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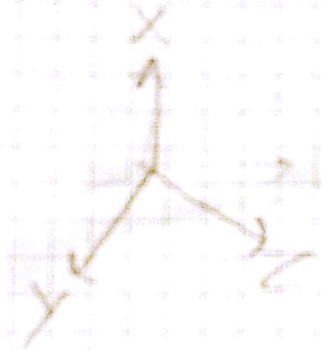
1 a) The matrix $R_b^a$ have columns for the x, y and z axis in the ~~start~~ frame for the rotation



7b)

7.

No time

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1c)

$R_b^a \in R^{3\times 3}$

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\det(R_b^a) = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{-1}{2} \cdot \frac{-1}{2}$$

$$\underline{\underline{= 1}}$$

$$\Rightarrow \underline{\underline{SO(3)}}$$

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1 d)

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\Rightarrow$ Rotation around the y-axis.

~~cos~~

$$\Rightarrow R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$\Rightarrow$ -30 degrees around y-axis

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1e) $R_a^b = (R_b^a)^T = (R_b^a)^{-1}$

1f) $u^b = R_a^b u^a =$

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} + \frac{3}{2} \\ 2 \\ -\frac{1}{2} + \frac{3\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3+\sqrt{3}}{2} \\ 2 \\ \frac{3\sqrt{3}-1}{2} \end{bmatrix}$$

$$W_a = R_b^a W^b = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} - 1 \\ -1 \\ \frac{1}{2} + \sqrt{3} \end{bmatrix} = \begin{bmatrix} -\frac{2+\sqrt{3}}{2} \\ -1 \\ \frac{1+2\sqrt{3}}{2} \end{bmatrix}$$

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1g)

$$\begin{bmatrix} æ \\ ø \\ å \end{bmatrix}^\times = \begin{pmatrix} 0 & -å & ø \\ å & 0 & -æ \\ -ø & æ & 0 \end{pmatrix} = a^\times$$

$$a = \begin{bmatrix} æ \\ ø \\ å \end{bmatrix}$$

$$R = \begin{bmatrix} x_{00} & x_{01} & x_{02} \\ x_{10} & x_{11} & x_{12} \\ x_{20} & x_{21} & x_{22} \end{bmatrix}$$

$$Ra = \begin{bmatrix} x_{00} & x_{01} & x_{02} \\ x_{10} & x_{11} & x_{12} \\ x_{20} & x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} æ \\ ø \\ å \end{bmatrix}$$

$$= \begin{bmatrix} æx_{00} + øx_{01} + åx_{02} \\ æx_{10} + øx_{11} + åx_{12} \\ æx_{20} + øx_{21} + åx_{22} \end{bmatrix}$$

$$(Ra)^\times = \begin{bmatrix} 0 & -(æx_{20} + øx_{21} + åx_{22}) & (æx_{10} + øx_{11} + åx_{12}) \\ (æx_{20} + øx_{21} + åx_{22}) & 0 & -(æx_{20} + øx_{21} + åx_{22}) \\ -(æx_{10} + øx_{11} + åx_{12}) & (æx_{20} + øx_{21} + åx_{22}) & 0 \end{bmatrix}$$

$$Ra^\times R^T = \begin{bmatrix} x_{00} & x_{01} & x_{02} \\ x_{10} & x_{11} & x_{12} \\ x_{20} & x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} 0 & -å & ø \\ å & 0 & -æ \\ -ø & æ & 0 \end{bmatrix} \begin{bmatrix} x_{00} & x_{10} & x_{20} \\ x_{01} & x_{11} & x_{21} \\ x_{02} & x_{12} & x_{22} \end{bmatrix}$$

$$= (Ra)^\times$$

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1g) ii) $a = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$ $b = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$

It is equavilent to rotate an orthogonal vector and to rotate two vectors and then find their orthogonal.

$$(Ra) \times (Rb) = (Ra)^{\times} Rb$$

$$= Ra^{\times} R^{T} Rb = Ra^{\times} R^{-1} Rb$$

$$= Ra^{\times} b = R(a \times b)$$

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1b) $R_b^a = R_y(\theta)\, R_z(\psi)\, R_x(\phi)$

$$= \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi & \cos\psi\sin\theta \\ \sin\psi\cos\theta & \cos\psi & \sin\psi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ \sin\psi\cos\theta & \cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi & -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ -\sin\theta & \cos\theta\cos\phi & \cos\theta\cos\phi \end{bmatrix}$$

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1 i) $R_1 = \begin{bmatrix} * & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ column and row is unit vectors

$\det(R_1) = 1 \Rightarrow$

$R_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$R_2 = \begin{bmatrix} -\frac{3}{5} & a & b \\ \frac{4}{5} & c & d \\ 0 & e & 1 \end{bmatrix}$

from 1h)

$-\sin\theta = 0 \Rightarrow \theta = 0$

$\Rightarrow 1 = \cos\phi \Rightarrow \phi = 0$

$\Rightarrow e = \cos\theta \sin\phi = 0$

$d = \cos\psi \sin\phi + \sin\psi \sin\theta \cos\phi = 0$

$-\frac{3}{5} = \cos\psi \cos\theta = \cos\psi \Rightarrow \psi = \cos^{-1}\left(-\frac{3}{5}\right)$

$\Rightarrow R_2 = \begin{bmatrix} -\frac{3}{5} & -\frac{4}{5} & 0 \\ \frac{4}{5} & -\frac{3}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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1;

$$R_3 = \begin{bmatrix} \frac{1}{2} & a & b \\ c & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{3}}{2} & d & e \end{bmatrix}$$

(from b)

$$-\sin\theta = -\frac{\sqrt{2}}{2} \Rightarrow \underline{\theta = \frac{1}{3}\pi}$$

$$\Rightarrow \cos\psi \cos\theta = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2}\cos\psi = \frac{1}{2} \Rightarrow \underline{\psi = 0}$$

$$\Rightarrow \frac{\sqrt{2}}{2} = \cos\phi \Rightarrow \underline{\phi = \frac{1}{4}\pi}$$

$$\Rightarrow R_3 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{6}}{4} & \frac{\sqrt{6}}{4} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \end{bmatrix}$$

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2a) $R^a_b k = k \cos\theta I + \sin k (k)^\times + \cancel{k}$
$+ (1-\cos\theta) k k^{*} (k^{*})^T$

$= k\cos\theta I + kk(k)^T - kk(k)^T \cos\theta$

$= k$

3 a) $T^{i}_{i+1} = Rot_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, Rot_{x,\alpha_i}$

$$Rot_{z,\theta_i} = \cos\theta_i \left( I \right) + \sin\theta_i \left( \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right)^{\times} + (1-\cos\theta_i) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_i & 0 & 0 \\ 0 & \cos\theta_i & 0 \\ 0 & 0 & \cos\theta_i \end{bmatrix} + \begin{bmatrix} 0 & -\sin\theta_i & 0 \\ \sin\theta_i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ (1-\cos\theta_i) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} = Rot_{z,\theta_i}$$

but $T = \begin{matrix} R^a_b & r^a \\ 0 & 1 \end{matrix}$

$$\Rightarrow \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3a) $Rot_{x,\alpha_i} = \cos\alpha_i I + \sin\theta_i \left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)^{\times}$

$+ (1-\cos\alpha_i)\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\end{bmatrix}$

$$= \begin{bmatrix}\cos\alpha_i & 0 & 0\\ 0 & \cos\alpha_i & 0\\ 0 & 0 & \cos\alpha_i\end{bmatrix} + \sin\alpha_i \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}$$

$$+ \begin{bmatrix}(1-\cos\alpha_i) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha_i & -\sin\alpha_i\\ 0 & \sin\alpha_i & \cos\alpha_i\end{bmatrix} = Rot_{x,\alpha_i}$$

$$= \begin{matrix}1 & 0 & 0 & 0\\ 0 & \cos\alpha_i & -\sin\alpha_i & 0\\ 0 & \sin\alpha_i & \cos\alpha_i & 0\\ 0 & 0 & 0 & 1\end{matrix}$$

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3a) $\text{Trans}_{z,d_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\text{Trans}_{x_i,a_i} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \text{Rot}_{z,\theta_i}\, \text{Trans}_{z,d_i}\, \text{Trans}_{x,a_i}\, \text{Rot}_{x,\alpha_i}$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3b) Rigth hand rule:
thumb, with thumb
in arm direction.

$\Rightarrow -q_3$

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3b)

c) $T_1^0 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & l_1 \cos q_1 \\ 0 & \cos q_1 & 0 & l_1 \sin q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T_2^1 = \begin{bmatrix} 1 & 0 & 0 & q_2 \cos(q_3) \\ 0 & \cos q_3 & -\cos q_3 & 0 \\ 0 & -\sin q_3 & \cos q_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3d) $T_1^0 T_2^1 = T_2^0$

$= \begin{bmatrix} \cos q_1 & -\sin q_1 \cos q_3 & \sin q_1 \cos q_3 & q_2 \cos q_1 \cos q_3 + l \cos q_1 \\ 0 & \cos q_1 \cos d_3 & \cos q_1 \cos q_3 & q_2 \cos q_1 \cos q_3 + l_1 \sin q_1 \\ 0 & -\sin q_3 & \cos q_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3e) $^{1}g^{2} = {}^{1}[\,l_1 \cos q_1 \quad -l_1 \sin q_1 \quad 0 \quad 1\,]^T$

$T_2^0\, g^2 = g^0$

$$= \begin{bmatrix} l_1 \cos^2 q_1 \\ -l_1 \cos q_1 \sin q_1 \cos q_3 - l_1 \sin q_1 \cos q_1 \cos q_3 \\ l_1 \cos q_1 \sin q_1 \cos q_3 - l_1 \sin q_1 \cos q_1 \cos q_3 = 0 \\ l_1 q_2 \cos^2 q_1 \cos q_3 + l_1^2 \cos^2 q_1 - l_1 q_2 \cos q_1 \sin q_1 \cos q_3 - l_1^2 \cos q_1 \sin q_1 + 1 \end{bmatrix}$$

=

3 b) for manipulator 2:

$$T_1^0 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & l_1 \cos q_1 \\ \sin q_1 & \cos q_1 & 0 & l_1 \sin q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \cos q_1 \\ \sin q_1 & \cos q_1 & 0 & l_1 \sin q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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3 c) Man 2:

$T_2^0 =$

$$\begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & 0 & l_2\cos(q_1+q_2) + l_1\cos q_1 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & 0 & l_2\sin(q_1+q_2) + l_1\sin(q_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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3d For man B

Altså, eg har vist at eg kan rekne på desse matrisene med manipulator A. Formelen er

$g^0 = T^0_2 \, g^2$

så eg gjer ikkje denne, fordi eg må jobbe med andre fag også.

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```
function [ theta,k ] = rot_to_eul( rot_mat )

z=zeros(1,4)
r_ii=max(diag(rot_mat));
i=size(find(diag(rot_mat),r_ii),1);
T=trace(rot_mat);
r_ii=max(r_ii,T);
if(r_ii=T),
    i=0;
end
z(i+1)=sqrt(1+2*r_ii-T);
if((-1 + (1+1)*rand(1,1))<0),
    z(i+1)=-z(i+1);
end

if i == 0,
z0 = sqrt(1 + 2*r_ii - T);
    z(1+1) = (rot_mat(3,2)-rot_mat(2,3))/z(1);
    z(2+1) = (rot_mat(1,3)-rot_mat(3,1))/z(1);
    z(3+1) = (rot_mat(2,1)-rot_mat(1,2))/z(1);
elseif i == 1,
    z(1+1) = sqrt(1 + 2*r_ii - T);
    z(1) = (rot_mat(3,2)-rot_mat(2,3))/z(1+1);
    z(2+1) = (rot_mat(2,1)+rot_mat(1,2))/z(1+1);
    z(3+1) = (rot_mat(1,3)+rot_mat(3,1))/z(1+1);
elseif i == 2,
    z(2+1) = sqrt(1 + 2*r_ii - T);
    z(1) = (rot_mat(1,3)-rot_mat(3,1))/z(2+1);
    z(1+1) = (rot_mat(2,1)+rot_mat(1,2))/z(2+1);
    z(3+1) = (rot_mat(3,2)+rot_mat(2,3))/z(2+1);
elseif i == 3,
    z(3+1) = sqrt(1 + 2*r_ii - T);
    z(1) = (rot_mat(2,1)-rot_mat(1,2))/z(3);
    z(1+1) = (rot_mat(1,3)+rot_mat(3,1))/z(3);
    z(2+1) = (rot_mat(3,2)+rot_mat(2,3))/z(3);
end

nj=z(1)/2
e=zeros(3);
e(1)=z(2)/2;
e(2)=z(3)/3;
e(3)=z(4)/4;

theta= 2*acos(nj);
k = e/(sin(theta/2));
```