TTK4215 Assignment 6

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Problem 1 I&S 4.10

c)

After several attempts at a single parametrisation of the system, two separate parametrizations were chosen to estimate the parameters k as well as β and m respectively. For k, this is simply

$$u = k(y_1 - y_2) \tag{1}$$

where $z = u$, $\phi = y_1 - y_2$ and $\theta^* = k$. Since there is no differentiation involved, there is no need for filtering in this case. For β and m, we haev

$$u = ms^2 y_2 + \beta s y_2 \tag{2}$$

which we filter with a stable Hurwitz second order polynomial Λ_0 obtaining

$$z = \frac{u}{\Lambda_0(s)} = \begin{bmatrix} \beta & m \end{bmatrix} \frac{1}{\Lambda_0(s)} \begin{bmatrix} s \\ s^2 \end{bmatrix} y_2 = \theta^{*\top} \phi \tag{3}$$

where $\theta^{*\top} = \begin{bmatrix} \beta & m \end{bmatrix}$. Since we have some a priori knowledge we can use a more sophisticated parameter estimation scheme than simple gradient descent. We define the feasible set

$$\mathcal{S} = \{\theta \in \mathcal{R}^n | g(\theta) \geq 0\}. \tag{4}$$

For k, g becomes

$$g_k(\theta) = \theta - 0.1 = k - 0.1. \tag{5}$$

Likewize, for β and m we get

$$g_{\beta m}(\theta) = \begin{bmatrix} \frac{1}{2} - |\beta - \frac{1}{2}| \\ m - 10 \end{bmatrix}. \tag{6}$$

Furthermore, the gradients are

$$\nabla g_k(\theta) = 1 \tag{7}$$

and

$$\nabla g_{\beta m} = \begin{bmatrix} -\frac{\beta - \frac{1}{2}}{|\beta - \frac{1}{2}|} \\ 1 \end{bmatrix}. \tag{8}$$

With that, we can apply the adaptive law (4.4.5) from I&S.

d)

From matlab simulations we see that the system behaves with dynamics shown in 1 below.

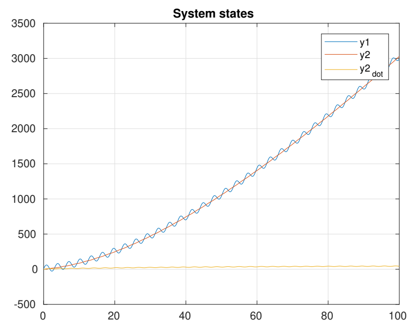

Figure 1: System simulation

Notice how the y_1 and y_2 positions diverge. This is due to the constant component of the input. In a regular mass-spring-damper configuration, a spring is connected directly to the wall which will balance any constant applied force. When a damper is placed inbetween this spring an the wall on the other hand, it can in theory be stretched to infinity as damping force for a theoretical damper is only dependent on speed. As for the input, it should also be noted that the sine component has been scaled with a significantly higher gain (close to 100) than the 5 gain that was mentioned in the assignment. When the adaptive law from (4.4.5) is implemented, this results in parameter convergence as shown in figure 2 below.

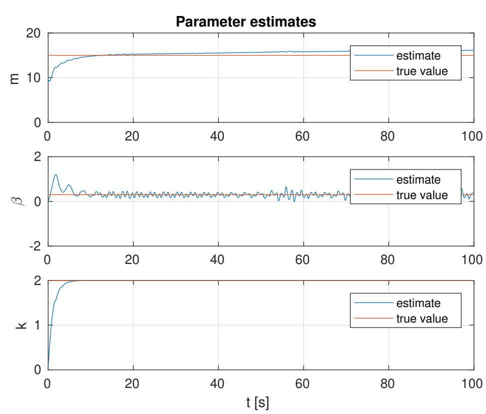

Figure 2: Parameter estimation

Problem 2 I&S 4.11

a)

Starting with $i = 0$ we have for the first closed loop

$$\theta_p = G_0(r - \theta_o) \tag{9}$$

which yields

$$\begin{aligned} s^2\theta_p &= k_0\omega_0^2(r - \theta_p) - \omega_0^2(1 - k_0)\theta_p - 2\xi_0\omega_0 s\theta_p \\ &= k_0\omega_0^2 r - k_0\omega_0^2\theta_p - \omega_0^2\theta_p + k_0\omega_0^2\theta_p - 2\xi_0\omega_0 s\theta_p \\ &= k_0\omega_0^2 r - \omega_0^2\theta_p - 2\xi_0\omega_0 s\theta_p \\ &= \begin{bmatrix} k_0\omega_0^2 & \omega_0^2 & \xi_0\omega_0 \end{bmatrix} \begin{bmatrix} r \\ -\theta_p \\ -2s\theta_p \end{bmatrix} \end{aligned} \tag{10}$$

At this point we have something resenbling a parametrization, but to make the s^2 on the rhs realizable, we need to filter it with a second order stable filter. Choosing Λ_0 to be a second order Hurwitz polynomial in s we get the parametrization $z_0 = \theta_0^{*\top}\phi$ with

$$z_0 = \frac{s^2\theta_p}{\Lambda_0}, \quad \phi_0 = \frac{1}{\Lambda_0}\begin{bmatrix} r \\ -\theta_p \\ -2s\theta_p \end{bmatrix}, \quad \theta_0^{*\top} = \begin{bmatrix} k_0\omega_0^2 \\ \omega_0^2 \\ \xi_0\omega_0 \end{bmatrix}. \tag{11}$$

From this we will find ω_0^2, which lets us calculate the values of k_0 and ξ_0. For $i = 1$ the closed loop yields

$$\dot{\theta} = G_1\theta_p \tag{12}$$

which becomes

$$\begin{aligned} s^2\dot{\theta} &= k_1\omega_1^2\theta_p - 2\xi_1\omega_1 s\dot{\theta} - \omega_1^2\dot{\theta} \\ &= \begin{bmatrix} k_1\omega_1^2 & \omega_1^2 & 2\xi_1\omega_1 \end{bmatrix} \begin{bmatrix} \theta_p \\ -\dot{\theta} \\ -2s\dot{\theta} \end{bmatrix} \end{aligned} \tag{13}$$

Filtering with a filter with the same properties as Λ_0, we can obtain the parametrization $z_1 = \theta_1^{*\top}\phi$ with

$$z_1 = \frac{s^2\theta_p}{\Lambda}, \quad \phi_1 = \frac{1}{\Lambda_1}\begin{bmatrix} \theta_p \\ -\dot{\theta} \\ -2s\dot{\theta} \end{bmatrix}, \quad \theta_1^{*} = \begin{bmatrix} k_1\omega_1^2 \\ \omega_1^2 \\ 2\xi_1\omega_1 \end{bmatrix}. \tag{14}$$

As with $i = 0$ we can here too solve for k_1 and ξ_1 using ω_1.