Time Reversal Compared To Inverse Filtering

Walid M. Dyab and Tapan K. Sarkar
L. C. Smith College of Engineering and Computer Science
Syracuse University
Syracuse, New York, USA
wmdyab@syr.edu

Magdalena Salazar-Palma
Department of Signal Theory & Communications
Universidad Carlos III de Madrid
Madrid, Spain

***Abstract*—Inverse filtering is the compensation of the magnitude and phase distortion caused by a linear time invariant system. Recently, time reversal is proposed as a new technique suitable for UWB systems. In this paper, the theory of electromagnetic time reversal is compared to the process of inverse filter design. From a signal processing point of view, time reversal is equivalent to a phase equalizer. Time reversal can compensate for the phase distortion in UWB, but at the same time it doubles the magnitude distortion. This fact is illustrated experimentally.**

I. Introduction

The first aim in this paper is to expose the real history of time reversal (TR) in electrical engineering. In the year 1957, TR was proposed by B. P. Bogert, from Bell Labs, as an automatic technique for the correction of delay-distortion of transmission networks [1]. A block diagram of the technique presented in that paper is shown in Fig. 1. Experiments were performed on data transmissions on a 5 KHz loop from Murray Hill, NJ, to Los Angeles, CA and back. The data was received in LA, recorded, reversed in time and retransmitted back to Murray Hill. Picture quality enhancement was achieved using this technique. A paper which is very similar to Bogert's paper appeared in the IBM journal in 1965 [2]. That paper deals with the problem of automatic distortion correction for efficient pulse transmission over telephone networks. In the abstract of [2], it is mentioned explicitly that time reversal systems compensate for the distortion in the *phase characteristic only*. The same idea appeared in a paper in the IEEE transactions on acoustics, speech and signal processing in 1974 [3]. In [3], the idea of time reversal was used to design digital filters with zero phase shifts. This idea was called "*noncausal digital filtering*". Successive papers based on this type of filtering appeared in the literature during the period 1982 to 1991 [4–7]. In [5], Daniel Harasty and Alan Oppenheim presented an approach based on non-causal filtering to solve the problem of *Television Signal De-ghosting*. It is not of much importance here to know whether the approach they presented was successful or not. However, the important thing to note is that there were attempts, long time ago, to use time reversal (inherent in non-causal filtering) to compensate for television signal impairments due to multipath propagation. This is exactly what many researchers are trying to introduce currently in personal wireless communications [8-9]. However, they introduce it as a new technology which was originated in acoustics in the early 1990's, and almost none of the new papers give any credit to the earlier work on time reversal such as Bogert's work [1]. The current research efforts applying TR in wireless communications even miss a simple definition of the procedure in the frequency domain. This is given in the next section.

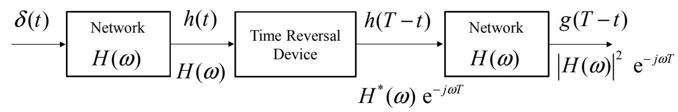

Figure 1: Block diagram of Bogert's experiment in 1957.

II. Inverse Filtering and Time Reversal

Assume a linear time invariant system with an impulse response $h(t)$ as shown in Fig. 1. In the frequency domain, an input message is modified by the complex function $H(\omega)$. To design an inverse filter that compensates for the magnitude and phase distortions caused by $H(\omega)$, one can simply design a filter whose frequency response is given by:

$$\begin{aligned} H^{-1}(\omega) &= \frac{1}{\Im\{h(t)\}} = \frac{1}{H(\omega)} = \frac{1}{R(\omega)+jX(\omega)} \\ &= \frac{1}{R^2(\omega)+X^2(\omega)}\left(R(\omega)-jX(\omega)\right) \end{aligned} \tag{1}$$

where $R(\omega)$ and $X(\omega)$ are the real and imaginary parts of $H(\omega)$ respectively, and $\Im\{\bullet\}$ denotes the Fourier transform. In time domain, the inverse filter response is given by:

$$h^{inv}(t) = \Im^{-1}\left\{H^{-1}(\omega)\right\} = h(-t) \otimes \Im^{-1}\left\{\frac{1}{|H(\omega)|^2}\right\} \tag{2}$$

where $|H(\omega)| \neq 0 \;\forall\omega$. Practically, $h(t)$ is causal, and this directly implies that $h(-t)$ is non-causal. Therefore a delay is usually added to $h(-t)$ as shown in Fig. 1, so it becomes $h(T-t)$. If this delay is acceptable in the real-time operation of the system, there will be no problem inverting any function even if it is a non-minimum phase function [10]. Therefore for a signal $x(t)$ input to the original filter $h(t)$, the output of the inverse filter is given by:

$$\hat{x}(t) = x(t) \otimes h(t) \otimes h(T-t) \otimes \Im^{-1}\left\{\frac{1}{|H(\omega)|^2}\right\} \tag{2}$$

Whereas the output of the TR system is given by:

$$\hat{x}_{TR}(t) = x(t) \otimes h(t) \otimes h(T-t) \tag{3}$$

Comparing equations (2), and (3) simply proves the fact that time reversal is an inverse filter which compensates for the phase-distortions only, exactly as stated in the literature more than 50 years ago. Therefore, it is not clear why there exist currently research efforts on TR in wireless communications which expect more than just phase equalization!

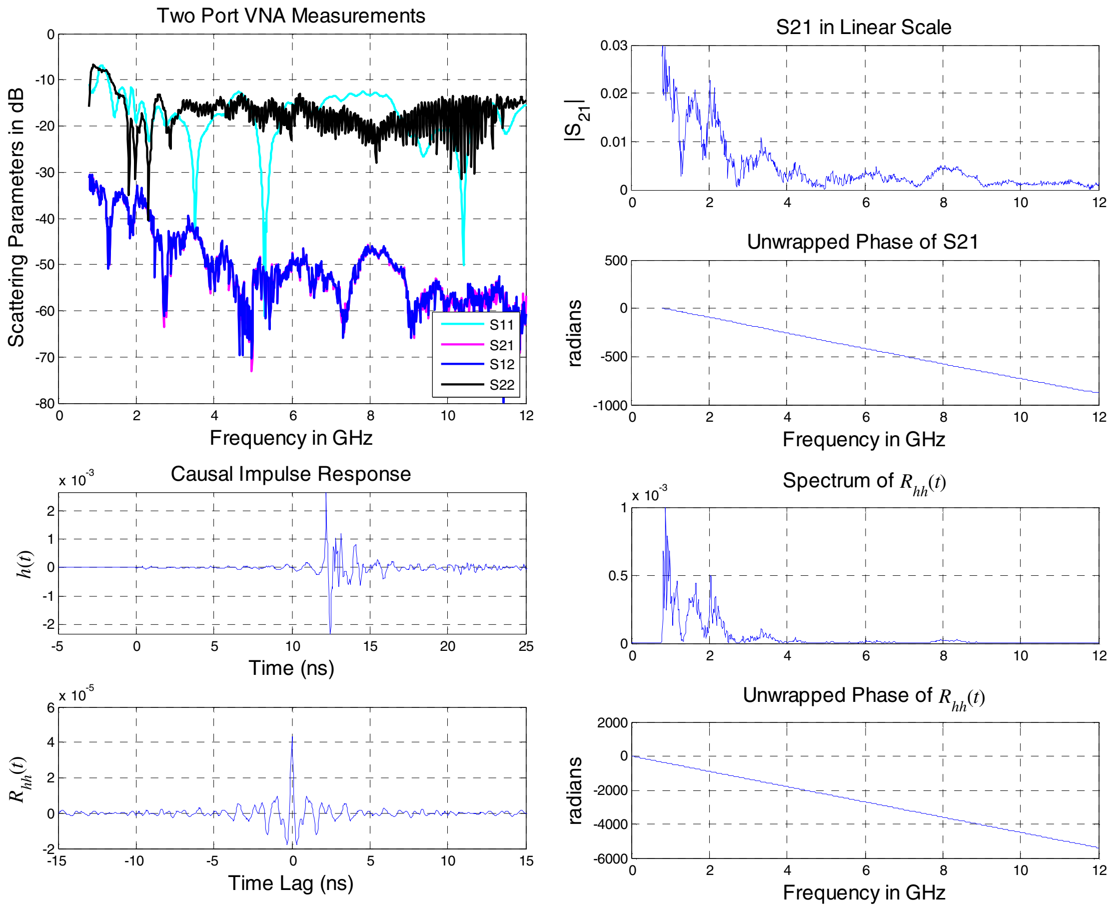

Figure 2: Experiment results of a TR procedure applied on an indoor wireless channel with century BW Cone-Blade antennas.

III. EXPERIMENTAL RESULTS

The conclusion presented in the last section is verified experimentally. A vector network analyzer with two ports is connected to two century bandwidth Cone-Blade antennas resulting in a 175% BW channel. The antennas are located in a lab room as an indoor channel. A frequency sweep is done from 800 MHz to 12 GHz. The causal time domain impulse response is calculated from the complex S_{21} and S_{12} data (which are equal due to the system reciprocity) using inverse Fourier transform. Causality of the calculated impulse responses is enforced by relating the real and imaginary parts of the measured data by the Hilbert transform. The results are shown in Fig. 2. The time domain response $h(t)$ is calculated from the S_{21}, then flipped in time, and convolved with the reponse calculated from S_{12}. This represents the TR output $R_{hh}(t)$ in Fig. 2. The function $R_{hh}(t)$ in the frequency domain has a perfectly linear phase response, but at the same time has a magnitude which is equal to the square of S_{21}.

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