Modsim cheat sheet

Simple Rotations

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Positive Real

$H(s)$ positive real iff

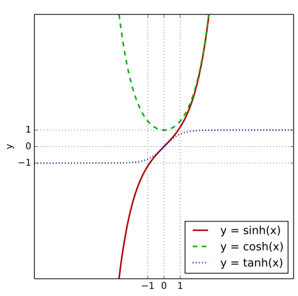
1. No poles with real part > 0
2. $H(s)$ real for all positive and real s
3. $\mathrm{Re}[H(s)] \geq 0$ for all $\mathrm{Re}[s] > 0$.

Euler's Formula

$$e^{ix} = \cos(x) + i\sin(x)$$

Hyperbolic Trig

$$\sinh(\sigma + j\omega) = \sinh\sigma\cos\omega + j\cosh\sigma\sin\omega$$
$$\cosh(\sigma + j\omega) = \cosh\sigma\cos\omega + j\sinh\sigma\sin\omega$$

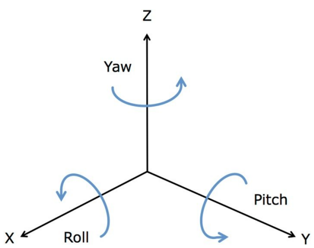


Skew Symmetric Matrix

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \iff \mathbf{a}^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

- $u \times v$ corresponds to $\mathbf{u}^\times \mathbf{v}$
- $(\mathbf{a}^\times)^\top = -(\mathbf{a}^\times)$

Roll Pitch Yaw



Dot Product

- $\vec{v} \cdot \vec{u} = |v||u|\cos\theta$
- $\mathbf{v} \cdot \mathbf{u} = \mathbf{u}^\top \mathbf{v}$

Transpose

1. $(\mathbf{A} + \mathbf{B})^\top = \mathbf{A}^\top + \mathbf{B}^\top$
2. $(\mathbf{AB})^\top = \mathbf{B}^\top \mathbf{A}^\top$
3. $(c\mathbf{A})^\top = c\mathbf{A}^\top$
4. $\det(\mathbf{A}^\top) = \det(\mathbf{A})$
5. $(\mathbf{A}^\top)^{-1} = (\mathbf{A}^{-1})^\top$

Stability Function

- Explicit methods
  $R(h\lambda) = \det\left[\mathbf{I} - \lambda h(\mathbf{A} - \mathbf{1}\mathbf{b}^\top)\right]$
- Other methods
  $R(h\lambda) = \left[1 + \lambda h \mathbf{b}^\top (\mathbf{I} - \lambda h \mathbf{A})^{-1}\mathbf{1}\right]$
  $R(h\lambda) = \dfrac{\det\left[\mathbf{I} - \lambda h(\mathbf{A} - \mathbf{1}\mathbf{b}^\top)\right]}{\det\left[\mathbf{I} - \lambda h \mathbf{A}\right]}$

Matrix Inverse

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ab - cd}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Matrix Fun Facts

- $\det(c\mathbf{A}) = c^n \det(\mathbf{A})$
- For 3-by-3 matrices, $(\mathbf{I} + \mathbf{A})$ and $(\mathbf{I} - \mathbf{A})$ commute.

Homogenous Transformation Matrix

$$\mathbf{T}_b^a = \begin{bmatrix} \mathbf{R}_b^a & \mathbf{r}_{ab}^a \\ \mathbf{0}^\top & 1 \end{bmatrix} \in SE(3)$$

$$(\mathbf{T}_b^a)^{-1} = \begin{bmatrix} (\mathbf{R}_b^a)^\top & -(\mathbf{R}_b^a)^\top \mathbf{r}_{ab}^a \\ \mathbf{0}^\top & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_a^b & \mathbf{r}_{ba}^a \\ \mathbf{0}^\top & 1 \end{bmatrix} = \mathbf{T}_a^b$$

Note that the positon of the a-frame in the transformed frame (the b-frame) is $\mathbf{r}_{ba}^a = -(\mathbf{R}_b^a)^\top \mathbf{r}_{ab}^a$.

A Fancy Integral

$$\int_0^h (1-\tfrac{z}{h})^n P(z)dz = \tfrac{h}{n+1}(P(0) + \int_0^h (1-\tfrac{z}{h})^{n+1} P'(z)dz)$$