Sigurd Hellesvik MTTK

Modsim øv 1 vår 19

1a) $\dot{x} = \alpha x - \beta x y$

$\dot{y} = -\gamma y + \delta x y$

$\Rightarrow \dot{H} = (bH - b_d H^2 - dH) - iHZ$

$\dot{Z} = rD + aI - nZH$

$\dot{D} = dH + dI + nZH - rD$

$\dot{I} = iHZ - dI - aI$

$\dot{H} + \dot{I} + \dot{Z} + \dot{D} = bH - b_d H^2$

---

b, b_d, d, i, r, a, n

1b) initial equation gives start values to ~~values~~ variables.

If there are as many dead, zombies and infected as there are humans, humanity is dead real fast.

If there are no humans the Z, I and D can only become ~~Z or~~ Z, because no extra input is given to the system (no birth).

1g) Modelica can model flow and with somé input and output code we would not need to duplicate code.

1f) The model could for example how many students join and leave a course. or literally any ecosystem.

2 a) $\ddot{x} + c\dot{x} + g\left(1 - \left(\frac{x_d}{x}\right)^k\right) = 0$

$x_1 = x \quad , x_2 = \dot{x}$

$\dot{x}_1 = x_2$

$\dot{x}_2 + c x_2 + g\left(1 - \left(\frac{x_d}{x_1}\right)^k\right) = 0$

$\dot{x}_2 = -c x_2 - g\left(1 - \left(\frac{x_d}{x_1}\right)^k\right)$

---

b) Euler's method.

$y_{n+1} = y_n + h f(y_n, t) + \frac{h^2}{2} \frac{d f(y_n, t)}{dt}$

$+ \dots + \frac{h^p}{p!} + \frac{d^{p-1} f(y_n, t)}{dt^{p-1}} + O(h^{p+1})$

where $\Delta g = \frac{y_{n+1} - y_n}{h} = f(y_n, t_n)$

~~Our slope:~~ $\cancel{\nabla \bar{x} = \nabla \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}}$

2b) $y_{1,n+1} = y_{1,n} + h y_{1,n}' = y_{1,n} + y_{2,n}$

$y_{2,n+1} = y_{2,n} + h y_{2,n}'$

$= y_{2,n} + h\left(c y_{2,n} - g\left(1 - \left(\frac{x_d}{y_{1,n}}\right)^k\right)\right)$

2c) Heuns method:

$k_1 = f(y_n, t_n)$

$k_2 = f(y_n + \frac{1}{3} h k_1, t_n + \frac{1}{3} h)$

$$\Rightarrow k_1 = \begin{cases} k_{1,1} = y_2 \\ k_{1,2} = -c y_{2,n} - g\left(1 - \left(\frac{x_d}{y_{1,n}}\right)^k\right) \end{cases}$$

$$k_2 = \begin{cases} k_{2,1} = y_{2,n} + \frac{1}{3} h k_1 \\ k_{2,2} = -c\left(y_{2,n} + \frac{1}{3} h k_1\right) - g\left(1 - \frac{x_d}{y_{1,n} + \frac{1}{3} h k_{1,1}}\right) \end{cases}$$

2d) $0 + 0 + g\left(1 - \left(\frac{x_d}{x}\right)^k\right) = 0$

$$\Rightarrow 1 - \left(\frac{x_d}{x}\right)^k = 0$$

$$\sqrt[k]{1} = \frac{x_d}{x}$$

$$x_1 = \frac{x_d}{\sqrt[k]{1}} = x_d$$

equilib. point

$$x_2 = 0$$

$$\begin{pmatrix} \frac{\partial \dot{y}_1}{\partial y_1} & \frac{\partial \dot{y}_1}{\partial y_2} \\ \frac{\partial \dot{y}_2}{\partial y_1} & \frac{\partial \dot{y}_2}{\partial y_2} \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ \left(-kg \frac{x_d}{y_1}\right)^{k+1} & -c \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -kg & -c \end{bmatrix}$$

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2e) We can use the eigenvalues of the linearized $\bar{y}$ to find if euler is stable; because we know that $|1+h\lambda| \leq 1$

$$\bar{y} = \begin{bmatrix} 0 & 1 \\ -kg & -c \end{bmatrix} \quad 0 = \det\begin{pmatrix} \lambda & -1 \\ kg & \lambda + c \end{pmatrix}$$

$$= \lambda^2 + \lambda c + kg = 0$$

$$\lambda = \frac{-c \pm \sqrt{c^2 - 4kg}}{2} =$$

$$= \frac{0 \pm \sqrt{-94{,}76}}{2}$$

$\Rightarrow$ imaginary $\Rightarrow$ Euler not stable for any h.

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2e) ii $\lambda = \frac{c \pm \sqrt{c^2 - 4kg}}{2}$

$\lambda = \frac{2{,}927 \pm \sqrt{-14{,}48}}{2}$

$\Rightarrow$ still imaginary values $\Rightarrow$ not stable for any h

3a) $L\frac{di}{dt} + Ri = u$

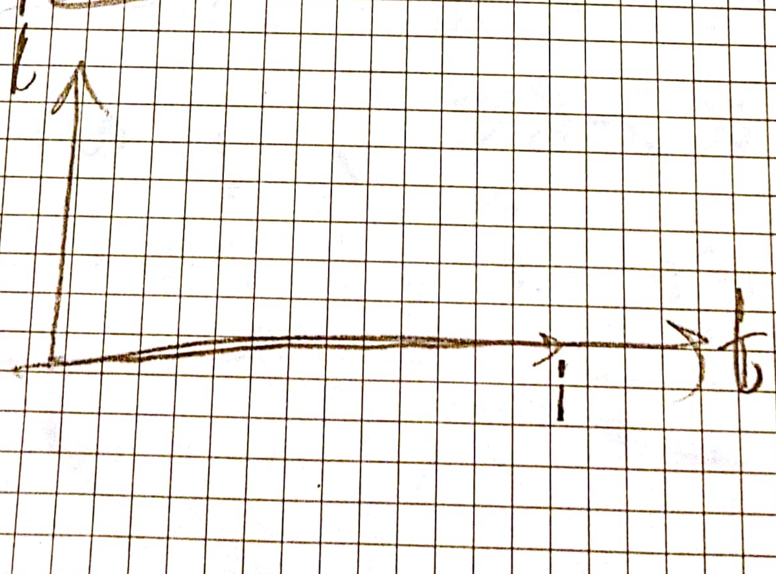
$\Rightarrow \frac{di}{dt} = -\frac{R}{L} i$

$\frac{dE}{di}\frac{di}{dt} = -\frac{R}{L} i \frac{dE}{di}$

$\frac{dE}{di} = \frac{d}{di}\left(\frac{1}{2}Li^2\right) = Li$

$\Rightarrow \frac{dE}{dt} = -Ri^2 < 0$

$\Rightarrow$ stable



3b)

$e = i - i_{ref}$

$E = \frac{1}{2} L e^2$

$$\frac{de}{dt} = -\frac{R}{L} e - \frac{1}{L} u$$

$$\dot{E} = \frac{de}{dt} \frac{dE}{de} = -Re^2 - eu < 0$$

~~now if~~ $-Re^2 - eu < 0$

the diff between i and i_{ref} will go to zero

$\Rightarrow$ if $Re^2 < -eu$

$u > -Re$, i will go to i_{ref} when $t \to \infty$.

TTK4130 Modellering | Lecture 1 – TTK4130 Modellering | Ingen lysbildetittel | main.dvi | Ex3_sol.pdf - OneDrive | TTK4130_assignment_3(3).pdf

https://ntnu.blackboard.com/bbcswebdav/pid-581805-dt-content-rid-19498505_1/courses/194_TTK4130_1_2019_V_1/TTK4130_assignment_3%283%29.pdf

NB: This is a computer exercise, and can therefore be solved in groups of 2 students. If you do so, please write down the name of your group partner in your answer.

The doomsday is upon us!

Zombies have begun to rise from the dead, and violently and indiscriminately kill and infest the living. In order to save humanity, you have to first model and simulate the zombie infestation using the little information available on these abominations.

There are 4 well-defined and non-overlapping populations:

IncompleteZombieApocalypse - IncompleteZombieApocalypse

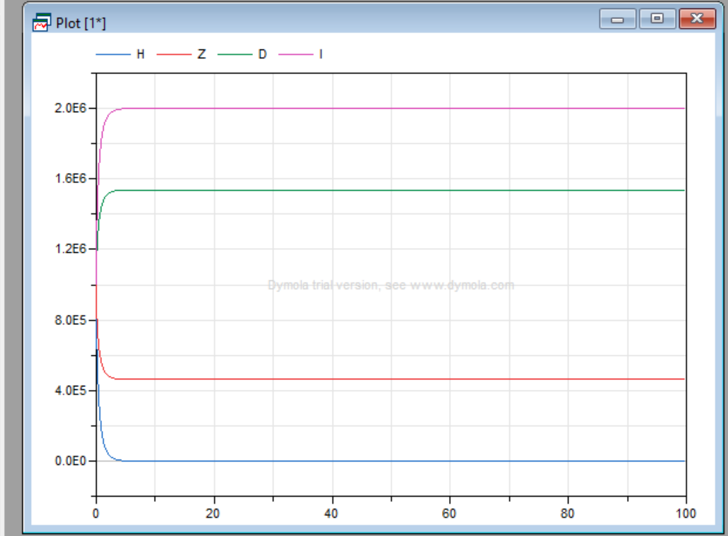
File Edit Simulation Plot Animation Commands Window Help Linear analysis

Time: 0 s Speed: 1

Variable Browser

Variable	Value
IncompleteZombieApocalypse 1	
IncompleteZombieApocalypse 2	
IncompleteZombieApocalypse 3	
☑ H	
☐ der(H)	
☑ Z	
☐ der(Z)	
☑ I	
☐ der(I)	
☑ D	
☐ der(D)	
☐ a	
☐ b	
☐ bd	
☐ d	
☐ i	
☐ n	
☐ r	

filter variables More >>



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There are 4 well-defined and non-overlapping populations:

Package Browser

Packages

- Dymola Commands
- Modelica Reference
- Modelica
- Unnamed
- IncompleteZombieApocalypse

Component Browser

IncompleteZombieApocalypse

```
model IncompleteZombieApocalypse "Incomplete zombie apocalypse model"
  // Define types, parameters and variables, as well as start values
  // ...

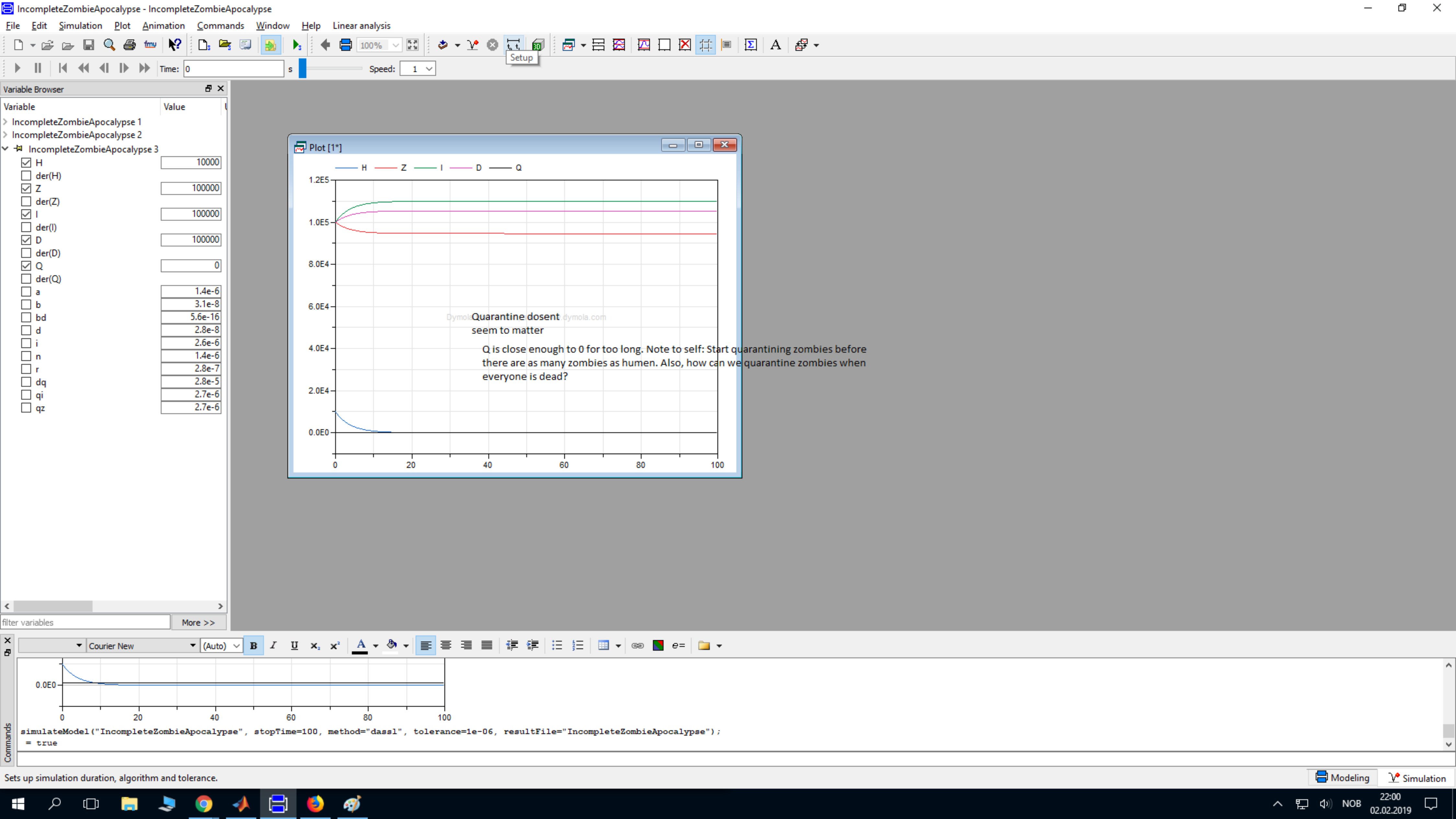
  Real H(start=10^6);
  Real Z(start=10^6);
  Real I(start=10^6);
  Real D(start=10^6);

  parameter Real a=1.4*10^(-6);
  parameter Real b=3.1*10^(-8);
  parameter Real bd=5.6*10^(-16);
  parameter Real d=2.8*10^(-8);
  parameter Real i=2.6*10^(-6);
  parameter Real n=1.4*10^(-6);
  parameter Real r=2.8*10^(-7);



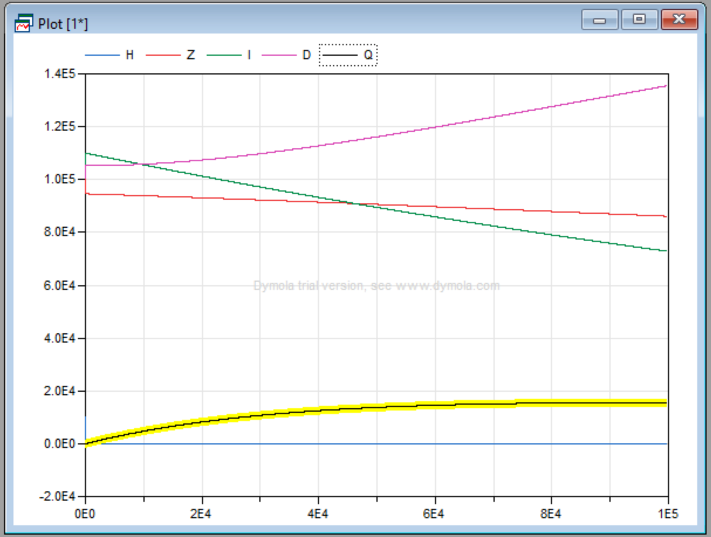

equation
  der(H)=b*H-bd*H^2-d*H-i*H*Z;
  der(Z)=r*D+a*I-n*Z*H;
  der(D)=d*H+d*I+n*Z*H-r*D;
  der(I)=i*H*Z-d*I-a*I;
  //der(H)+der(Z)+der(D)+der(I)=b*H-bd*H^2;

end IncompleteZombieApocalypse;
```


IncompleteZombieApocalypse - IncompleteZombieApocalypse
File Edit Simulation Plot Animation Commands Window Help Linear analysis
Setup
Time: 0 s Speed: 1
Variable Browser
Variable Value
IncompleteZombieApocalypse 1
IncompleteZombieApocalypse 2
IncompleteZombieApocalypse 3
H 10000
der(H)
Z 100000
der(Z)
I 100000
der(I)
D 100000
der(D)
Q 0
der(Q)
a 1.4e-6
b 3.1e-8
bd 5.6e-16
d 2.8e-8
i 2.6e-6
n 1.4e-6
r 2.8e-7
dq 2.8e-5
qi 2.7e-6
qz 2.7e-6
filter variables
More >>
Plot [1*]
H Z I D Q
1.2E5
1.0E5
8.0E4
6.0E4
4.0E4
2.0E4
0.0E0
0 20 40 60 80 100
Quarantine dosent seem to matter
Q is close enough to 0 for too long. Note to self: Start quarantining zombies before there are as many zombies as humen. Also, how can we quarantine zombies when everyone is dead?
Courier New (Auto)
simulateModel("IncompleteZombieApocalypse", stopTime=100, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
= true
Commands
Sets up simulation duration, algorithm and tolerance.
Modeling Simulation
22:00 02.02.2019

IncompleteZombieApocalypse - IncompleteZombieApocalypse

Variable	Value
H	10000
der(H)	
Z	100000
der(Z)	
I	100000
der(I)	
D	100000
der(D)	
Q	0
der(Q)	
a	1.4e-6
b	3.1e-8
bd	5.6e-16
d	2.8e-8
i	2.6e-6
n	1.4e-6
r	2.8e-7
dq	2.8e-5
qi	2.7e-6
qz	2.7e-6



If we simulate for several years, we can see that the infected are still living on healthily.
Conclusions: The infected must be the ones qurantining the zombies when all healthy are dead.
Also, since this simulation is lasting 10000/365=273 years, we can say that human civilization will continue to exist for centuies after all dead healthy men are dead. And with a positive look at science, we can also hope that the infected will find a cure to the main problem of humanity over there years. But if we compare it to global warming, maybe noone will care after all.

```
simulateModel("IncompleteZombieApocalypse", stopTime=100, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
simulateModel("IncompleteZombieApocalypse", stopTime=100, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
simulateModel("IncompleteZombieApocalypse", stopTime=100, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
simulateModel("IncompleteZombieApocalypse", stopTime=100000, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
```

https://ntnu.blackboard.com/bbcswebdav/pid-581805-dt-content-rid-19498505_1/cour...

The quarantined approach only seems to delay the inevitable. Luckily, news of a cure for "zombie-ism" arrive. This treatment converts zombies and infected individuals to healthy individuals: If the individual was infected, it returns back to its healthy human form; and if it was resurrected from the dead, it returns to its healthy human form before death. Note however that this treatment does not provide immunity, i.e. individuals that are given the treatment may be converted to zombies in the future.

In order to model the effects of the cure, add the following changes to the previous model:

- Quarantine is no longer needed, i.e. the population Q is no longer part of the model.
- Zombies and infected individuals move to the healthy population with rate $c > 0$.

(e) Model the new dynamics of the populations H, I, Z and D.

(f) Extend the Modelica model from part c. to represent the new system $[H, I, Z, D]^T$. Do not change the parameter values for a, b, b_d, d, i, n and r, and do not change the start values for H, I, Z and D. Furthermore, use the parameter value $c = 2.7 \cdot 10^{-3}$.

Simulate the new model for 100 days. Add the Modelica model and a plot with the obtained results for all the populations to your answer. What is the stationary value of H? Comment on the results.

Hint: Not sure about the stationary value of H? Simulate the model for some years.

Your are a champion of humanity! But before you can relax and think back with pride of the many lives your work saved, there is still one task to do: to clean up your code.

(g) Explain how you would use the language features of Modelica to reduce the amount of repeated code in the 3 models developed in parts (b), (d) and (f).

Feel free to rewrite the model codes so that the number of repeated code lines is minimized. If you do so, you only need to add the newest versions of the model codes to your answer. However, no code implementation is needed to solve this task.

A zombie outbreak is of course an unrealistic scenario if taken literally. However, the structure of the models developed here can be used for real-life applications.

(h) Give some examples of such applications.

Problem 2 (Euler's method, linearization, stability. 25 %)

In this problem we will study the system

IncompleteZombieApocalypse - IncompleteZombieApocalypse - [Modelica Text - Editing]

File Edit Simulation Plot Animation Commands Window Help Linear analysis

Package Browser

Packages
- Dymola Commands
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IncompleteZombieApocalypse

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  // ...

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  Real Z(start=10^6);
  Real I(start=10^6);
  Real D(start=10^6);
  Real Q;

  parameter Real a=1.4*10^(-6);
  parameter Real b=3.1*10^(-8);
  parameter Real bd=5.6*10^(-16);
  parameter Real d=2.8*10^(-8);
  parameter Real i=2.6*10^(-6);
  parameter Real n=1.4*10^(-6);
  parameter Real r=2.8*10^(-7);
  parameter Real dq=2.8*10^(-5);
  parameter Real qi=2.7*10^(-6);
  parameter Real qz=2.7*10^(-6);



equation
  der(H)=b*H-bd*H^2-d*H-i*H*Z;
  der(Z)=r*D+a*I-n*Z*H-qz*Z;
  der(D)=d*H+d*I+n*Z*H-r*D+dq*Q;
  der(I)=i*H*Z-d*I-a*I-qi*I;
  der(Q)=qz*Z+qi*I-dq*Q;

  //der(H)+der(Z)+der(D)+der(I)=b*H-bd*H^2;
  a
end IncompleteZombieApocalypse;
```

Line: 30 Modeling Simulation

NOB 22:11 02.02.2019

The quarantined approach only seems to delay the inevitable. Luckily, news of a cure for "zombie-ism" arrive. This treatment converts zombies and infected individuals to healthy individuals: If the individual was infected, it returns back to its healthy human form; and if it was resurrected from the dead, it returns to its healthy human form before death. Note however that this treatment does not provide immunity, i.e. individuals that are given the treatment may be converted to zombies in the future.

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IncompleteZombieApocalypse - IncompleteZombieApocalypse - [Modelica Text]

```
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  // Define types, parameters and variables, as well as start values
  // ...

  Real H(start=10^6);
  Real Z(start=10^6);
  Real I(start=10^6);
  Real D(start=10^6);

  parameter Real a=1.4*10^(-6);
  parameter Real b=3.1*10^(-8);
  parameter Real bd=5.6*10^(-16);
  parameter Real d=2.8*10^(-8);
  parameter Real i=2.6*10^(-6);
  parameter Real n=1.4*10^(-6);
  parameter Real r=2.8*10^(-7);
  parameter Real c=2.7*10^(-7);

equation
  der(H)=b*H-bd*H^2-d*H-i*H*Z+c*Z+c*I;
  der(Z)=r*D+a*I-n*Z*H-c;
  der(D)=d*H+d*I+n*Z*H-r*D;
  der(I)=i*H*Z-d*I-a*I-c*I;

  //der(H)+der(Z)+der(D)+der(I)=b*H-bd*H^2;
  a
end IncompleteZombieApocalypse;
```

IncompleteZombieApocalypse - IncompleteZombieApocalypse

File Edit Simulation Plot Animation Commands Window Help Linear analysis

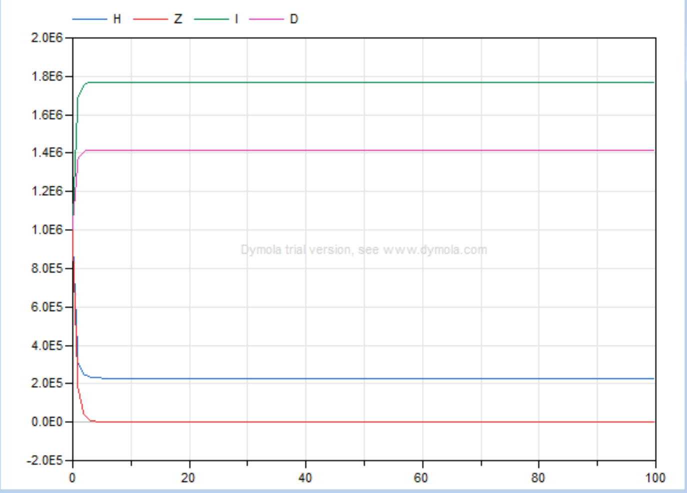
Time: 0 s Speed: 1

Variable Browser

Variable	Value
IncompleteZombieApocalypse 1	
IncompleteZombieApocalypse 2	
IncompleteZombieApocalypse 3	
H	1e+6
der(H)	
Z	1e+6
der(Z)	
I	1e+6
der(I)	
D	1e+6
der(D)	
a	1.4e-6
b	3.1e-8
bd	5.6e-16
d	2.8e-8
i	2.6e-6
n	1.4e-6
r	2.8e-7
c	2.7e-7

filter variables More >>

Plot [1*]



EXTRA:
If the infected kill zombies half as often as healthy people, we also fix the apocalypse. It goes to show that is everyone is helping, the world become a better place.

```
simulateModel("IncompleteZombieApocalypse", stopTime=10000, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
simulateModel("IncompleteZombieApocalypse", stopTime=10000, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
simulateModel("IncompleteZombieApocalypse", stopTime=10000, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
simulateModel("IncompleteZombieApocalypse", stopTime=100, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
```

Modeling Simulation

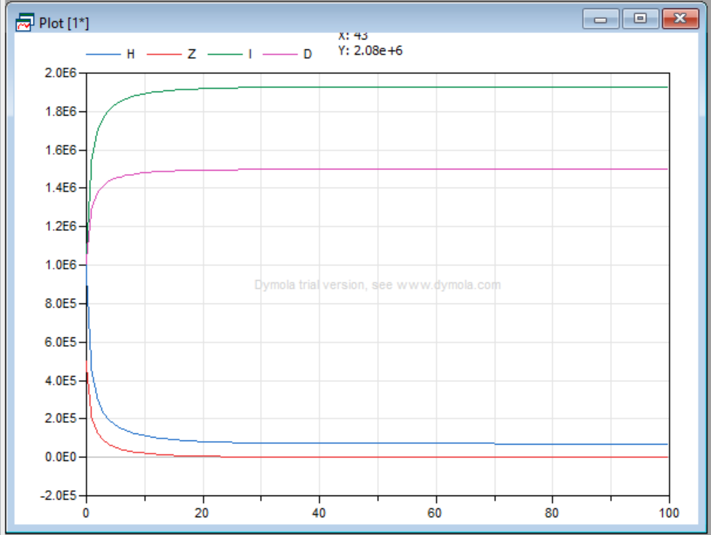
22:28 02.02.2019

IncompleteZombieApocalypse - IncompleteZombieApocalypse

Variable Browser

Variable	Value
IncompleteZombieApocalypse 1	
IncompleteZombieApocalypse 2	
IncompleteZombieApocalypse 3	
H	1e+6
der(H)	
Z	500000
der(Z)	
I	1e+6
der(I)	
D	1e+6
der(D)	
a	1.4e-6
b	3.1e-8
bd	5.6e-16
d	2.8e-8
i	2.6e-6
n	1.4e-6
r	2.8e-7
c	2.7e-7

Plot [1*]



Afte ronly halving the start value of zombies, men manage to kill them off yey.

The stationary value of H is low, but there is still a lot of very functional infected people. Humanity will be fine

```
simulateModel("IncompleteZombieApocalypse", stopTime=100, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
Z = 1000000;
simulateModel("IncompleteZombieApocalypse", stopTime=100, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
Z = 5e+5;
simulateModel("IncompleteZombieApocalypse", stopTime=100, numberOfIntervals=0, outputInterval=1, method="dassl", tolerance=1e-06, resultFile="IncompleteZombieApocalypse");
 = true
```