space-time signal processing is that the average channel capacity grows linearly with the number of antennas if the fades between pairs of transmit-receive antenna elements are independent and identically Rayleigh-distributed (Rayleigh i.i.d.). In practice, however, the MIMO channel can deviate significantly from the i.i.d. assumption, namely because channels can be correlated [1]. In the last years, many papers[1] have dealt with MIMO signal processing and information theory for correlated channels, using the extraordinarily popular Kronecker model as a general model for correlation. This leads to believe that the Kronecker model is representative of many typical cases, irrespective of the channel and the antenna arrays.

Most papers using the Kronecker model cite [2]–[5] as a justification. However, the discussion in [3], based on the one-ring model, is not as conclusive as it may seem at first sight. The same remark holds for the experimental results of [4]. These points are discussed in this paper. Then, [6] illustrated through experimental results that the Kronecker model might provide large underestimations of the mutual information.

The goal of the present contribution is to evaluate the validity of the Kronecker model on a theoretical point of view, with regard to several applications and experimental results. Incidentally, we show that previous experimental or simulation results, obtained in particular cases, have been questionably extrapolated to a general rule. Section II details the mathematical and propagation-related conditions of validity of the Kronecker assumption. Section III addresses several applications, investigating whether the Kronecker model applies or not, and compares the derived conclusions with experimental results. In Section III, we also investigate a popular geometry-based model. This investigation is by no means an analysis of the validity of the one-ring model, but addresses the validity of the Kronecker representation for the geometry-based model under study. Indeed, one fact should be clear: analytical models of the MIMO channel are not *self-sufficient* (by contrast to physical models). They are convenient matrix representations of the channel which can be used e.g. in the analytical design of space-time codes. However, to use an analytical model in simulations, physical models or data to be used as input are still required (this is exemplified in Section II.A). Finally, we summarize a few preliminary guidelines relative to the use of the Kronecker model in the conclusion.

II. Theoretical Background

A. Covariance Matrix

In this paper, we deal only with identically Rayleigh distributed channels. That means that there is no coherent fixed component, and that all channels convey the same average energy, normalized to unity. Furthermore, to reduce the number of parameters, we restrict the discussion to 2×2 MIMO channels. This restriction is only made to simplify notations, and all considerations below apply similarly to arbitrary array sizes. As a consequence, the channel matrix $\mathbf{H}$ is expressed as:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}, \qquad (1)$$

and its *average* Frobenius norm is equal to 4 (this is not the case of the instantaneous Frobenius norm). Since, for Rayleigh fading, second-order statistics fully describe the multi-antenna channel, a general model of $\mathbf{H}$ is given by:

$$\mathrm{vec}(\mathbf{H}) = \mathbf{R}^{1/2}\,\mathrm{vec}(\mathbf{H}_w) \qquad (2)$$

where $\mathbf{H}_w$ is the spatially white (Rayleigh i.i.d.) MIMO channel; $\mathbf{R} = \mathbf{R}^{1/2}\mathbf{R}^{H/2}$ is the covariance matrix defined as $\mathbf{R} = E\left\{\mathrm{vec}(\mathbf{H})\,\mathrm{vec}(\mathbf{H})^H\right\}$, with "vec" being the operator stacking the matrix $\mathbf{H}$ into a vector columnwise; and the superscript ^{H} designates the conjugate transposition. Because all four

[1] The list of papers would be too long to cite them here. We invite the reader to refer to [1] for some of these papers.

channels are identically distributed (and normalized to provide unitary average energy), the covariance matrix **R** contains 6 different parameters, as defined below:

$$\mathbf{R} = \begin{bmatrix} 1 & r_1 & t_1 & s_1 \\ r_1^* & 1 & s_2 & t_2 \\ t_1^* & s_2^* & 1 & r_2 \\ s_1^* & t_2^* & r_2^* & 1 \end{bmatrix} \tag{3}$$

In (3), parameters r_1, r_2, t_1, t_2 have been known for many years, since they represent the correlations between channels at two receive (resp. transmit) antennas at one side, but originating from (resp. impinging on) the same transmit (resp. receive) antenna at the other side of the link. They are the classical correlation coefficients of diversity-based systems (MISO and SIMO). Hence, they are referred to in the following as *antenna* correlations. The two remaining parameters s_1 and s_2 are defined as the *diagonal* correlations, or *cross*-correlations. They represent the correlations between channels originating from and impinging on different antennas at each side of the link [7].

The model outlined by equations (2) and (3) is not self-sufficient, as it does not give any indication on how to fill the matrix **R** with quantitative parameters. To this end, experimental data or physical models (ray-tracing, geometry-based models, etc.) must be used in combination with (3). This is true for any analytical model, since, by essence, they only provide an analytical formalism to represent the MIMO channel matrix. Therefore, it does make sense to question the validity of a given analytical model with respect to a data source (experimental data, physical model, etc.), as carried out in Sections III.C and III.D.

A major drawback of expression (3) is that it is not easily tractable. As a first step towards simplification, it may be considered that, in many cases to be identified in the sequel, $r_1 = r_2$ and $t_1 = t_2$. Therefore, they can be denoted as r and t, and it is then possible to define 2×2 transmit (Tx) and receive (Rx) correlation matrices, $\mathbf{R}_t$ and $\mathbf{R}_r$, as to decompose any MIMO system into two "interconnected" MISO/SIMO subsystems. This decomposition has naturally lead to development of a simpler and less general model of the covariance matrix,

$$\mathbf{R}_K = \mathbf{R}_t \otimes \mathbf{R}_t \tag{4}$$

where ⊗ designates the Kronecker product, and

$$\mathbf{R}_t = \begin{bmatrix} 1 & t \\ t^* & 1 \end{bmatrix} \text{ and } \mathbf{R}_r = \begin{bmatrix} 1 & r \\ r^* & 1 \end{bmatrix}.$$

Replacing **R** by the expression of $\boldsymbol{R_K}$ in (2), the correlated channel matrix can be written as:

$$\mathbf{H} = \mathbf{R}_r^{1/2}\, \mathbf{H}_w\, \mathbf{R}_t^{1/2} \tag{5}$$

Expressions (4) and (5) are referred to as the Kronecker model. It can be seen that (5) is more tractable than (2), since the "vec" operator has been replaced by matrix multiplications.

B. Mathematical Validity Conditions

Mathematically, the Kronecker model is valid if and only if two conditions are jointly met, although contradictory statements can be found in the literature [5].

As stated above, the first condition is that the Tx (resp. Rx) correlation coefficients are (in magnitude) independent from the considered Rx (resp. Tx) antenna, so that $r_1 = r_2$ and $t_1 = t_2$. In [5], it is claimed that this first condition is the only one necessary for the Kronecker model to be valid. Yet, there is an additional condition, which is that the cross correlations must be equal to the product of Tx and Rx correlations, so **R** is further simplified by considering that $s_1 = r\,t$ and $s_2 = r^* t$, which yields (4). Note that for real-valued correlations, a single diagonal correlation can be defined as $s = s_1 = s_2 = r\,t$.

C. Propagation Validity Conditions

Both conditions identified above can be translated into propagation-related conditions. The first condition on the antenna correlations can be interpreted as the following: $r_1 = r_2$ if both antenna of the Tx array are located not too far from each other, and have the same radiation pattern and the same orientation. A similar condition should be fulfilled by the Rx array to ensure that $t_1 = t_2$. Once the first condition is met, the second condition is equivalent of having all angles-of-departure (AoDs) coupled with all angles-of-arrival (AoAs), so that the joint AoA-AoD spectrum is the product of the marginal spectra. In other words, AoDs and AoAs are statistically independent. It is often affirmed that this occurs when the immediate surroundings to each array are responsible for the correlation between its antennas, but have no impact on the correlation at the other end of the link. Note that this affirmation is not necessarily natural, although this might not be clear at first sight. Indeed, it implies more than a physical separation between both surroundings, but implicitly rules out any unique coupling between them.

D. Comparison between Mathematical and Propagation Validity Conditions

We have shown that the propagation-related conditions imply that mathematical conditions are met, irrespective of the antenna arrays, provided that the array dimensions are not unreasonably large. In other words, any environment with not too small spatial stationarity regions (so the first condition will be met for all realistic arrays) and independent DoA and DoD spectra will therefore yield a Kronecker channel matrix for all array sizes and/or configurations. Such propagation environments re defined as Kronecker-structured.

By contrast, the mathematical conditions do not imply that the environment is Kronecker-structured. That means that an environment providing a Kronecker channel matrix for given array configurations, might not provide this structure for all configurations. This is for example the case of any single-bounce channel, as discussed in Section III.

III. APPLICATIONS

A. MIMO with Different Antenna Patterns

This scenario is found for macroscopic arrays, or when antenna patterns differ (although antennas are co-located), i.e. for narrow-beam antennas or when mutual coupling is accounted for. In the first case, when the antennas at one link end are separated by a large distance, each will experience different shadow fading conditions. The antenna correlation at that link end (say, at the terminal) can clearly be zero, as both transmitted signals will experience very different channels. Yet, the Kronecker representation is not valid in this case, because the correlation at the other end, is not the same whether calculated from the first or the second terminal. For such systems, the first condition is evidently not met.

In the second case, when several antenna elements are located closely to each other, the electrical field generated by one antenna alters the current distribution on the other antennas. As a consequence, the radiation pattern and input impedance of each array element are disturbed because of the other elements. This effect is known as antenna or mutual coupling. Intuitively, antenna coupling can be seen as a distortion of the original radiation patterns. Because these patterns will be different (though symmetrical, see [8]) when antenna coupling is significant, the first condition is again not met. Note that, even if the channel without coupling is Kronecker-structured, the channel with coupling is not necessarily separable. Therefore, when antenna coupling is considered or, more generally, when antenna patterns differ across the array at one side of the link, the Kronecker model might not be applicable anymore.

B. Rayleigh-fading Dual-polarized Channels

Let us now consider a 2×2 dual-polarized slanted scheme, i.e. each array is made of two co-located antennas with orthogonal polarizations at $\pm$ 45°. It has been illustrated in [8] that a dual-polarized channel can be represented by the following matrix:

$$\mathbf{H} \approx \alpha \begin{bmatrix} 1 & \mu\,\mathrm{e}^{j\phi}/\chi \\ \mathrm{e}^{j\phi}/\chi & \mu \end{bmatrix} \tag{7}$$

where α is a random time-varying fading variable; ϕ is a random time varying angle uniformly distributed over $[0, 2\pi]$; μ and $\chi \in [0,1]$ are random time-varying variables which represent respectively the ratio of the propagation loss in orthogonal components and the cross-polar discrimination (XPD). In slanted schemes, it is evident that $\mu = 1$ due to the array symmetry. Furthermore, experimental results [9] suggest that, for large receive-to-transmit distances, or in large excess path-loss areas, α is Rayleigh distributed (the Ricean K-factor describing fading is very low), and that $\chi \approx 1$ (the XPD is about 0 dB). Note that since the K-factor and the XPD are strongly correlated [9], the Rayleigh behavior might be a sufficient condition to obtain both a low K-factor and $\chi \approx 1$. So, in these cases, the dual-polarized channel matrix reduces to:

$$\mathbf{H} \approx \alpha \begin{bmatrix} 1 & \mathrm{e}^{j\phi} \\ \mathrm{e}^{j\phi} & 1 \end{bmatrix} \tag{8}$$

In (8), the matrix elements are Rayleigh-fading. The receive and transmit antenna correlations are also equal to zero, owing to the random phase-shift between co- and cross-polar components ($E\{e^{j\phi}\} = 0$). By contrast, the diagonal correlations are both equal to 1. The channel covariance matrix is then equal to:

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

It is clear that such channels cannot be represented by the Kronecker representation (which would yield i.i.d. channels). Furthermore, it has been shown in [11] that covariance matrices such as (9) maximize the ergodic capacity, and provide higher capacity than i.i.d. channels at any given receive SNR. This interesting behaviour, which has been observed in experimental results [9] for large excess path-loss conditions[2], cannot be explained at all by the Kronecker assumption, but requires a full-covariance representation.

C. One-Ring Model

The one-ring model is a popular geometry-based stochastic model, which has been proposed in the context of MIMO systems by [3] and [12]. It is true that the first condition is met for usual antenna spacings and omnidirectional antennas. Still, the one-ring model is single-bounce, therefore clearly violating the second condition. To illustrate this affirmation, let us apply the one-ring model of [3] to a 2×2 uplink scenario, with broadside antennas at the base station (BS). We denote by δ the ratio of the range to the radius of the ring. The set of equations (10) details the different correlation coefficients:

$$\begin{aligned} r &= J_0\left[\frac{2\pi}{\lambda}\Delta_r\right] & s_1 &= J_0\left[\frac{2\pi}{\lambda}\left(\frac{\Delta_t}{\delta}+\Delta_r\right)\right] \\ t &= J_0\left[\frac{2\pi}{\lambda}\frac{\Delta_t}{\delta}\right] & s_2 &= J_0\left[\frac{2\pi}{\lambda}\left(\frac{\Delta_t}{\delta}-\Delta_r\right)\right] \end{aligned} \tag{10}$$

where λ is the wavelength, and Δ_t and Δ_r are the Tx/Rx antenna spacings. Only 4 parameters are needed, as we assume that the first condition is fulfilled. As far as the second one is concerned, the single-bounce assumption naturally violates it. When Δ_t and Δ_r are chosen as the smallest possible values to force the Tx and Rx correlations to zero, the cross correlations s_1 and s_2 are respectively equal to – 0.24 and 1. Clearly, this channel is not Kronecker-structured, and the relative error on the Frobenius norm of the difference between $\mathbf{R}_K$ and $\mathbf{R}$ [13] is equal in this case to 34 %. Are these results in contradiction with [3]? Actually, [3] only validates the Kronecker model in a particular scenario, considering that δ is very large (so that s_1 and $s_2 \cong r$) and that Δ_r is significantly larger than 0.5 λ, so that $r \cong 0$, as are thus s_1 and s_2. For this particular scenario, the covariance matrix of the one-ring model has indeed a Kronecker structure. However, one side of the link is totally decorrelated

[2] Note that the capacity results in [6] are normalized as to remove any path-loss effect on the SNR, so that the comparison with i.i.d. channels using such normalization is totally fair.

through large antenna spacing at Rx. That highlights the paradox of the Kronecker model, in that, although developed to represent correlation, it fails to fit the one-ring channel with significant correlations at both Rx and Tx. Interestingly enough, the same conclusion is reached for any single-bounce model, and, by extrapolation, for any real-world scenario where uniquely-linked propagation mechanisms dominate[3]. Whether these mechanisms are dominant or not is generally an open question, despite a number of experimental results presented below.

D. Experimental Data

Few experiments have truly investigated the validation of the Kronecker model, contrarily to the general belief. Before detailing these results, it must also be mentioned that any comparison relies on one or several metrics (mutual information, eigenvalue distributions, joint AoA-AoD spectrum, diversity order, etc.). Although the Kronecker model might adequately reproduce one given metric, it may simultaneously fail on another metric. Therefore, experimental validations described in the literature must be taken with caution, as they often rely on one particular metric. There is another reason why experimental validations should be taken cautiously. The MIMO channel matrix does not only depend on the propagation channel (i.e. the environment) but also on the configuration of the antenna array. This explains namely why the one-ring model has been associated with the Kronecker model in the past, since it was shown to be Kronecker-structured for a particular array configuration. We have shown that the one-ring model is however not Kronecker *per se*. The same remark applies with regard to experimental validations. It may be well possible that the experimental results validate the Kronecker, but this validation should *a priori* not be considered as a general property of the environment (as it is often the case). The results only hold true for the given range of antenna spacings that has been used in the experiment. The consequence is that the physical channel might not be Kronecker structured, but that the combination of the channel with the experimental antenna spacings could produce the Kronecker structure.

In [5], the Kronecker model is successfully validated in indoor microcellular and picocellular scenarios for 4×4 MIMO systems, based on the distributions of the eigenvalues. However, the closeness between measured and simulated distributions is not quantified. A more global analysis, based on the median error on each eigenvalue, yields errors below 1 dB for the largest eigenvalue, and 3.5 dB for the smallest eigenvalue in 90 % of the cases.

In [13], the Kronecker model is validated in an indoor environment for 2×2 and 3×3 setups, using a metric measuring the closeness of $\mathbf{R}$ and $\mathbf{R}_K$. The error metric varies from 5 to 8 %, which is considered to be small enough to accept the separability assumption.

[3] Uniquely-linked modes are not necessarily single-bounce, but are such that any AoD is linked to a single AoA and vice-versa. Uniquely-linked modes are however different from keyholes. By contrast, double-bounce channels may also present uniquely-linked modes, namely when the angle-spread at one side is very narrow.

In [4], 16×16 MIMO downlink experimental results in downtown New York City enable to validate the Kronecker model using the mutual information as a metric. However, it must be pointed out that the measured Rx and Tx correlations seem pretty low in these measurements. As pointed out in the sequel, these results should not allow concluding on the general validity of the Kronecker model.

In [14], the validity of the Kronecker model is investigated for 2×2, 4×4 and 8×8 indoor Rayleigh channels, based on different metrics. These results are, to our knowledge, the only ones presenting a full investigation on the validity of the Kronecker model. An interesting conclusion is that the validity of the Kronecker model decreases as the array sizes increase, which nuances the validation of [13]. Simulations at 20 dB SNR show that the Kronecker model generally underestimates the mutual information by more than 10 % for large arrays (8×8) and low capacity values (i.e. at high correlation levels). Again, we observe that the Kronecker model fails when correlations are high, although it is supposed to model correlated channels. Also, the diversity order is never well reproduced, even for small arrays.

IV. Conclusions

This paper has clearly detailed the conditions of validity of the popular Kronecker model for MIMO correlated channels, both on mathematical and propagation viewpoints. It is pointed out that two propagation conditions must be fulfilled for the Kronecker model to be *rigorously* valid for any antenna configuration. If one of these conditions is not met, it is still possible that the Kronecker model still be valid in some particular scenarios, strongly depending on the antenna spacings and array configurations. Therefore, it is really dangerous to conclude on the general Kronecker structure of a physical channel, since what is true for a given array configuration might not be true for another configuration.

Some channels (macroscopic MIMO, mutual coupling) are clearly not Kronecker-structured. For other scenarios, if there are dominant uniquely-coupled propagation modes, the channel matrix will be Kronecker-structured when sufficiently large antenna spacings at one side allow for low antenna correlation across the array at that side. Note that this condition imposes the antenna spacing to "be in the tail" of the correlation-vs.-spacing curve, so this is a stronger condition than just cancelling the antenna correlation at that side. This constitutes the paradox of the Kronecker model: unless it is rigorously valid, it appears to be approximately valid only for low-correlated channels (at least at one side), although it is widely used to account precisely for correlation effects. This paradox can be thought of as the direct consequence of the fact that there is more in MIMO than the addition of SIMO and MISO.

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