NTNU – Trondheim
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For questions during the exam:
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Exam in TTK4130 Modeling and Simulation

Thursday, June 6th 2013
09:00 – 13:00

Permitted aids (code A): All written and handwritten examination support materials are permitted.

Note: A Norwegian text is appended.

Answers in English, Norwegian, or a mixture of the two accepted.

Grades available: As specified by regulations.

Problem 1 (15 %)

Consider the open tank system in Figure 1. Assume that the volumetric flow into the tank, q_i, is known, that the fluid is incompressible with (constant) density ρ, and that the flow exiting the tank is given by $q_o = C\sqrt{p - p_0}$, where C is a constant, p is the pressure in the bottom of the tank, and p_0 is the pressure outside the tank.

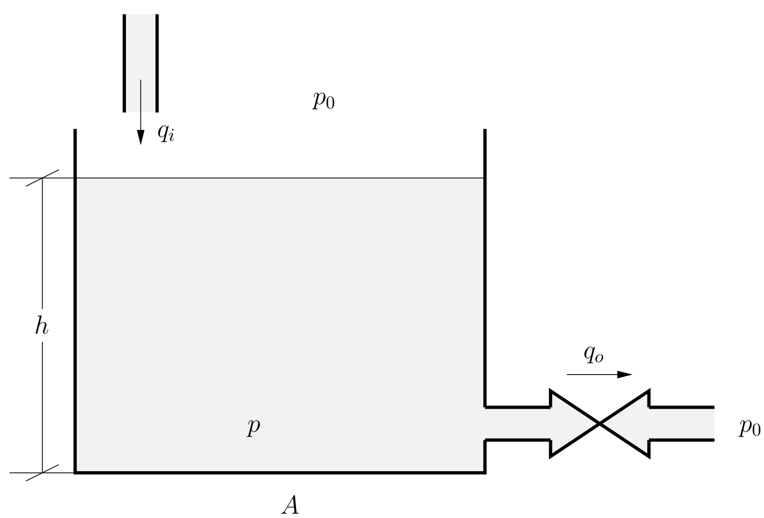

Figure 1: Open tank system

(5 %) (a) Set up a model for the level in the tank, h.

(10 %) (b) We now want to establish a passivity result for the tank model. Let the "liquid pressure" $y = p - p_0 = \rho g h$ be an output, and choose (half) the potential energy in the tank as a storage function,

$$V(h) = \frac{1}{2}mgh.$$

Choose an appropriate input u, and show that the tank model is passive with this input and $y = p - p_0$ as output.

Problem 2 (40 %)

(10 %) (a) A closed tank with constant volume V is completely filled with a *compressible* fluid with bulk modulus β. The fluid exits the tank through a valve, where we assume turbulent flow through the valve. Assume that the pressure p and density ρ are spatially constant in the tank, and that the density is related to the pressure as $\rho = ap + b$, where we assume a and b constant.

A model for the pressure is given as

$$\dot{p} = -C\sqrt{\frac{p - p_0}{ap + b}}.$$

Explain how this model is deduced from relevant balance laws, definitions and valve equations, and suggest an expression for the constant C.

(8 %) (b) Linearize the model around a pressure p^*. Explain how this model can give problems when using explicit solvers with automatic adjustment of step size.

(7 %) (c) Despite the potential problems, we want to use a method described by the following Butcher array to simulate the system:

$$\begin{array}{c|ccc} 0 & & & \\ 1/2 & 1/2 & & \\ 1 & -1 & 2 & \\ \hline & 1/6 & 2/3 & 1/6 \end{array}$$

Write "pseudo-code" for integrating the system in (a) one time-step using this method. Is this an implicit or explicit method?

(10 %) (d) Calculate the stability function for the Runge-Kutta method in (c).

(5 %) (e) What is the order of the Runge-Kutta method in (c)? Explain how you reason.

Problem 3 (20 %)

Consider a system consisting of three identical beads of mass m. The beads are sliding without friction on a fixed, horizontal circular hoop, with radius r, and are connected by four identical, massless springs with spring constant k. The springs are attached to the beads and to a fixed point as shown in Figure 2. Assume that the springs are unstretched/uncompressed in the positions in the figure, and that they stretch/compress along the hoop/circle.

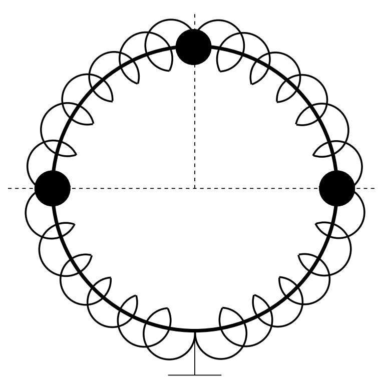

Figure 2: Three spring-connected beads on a hoop. The beads move along the horizontal, circular hoop. Two of the springs have one end attached to a fixed point.

(10 %) (a) Choose appropriate generalized coordinates. What are the kinetic and potential energy as function of these coordinates?

(10 %) (b) What are the equations of motion for this system?

Problem 4 (25 %)

(8 %) (a) Given the matrix

$$\mathbf{R} = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ -1 & 0 & 1 \end{pmatrix}.$$

Show that this is a rotation matrix. What simple rotation does this matrix correspond to?

(8 %) (b) A coordinate system b with orthogonal unit vectors $\vec{b}_1$, $\vec{b}_2$ and $\vec{b}_3$ along the coordinate axes, undergoes a rotation. The resulting coordinate system is called c and has orthogonal unit vectors $\vec{c}_1$, $\vec{c}_2$ and $\vec{c}_3$ along the coordinate axes. Write the rotation matrix as

$$\mathbf{R}_c^b = \begin{pmatrix} \mathbf{n} & \mathbf{s} & \mathbf{a} \end{pmatrix}.$$

Give an interpretation of the column vectors $\mathbf{n}$, $\mathbf{s}$ and $\mathbf{a}$ in terms of the orthogonal unit vectors mentioned above.

(9 %) (c) A rigid body rotates with angular velocity $\vec{\omega}_{ib}$ about the origin of an inertial frame (i). Two fixed points in the rigid body has coordinates $\mathbf{r}_1^b = (1, 0, 0)^\mathsf{T}$ and $\mathbf{r}_2^b = (0, 0, 1)^\mathsf{T}$ in the body-fixed coordinate system (b). Assume the velocity in the first point is $\mathbf{v}_1^b = (0, 3, 5)^\mathsf{T}$. What is the x-component of the velocity, $\mathbf{v}_2^b$, in the second point?