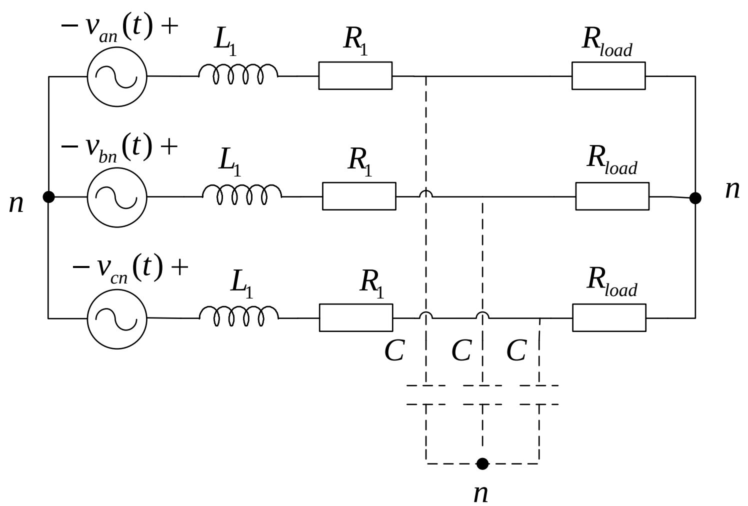
1 THREE-PHASE SYSTEMS

a) Explain two of the main advantages of using three-phase systems compared with single-phase systems for generation and transmission of electric energy.

Consider the circuit below. You can assume without proof that the neutral points marked "n" are at the same potential (this is always true for balanced three-phase systems). The capacitors are marked with dashed lines since they will only be used in problems d) and e).



$$v_{an}(t) = 100\sin(\omega t)$$

$$v_{bn}(t) = 100\sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{cn}(t) = 100\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$L_1 = 10\text{ mH} \quad R_1 = 1\,\Omega$$

$$R_{load} = 10\,\Omega \quad f = 50\text{ Hz}$$

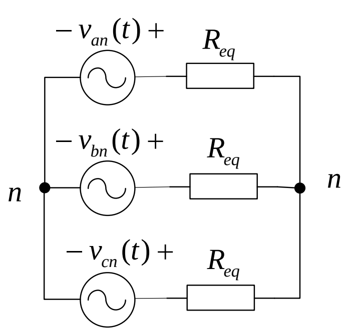
b) What is the magnitude of the line-to-line RMS voltage of the three-phase voltage source?

Assume in c) that the capacitors marked with dashed lines are disconnected.

c) Find the total active and reactive power delivered by the three-phase voltage source

Assume now that the capacitors are connected to the circuit

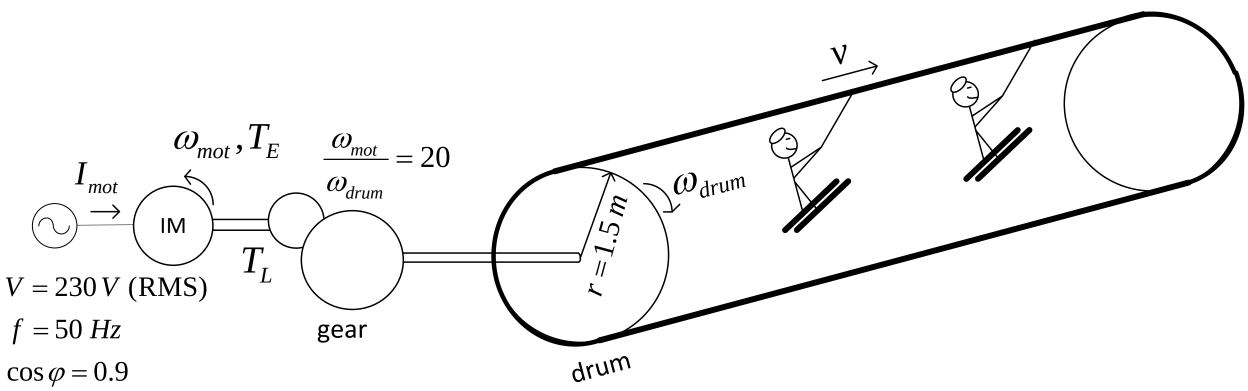
d) Find a value of the capacitance *C* that makes the reactive power from the three-phase voltage source equal to zero.

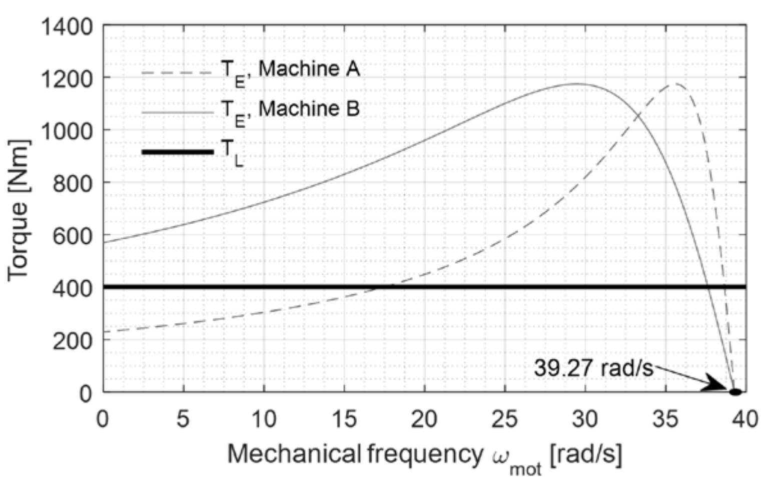


e) With your choice of *C* we can represent the three-phase load by an equivalent resistance R_{eq} as shown in the figure to the left. What is the numerical value of R_{eq} ?

2 SKI LIFT POWERED BY INDUCTION MOTOR

In this task we will analyze the ski lift in the following figure. The ski lift consists of a rotating drum with radius *r=1.5* m, where the ski lift wire is connected on the periphery as shown. The drum is rotating with rotational speed ω_{drum}. The drum is attached by a shaft to a mechanical gearbox, which again is connected to the motor shaft. The induction motor is connected to a 50 Hz voltage supply with constant voltage $V = 230\ \mathrm{V\ (rms)}$ (single-phase) and $\cos\varphi = 0.9$.





The load torque can be modelled as constant $T_L = 400\ \mathrm{Nm}$, referred to the motor side of the gear. We have two motors that we can use: A and B. Their torque-speed curves are shown to the left together with the load torque (thick line).

a) Mention one of the main challenges with start-up of induction machines. Which of the machines, A or B, would you recommend for the ski lift, and why?

b) With your choice of machine (A or B), find the following quantities (neglect all losses):

I. The number of poles of the induction machine

II. The power delivered to the ski lift

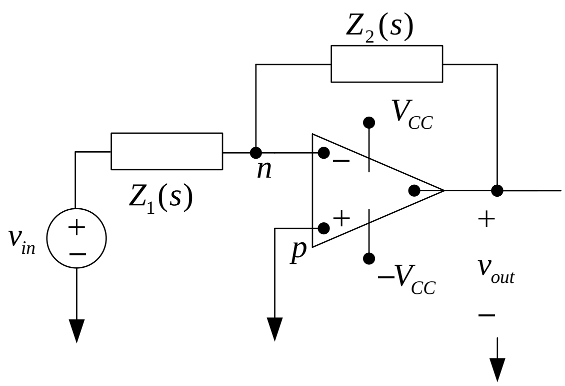
III. The current I_{mot} supplied to the motor

IV. The speed *v* of the persons taking the ski lift.

c) Assume the ski lift is operated in the condition from problem b) when $t < 0$. At $t = 0$ all persons leave the ski lift instantaneously, therefore the load torque T_L drops from 400 to 0 Nm. The system will then undergo a step response and stabilize at a new speed given by the torque-speed curve. Find the expression for $v(t)$ for $t \geq 0$. Assume the following:
- The motor torque-speed curve is linear at the nominal operation range.
- The total moment of inertia is $J_{tot} = 100\ kg \cdot m^2$ referred to the motor side of the gearbox.

d) The ski lift owner wants to control the speed *v* independent of the load torque. Suggest a modification to the system in order to achieve this (make a sketch). Explain your solution briefly.

3 OP-AMP BASED HIGH PASS FILTER DESIGN



Consider the active filter in the figure above with unknown impedances Z_1 and Z_2.

a) We want to design a first-order high-pass filter. What components must be included in Z_1 and Z_2 in this case? Redraw the circuit with these components.

b) The capacitor in the filter should be 250 *nF*. Find the parameter values of the other components if we want to have the cutoff frequency equal to $f_c = 4\ kHz$ and a passband gain of 8.

In problem c) and d), assume $v_{in}(t) = 2.5\cos(\omega t)$, where ω is unknown. Assume stationary conditions.

c) Assume the op.amp. operates in its linear region, i.e. is not saturated.

Find the output voltage $v_{out}(t)$ when: *I*) $\omega = \omega_c$, *II*) $\omega = \dfrac{\omega_c}{8}$ *III*) $\omega = 8\omega_c$

d) What is the smallest value of supply voltage V_{cc} that will cause the op amp to operate always in its linear region?

In case you do not solve a) and/or b), the following transfer function can be used to solve c) and d):

$\dfrac{V_{out}}{V_{in}}(s) = \dfrac{6}{3500s+1}$ (NB: this transfer function is not necessarily related to the answers in a) and b))

4 DC-DC CONVERTER DESIGN FOR PHOTOVOLTAIC MAXIMUM POWER POINT TRACKING

In this problem, we want to charge a battery from a photovoltaic (PV) module. The battery voltage is $V_{batt} = 10\,V$ (DC), while the PV-module voltage depends on the current and on the irradiation level. We will place a buck (step-down) DC/DC-converter between the PV-module and the battery. We will assume a converter without losses (no resistance).

The table below shows the average values of the power, current and voltage when the module is operated at the Maximum Power Point (MPP) for three different irradiation levels. The cells with question marks are related with problem b) below.

Irradiation level	P_{MPP}	I_{PV}	V_{PV}	D	I_{batt}
1000 W/m^2	2.416	0.154	15.69	?	?
660 W/m^2	1.627	0.114	14.27	?	?
370 W/m^2	1.238	0.081	13.6	?	?

a) Provide a schematic of the solution from the PV module to the load. Include a schematic of the buck converter (step-down converter).
b) If we operate the PV-module in Maximum Power Point condition at all irradiation levels, calculate the required duty cycle and average battery current. That is, fill in the cells with question mark in the table.
c) Draw the inductor voltage and current for one switching period (only waveforms, no numerical values needed).

Assume now that the irradiation level is 1000 W/m^2. We want to design the buck converter to have a peak-to-peak ripple current ΔI_L that is 10 % of I_{pv} at this condition. The switching frequency is *20 kHz*.

d) Calculate the required inductance of the buck converter

APPENDIX: FORMULAS

Inductance and capacitance

$$v_L = L\frac{di_L}{dt} \ , \ i_C = C\frac{dv_C}{dt} \ , \ X_L = \omega L \ , \ X_C = \frac{1}{\omega C}$$

Phasors and complex power

$$X\cos(\omega t+\theta) \Leftrightarrow Xe^{j\theta} \quad , \quad S = VI^* = P + jQ$$

Electromagnetism:

$$\varepsilon = N\frac{d\varphi}{dt} \ , \ NI = \Re\varphi \, , \ \Re = \frac{l}{\mu A} \ , \ \varphi = BA$$

Trigonometrics

$$\cos(2x) = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

Three-phase

$$|V_{LL}| = \sqrt{3}|V_{ph}|$$

Electrical machines

$$f_{el} = \frac{p}{2} f_{mech}$$

Cut-off frequency

$$|H(j\omega_c)| = \frac{|H_{\max}|}{\sqrt{2}}$$

DC-machine

$$E_a = K\varphi\omega \qquad T = K\varphi I_a$$

Induction (asynchronous) machine

$$\omega_{mech} = (1-s)\omega_s$$

DC-DC converters

Buck (step-down): $D = \frac{V_{out}}{V_{in}}$

Boost (step-up): $\frac{V_{out}}{V_{in}} = \frac{1}{1-D}$

Inverters

Single-phase: $m = \frac{\sqrt{2}|V_{ac}|}{V_{dc}}$

Three-phase: $m = \frac{2\sqrt{2}|V_{LL}|}{\sqrt{3}V_{dc}}$

Mechanics

$$P = T\omega \qquad P = F\cdot v \qquad v = \omega r \qquad E_k = \frac{1}{2}mv^2$$

$$T_{mot} - T_{load} = J\frac{d\omega}{dt}$$

Laplace transforms

Constant: $\mathcal{L}(K\cdot f(t)) = K\cdot F(s)$

Step response: $\mathcal{L}(u(t)) = \frac{1}{s}$

Exponential: $\mathcal{L}(e^{at}) = \frac{1}{s-a}$

s-shift: $\mathcal{L}\{e^{-at}\cdot f(t)\} = F(s+a)$

Sine: $\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2+\omega^2}$

Cosine: $\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$

Damped sine: $\mathcal{L}\{e^{-at}\sin(\omega t)\} = \frac{\omega}{(s+a)^2+\omega^2}$

Damped cosine: $\mathcal{L}\{e^{-at}\cos(\omega t)\} = \frac{s+a}{(s+a)^2+\omega^2}$