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Eksamen i TMA4175 Kompleks Analyse
Dato: Tirsdag 28. Mai, 2013
Tid: 09.00 - 13:00
Hjelpemidler: Kode A.
Sensur: 18. Juni 2013

Problem 1

Let f be an entire function and assume that $|f(z)| \leq |z|^{10}$ for all $z \in \mathbb{C}$.

a) Prove that $f^{(n)}(0) = 0$ for all $n \geq 11$.

b) Show that f is a polynomial of degree less than or equal to 10.

Problem 2

Let $p(z) = z^3 + 3z^2 + 17z + 50$. Show that p has at least one zero in $\{z \in \mathbb{C}; |z| \leq 10.\}$

Problem 3

Let $\gamma(t) = 4e^{it}, 0 \leq t \leq 2\pi$. Find

$$\int_\gamma \left(\frac{e^z - 1}{z}\right)\left(\frac{1}{z^4 + 3 + 3i}\right) dz$$

(Do not try to simplify the answer.)

Problem 4

Find

$$\int_0^\infty \frac{x \sin x}{x^4 + 1} dx$$

(Show all estimates)

Problem 5

Find a conformal map from $D = \{z \in \mathbb{C}; -1 < \mathrm{Re}(z) < 1\}$ to the unit disc.

Problem 6

Assume that f is analytic on the unit disc, $|f(z)| \leq 1, f(0) = 0$ and $f'(0) = 0$. Show that $|f(z)| \leq |z|^2$ for all z in the unit disc and if $|f(z_0)| = |z_0|^2$ for some $0 < |z_0| < 1$, then $f(z) = e^{i\theta} z^2$.

Problem 7

True or false? Give a short explanation.

a) Let $u(x, y) = x^3 - 2xy$. Can we find a function v such that $u + iv$ is analytic in $\mathbb{C}$?

b) Let $D = \mathbb{C} \backslash \gamma$, where $\gamma(t) = (t, t^2), 0 \leq t < \infty$. Does there exist a non constant bounded analytic function on D?

c) Can we find a non constant analytic function f in the unit disc such that $f(1/n) = 0, n = 2, 3, 4, \ldots$

d) Let $u(e^{it}) = |\cos t|$ when $0 \leq t \leq 2\pi$. Is it possible to find a harmonic function $\hat{u}$ on the unit disc, continuous on the closed disc, such that $\hat{u}(e^{it}) = u(e^{it})$?