Modsim

ØV 4

Sigurd Hellesvik
sigurhe

Vår 19
MTTK

Modsim 4

Relative tolerance to stabilize

1a) 1 round: $1e^{-3}$

2 rounds: $1e^{-5}$

3 rounds: $1e^{-8}$

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b) ~~0,1~~ 0,0005

is a step length that gives accurate solutions for several rounds.

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c) If RK45 misses on a fast diff in the graph, it will be off forever. RK5 will however be right all the time, IF the step-length is sufficient for the extremes. It will be slower though.

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2 a) $b_1 + b_2 = 1$ $\quad$ $b_2 c_2 = \frac{1}{2}$

$a_{21} = c_2$

$a_{21} = t$

$\Rightarrow c_2 = t$

$\Rightarrow b_2 = \frac{1}{2t}$

$$\Rightarrow \underline{\underline{b_1 = 1 - \frac{1}{2t}}}$$

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2b)

$$\begin{array}{c|cc} 0 & & \\ c_2 & a_{21} & \\ \hline & b_1 & b_2 \end{array}$$

$$y_{n+1} = R y_n \qquad \dot{y} = \lambda y$$

$$k_1 = f(y_n, t), \quad k_2 = f(y_n + h a_{21} k_1, t_n + h c_2)$$

$$y_{n+1} = y_n + h(b_1 k_1 + b_2 k_2)$$

$$y_{n+1} = y_n + h(b_1 f(y_n, t) + b_2 f(y_n + h a_{21} f(y_n, t), t_n + h c_2))$$

$$= y_n + h(b_1 \lambda y_n + b2 \lambda y + b_2 h a_{21} \lambda y)$$

λy

$$= y_n (1 + h\lambda b_1 + h\lambda b_2 + h^2 \lambda b_2 a_{21}) = y_n R(h\lambda)$$

$$|R(h\lambda)| \leq 1 \Rightarrow |1 + h\lambda b_1 + h\lambda b_2 + h^2 \lambda b_2 a_{21}| \leq 1$$

import from a: $\left|1 + h\lambda - \frac{h\lambda}{2t} + \frac{h\lambda}{2t} + \frac{h^2 \lambda t}{2t}\right| \leq 1$

$$\Rightarrow \left|1 + h\lambda + \frac{h^2 \lambda}{2}\right| \leq 1$$

t forsvant, kun polene λ og tidssteg h påvirker stabiliteten.

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3a) $\ddot{x} + g\left(1 - \left(\frac{x_d}{x}\right)^k\right) = 0$

3d) $E = \frac{mg}{k-1}\frac{x_d^k}{x^{k-1}} + mgx + \frac{1}{2}m\dot{x}^2$

$\dot{E} = mg\dot{x} \mp mg\left(1 - \left(\frac{x_d}{x}\right)^k\right)$

$\ddot{E} = 0 \Rightarrow$ konstant energi

Svart ~~Blå~~ = Explicit Euler

Grønn = Implicit —"—

Rød = Midpoint

Modsim Øv 4

4.1 a) $z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$

$$\dot{z} = \begin{bmatrix} -z_3 + 4z_4^3 \\ -z_1 + 2z_2 \end{bmatrix} \qquad 0 = \begin{bmatrix} -z_1 + z_4^3 - z_2 + q_1 \\ -z_1 + z_4 + z_3 - q_2 \end{bmatrix}$$

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$z_2 = x_1 \quad z_3 = x_2, \quad z_1 = y_1 \quad z_4 = y_2$

$u_1 = q_1 \quad u_2 = q_2$

$$\Rightarrow \dot{x} = \begin{bmatrix} -x_2 + 4y_2^2 \\ 2x_1 - y_1 \end{bmatrix}$$

$$0 = \begin{bmatrix} y_2^3 - x_1 - y_1 + u_1 \\ -y_1 + x_2 - y_2 - u_2 \end{bmatrix} = g$$

b) $$\frac{dg}{dt} = \begin{bmatrix} 3\dot{y}_2 - \dot{x}_1 - \dot{y}_1 \\ -\dot{y}_1 + \dot{x}_2 - \dot{y}_2 \end{bmatrix} \Rightarrow \underline{\text{index} = 1}$$

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4.2 a) $z_2' = q_1 - z_1$

$z_3' = q_2 - (1+a)z_2 - at(q_1 - z_1)$

$q_3 = at z_2 + z_3$

$z_2 = x_1 \quad z_3 = x_2 \quad z_1 = y_1 \quad z_4 = y_2$

$q_1 = u_1 \quad q_2 = u_2 \quad q_3 = u_3$

$$\dot{x} = \begin{bmatrix} -y_1 + u_1 \\ -(1+a)x_1 + at y_1 + u_2 - at u_1 \end{bmatrix}$$

$0 = at x_1 + x_2 - u_3 = g$

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b) $\dot{g} = at\dot{x}_1 + \dot{x}_2 = at y_1 + at u_1$
$- (1+a)x_1 + at y_1 + u_2 - at u_1$

$\ddot{g} = -at\dot{y}_1 - (1+a)\dot{x} + at\dot{y}_1 \Rightarrow \text{index} = 2$

4.3 a) $\dot{q} = v - G^T\eta$

$M\dot{v} = Fq - G^T\lambda$

$0 = Gv$, $r = Gq$

$q = x_1$, $v = x_2$, $\eta = y_1$, $\lambda = y_2$

$r = y_3$

$$\dot{x} = \begin{bmatrix} x_2 - G^T y_1 \\ M^{-1}Fx_1 - G^T y_2 \end{bmatrix}$$

$$0 = \begin{bmatrix} Gx_2 \\ Gx_1 - y_3 \end{bmatrix} = g$$

b) $$\dot{g} = \begin{bmatrix} G\dot{x}_2 \\ G\dot{x}_1 - \dot{y}_3 \end{bmatrix} = \begin{bmatrix} G(x_2 - G^T y_1) \\ G(M^{-1}Fx_1 - G^T y_2) - \dot{y}_3 \end{bmatrix}$$

$$\ddot{g} = \begin{bmatrix} G(\dot{x}_2 - G^T\dot{y}_1) \\ G(M^{-1}F\dot{x}_1 - G^T\dot{y}_2) - \ddot{y}_3 \end{bmatrix} \Rightarrow \text{index} = 2$$

This dosent work!

4.4 a) $m_1 \ddot{x}_1 = k(x_2 - x_1 - x_0) + F$

$$m_2 \ddot{x}_2 = -k(x_2 - x_1 - x_0)$$

$$x_2 = r$$

$a_1 = x_1$ $\qquad \mathring{a}_1 = x_2$

$a_2 = \dot{x}_1$ $\qquad \dot{\mathring{a}}_2 = \dot{x}_2$

$$\dot{a}_2 = \frac{k}{m_1}(\mathring{a}_1 - a_1 - x_0) + F$$

$$\dot{\mathring{a}}_2 = \frac{k}{m_2}(\mathring{a}_1 - a_1 - x_0)$$

$$\dot{a}_1 = a_2, \quad \dot{\mathring{a}}_1 = \mathring{a}_2$$

$$\mathring{a}_1 = r$$

$\Rightarrow a_2 = x_1,\ \mathring{a}_2 = x_2,\ a_1 = x_3,\ \mathring{a}_1 = x_4$

$x_0 = u_1 \qquad F = y_1$

$r = u_2$

$$\Rightarrow \dot{x} = \begin{bmatrix} \frac{k}{m_1}(x_4 - x_3 - u_1) + y_1 \\ \frac{k}{m_2}(x_4 - x_3 - u_1) \\ x_1 \\ x_2 \\ u_2 \end{bmatrix}$$

$0 = r - x_4 = g$

4.4 b) $\dot{y} = -\dot{x}_4 = x_2$

$\ddot{y} = \dot{x}_2 = \frac{k}{m_2}(x_4 - x_3 - u_1)$

$\dddot{y} = \frac{k}{m_2}(\dot{x}_4 - \dot{x}_3) = \frac{k}{m_2}(x_2 - x_1)$

$\ddddot{y} = \frac{k}{m_2}(\dot{x}_2 - \dot{x}_1) = \frac{k}{m_2}\Big(\frac{k}{m_2}(x_4 - x_3 - u_1) -$

$\frac{k}{m_1}(x_4 - x_3 - u_1) + y_1\Big)$

$y^{(5)} = \frac{k}{m_2}\Big(\frac{k}{m_2}(\dot{x}_4 - \dot{x}_3) - \frac{k}{m_1}(\dot{x}_4 - \dot{x}_3) + \dot{y}_1\Big)$

$\Rightarrow$ index = 5