



Contact during exam:

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Complex Analysis (TMA4175)

2012 May 29

Time: 09:00 – 13:00

Allowed materials:

One A5 yellow sheet stamped by the department with students notes. Calculator HP30S

**Problem 1** Find all values of  $(-2 + 2i)^{1/3}$  and plot them.

**Problem 2** For each real value of  $a$  plot the circle  $\{z = x + iy; x^2 + y^2 = ax\}$  and find its image under the mapping  $w = 1/z$ .

☐ **Problem 3** For each  $n = 0, \pm 1, \pm 2, \dots$  find the residues at infinity of the function  $z^n e^{1/z}$ .

**Problem 4** Find the integral

$$\int_0^\pi \frac{d\phi}{a + \cos \phi}, \quad a > 1.$$

**Problem 5** Find the integral

$$\int_0^\infty \frac{x^2 + 1}{x^4 + 1} dx.$$

**Problem 6** Given  $\alpha \in (0, 2\pi)$  find the image of the half strip  $\{z = x+iy; x < 0, 0 < y < \alpha\}$  under the mapping  $w = e^z$ .

**Problem 7** Find for which  $a$  the function  $u(x, y) = x^2 + ay^2 + 2ax$  is harmonic and find its harmonic conjugate.

**Problem 8** Let  $f$  be an entire function such that  $\operatorname{Re} f(z) > -1$ ,  $z \in \mathbb{C}$ . Prove that  $f(z) = \text{Const}$

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