Modsim øv 7 Vår 19
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1 a) model: new set of eqations to create a model,

equation: Here comes the next equation

end: closes block

parameter: variable dynde can change when solving equation.

Real: Declares that variable is a real number

der: derivative

b) Comments explain code.
"This is compiled a a comment"

~~b)~~ //This is ignored

d) Same if b = constant
~~cause~~ But we cannot change the offset inbetween simulations.

2.1 a) $0 = \frac{x}{\ln(\sqrt{x^2+y^2})}$

$0 = x \cdot \frac{1}{\ln(\sqrt{x^2+y^2})}$

[illegible]

b) $\frac{x}{\ln(\sqrt{x^2+y^2})}$ $\qquad \frac{u'v - uv'}{v^2}$

$u = x \qquad u' = 1$

$v = \ln(\sqrt{x^2+y^2})$

$v' = \quad g(h) = \ln(h) \qquad g'(h) = \frac{1}{h}$

$h = \sqrt{x^2+y^2} \qquad h' =$

$h' = j(i) = \sqrt{i} \qquad j'(i) = \frac{1}{2\sqrt{i}}$

$i = x^2 + y^2 \qquad i' = 2x$

$i' \cdot j'(i) = \frac{2x}{2\sqrt{i}} = \frac{x}{\sqrt{x^2+y^2}}$

$v' = g'(h) \cdot h' = \frac{x}{\sqrt{x^2+y^2}} \cdot \frac{1}{\sqrt{x^2+y^2}} = \frac{x}{x^2+y^2}$

21. b) $\frac{y^2 y - y x^2}{y^2}$

$= \frac{\ln(\sqrt{x^2+y^2}) - \frac{x^2}{x^2+y^2}}{\ln(\sqrt{x^2+y^2})}$

And now that I have shown that I can and/or can't do derivatives, it's probably best that I solve the rest in Wolfram alpha.

$\frac{\partial}{\partial x}\left(-x \frac{1}{\ln(\sqrt{x^2+y^2})}\right) \quad \frac{4xy}{(x^2+y^2)\ln^2(x^2+y^2)}$

$\frac{\partial}{\partial y}\left(-x - \frac{y}{\ln(\sqrt{x^2+y^2})}\right) =$

$\frac{y^2}{(x^2+y^2)\ln^2(\sqrt{x^2+y^2})} - \frac{1}{\ln(\sqrt{x^2+y^2})}$

2.1b)

$$\frac{\partial}{\partial y}\left(-y+\frac{x}{\ln(\sqrt{x^2+y^2})}\right)$$

$$= -\frac{4xy}{(x^2+y^2)\ln^2(x^2+y^2)} - 1$$

$$\frac{\partial}{\partial x}\left(-y+\frac{x}{\ln(\sqrt{x^2+y^2})}\right)$$

$$= \frac{1}{\ln(\sqrt{x^2+y^2})} - \frac{x^2}{(x^2+y^2)\ln^2(\sqrt{x^2+y^2})}$$

$$= \frac{1}{\ln(\sqrt{r^2\cos^2\theta+r^2\sin^2\theta})} - \frac{r^2\cos^2\theta}{(r^2\cos^2\theta+r^2\sin^2\theta)\ln^2(\sqrt{r^2\cos^2\theta+r^2\sin^2\theta})}$$

L-hopitals

$\lim_{r\to 0}$ 0 - What, no, impossible

2.1.c) probably unstable

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2.2a) $0 = a - x - \frac{4xy}{1+x^2}$ } $\emptyset$

$0 = bx\left(1 - \frac{y}{1+x^2}\right)$ } α

$$1 - \frac{y}{1+x^2} = 0 \Rightarrow y = 1+x^2$$

$$0 = a - x - 4x$$

$$\Rightarrow 0 = a - 5x \Rightarrow x = \frac{a}{5}$$

$$y = 1 + \frac{a^2}{25}$$

b)

$$\frac{d\emptyset}{dx} = \frac{8x^2y}{(x^2+1)^2} - \frac{4y}{(x^2+1)} - 1$$

$$\frac{d\emptyset}{dy} = -\frac{4x}{x^2+1}$$

$$\frac{d\alpha}{dx} = \frac{b(x^4+x^2(y+2)-y+1)}{(x^2+1)^2}$$

$$\frac{d\alpha}{dy} = -\frac{bx}{x^2+1}$$

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2.2 b)

$$\Delta\phi = \left(\frac{8\frac{a^2}{25}\left(1+\frac{a^2}{25}\right)}{\left(\frac{a^2}{25}+1\right)^2} - \frac{4\left(1+\frac{a^2}{25}\right)}{\frac{a^2}{25}+1} - 1 \right)$$

$$\Delta x + \left(- \frac{4\frac{a}{5}}{\frac{a^2}{25}+1} \right) \Delta y$$

$$\Delta Q = \left(\frac{6\frac{a^4}{5^4} + \frac{a^2}{25}\left(1+\frac{a^2}{25}+2\right)}{\left(\frac{a^2}{25}+1\right)^2} - \left(1+\frac{a^2}{25}\right) \right) \Delta x$$

$$+ \frac{6\frac{a}{5}}{\frac{a^2}{25}+1} \Delta y$$

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2.2 c)

$$\begin{bmatrix} \dfrac{\frac{8a^2}{50}\left(1+\frac{a^2}{25}\right)}{\left(\frac{a^2}{25}+1\right)^2} - \dfrac{4\left(1+\frac{a^2}{25}\right)}{\frac{a^2}{25}+1} - 1 & \dfrac{-\frac{4a}{5}}{\frac{a^2}{25}+1} \\ \dfrac{\frac{8a^4}{50}\cdot\frac{a^2}{25}\left(1+\frac{a^2}{25}+1\right)-\left(1+\frac{a^2}{25}\right)}{\left(\frac{a^2}{25}+1\right)^2} & \dfrac{\frac{4a}{5}}{\frac{a^2}{25}+1} \end{bmatrix}$$

Da eg berre er eit vanleg dødleg menneske, klarer eg ikkje å finne eigenvektorane til denne matrisa. Eg kunne sjekka eigenverdiane til denne, og diskutert stabilitet ut frå dei.

[illegible]

$= (\ldots - \lambda)$ [illegible] $- 2 - \lambda]$

[illegible] $= 0$

$(\ldots - \lambda)(\ldots - \lambda)(\ldots - \lambda) - 2 - \lambda = 0$

$\Rightarrow \lambda_1 = 4$

[illegible] $(\ldots - \lambda) - 2 - \lambda = 0$

[illegible] $- \lambda^3 - 2 - \lambda = 0$

$\lambda^3 - 6\lambda^2 + 32 = 0$

[illegible] ion norme 0.

$0 \Rightarrow 32,\ 1 \Rightarrow 27,\ -1 \Rightarrow 25,\ 2 \Rightarrow 16$

$-2 \Rightarrow 0$

$\Rightarrow \lambda_2 = -2$

$$
\begin{array}{l}
(\lambda^3 - 6\lambda^2 + 32) / (\lambda + 2) = \lambda^2 - 8\lambda + 16 \\
-(\lambda^3 + 2\lambda^2) \\
0 - 8\lambda^2 + 32 \\
-(-8\lambda^2 - 16\lambda) \\
\quad 16\lambda + 32 \\
\quad -(16\lambda + 32) \\
\quad\quad 0
\end{array}
$$

$\Downarrow$

$(\lambda - 4)^2$

$\Downarrow$

$\lambda_3 = \lambda_1 = 4$

3(a) 1. $\lambda_1 = -2 \quad \lambda_4 = \lambda_2 = \lambda_3 = 4$

$[A - \lambda I][V] = 0$

$\lambda_1 \Rightarrow \begin{bmatrix} 4+2 & 2 & 0 & 0 \\ 0 & 3+2 & 0 & 1 \\ 0 & 0 & -2+2 & 0 \\ 0 & -1 & 0 & 5+2 \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \\ V_{13} \\ V_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 6 & 2 & 0 & 0 \\ 0 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 7 \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \\ V_{13} \\ V_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$6V_{11} + 2V_{12} = 0$

$5V_{12} = 0 \Rightarrow V_{12} = 0$

$-V_{12} + 7V_{14} = 0$

$V_{11} = 0, \; V_{14} = 0$

Choose $V_{13} = 1$

$\Rightarrow V_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda_2 \Rightarrow \begin{bmatrix} 4-4 & 2 & 0 & 0 \\ 0 & 3-4 & 0 & 1 \\ 0 & 0 & -2-4 & 0 \\ 0 & -1 & 0 & 5-4 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \\ V_{23} \\ V_{24} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \\ V_{23} \\ V_{24} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$R_4 - R_2 \rightarrow R_4$

$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \\ V_{23} \\ V_{24} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$2V_{22} = 0 \Rightarrow V_{22} = 0$

$-V_{22} + V_{24} = 0 \Rightarrow V_{24} = 0$

$-2V_{23} = 0 \Rightarrow V_{23} = 0$

choose $V_{11} = 1$

$\Rightarrow V_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

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3a) 2 $\begin{pmatrix} 0 & 8 & 1 & 5 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \Rightarrow \begin{vmatrix} \lambda & 8 & 1 & 5 \\ 0 & \lambda & 1 & 0 \\ 1 & 0 & \lambda & 0 \\ 0 & 1 & 0 & \lambda \end{vmatrix} = \lambda \begin{vmatrix} [illegible] \end{vmatrix} - 8 \begin{vmatrix} [illegible] \end{vmatrix} + 1 \begin{vmatrix} [illegible] \end{vmatrix}$

$-5 \begin{vmatrix} [illegible] \end{vmatrix} = \lambda(\lambda^3) - 8(-\lambda) + 1 \cdot (\lambda \cdot (-\lambda)) + 5(\lambda(-1 \cdot (-\lambda))$

$-1(-1)) = \lambda^4 + 8\lambda - 2\lambda^2 + 5(\lambda^2 - 1)$

$= \lambda^4 + 2\lambda^2 + 8\lambda + 5 = 0$

Testing numbers close to 0.

$0 \Rightarrow 5 \quad 1 \Rightarrow 16 \quad -1 \Rightarrow 0 \Rightarrow \underline{\lambda_1 = -1}$

$(\lambda^4 + 2\lambda^2 + 8\lambda + 5)/(\lambda + 1) = \lambda^3 - \lambda^2 + 3\lambda + 5$

$-(\lambda^4 + \lambda^3)$

$= 0 - \lambda^3 + 2\lambda^2 + 8\lambda + 5$

$-(-\lambda^3 - \lambda^2)$

$= 0 + 3\lambda^2 + 8\lambda + 5$

$-(3\lambda^2 + 3\lambda)$

$= 0 + 5\lambda + 5$

$-(5\lambda + 5)$

0

$\Downarrow$

testing values close to 0.

$0 \Rightarrow 5, 1 \Rightarrow 8, -1 \Rightarrow 0$

$\Rightarrow \underline{\lambda_2 = -1}$

$(\lambda^3 - \lambda^2 + 3\lambda + 5)/(\lambda + 1) = \lambda^2 - 2\lambda + 5$

$-(\lambda^3 + \lambda^2)$

$= 0 - 2\lambda^2 + 3\lambda + 5$

$-(-2\lambda^2 - 2\lambda)$

$0 + 5\lambda + 5$

$-(5\lambda + 5)$

0

$\Downarrow$

$\lambda = \dfrac{2 \pm \sqrt{4 - 4 \cdot 5}}{2} = \dfrac{2 \pm \sqrt{-16}}{2}$

$= \dfrac{2 \pm 4i}{2} = \underline{1 \pm 2i}$

$\Rightarrow \lambda_3 = 1 + 2i$

$\lambda_4 = 1 - 2i$

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2a) 2 $\lambda_2 = -1$, $\lambda_3 = 1+2i$, $\lambda_4 = 1-2i$

$[\lambda I - A]v = 0$

$$\Rightarrow \begin{bmatrix} -1 & 3 & 2 & 5 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \cancel{\begin{bmatrix} 1 & 3 & 2 & 5 \\ 0 & -1 & 1 & 0 \\ 0 & 3 & [illegible] & 0 \\ [illegible] & [illegible] & 0 & -1 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ -1 & 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{bmatrix} - \begin{bmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$-v_{11} - v_{13} = 0$, $-v_{12} - v_{13} = 0$, $-v_{13} - v_{14} = 0$

choose $v_{11} = 1$

$\Rightarrow v_{13} = -1$, $v_{12} = 1$, $v_{14} = 1$

$$\Rightarrow v_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

aaand im done, rest of eigenvalues and vectors can be found using a computer. Already used 2 hours finding the first ones, and im not gonna wast more time doing this.

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3a)2. $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} \text{[illegible]} \\ \text{[illegible]} \\ 1 \end{bmatrix}$ $v_3 = \begin{bmatrix} \text{[illegible]} \\ \text{[illegible]} \\ 1 \end{bmatrix}$

3a)3. $\lambda_1 = i$ $\lambda_2 = -i$

$v_1 = \begin{bmatrix} -\frac{1}{2} - \frac{i}{2} \\ 1 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} -\frac{1}{2} + \frac{i}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix}$

3a)4. $\lambda_{1,2,3,4} = 3$ $v_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \dot{\vec{x}} = A\vec{x} + Bu$

3c) $\dot{x} = x + 3z + 4y$

$\dot{y} = -4y - 3z - x$

$\dot{z} = -3z - 2y + x$ 3z 0

$\Rightarrow \begin{bmatrix} 1 & 4 & 3 \\ -1 & -4 & -3 \\ 1 & -2 & -3 \end{bmatrix} = A$

$\Rightarrow \lambda_1 = -0.99, \lambda_2 = 1.85, \lambda_3 = -6.9$[illegible]

$\Rightarrow$ negative eigenvalues

$\Rightarrow$ <u>unstable</u>

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4a) $N_i = \phi\,(R_a + R_c + R_b + R_r)$

$N_i = \phi\,(R_a + R_r)$

$N_i = \phi\left(\frac{z}{A\mu_0}\right) + R_r)$

$= \phi\left(\frac{z}{\pi R^2 \mu_0} + R_r\right)$

$R_r = \frac{z_0}{A\mu_0}$

$\Rightarrow = \phi\left(\frac{z}{\pi R^2 \mu_0} + \frac{z_0}{A\mu_0}\right)$

$= \frac{\phi(z + z_0)}{\mu_0 \pi R^2}$

4b) $F = \frac{i^2}{2} \frac{\partial L(z)}{\partial z}$

$L(z) = \frac{N\phi}{i} = \frac{N^2 A \mu_0}{z+z_0}$

$\frac{\partial}{\partial z} \frac{1}{z+z_0} = -\frac{1}{(z+z_0)^2}$

$g(u) = \frac{1}{u} = u^{-1} \quad g'(u) = -u^{-2}$

$u = z+z_0 \quad u' = 1$

$F = -\frac{i^2}{2} \frac{N^2 A \mu}{(z+z_0)^2}$

$m\ddot{z} = -\frac{i}{2} \frac{N^2 A \mu}{(z+z_0)^2} + gm$

$\ddot{z} = -\frac{i}{2m} \frac{N^2 A \mu}{(z+z_0)^2} + g$

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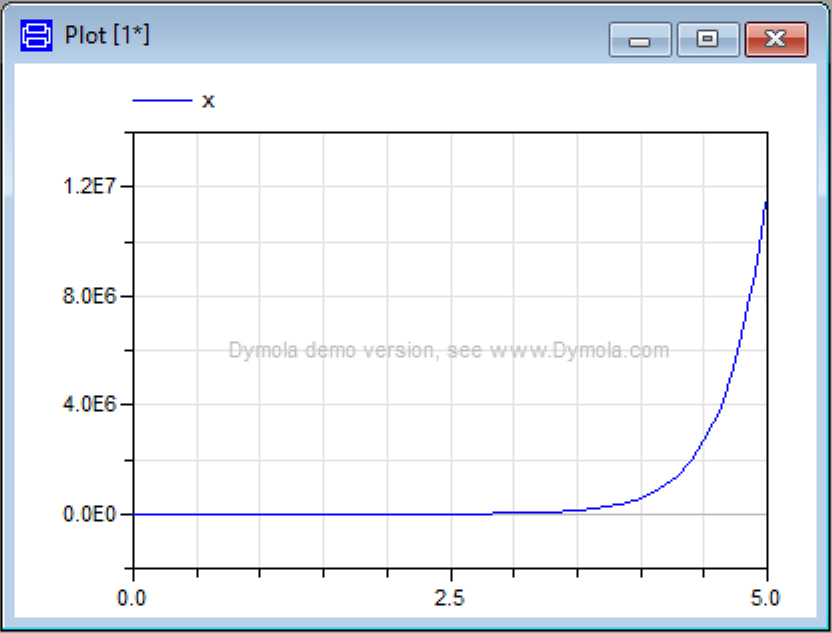
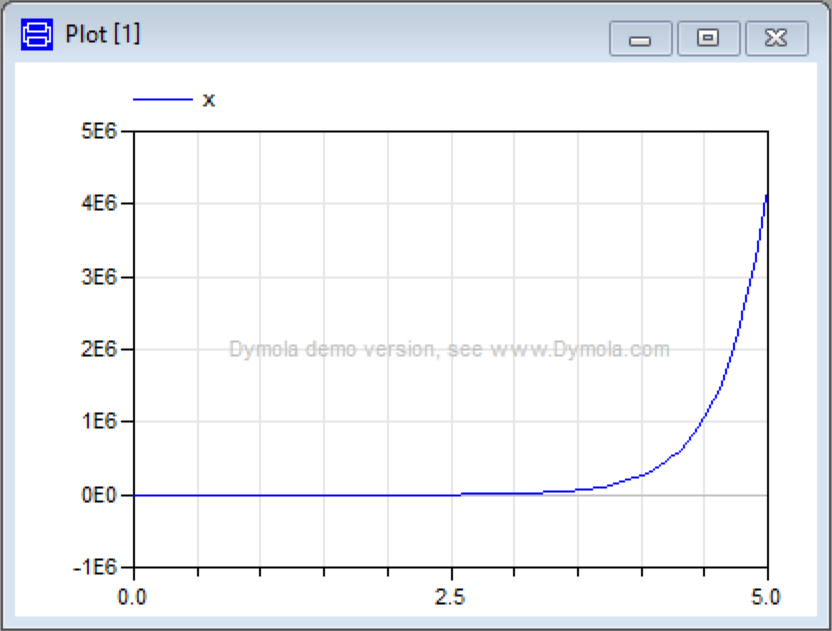
4c) $\Delta \ddot{z} = \frac{\partial}{\partial z}\left(-\frac{i^2}{2m}\frac{NA\mu}{z+z_0}+g\right)\Big|_{i=i_d, z=z_d}\Delta z$

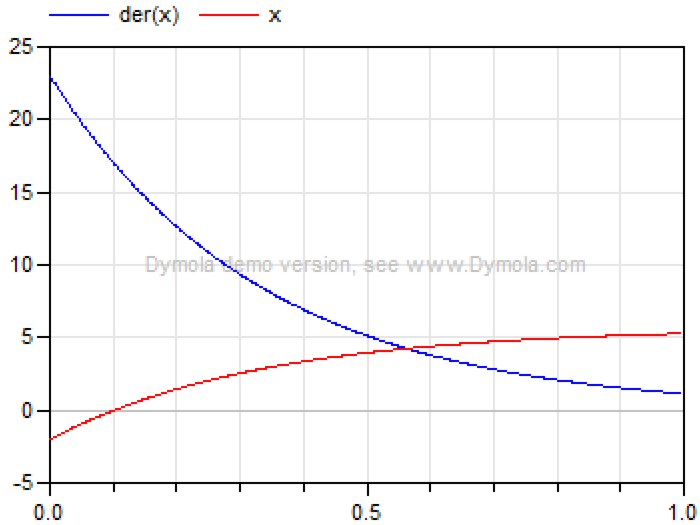
$+\frac{\partial}{\partial i}\left(-\frac{i^2}{2m}\frac{NA\mu}{z+z_0}+g\right)\Big|_{i=i_d, z=z_d}\Delta i$

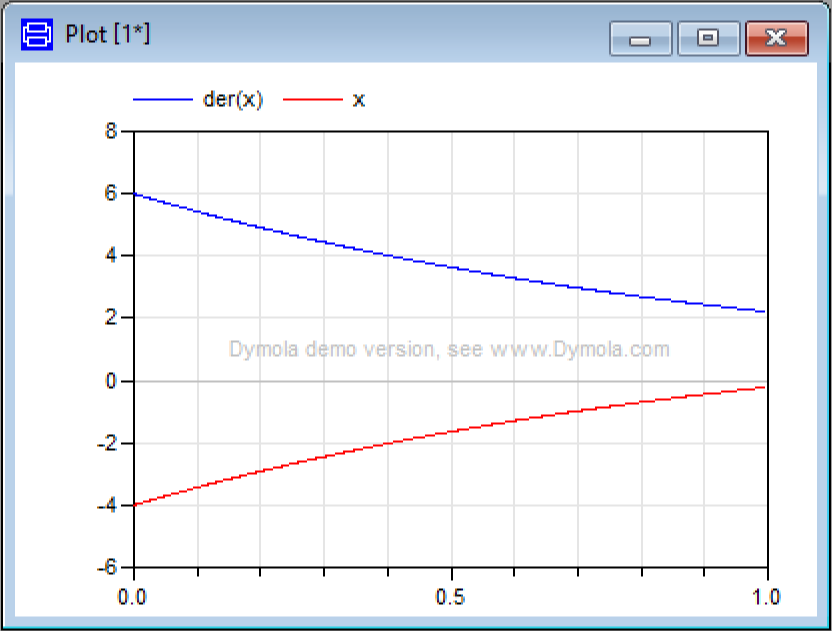
$= \frac{i_d^2}{2m}\frac{NA\mu}{(z_d+z_0)^2}\Delta z$

$- \frac{i_d}{m}\frac{NA\mu}{z_d+z_0}\Delta i$


Plot [1*]
x
1.2E7
8.0E6
4.0E6
0.0E0
0.0
2.5
5.0
Dymola demo version, see www.Dymola.com


Plot [1]
x
5E6
4E6
3E6
2E6
1E6
0E0
-1E6
0.0
2.5
5.0
Dymola demo version, see www.Dymola.com


der(x)
x
25
20
15
10
5
0
-5
0.0
0.5
1.0
Dymola demo version, see www.Dymola.com


Plot [1*]
der(x)
x
8
6
4
2
0
-2
-4
-6
0.0
0.5
1.0
Dymola demo version, see www.Dymola.com