Modsim
Vår 19

Øv 9 Sigurhe Sigurd Hellesvik

1a)

$$C_r^i = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \quad C_\theta^i = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} \quad C_z^i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_r^i \cdot C_\theta^i = \cos\theta\sin\theta - \cos\theta\sin\theta = 0$$

$$C_r^i \cdot C_z^i = 0+0+0 = 0$$

$$C_\theta^i \cdot C_z^i = 0+0+0 = 0$$

$$C_z^i \times C_r^i = \begin{vmatrix} i & 0 & \cos\theta \\ j & 0 & \sin\theta \\ k & 1 & 0 \end{vmatrix}$$

$$= \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} = C_\theta^i \Rightarrow \text{right hand}$$

$$\left| \begin{matrix} \cos\theta \\ \sin\theta \\ 0 \end{matrix} \right| = \cos^2\theta + \sin^2\theta = \underline{\underline{1}}$$

$$|C_\theta^i| = \cos^2\theta + \sin^2\theta = \underline{\underline{1}}$$

$$|C_z^i| = 1^2 = \underline{\underline{1}}$$

1 (a)

$$s^i_r = \begin{bmatrix} \sin\phi\cos\theta \\ \sin\phi\sin\theta \\ \cos\phi \end{bmatrix} \qquad s^i_\theta = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}$$

$$s^i_\phi = \begin{bmatrix} \cos\phi\cos\theta \\ \cos\phi\sin\theta \\ -\sin\phi \end{bmatrix}$$

$$s^i_r s^i_\theta = \sin\phi\cos\theta\sin\theta - \sin\phi\cos\theta\sin\theta$$
$$= \underline{\underline{0}}$$

$$s^i_\theta s^i_\phi = \cos\phi\cos\theta\sin\theta - \cos\phi\cos\theta\sin\theta$$
$$= \underline{\underline{0}}$$

$$s^i_r s^i_\phi = \sin\phi\cos\phi\cos^2\theta + \sin\phi\cos\phi\sin^2\theta$$
$$- \cos\phi\sin\phi = \underline{\underline{0}}$$

(b)

1 b)

$$\cancel{\dot{c}_r^i = \begin{bmatrix} -\dot{\theta}\cos\theta \\ -\dot{\theta}\sin\theta \\ 0 \end{bmatrix} \qquad \dot{c}_z^i = \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix}}$$

$$\cancel{\dot{c}_\theta^i = \begin{bmatrix} -\dot{\theta}\cos\theta \\ -\dot{\theta}\sin\theta \\ 0 \end{bmatrix}}$$

$$\dot{u}^a_I = R^a_b \left[\dot{u}^b + (\omega^b_{ab}) \times u^b \right]$$

$$R^c_i = R^{i\,T}_c = \left[c^i_r ,\ c^i_\theta ,\ c^i_z \right]^T$$

$$\dot{u}^i = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[

1b)

$$\dot{c}_r^i = \begin{bmatrix} -\dot{\theta}\sin\theta \\ \dot{\theta}\cos\theta \\ 0 \end{bmatrix} \qquad \dot{c}_\theta^i = \begin{bmatrix} \dot{\theta}\cos\theta \\ \dot{\theta}\sin\theta \\ 0 \end{bmatrix}$$

$$\dot{c}_z^i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\cancel{R_i^g}\ R_i^c = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{R}_i^c = \begin{bmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta & 0 \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\omega_{ic}^c)^\times = \dot{R}_i^c (R_i^c)^T$$

$$= \begin{bmatrix} -\dot{\theta}\sin\theta & \dot{\theta}\cos\theta & 0 \\ \dot{\theta}\cos\theta & \dot{\theta}\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\dot{\theta}\sin^2\theta - \dot{\theta}\cos^2\theta & 0 \\ \dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}}}$$

Scanned by CamScanner

1c) $\dot{s}_r^i = \begin{bmatrix} \dot{\phi}\cos\phi\cos\theta - \dot{\theta}\sin\phi\sin\theta \\ \dot{\phi}\cos\phi\sin\theta + \dot{\theta}\sin\phi\cos\theta \\ -\dot{\phi}\sin\phi \end{bmatrix}$

$$\dot{s}_\phi^i = \begin{bmatrix} -\dot{\phi}\sin\phi\cos\theta - \dot{\theta}\cos\phi\sin\theta \\ -\dot{\phi}\sin\phi\sin\theta + \dot{\theta}\cos\phi\cos\theta \\ -\dot{\phi}\cos\phi \end{bmatrix}$$

$$\dot{s}_\theta^i = \begin{bmatrix} -\dot{\theta}\cos\theta \\ -\dot{\theta}\sin\theta \\ 0 \end{bmatrix}$$

$$R_i^c = \begin{bmatrix} \sin\phi\cos\theta & \cos\phi\cos\theta & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi\sin\theta & \cos\theta \\ \cos\phi & -\sin\phi & 0 \end{bmatrix}$$

$$(R_i^c)^T = \begin{bmatrix} \sin\phi\cos\theta & \sin\phi\sin\theta & \cos\phi \\ \cos\phi\cos\theta & \cos\phi\sin\theta & -\sin\phi \\ -\sin\theta & \cos\theta & 0 \end{bmatrix}$$

$$\dot{R}_i^c = \begin{bmatrix} \dot{\phi}\cos\phi\cos\theta - \dot{\theta}\sin\phi\sin\theta & -\dot{\phi}\sin\phi\cos\theta - \dot{\theta}\cos\phi\sin\theta & -\dot{\theta}\cos\theta \\ \dot{\phi}\cos\phi\sin\theta + \dot{\theta}\sin\phi\cos\theta & -\dot{\phi}\sin\phi\sin\theta + \dot{\theta}\cos\phi\cos\theta & -\dot{\theta}\sin\theta \\ -\dot{\phi}\sin\phi & -\dot{\phi}\cos\phi & 0 \end{bmatrix}$$

Scanned by CamScanner

1b) $\dot{\underline{e}}_r^i = \begin{bmatrix} -\dot\theta\sin\theta \end{bmatrix} \quad \dot{\underline{e}}^i_\theta \begin{bmatrix} \dot\theta\cos\theta \end{bmatrix}$

d)

$$s^i_\theta \times s^i_r = \det \begin{bmatrix} i & -\sin\theta & \sin\phi\cos\theta \\ j & \cos\theta & \sin\phi\sin\theta \\ k & 0 & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi\cos\theta \\ \cos\phi\sin\theta \\ -\sin\phi \end{bmatrix} = \underline{s^i_\phi}$$

$$\sin^2(x) = 1-\cos^2(x)$$

$$|s^i_r| = \sqrt{\sin^2\phi\cos^2\theta + \sin^2\phi\sin^2\theta + \cos^2\phi}$$

$$= \sqrt{\cos^2\theta - \cos^2\theta\cos^2\phi + (1-\sin^2\phi)(1-\cos^2\theta) + \cos^2\phi}$$

$$= \sqrt{\cos^2\theta - \cos^2\theta\cos^2\phi + \underbrace{\sin^2\phi + \cos^2\phi}_{1} - \sin^2\phi\cos^2\theta}$$

$$= \sqrt{\cos^2\theta(1-\cos^2\phi - \sin^2\phi) + 1} = \underline{\underline{1}}$$

$$|s^i_\phi| = \sqrt{\cos^2\phi\cos^2\theta + \cos^2\phi\sin^2\theta + \sin^2\phi}$$

$$= \sqrt{\cos^2\phi(\underbrace{\cos^2\theta + \sin^2\theta}_{1}) + \sin^2\phi} = \underline{\underline{1}}$$

$$|s^i_r = \sqrt{+\sin^2\theta + \cos^2\theta} = \underline{\underline{1}}$$

Scanned by CamScanner

1c) $(\omega_{ic}^{c})^{\times} = \dot{R}_{i}^{c} (R_{i}^{c})^{T} = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$$a = (\dot{\phi} \cos\phi \sin\theta + \dot{\theta} \sin\phi \cos\theta)(\sin\phi \cos\theta)$$
$$+ (-\dot{\phi} \sin\phi \sin\theta + \dot{\theta} \cos\phi \cos\theta)(\cos\phi \cos\theta)$$
$$+ (-\dot{\theta} \sin\theta)(-\sin\theta)$$
$$= \dot{\phi} \cos\phi \sin\phi \cos\theta \sin\theta + \dot{\theta} \sin^2\phi \cos^2\theta$$
$$- \dot{\phi} \cos\phi \sin\phi \cos\theta \sin\theta + \dot{\theta} \cos^2\phi \cos^2\theta$$
$$+ \dot{\theta} \sin^2\theta$$
$$= \dot{\theta} \sin^2\phi \cos^2\theta + \dot{\theta} \cos^2\phi \cos^2\theta = \underline{\underline{\dot{\theta} \cos^2\theta}}$$

$$b = -\dot{\phi} \sin^2\phi \cos\theta - \dot{\phi} \cos^2\phi \cos\theta = \underline{\underline{\dot{\phi} \cos\theta}}$$

$$c = -\dot{\phi} \sin^2\phi \sin\theta - \dot{\phi} \cos^2\phi \sin\theta = \underline{\underline{\dot{\phi} \sin\theta}}$$

$$\Rightarrow \omega_{ic}^{c} = \begin{bmatrix} \dot{\theta} \cos^2\theta \\ \dot{\phi} \cos\theta \\ \dot{\phi} \sin\theta \end{bmatrix}$$

Scanned by CamScanner

1d)

$$\vec{r}_m = r\vec{c}_r + z\vec{c}_z$$

$$\vec{V}_m = \dot{\vec{r}} = \dot{r}\vec{c}_r + r\dot{\vec{c}}_r + \dot{z}\vec{c}_z + z\dot{\vec{c}}_z$$

$$= \dot{r}\begin{bmatrix}\cos\theta\\ \sin\theta\\ 0\end{bmatrix} + r\underbrace{\begin{bmatrix}-\dot{\theta}\sin\theta\\ \dot{\theta}\cos\theta\\ 0\end{bmatrix}}_{\dot{\theta}\, c_\theta^{i}} + \begin{bmatrix}0\\ 0\\ \dot{z}\end{bmatrix}$$

$$= \dot{r}\vec{c}_r + r\dot{\theta}c_\theta^{i} + \dot{z}\vec{c}_z$$

$$\vec{a}_m = \ddot{\vec{r}} =$$

$$\ddot{r}\vec{c}_r + \dot{r}\dot{c}_r + \dot{r}\dot{\theta}c_\theta^{i} + r\ddot{\theta}c_\theta^{i} + r\dot{\theta}\dot{c}_\theta^{i} + \ddot{z}\vec{c}_z + \dot{z}\dot{\vec{c}}_z$$

$$= \ddot{r}\vec{c}_r + \dot{r}\dot{\theta}\vec{c}_\theta + \dot{r}\dot{\theta}\vec{c}_\theta^{i} + r\ddot{\theta}\vec{c}_\theta^{i} + r\dot{\theta}\underbrace{\begin{bmatrix}\dot{\theta}\cos\theta\\ \dot{\theta}\sin\theta\\ 0\end{bmatrix}}_{\dot{\theta}\, c_r^{i}} + \ddot{z}\vec{c}_z + 0$$

$$= \vec{c}_r(\ddot{r} + r\dot{\theta}^2) + \vec{c}_\theta(2\dot{r}\dot{\theta} + r\ddot{\theta}) + \ddot{z}\vec{c}_z$$

Scanned by CamScanner

1e) $\vec{r}_m = r\vec{s}_r$

$$\vec{v}_m = \dot{\vec{r}} = \dot{r}\vec{s}_r + r\dot{\vec{s}} = \dot{r}\vec{s}_r + r\underbrace{\begin{bmatrix} \dot\phi\cos\phi\cos\theta - \dot\theta\sin\phi\sin\theta \\ \dot\phi\cos\phi\sin\theta + \dot\theta\sin\phi\cos\theta \\ -\dot\phi\sin\phi \end{bmatrix}^{1}}_{\dot\phi\, s^1_\phi + \dot\theta\sin\phi\, s^1_\theta}$$

$$= \dot{r}\vec{s}_r + \dot\phi\, s^1_\theta + \dot\theta\sin\phi\, s^1_\theta\, r \Longleftarrow \dot\phi\, s^1_\phi + \dot\theta\sin\phi\, s^1_\theta$$

$$\vec{a}_m = \ddot{\vec{r}}_m = \ddot{r}\vec{s}^1_r + \dot{r}\dot{\vec{s}}^1_r + \ddot\phi\vec{s}^1_\theta + \dot\phi\dot{\vec{s}}^1_\theta + \ddot\theta\sin\phi\,\vec{s}^1_\theta + \dot\theta\dot\phi\cos\phi\,\vec{s}^1_\theta + \dot\theta\sin\phi\,\dot{\vec{s}}^1_\theta$$

$$= \ddot{r}\vec{s}_r + \dot{r}\left(\dot\phi\vec{s}^1_\theta + \dot\theta\sin\phi\,\vec{s}^1_\theta\right) + \ddot\phi\vec{s}^1_\theta + \dot\phi\underbrace{\begin{bmatrix} -\dot\phi\sin\phi\cos\theta - \dot\theta\cos\phi\sin\theta \\ -\dot\phi\sin\phi\sin\theta + \dot\theta\cos\phi\cos\theta \\ -\dot\phi\cos\phi \end{bmatrix}^{r}}_{-\dot\phi\,\vec{s}^1_r + \dot\theta\cos\phi\,\vec{s}^1_\theta\, r}$$

$$+ \ddot\theta\sin\phi\,\vec{s}^1_\theta + \dot\theta\dot\phi\cos\phi\,\vec{s}^1_\theta + \dot\theta\sin\phi\begin{bmatrix} -\dot\theta\cos\theta \\ -\dot\theta\sin\theta \\ 0 \end{bmatrix}$$

$$= \vec{s}_r\left(\ddot{r} + r\dot\phi^2 - r\sin^2\phi\,\dot\theta^2\right)$$

$$+ \vec{s}_\phi\left(r\ddot\phi + 2\dot{r}\dot\phi - r\sin\phi\cos\phi\,\dot\theta^2\right)$$

$$+ \vec{s}_\theta\left(\ddot\theta r\sin\phi + \dot\theta\, 2\dot{r}\sin\phi + 2r\dot\phi\dot\theta\cos\phi\right)\vec{s}_\theta$$

Scanned by CamScanner

f) $\Sigma F = ma = \vec{N} - \vec{G}$

$$\Rightarrow \vec{N} = m\vec{a} + \vec{G}$$

$$= m\Big((\ddot{r} - r\dot{\phi}^2 - r\sin^2\phi\,\dot{\theta}^2)\vec{e}_r$$

$$+ (r\ddot{\phi} + 2\dot{r}\dot{\phi} - r\sin\phi\cos\phi\,\dot{\theta}^2)\vec{e}_\phi$$

$$+ (r\sin\phi\,\ddot{\theta} + 2\dot{r}\sin\phi\,\dot{\theta} + 2r\dot{\phi}\cos\phi\,\dot{\theta})\vec{e}_\theta$$

$$- g e_3 \Big)$$

Scanned by CamScanner

1g) $E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\dot{r} \vec{s}_r + r \dot{\phi} \vec{s}_\phi + r \sin\phi \, \theta \vec{s}_\theta \right)$

$E_p = mgh = mg r \vec{s}_r$

$L = E_k - E_p = m \left((gr + \dot{r}) \vec{s}_r + r \dot{\phi} \vec{s}_\phi + r \sin\phi \, \theta \vec{s}_\theta \right)$

$$= m \left((gr + \dot{r}) \begin{bmatrix} \sin\phi\cos\theta \\ \sin\phi\sin\theta \\ \cos\phi \end{bmatrix} + r\dot{\phi} \begin{bmatrix} \cos\phi\cos\theta \\ \cos\phi\sin\theta \\ -\sin\phi \end{bmatrix} + r \begin{bmatrix} -\theta\sin^2\phi \\ \theta\sin\phi\cos\theta \\ 0 \end{bmatrix} \right)$$

$$\frac{\partial L}{\partial r} = \underline{\underline{\left(g\vec{s}_r + \dot{\phi}\vec{s}_\phi + \sin\phi \, \theta \vec{s}_\theta \right) m}}$$

$$\cancel{\frac{\partial L}{\partial \phi} = (gr + \dot{r}) \underbrace{\begin{bmatrix} \cos\phi\cos\theta \\ \cos\phi\sin\theta \\ -\sin\phi \end{bmatrix}}_{\vec{s}_\phi} + r \underbrace{\begin{bmatrix} (\ddot{\phi}\cos\phi - \dot{\phi}\sin\phi)\cos\theta \\ (\ddot{\phi}\cos\phi - \dot{\phi}\sin\phi)\sin\theta \\ -(\ddot{\phi}\sin\phi + \dot{\phi}\cos\phi) \end{bmatrix}}_{\ddot{\phi} s^i_\phi - \dot{\phi} s^i_r}}$$

$$\cancel{+ r \underbrace{\begin{bmatrix} -\theta 2\sin\phi\cos\phi \\ \theta\cos\phi\cos\theta \\ 0 \end{bmatrix}}_{\theta\cos\phi \, s^i_\theta}}$$

$$\cancel{= \underline{\underline{m\left((gr + \dot{r} + \ddot{\phi}) s^i_\phi + r\theta\cos\phi \, s^i_\theta - \phi s^i_r \right)}}}$$

1 ↪ 1 j)

$$\frac{\partial L}{\partial \theta} = m\left((gr+\dot{r})\begin{bmatrix}-\sin\phi\sin\theta\\ \sin\phi\cos\theta\\ 0\end{bmatrix} - r\dot{\phi}\begin{bmatrix}-\cos\phi\sin\theta\\ +\cos\phi\cos\theta\\ 0\end{bmatrix}\right.$$

$$\left. + r\begin{bmatrix}-\sin^2\phi\\ \sin\phi\cos\theta - \theta\sin\phi\sin\theta\\ 0\end{bmatrix}\right)$$

$$\frac{\partial L}{\partial \phi} = m\left((gr+\dot{r})\begin{bmatrix}\cos\phi\cos\theta\\ \cos\phi\sin\theta\\ -\sin\phi\end{bmatrix} + r\dot{\phi}\begin{bmatrix}-\sin\phi\cos\theta\\ -\sin\theta\sin\theta\\ -\cos\theta\end{bmatrix}\right.$$

$$\left. + r\theta\begin{bmatrix}2\sin\phi\cos\phi\\ \theta\cos\phi\cos\theta\\ 0\end{bmatrix}\right)$$

$$\frac{\partial L}{\partial \dot{r}} = m\vec{s}_r$$

$$\frac{\partial L}{\partial \dot{\phi}} = r\vec{s}_\phi$$

$$\frac{\partial L}{\partial \dot{\theta}} = 0$$

Scanned by CamScanner

$1 \to 1g)$ $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right)$ $m\,\dot{ }$

R constant

$$\cancel{\tau_r = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = m\dot{\vec{s}}_r -}$$

$$\cancel{m\left(g\vec{s}_r + \dot{\phi}\,\dot{\vec{s}}_\phi + \sin\phi\,\theta\,\vec{s}_\theta\right)}$$

$$\cancel{= m\left((g+1)\vec{s}_r + \dot{\phi}\,\vec{s}_\phi + \theta\sin\phi\,\vec{s}_\theta\right)}$$

$$\tau_\phi = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = r\vec{s}_\phi$$

$$- m\left((g r + \dot{r})\,\vec{s}_\phi - r\ddot{\phi}\begin{bmatrix} \sin\phi\cos\theta \\ \sin\theta\sin\theta \\ -\cos\theta \end{bmatrix}\right.$$

$$\left. + r\theta\begin{bmatrix} z\sin\phi\cos\phi \\ \theta\cos\phi\cos\theta \\ 0 \end{bmatrix}\right)$$

$$\tau_\theta = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial \theta}$$

Allereie skrive denne.

1h) $(7a) = x = R\sin\phi\cos\theta$

$(8a) = X = \dfrac{R}{\sqrt{1+a^2\theta^2}}\cos\theta$

$\Rightarrow \sin\phi = \dfrac{1}{\sqrt{1+a^2\theta^2}} \quad (*)$

$(7c) = z = R\cos\phi$

$(8c) = z = \dfrac{Ra\theta}{\sqrt{1+a^2\theta^2}}$

$\Rightarrow \cos\phi = \dfrac{a\theta}{\sqrt{1+a^2\theta^2}}$

$\Rightarrow \dfrac{1}{\sqrt{1+a^2\theta^2}} = \dfrac{a\theta}{\cos\phi}$

$\Rightarrow (*) = \sin\phi = \dfrac{\cos\phi}{a\theta}$

$\Rightarrow \theta = \tan^{-1}(a\theta)$

Scanned by CamScanner

1 i) $(6b) = \vec{V}_m = \dot{r}\vec{s} + r\dot{\phi}\vec{s}_\phi + r\sin\phi\,\dot{\theta}\vec{s}_\theta$

$r = R \Rightarrow \dot{r} = 0$

$$\Rightarrow \vec{V}_m = r\dot{\phi}\vec{s}_\phi + \sin\phi\,\dot{\theta}\vec{s}_\theta$$

$$\phi = \tan^{-1}(a\theta) \Rightarrow s^i_\phi = \begin{bmatrix} \cos(\tan^{-1}(a\theta))\cos\theta \\ \cos(\tan^{-1}(a\theta))\sin\theta \\ -\sin(\tan^{-1}(a\theta)) \end{bmatrix}$$

$$s^i_\theta = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}$$

$$E_k = \frac{1}{2}mv^2$$

$$\vec{V}_m^{\,2} = r^2\dot{\phi}^2\vec{s}_\phi^{\,2} + 2r\dot{\phi}\sin\phi\,\dot{\theta}\,\vec{s}_\phi\vec{s}_\theta + \sin^2\phi\,\dot{\theta}^2\vec{s}_\theta^{\,2}$$

$$(6a) = \vec{r}_{mr} = r\vec{s}_r = r\begin{bmatrix} \sin(\tan^{-1}(a\theta))\cos\theta \\ \sin(\tan^{-1}(a\theta))\sin\theta \\ -\cos(\tan^{-1}(a\theta)) \end{bmatrix}$$

$$E_p = mg\vec{r}_m$$

Scanned by CamScanner

1i) $L = E_k - E_p = r^2 \dot{\phi}^2 \begin{bmatrix} \cos^2(\tan^{-1}(a\theta))\cos^2\theta \\ \cos^2(\tan^{-1}(a\theta))\sin^2\theta \\ -\sin^2(\tan^{-1}(a\theta)) \end{bmatrix}$

$+ 2r\dot{\phi}\sin\phi\,\dot{\theta} \begin{bmatrix} \cos(\tan^{-1}(a\theta))\cos\theta \\ \cos(\tan^{-1}(a\theta))\sin\theta \\ -\sin(\tan^{-1}(a\theta)) \end{bmatrix} \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}^T$

$+ \sin^2\phi\,\dot{\theta}^2 \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}$

this is impossible, right? wrong dimentions.

1i) not finished

1j) not done

2 a) Variable time-step gives zero-crossing dectection stop. Too many zero crossings.

Fixed step works, we can see that the friction force zero crosses at the start, but the detection dos ent stop it. Connot turn off zero-crossing detection when fixed step, so matlab probably fixes our problem. After ~1 second the force is normal, so the block is moving, giving an understandable friction. Probably wrong simulation, but fixed-step dosen't know that.

Scanned by CamScanner

2b) When using Karnopp's, we first get friction when the force pushes harder than the friction would have done.
This is more correct than in 2a).
If/else changes when simulink detects event $u1 = 0$. Change friction model.

2c) A deadzone works as in 2b), ~~er~~ cause you say there'e is no forces acting on the block when too little force tries to. But this dosent work if we ~~start~~ in ~~the wrong~~ direct cross over the deadzone. Then we will glide on the deadzone. This could be fixed by setting a small enough deadzone.

Scanned by CamScanner