

1 RLC CIRCUIT

Figure 1 shows a RLC-circuit. The input voltage is $v_{in}(t)$, while the output voltage $v_o(t)$ is defined as the voltage across the LC-branch.

Assume $v_{in}(t) = 10\cos(\omega t)$ throughout the entire problem. $R = 1\Omega$ and $L = 1\text{ mH}$, while C is unknown.

We will first assume this circuit is used as a filter with input $v_{in}(t)$ and output $v_o(t)$, and $H(s) = \frac{V_o(s)}{V_{in}(s)}$

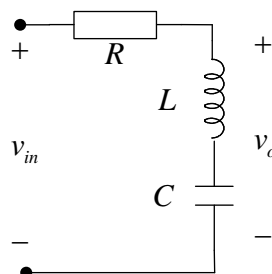


Figure 1 - RLC-circuit

- What kind of filter does $H(s)$ represent? (lowpass, highpass, band-pass or band-stop)? Give reasons for your answer.
- We want $v_o(t)$ to have as small amplitude as possible when the frequency is 250 Hz. Find the value of C that minimizes the amplitude of $v_o(t)$ at this frequency. What is the value of the minimum amplitude?

Assume now the frequency is 50 Hz. If you did not solve b), use $C = 1\text{ mF}$ in problems c) and d).

- How much active and reactive power is supplied from the voltage source? Is the circuit inductive or capacitive (give reasons for your answer)?

Assume now we connect another impedance Z_{load} in parallel to the LC branch as shown in Figure 2.

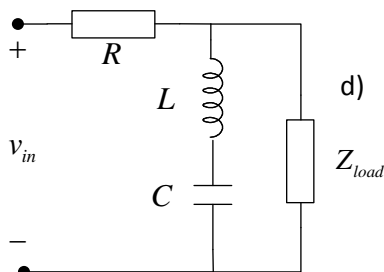
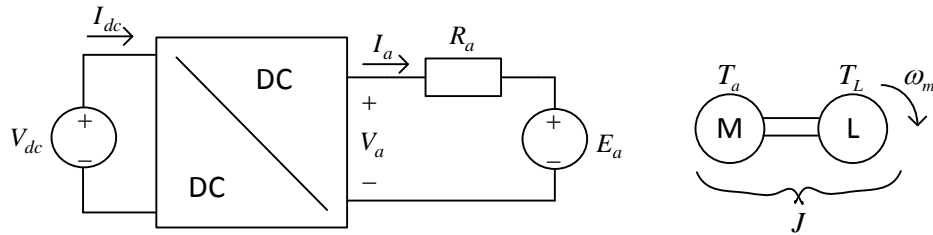


Figure 2 – Adding load to Figure 1

- Find the value of Z_{load} that maximizes the active power consumption in Z_{load} .

2 DC-MOTOR DRIVEN BY BATTERY AND DC/DC CONVERTER



The figure shows a battery with voltage V_{dc} connected to a DC-motor with resistance R_a and induced voltage E_a through a DC/DC-converter. The DC/DC-converter control system can set the required duty-cycle D defined as $D = \frac{V_a}{V_{dc}}$. The mechanical load has a load torque T_L .

- a) The field winding is not shown in the figure. Make a complete schematic of a shunt-excited DC-machine, including the field winding resistance R_f and field winding inductance L_f .

Assume in b) and c) that $T_L = k_L \omega_m$, where ω_m is the rotational speed of the shaft. The system has no mechanical gearbox. In problems b) and c), assume the motor operates in steady-state, i.e. no acceleration. Some useful formulas for the DC-motor is included in appendix.

Numerical values: $R_a = 0.1 \Omega$, $k\phi = 0.3 \frac{Vs}{rad}$, $k_L = 0.1 \frac{Nms}{rad}$ and $V_{dc} = 100 V$

- b) What duty-cycle is needed in order to have a shaft speed of $\omega_m = 200 \text{ rad/s}$?
 c) Find an expression for the motor speed ω_m expressed by k_L, D, V_{dc}, k, R_a , as well as the motor constant k and the flux ϕ (these are the only variables you should include in the expression)

Assume now that the load torque is zero, $T_L = 0$. Assume also the system has a total moment of inertia equal to J . At $t = 0$, the DC/DC-converter makes a step in duty cycle from $D = 0$ to $D = D_0$.

- d) Find the expression for the motor speed $\omega_m(t)$ for $t \geq 0$ (only variables, no numerical values).

3 CIRCUIT CALCULATIONS INCLUDING IRON CORE

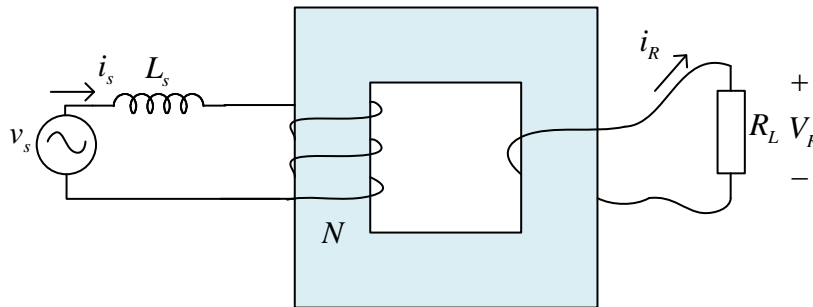


Figure 3 – Voltage source, iron core and load resistance

A resistor is connected to a voltage source through an iron core as presented in Figure 3. The core has $N = 3$ windings on the voltage source side, and a single winding at the load resistor side. We make the following assumptions:

- All induced magnetic field is kept within the core
- The iron is ideal, i.e. $\mu_r = \infty$
- The core will not saturate

- Explain why we can treat the iron core as an ideal transformer with ratio N .
- If i_s is positive, is i_R positive or negative? Explain why.

If the frequency is $f = 50 \text{ Hz}$, $v_s = 10 \text{ V (RMS)}$ and $R_L = 0.5 \Omega$, the current i_s has an RMS-value of 2 A .

- Find the numerical value of the inductor L
- Derive an expression for the power consumed by the resistor R_L . No numerical values, only with symbols (v_s, L_s, N, R_L, f)

4 DIODE RECTIFIER

Figure 4 shows a diode rectifier where the AC-voltage is a triangular wave. In problem a) and b), the DC-side is a constant current source I_{dc} , while in problem c) and d) it is represented by a resistor R .

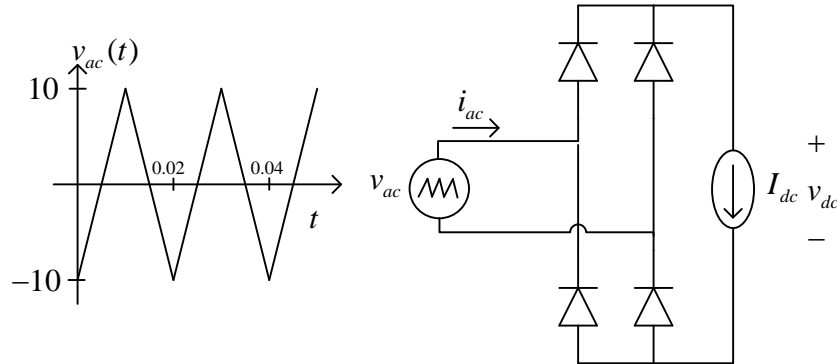


Figure 4: Diode rectifier supplied by a triangular voltage source. Constant DC-current

Assume $I_{dc} = 5 \text{ A}$

- Find and sketch the DC-side voltage $v_{dc}(t)$ and the AC-side current $i_{ac}(t)$ for $0 \leq t \leq 0.04 \text{ s}$.
- What is the average power delivered from the AC voltage source?

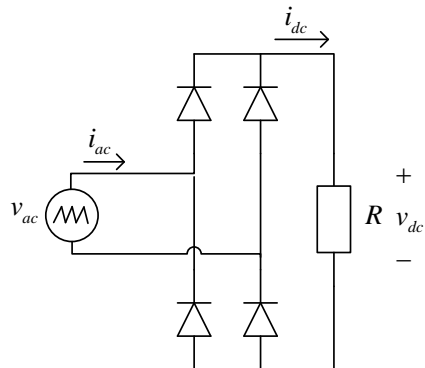


Figure 5 - Diode rectifier supplied by a triangular voltage source. Resistive DC-load

Now the constant current source is replaced with a resistor $R = 2 \Omega$ as shown in Figure 5.

- Find and sketch the DC-side current $i_{dc}(t)$ and AC-side current $i_{ac}(t)$ for $0 \leq t \leq 0.04 \text{ s}$.
- What is the average power delivered from the AC voltage source?

APPENDIX: FORMULAS

Inductance and capacitance

$$v_L = L \frac{di_L}{dt}, \quad i_C = C \frac{dv_C}{dt}, \quad X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$

Phasors and complex power

$$X \cos(\omega t + \theta) \Leftrightarrow X e^{j\theta}, \quad S = VI^* = P + jQ$$

Electromagnetism:

$$\varepsilon = N \frac{d\varphi}{dt}, \quad NI = \Re \varphi, \quad \Re = \frac{l}{\mu_0 \mu_r A},$$

$$\varphi = BA, \quad \mu_0 = 4\pi \cdot 10^{-7}$$

Trigonometrics

$$\cos(2x) = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

Three-phase

$$|V_{LL}| = \sqrt{3} |V_{ph}|$$

Electrical machines

$$f_{el} = \frac{p}{2} f_{mech}$$

Cut-off frequency

$$|H(j\omega_c)| = \frac{|H_{\max}|}{\sqrt{2}}$$

DC-machine

$$E_a = K\varphi\omega \quad T_a = K\varphi I_a$$

Induction (asynchronous) machine

$$\omega_{mech} = (1 - s)\omega_s$$

DC-DC converters

$$\text{Buck (step-down): } D = \frac{V_{out}}{V_{in}}$$

$$\text{Boost (step-up): } \frac{V_{out}}{V_{in}} = \frac{1}{1 - D}$$

Inverters

$$\text{Single-phase: } m = \frac{\sqrt{2} |V_{ac}|}{V_{dc}}$$

$$\text{Three-phase: } m = \frac{2\sqrt{2} |V_{LL}|}{\sqrt{3} V_{dc}}$$

Mechanics

$$P = T\omega \quad P = F \cdot v \quad v = \omega r \quad E_k = \frac{1}{2}mv^2$$

$$T_{mot} - T_{load} = J \frac{d\omega}{dt}$$

Laplace transforms

$$\text{Constant: } \mathcal{L}(K \cdot f(t)) = K \cdot F(s)$$

$$\text{Step response: } \mathcal{L}(u(t)) = \frac{1}{s}$$

$$\text{Exponential: } \mathcal{L}(e^{at}) = \frac{1}{s - a}$$

$$\text{s-shift: } \mathcal{L}\{e^{-at} \cdot f(t)\} = F(s + a)$$

$$\text{Sine: } \mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2 + \omega^2}$$

$$\text{Cosine: } \mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2 + \omega^2}$$

$$\text{Damped sine: } \mathcal{L}\{e^{-at} \sin(\omega t)\} = \frac{\omega}{(s + a)^2 + \omega^2}$$

$$\text{Damped cosine: } \mathcal{L}\{e^{-at} \cos(\omega t)\} = \frac{s + a}{(s + a)^2 + \omega^2}$$

