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Sgube MTTK Vår 19

Oppdveg

1a) $|I\lambda - A| = 0$

$$\Rightarrow \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ -9,1 & 9,75 & \lambda - 1,78 \end{vmatrix} = 0$$

$$\lambda^2(\lambda - 1,78) = 0$$

$$\lambda_1 = \lambda_2 = 0$$

$$\lambda_3 = 1,78$$

$\Rightarrow$ Unstable system

1b)

u_t is a scalar

$x_t = \begin{bmatrix} x_{t0} \\ x_{t1} \\ x_{t2} \end{bmatrix}$; 3×1 vector

$$f(z) = \sum_{t=0}^{N-1} \left\{ y_{t+1}^2 + r u_t^2 \right\}, \quad r > 0$$

$$= \sum_{t=0}^{N-1} \left(C x_{t+1} \right)^2 + r u_t^2$$

$$= \frac{1}{2} \sum_{t=0}^{N-1} \left(x_{t+1}^T \underbrace{\left(2 C^T C \right)}_{Q} x_{t+1} + u_t^T \underbrace{2r}_{R} u_t \right)$$

$$Q = 2 C^T C = 2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$R = 2r = 2$$

1c) The convexity of the QP depends on ~~whether~~ if Q and R are positive (semi) definite. R ~~is~~ is positive definite and Q is positive semidefinite.

⇒ Convex QP

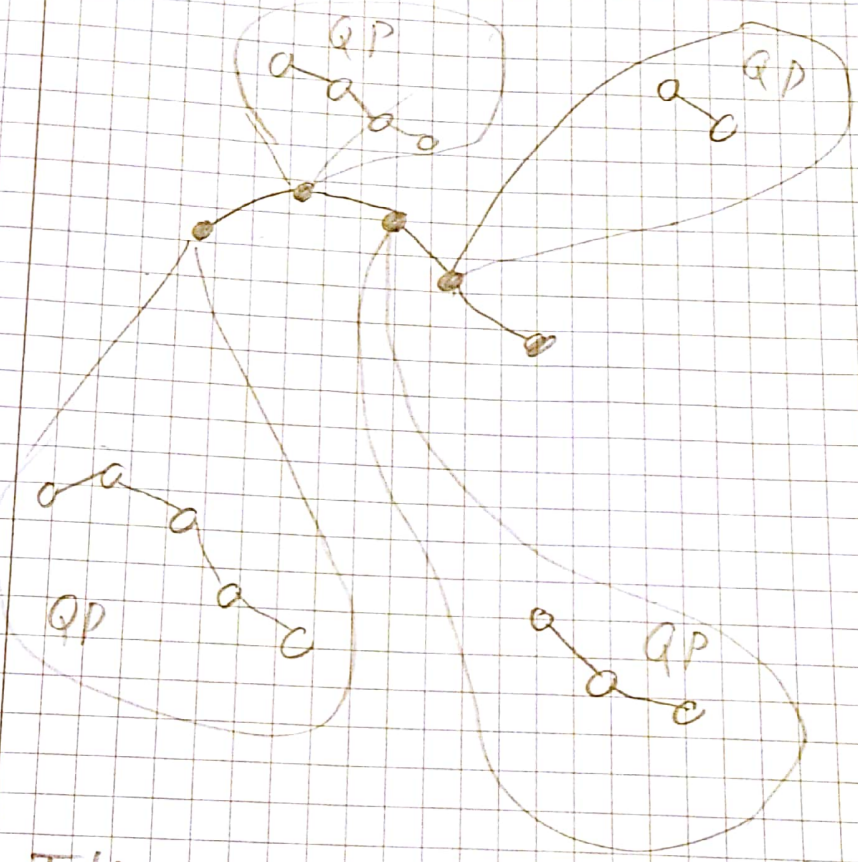
1d)

$$G = \begin{bmatrix} q & & & \\ & q & & \\ & & \ddots & \\ & & & r_k \end{bmatrix}$$

(16.4)
$$\begin{bmatrix} G & -A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} -c \\ b \end{bmatrix}$$

2a)

On each timestep MPC solves an QP and finds the perfect path, but only uses this ~~n~~ steps input signal.



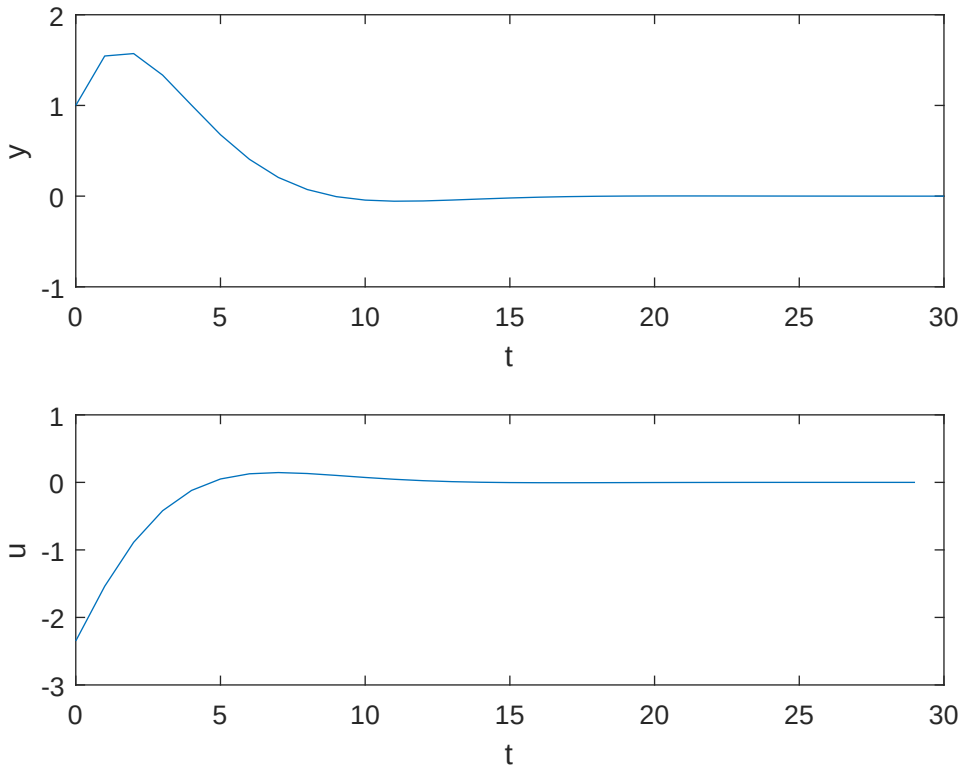
This way it will always chose an optimal input, even if the journey dosent go as planned all the way.

2b) The MPC gives exactly the same result as the QP when we use an correct model.

2c) We see that the MPC uses more input u and gets a more oscillatory y when the system are inaccurately modelled. Nevertheless it manages to regulate to the value of y we want.

Figurer er i rekkefølgen:

1d,1e,1f,2b,2c,2c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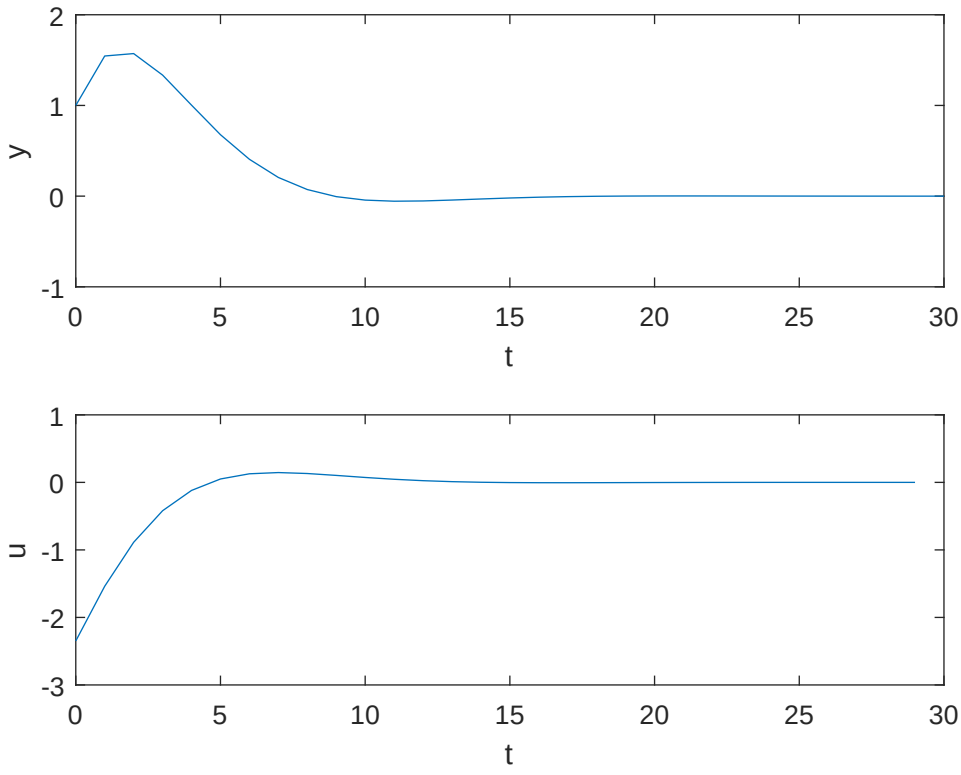
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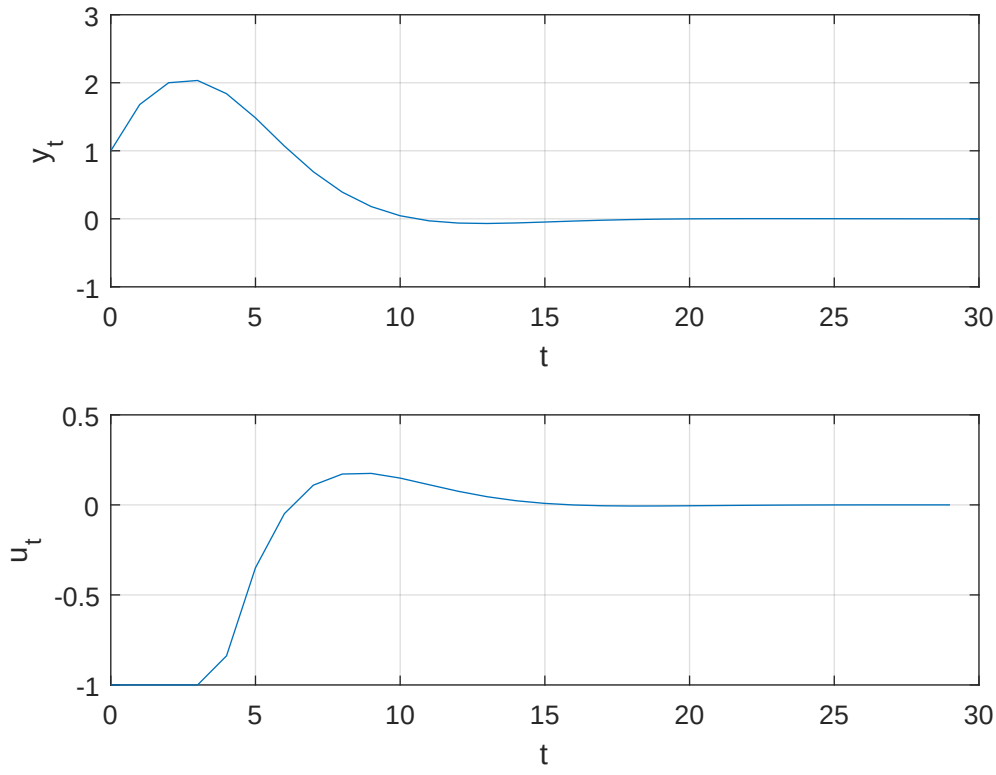
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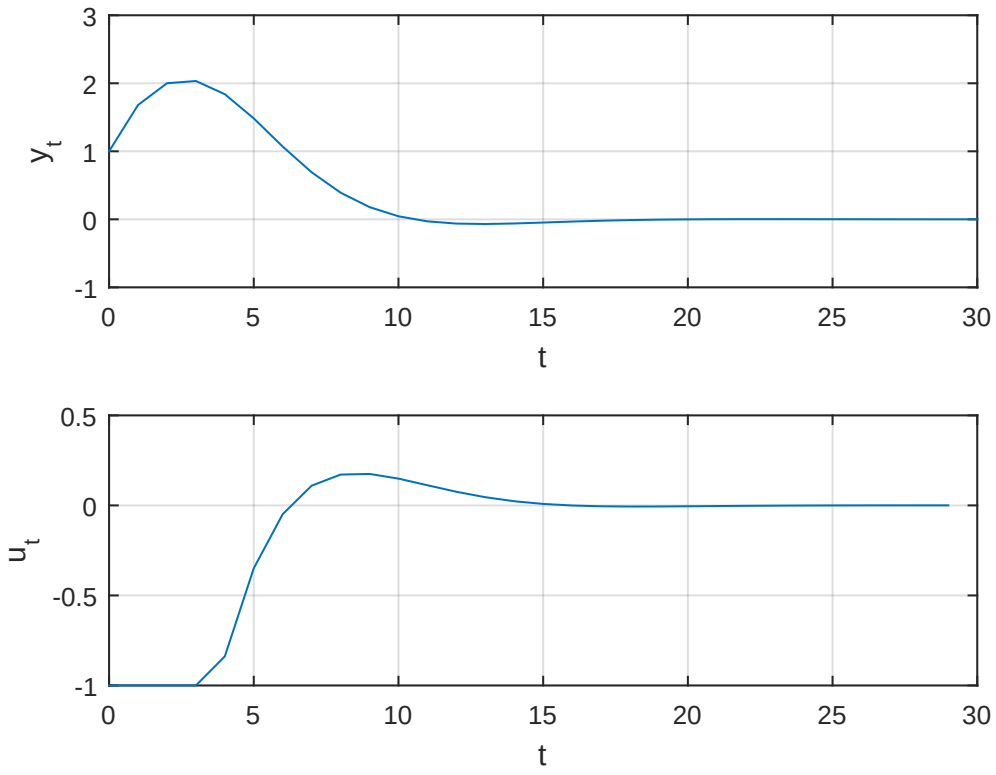
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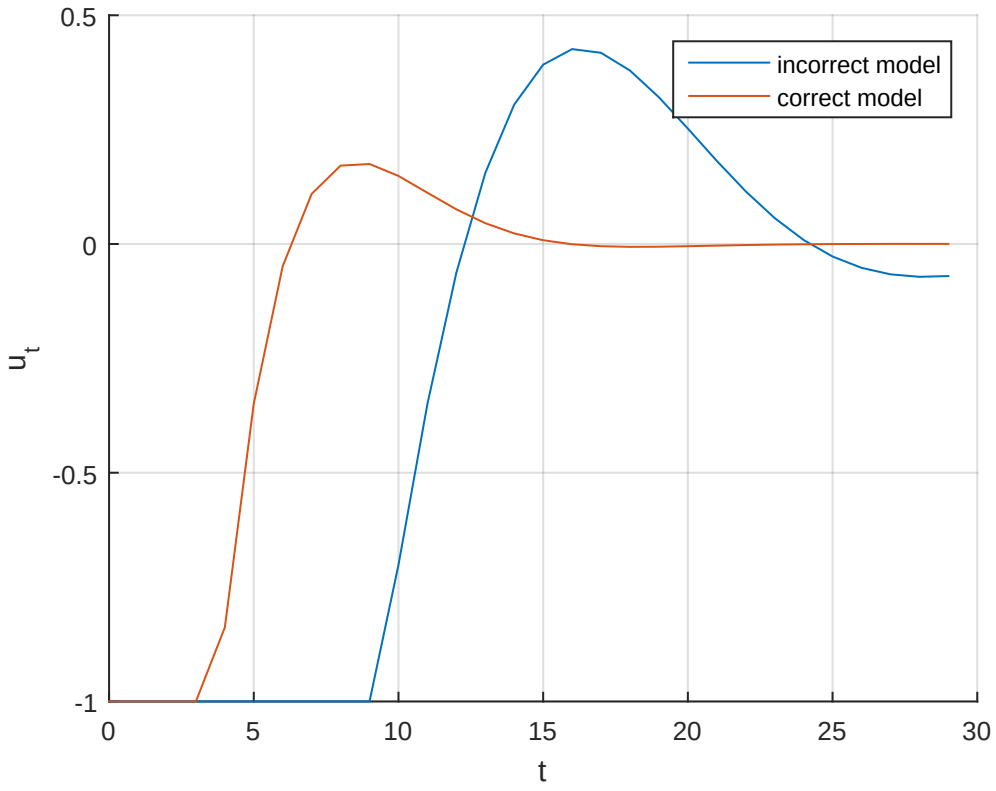
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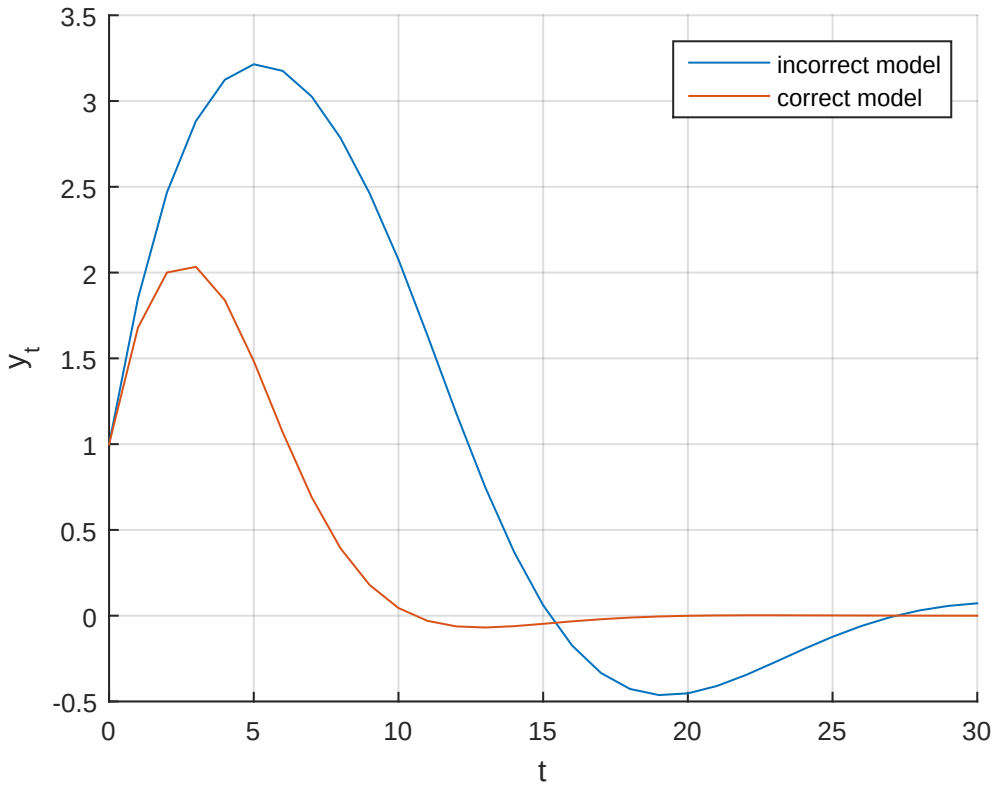
incorrect model
correct model


incorrect model
correct model
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