TTK4215 – Assignment 8

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Problem 1 I&S 6.1

We have the system

$$y = \frac{b}{s-1}u \tag{1}$$

that we wish to control using direct model reference adaptive control using the model

$$y_m = \frac{2}{s+2}r. \tag{2}$$

This design corresponds to the one presented in I&S example 6.2.2, with

$$x = y, \quad a = 1, \quad a_m = 2, \quad b_m = 2, \tag{3}$$

and $b > 0$ unknown, but with known sign. The signals y, y_m and the reference r are known signals. Since this is direct control, we wish to estimate the controller parameters directly. We define

$$k^* = \frac{a_m + a}{b} = \frac{3}{b}, \quad \text{and} \quad l^* = \frac{b_m}{b} = \frac{2}{b} \tag{4}$$

and wish to use the control law

$$u = -k^* y + l^* r. \tag{5}$$

We then see the need for the requirement $b > 0$, as we cannot allow b to cross zero and cause the controller parameters to blow up. However, since k^* and l^* are unknown parameters, we cannot use this controller, so we instead define the control law

$$u = -k(t)y + l(t)r, \tag{6}$$

where k and l are corresponding estimates. We define the tracking error

$$e = y - y_m, \tag{7}$$

as well as the estimation error

$$\epsilon_1 = e - \hat{e}, \tag{8}$$

but since we can measure y, $\hat{e} = 0$ and so $\epsilon_1 = e$. We then get the adaptive laws according to (6.2.29), i.e.

$$\begin{aligned} \dot{k} &= \gamma_1 \epsilon_1 y sgn(b) = \gamma_1 \epsilon_1 y, \\ \dot{l} &= \gamma_2 \epsilon_1 r sgn(b) = \gamma_2 \epsilon_1 r. \end{aligned} \tag{9}$$

We can write up a block diagram for this like shown in figure 1 below.

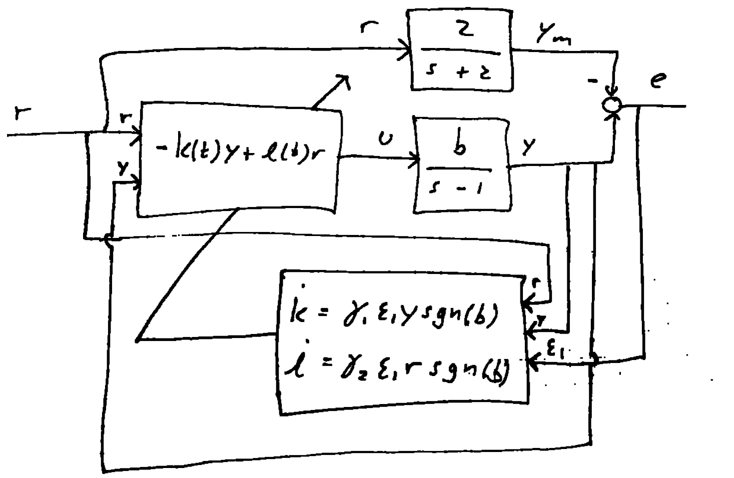

Figure 1: MRAC block diagram

Problem 2 I&S 6.2

a)

Given the system

$$V = \frac{b}{s+a}\theta + d \tag{10}$$

where d is constant a disturbance, and a reference model

$$V_m = \frac{0.5}{s = 0.5} V_s \tag{11}$$

i.e., unknown parameters $b > 0$, a and d and known model parameters $a_m = 0.5$ and $b_m = 0.5$. In the first case, we assume the unknown parameters are actually known. We rewrite the system to

$$V(s+a) = b\theta + ds + da \tag{12}$$

and note that since d is a constant disturbance, $ds = 0$, meaning we get

$$Vs = -aV + b\theta + da. \tag{13}$$

With MRC, we want

$$V = V_m. \tag{14}$$

We want a controller with a feed forward term, a feedback term and a constant term to combat the constant disturbance, i.e. we want

$$\theta = k^* V - l^* V_s - m^* \tag{15}$$

where k^*, l^* and m^* are constants. We insert this into (13) and obtain

$$Vs = -aV + b(k^* - l^* - m^*) + da \tag{16}$$

$sd = 0$ since the disturbance is constant. We then get

$$\begin{aligned} sV &= -a(a - bk^*)V + bl^* V_s - bm^* + ad \\ &= -(a - bk^*)V + \frac{bl^*}{b_m}(V_m s + a_m V_m) - bm^* + ad \end{aligned} \tag{17}$$

We rewrite this to

$$sV + (a - bk^*)V = \frac{bl^*}{b_m} V_m s + \frac{bl^* a_m}{b_m} V_m - b_m m^* + ad. \tag{18}$$

Now, to make $V = V_m$ as well as $sV = sV_m$, we must require

$$m^* = \frac{ad}{b}, \quad l^* = \frac{b_m}{b} \tag{19}$$

and with

$$a - bk^* = \frac{bl^* a_m}{b_m} \implies bk^* = a - \frac{bl^* a_m}{b_m} = a - a_m \tag{20}$$

we obtain

$$k^* = \frac{a - a_m}{b}. \tag{21}$$

Of course, we must know the sign of $b > 0$ for these parameters not to go to infinity.