POLITECNICO DI MILANO

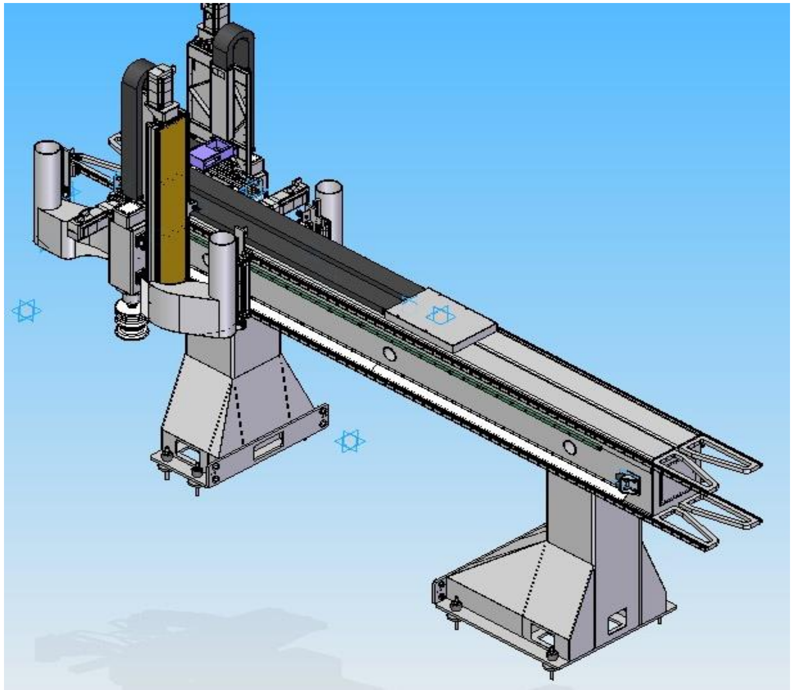
Finite Element Simulation For Mechanical Design

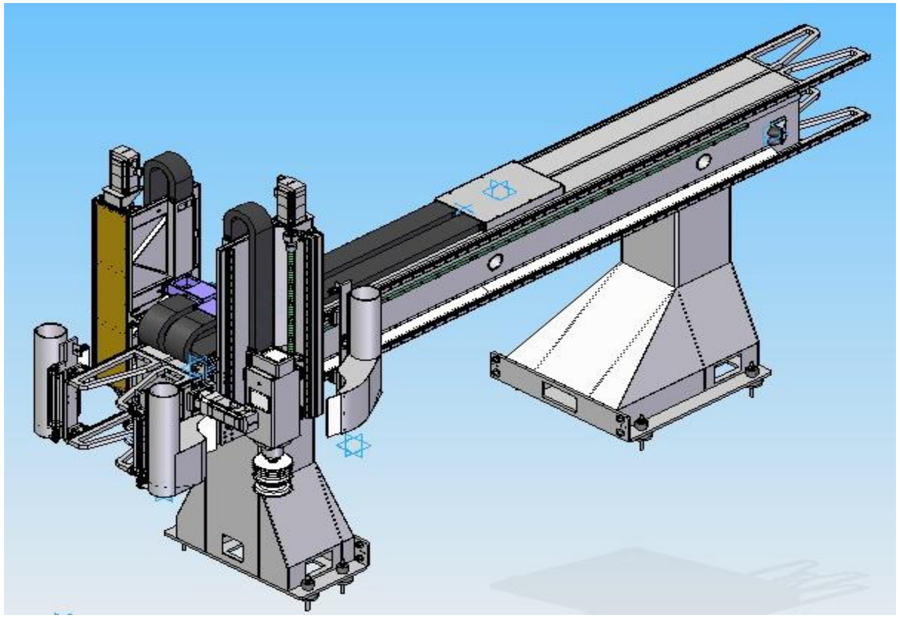
FE Analysis of the structure of a machine tool:

Beam elements

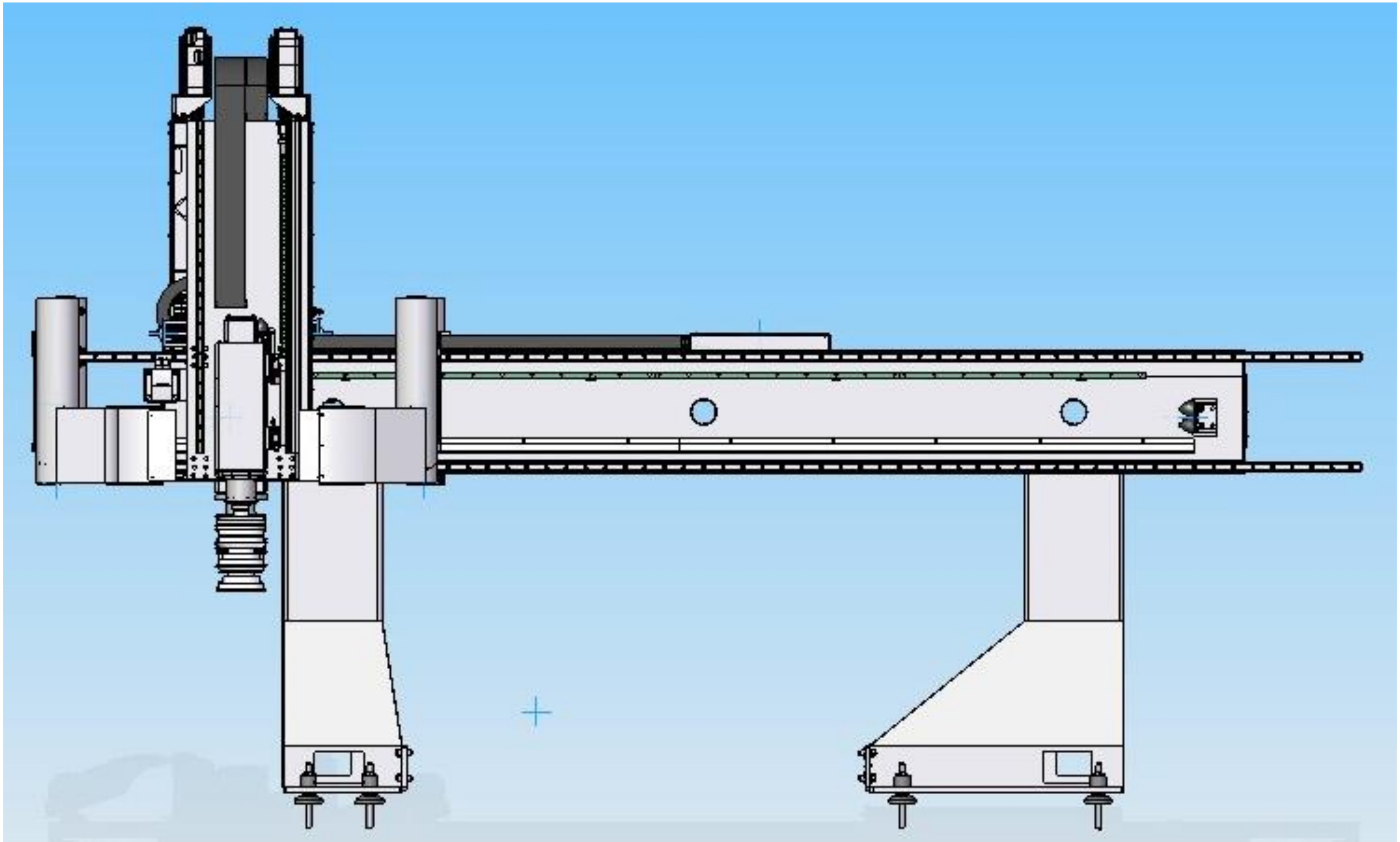
A. Bernasconi

Introduction

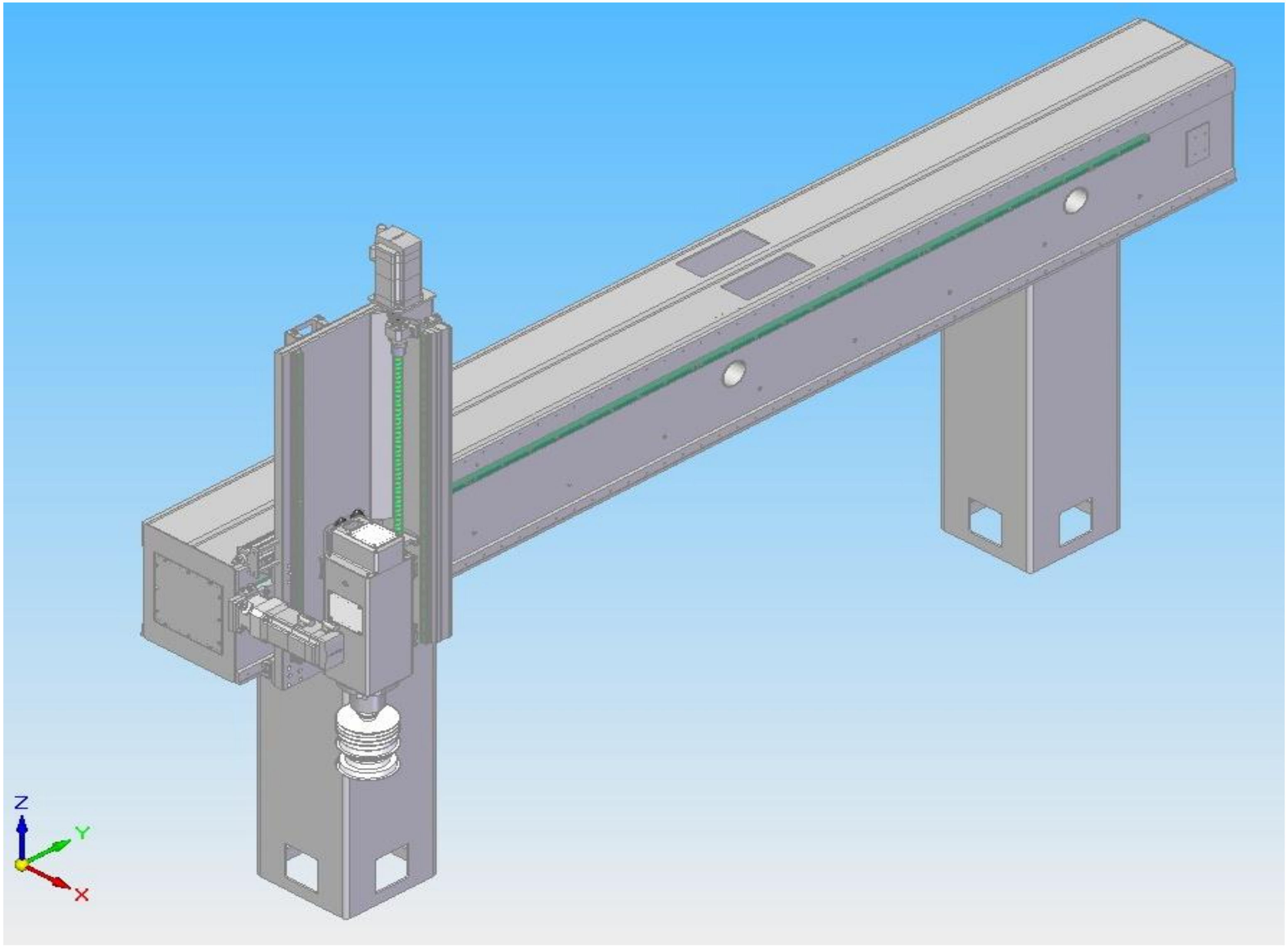




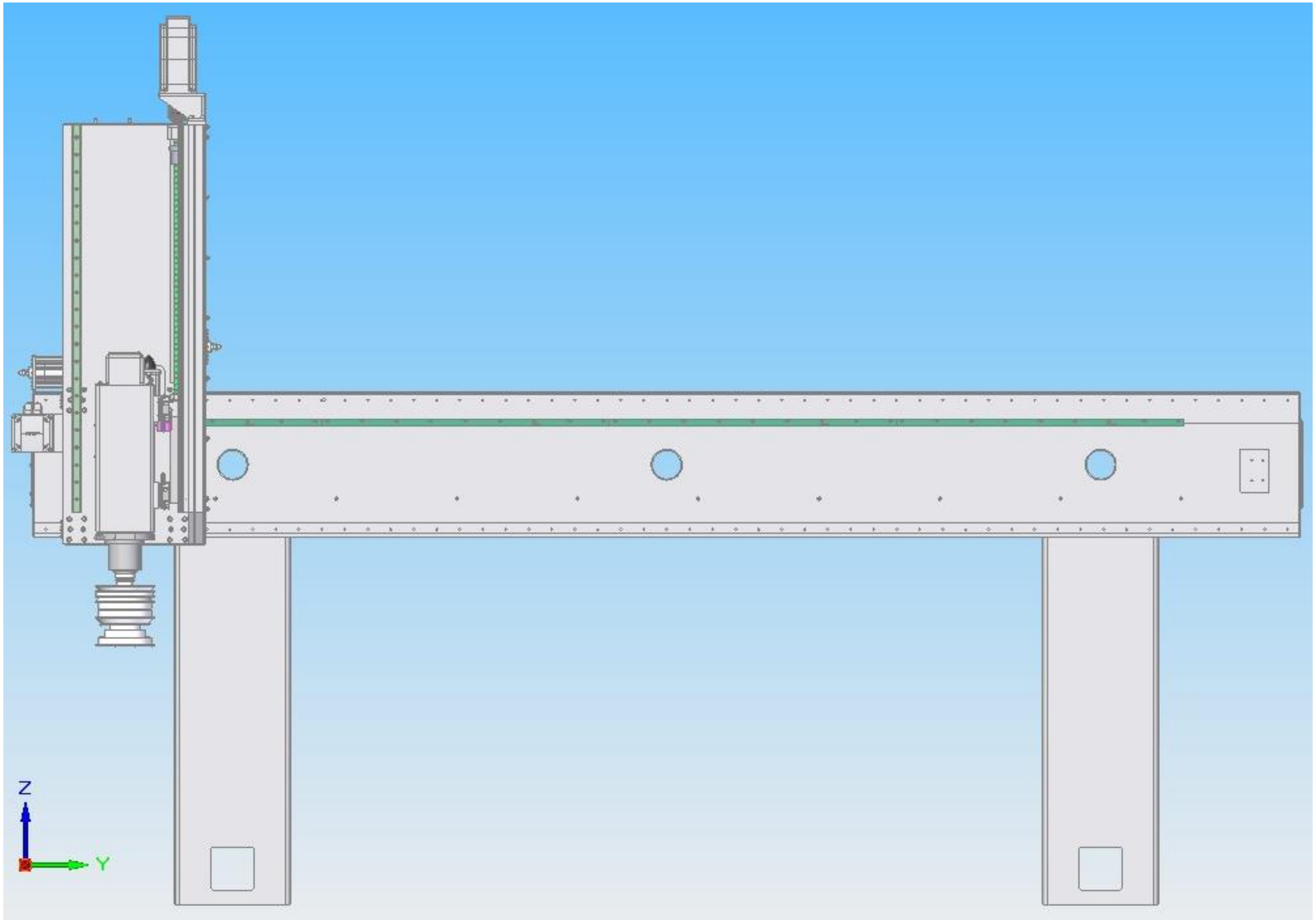
Introduction



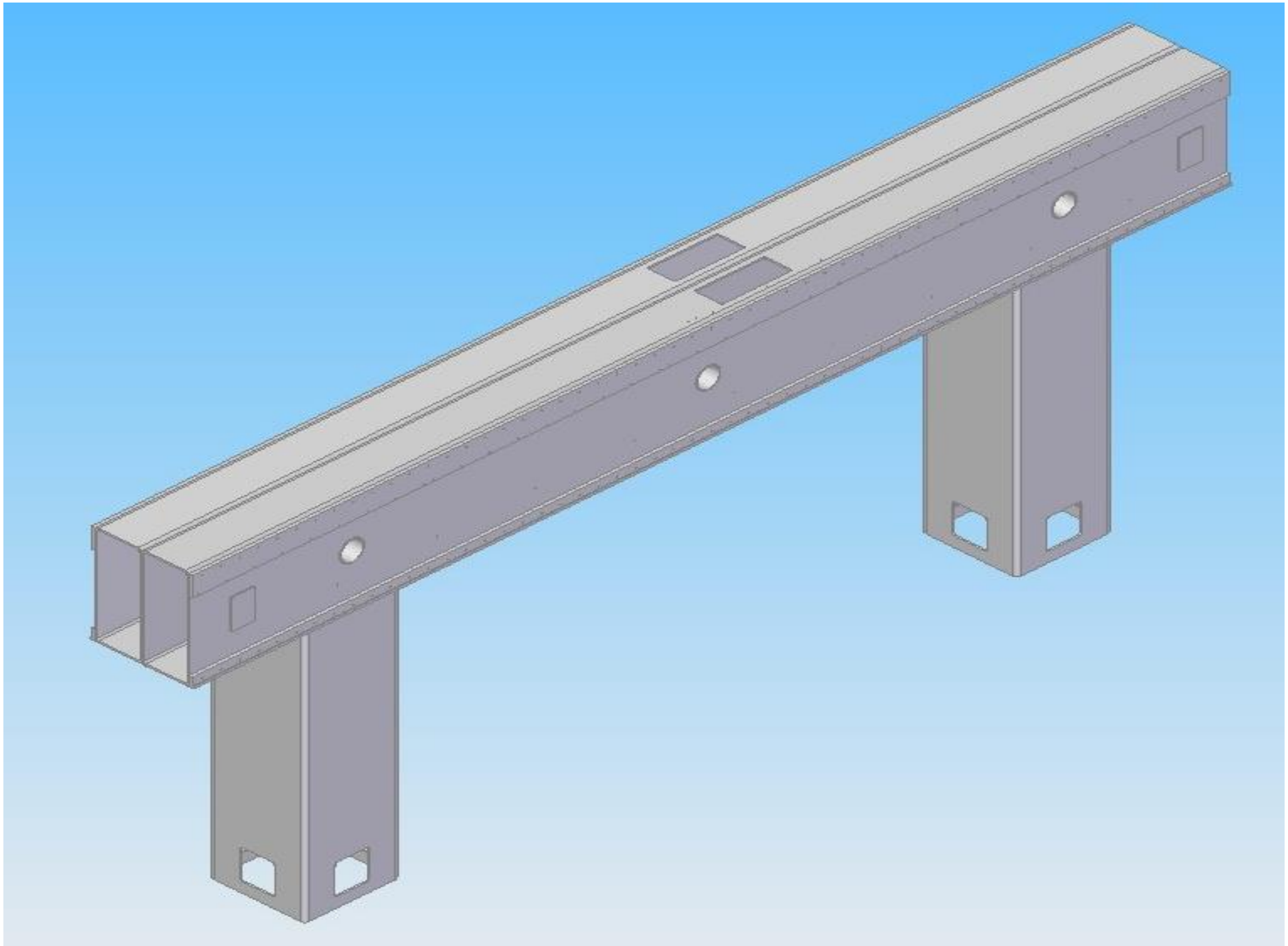
Simplified model



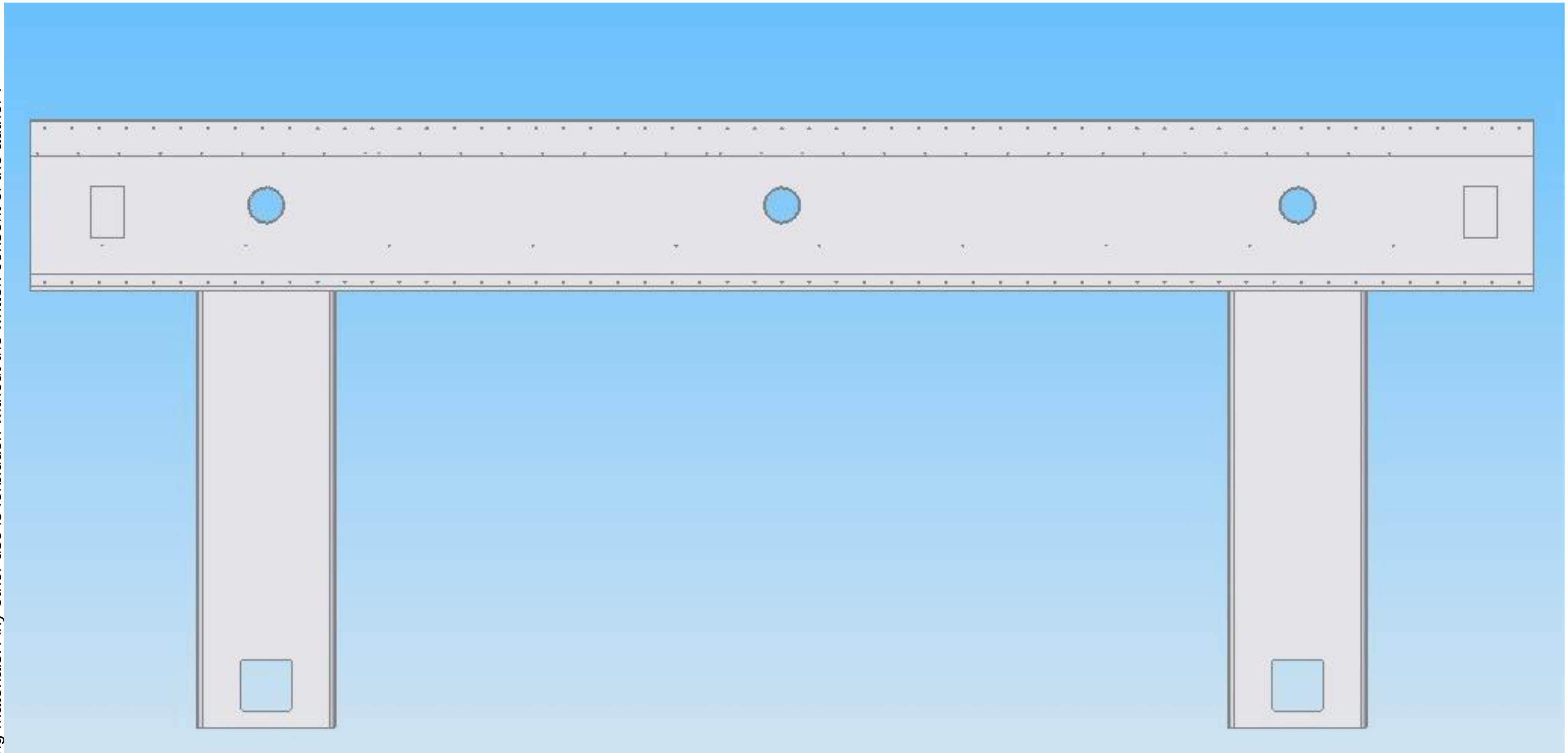
Simplified model



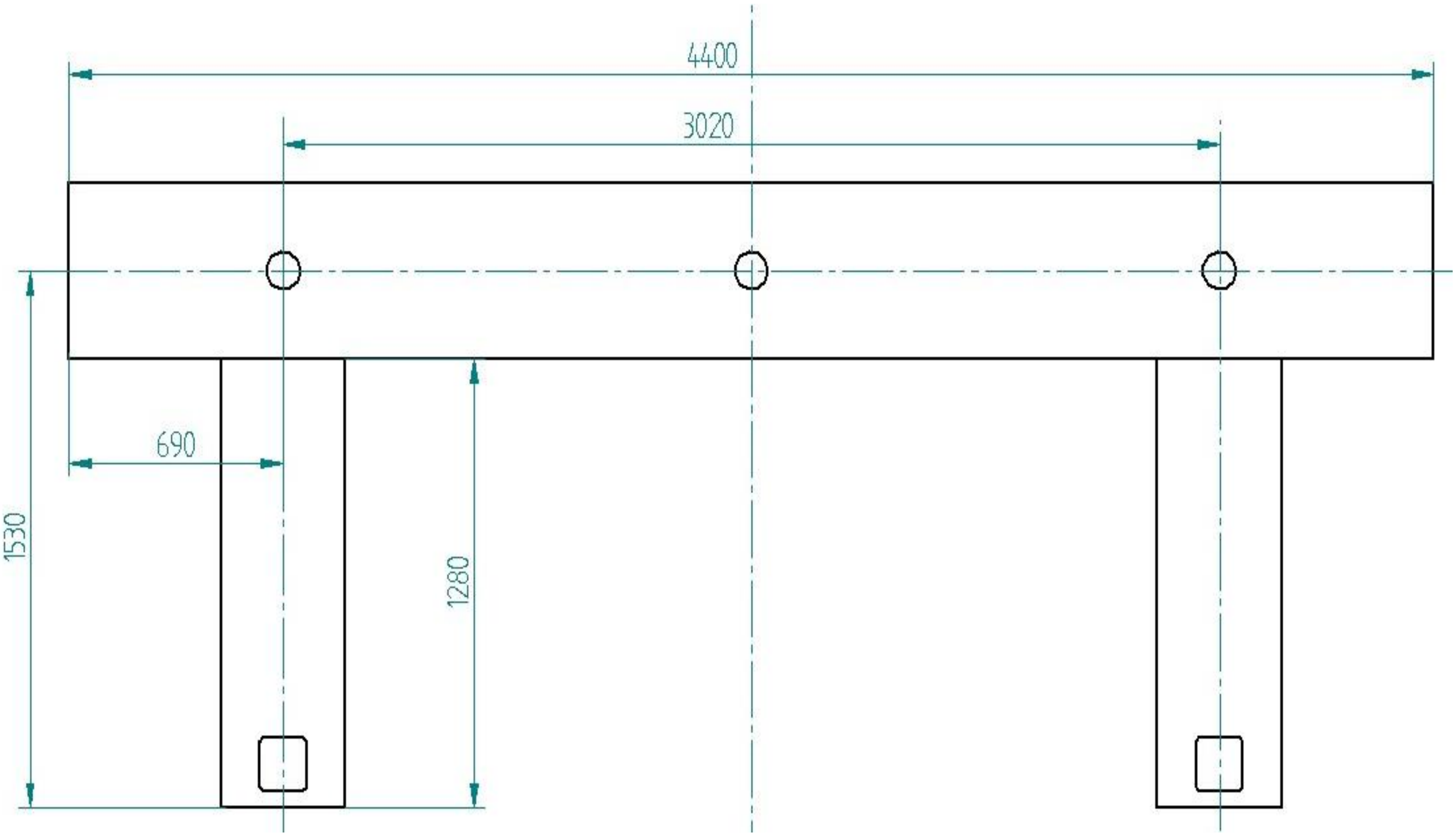
Simplified model



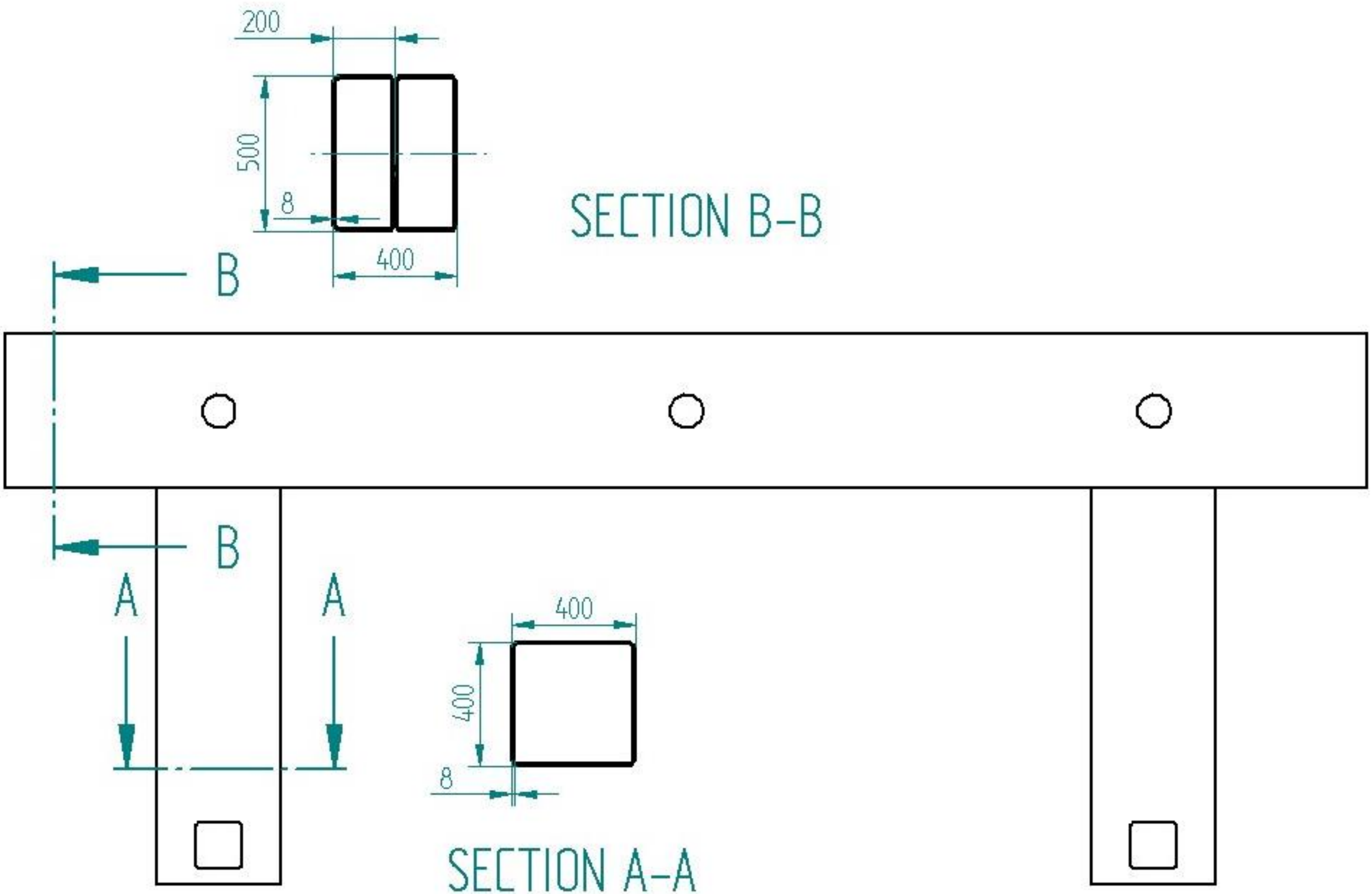
Simplified model



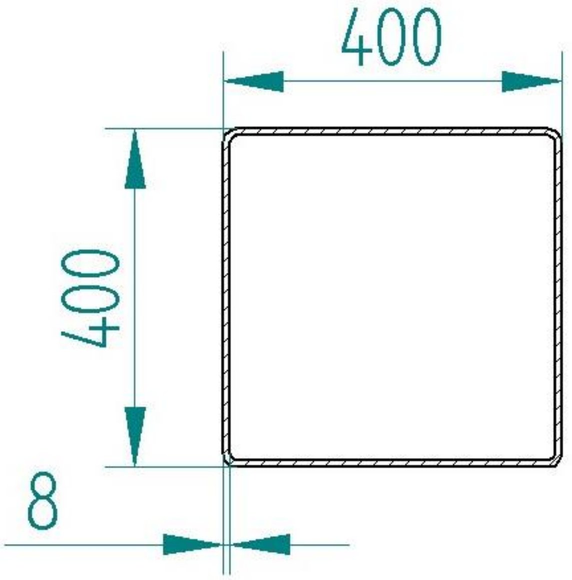
Dimensions of the simplified model



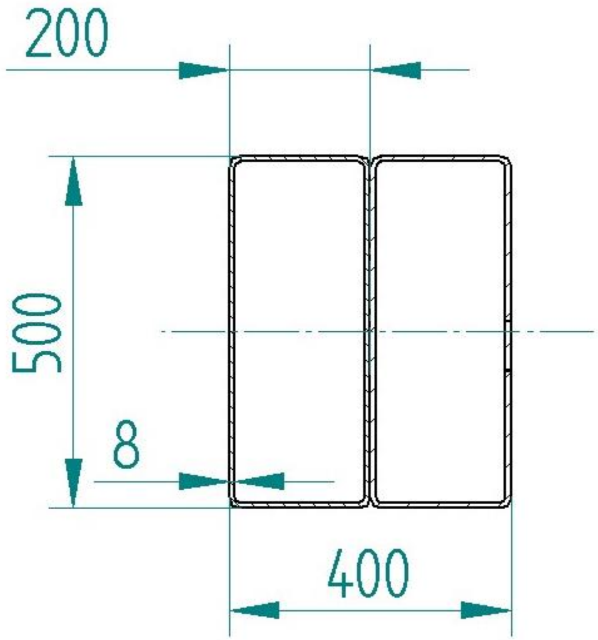
Dimensions of the simplified model



Dimensions of the columns (sx) and crosshead (dx)



SECTION A–A

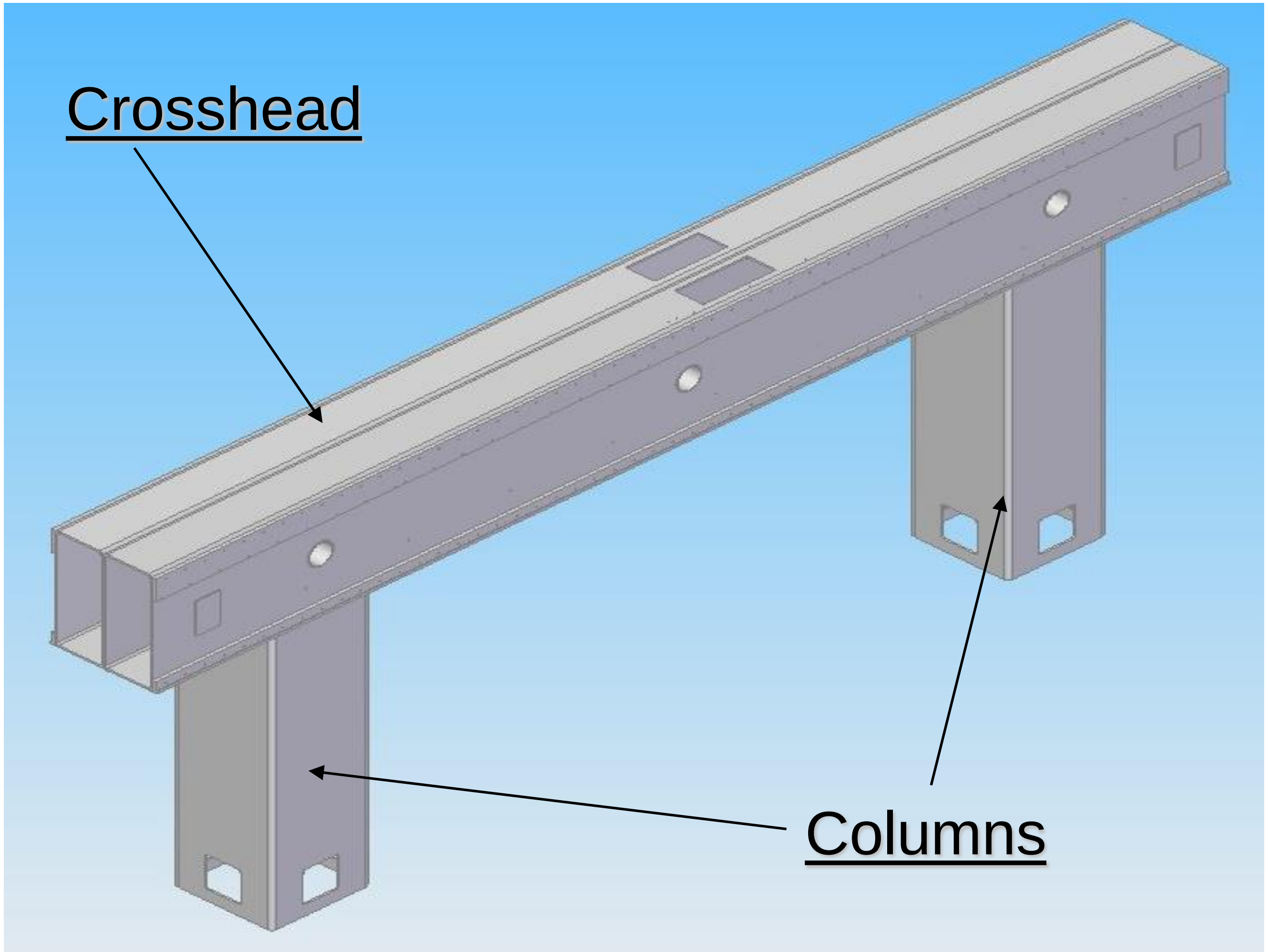


SECTION B–B

NB: Remove fillet radiuses

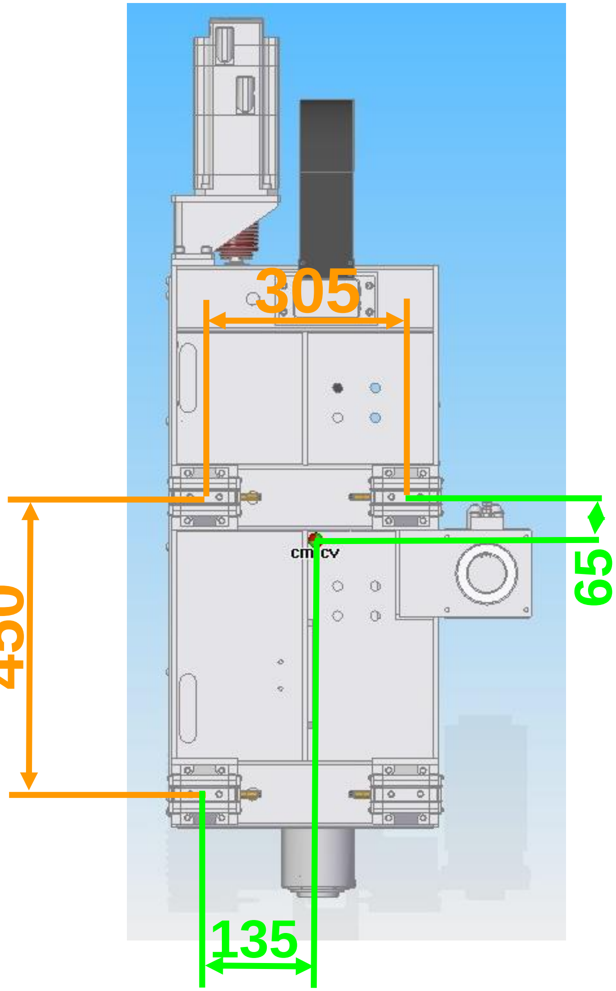
FE model

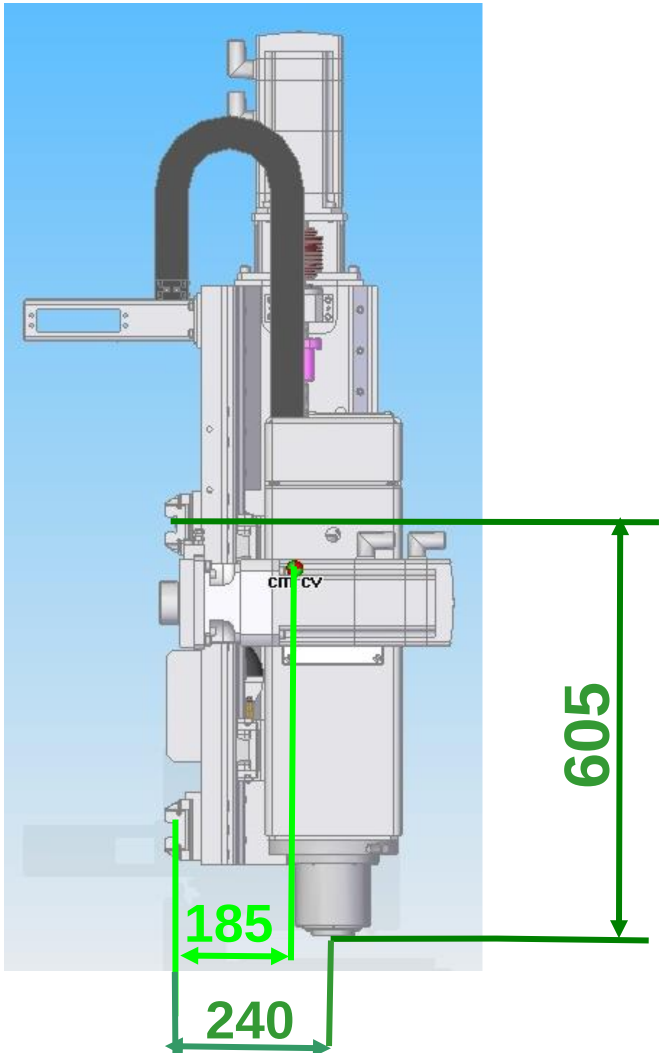
Parts to be modeled



FE model

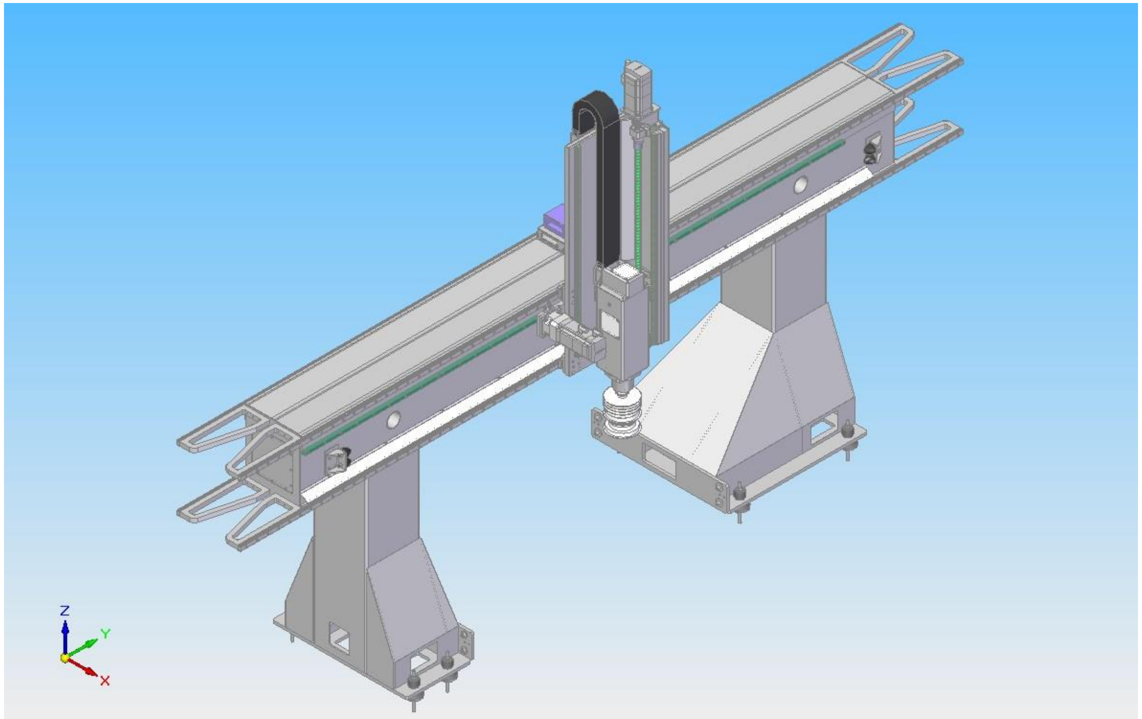
Position of the center of mass and spindle nose





FE model

Two cutting forces are applied to the spindle nose



F1 = 500 N

Perpendicular to the crosshead

F2 = 600 N

Parallel to the crosshead

Simplifications:

Forces are supposed to be applied to spindle nose, neglecting the stiffness of the tool and of the coupling between tool and spindle

Stiffness of the carriage and mandrel are not considered

Stiffness of guideways and cross slide are neglected

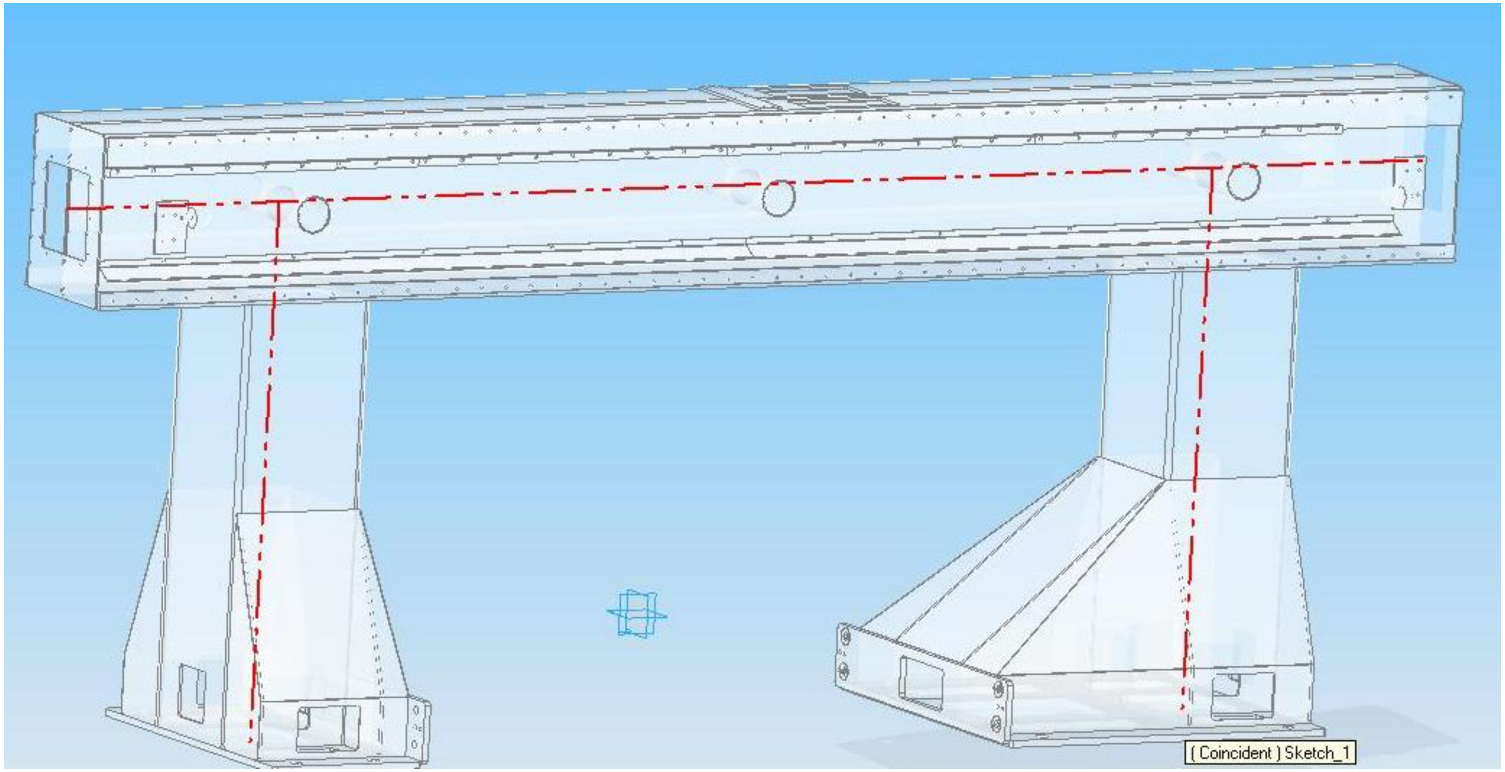
Stiffness of the splindle is neglected

FE model

Beam model

Wire parts need to be buildt

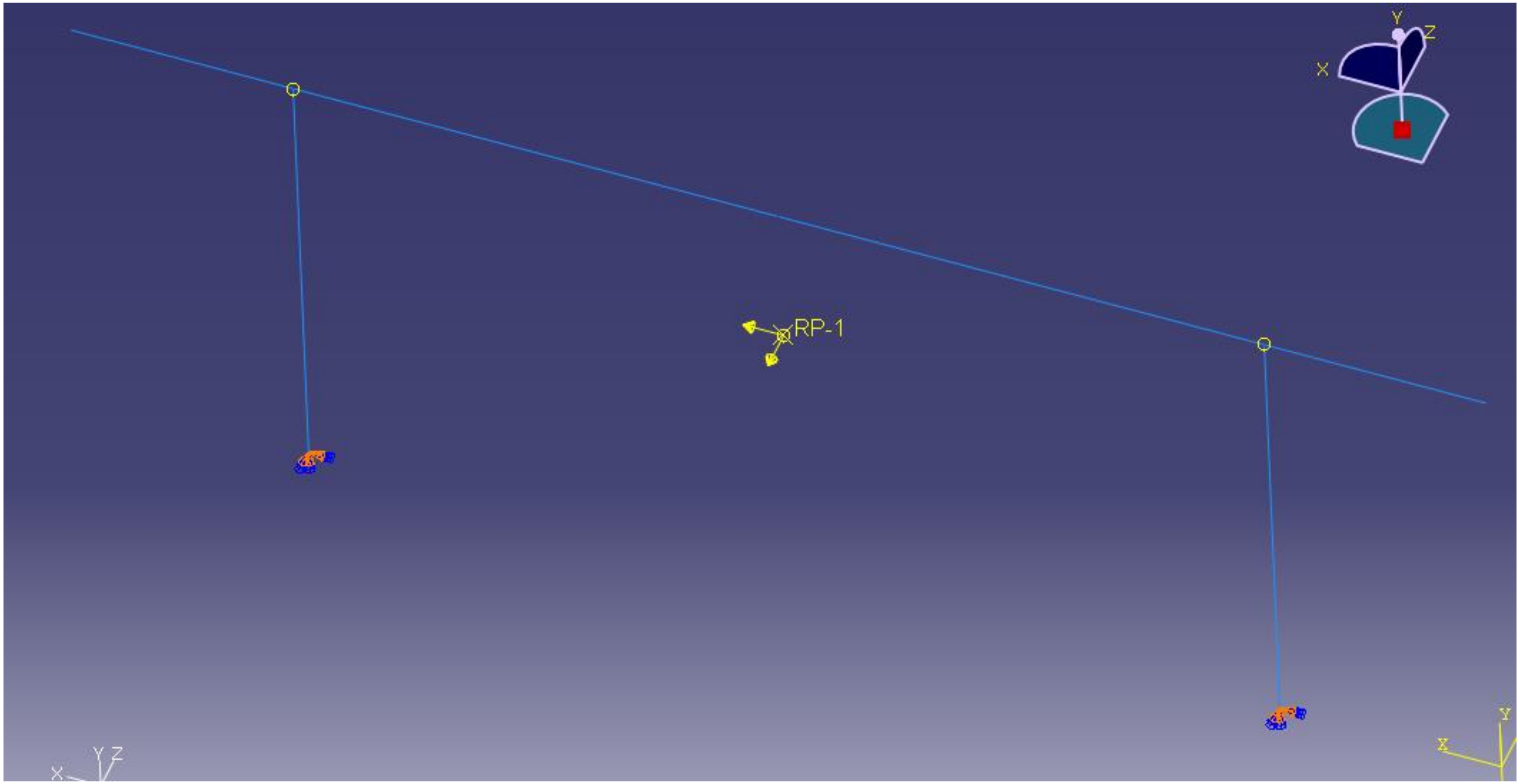
Geometry of the wire features forms a sort of backbone of the structure



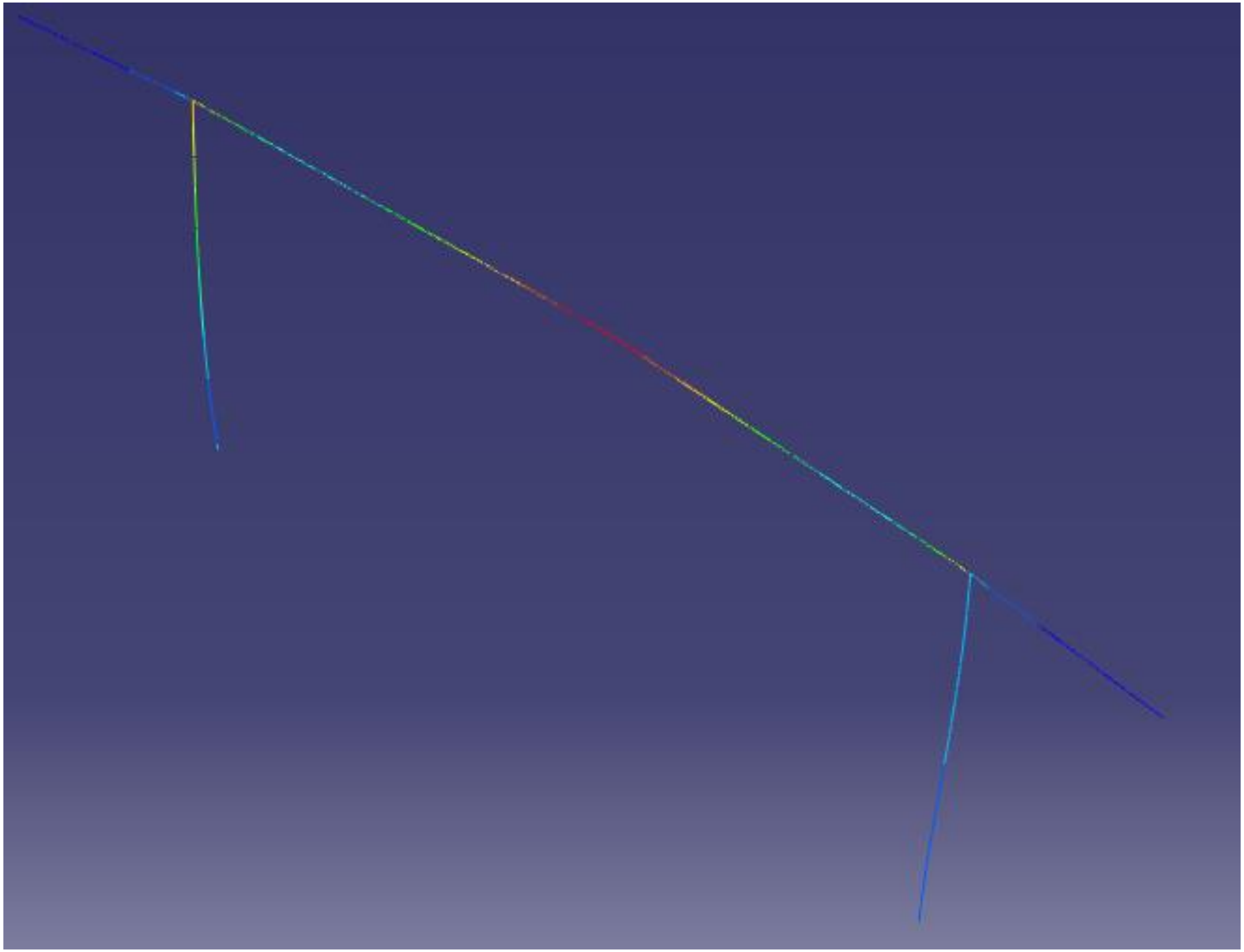
FE beam model

Forces are applied to a reference point (RP)

RP is connected to the crosshead using a rigid kinematic coupling

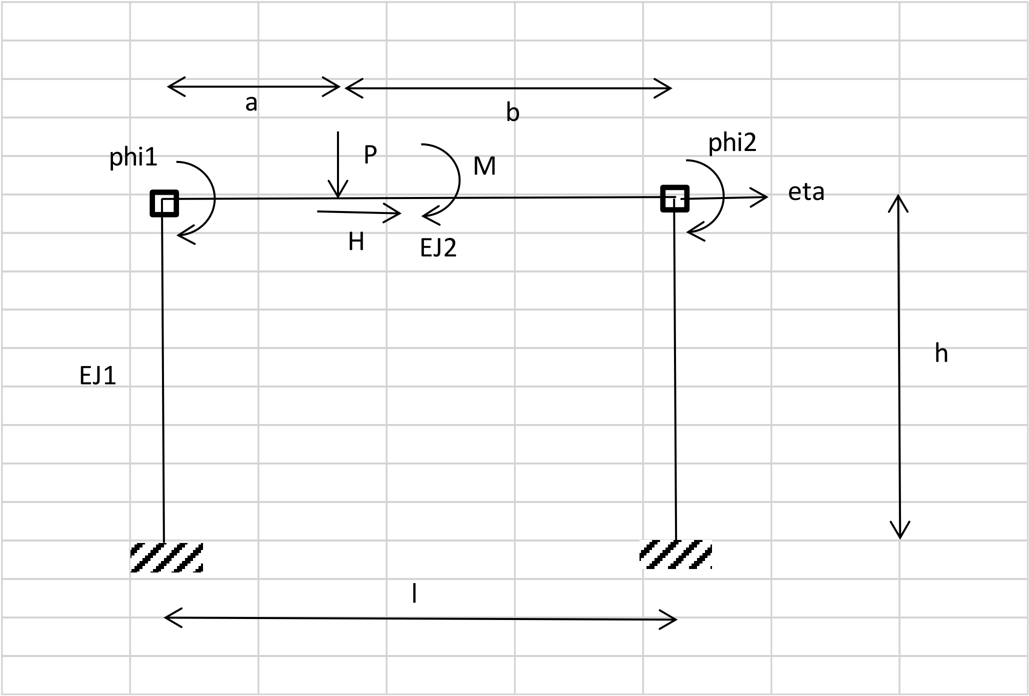


FE beam model: results

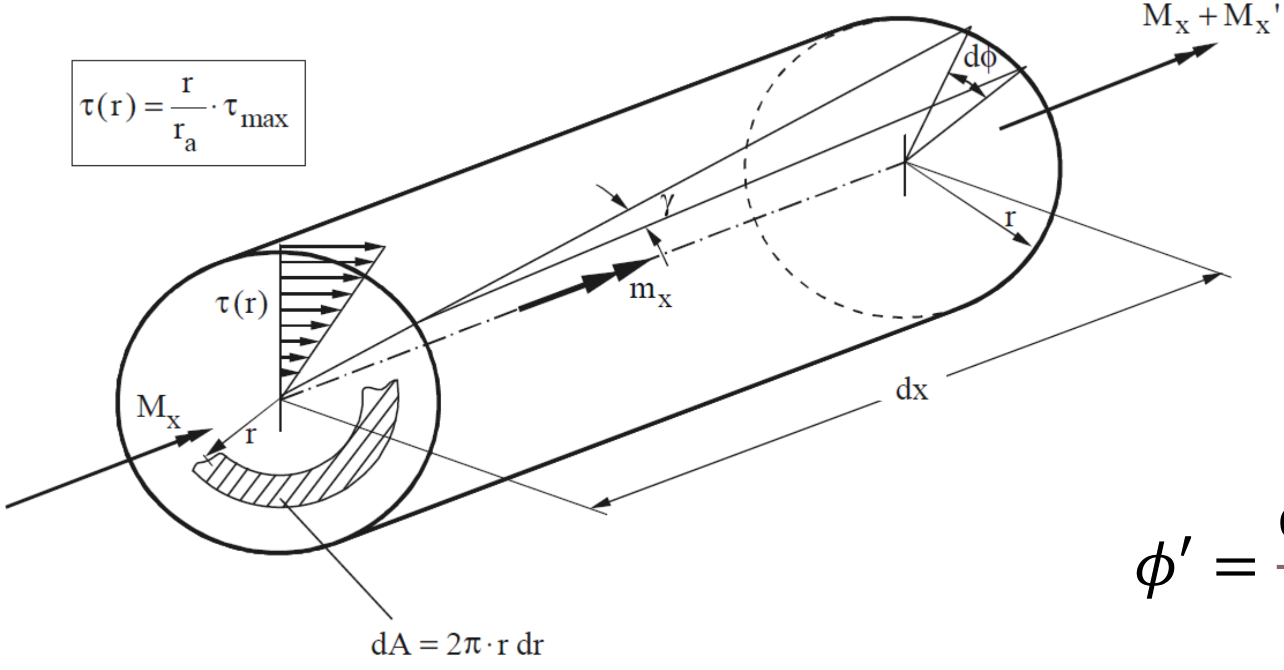


Check results against another method

Stiffness method with vertical load
Vertical load is applied for validation purposes only



Torsion of circular cross section beams



$$\phi' = \frac{d\varphi}{dx}$$

Angle of twist per unit length

$$\tau(r) = G \cdot \gamma = G \cdot \phi' \cdot r \equiv G \cdot D \cdot r$$

$$M_x \equiv \int_0^r \tau(r) \cdot dA \cdot r = G \cdot \phi' \underbrace{\int_0^r r^2 \cdot dA}_{J_p} = G \cdot J_p \cdot \phi'$$

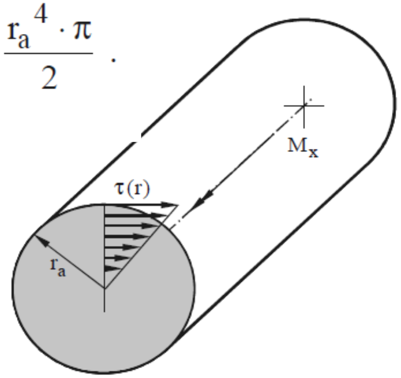
Torsion of solid and hollow cross sections

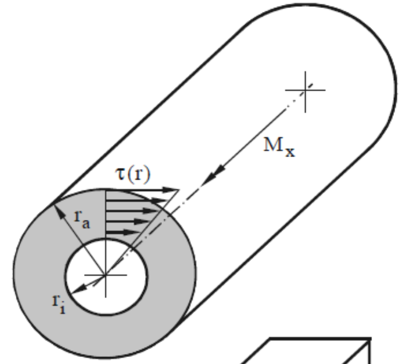
$$\tau_{max} = G \cdot r_a \cdot \phi' = \frac{M_x \cdot r_a}{J_t}$$

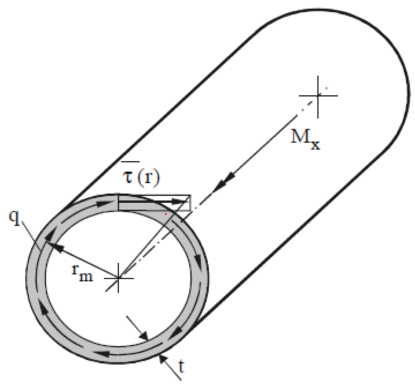
$$J_t \equiv J_p = \int_0^{r_a} r^2 dA = 2\pi \int_0^{r} r^3 dr = \frac{r_a^{\,4} \cdot \pi}{2} \, .$$

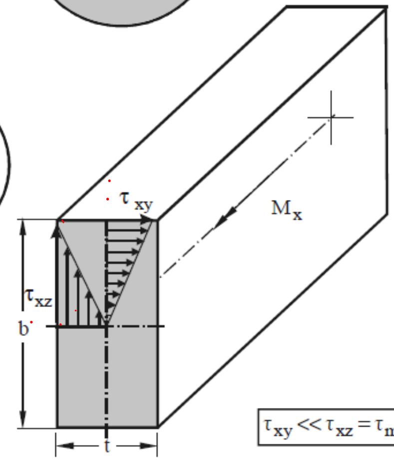
$$\tau_{max} = \frac{M_x \cdot r_a}{J_t}, \qquad \tau_{min} = \frac{M_x \cdot r_i}{J_t} \, ,$$

$$J_t \equiv J_p = \frac{\pi}{2}\left(r_a^{\,4} - r_i^{\,4}\right)$$









b/t	1	1,5	2	3	4	5	6	8	10	∞
ζ_1	0,424	0,588	0,687	0,789	0,843	0,875	0,897	0,920	0,938	1
ζ_2	0,625	0,664	0,737	0,801	0,845	0,873	0,894	0,919	0,936	1

$\boxed{\tau_{xy} << \tau_{xz} = \tau_{max}}$

$$J_t \equiv J_p = 2\,\pi \cdot r_m^{\,3} \cdot t \left[1 + \left(\frac{t}{2\,r_m}\right)^2\right] \approx 2\,\pi \cdot r_m^{\,3} \cdot t,$$

$$J_t = \frac{1}{3} t^3 \cdot b \left\{1 - \frac{192}{\pi^5} \cdot \frac{t}{b}\left[\tanh\left(\frac{\pi \cdot b}{2\,t}\right) + \frac{1}{243}\tanh\left(\frac{3\pi \cdot b}{2\,t}\right)\right]\right\} \approx \frac{1}{3}\zeta_1 \cdot t^3 \cdot b$$

Closed hollow sections: Bredt's formula

q, shear flow; $q = \tau(s)\, t(s)$

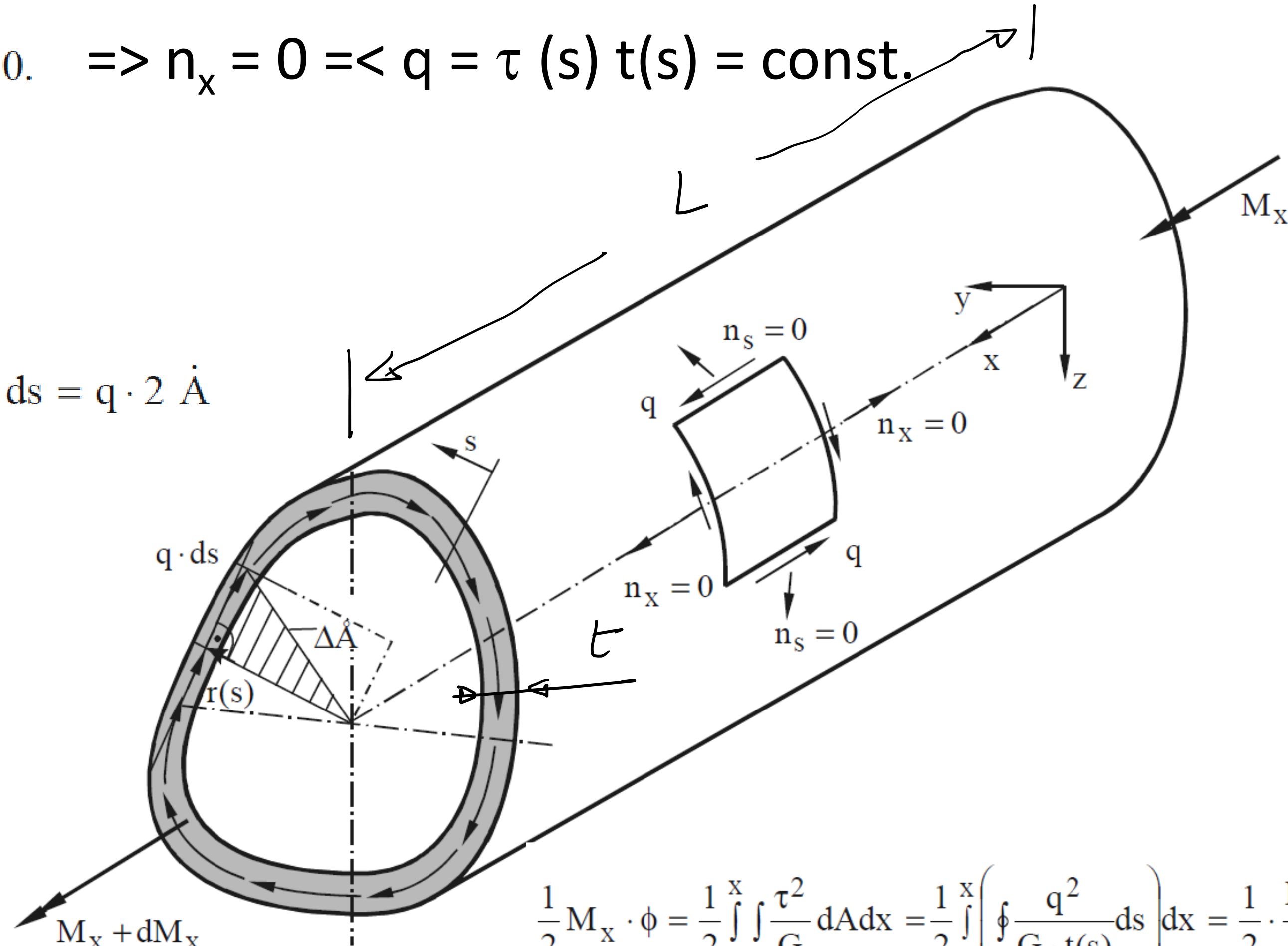
$$\frac{\partial n_x}{\partial x} + \frac{\partial q}{\partial s} = 0.$$

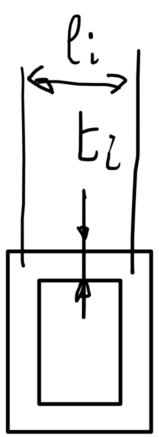
$\Rightarrow n_x = 0 \Leftarrow q = \tau(s)\, t(s) = \text{const.}$

$$\tau(s) = \frac{q}{t(s)}$$

$$M_x = q \oint_0^s r \cdot ds = q \cdot 2\, \dot{A}$$

$$q = \frac{M_x}{2\, \dot{A}}\,.$$





$$\oint \frac{ds}{t} = \sum_i \frac{l_i}{t_i}$$

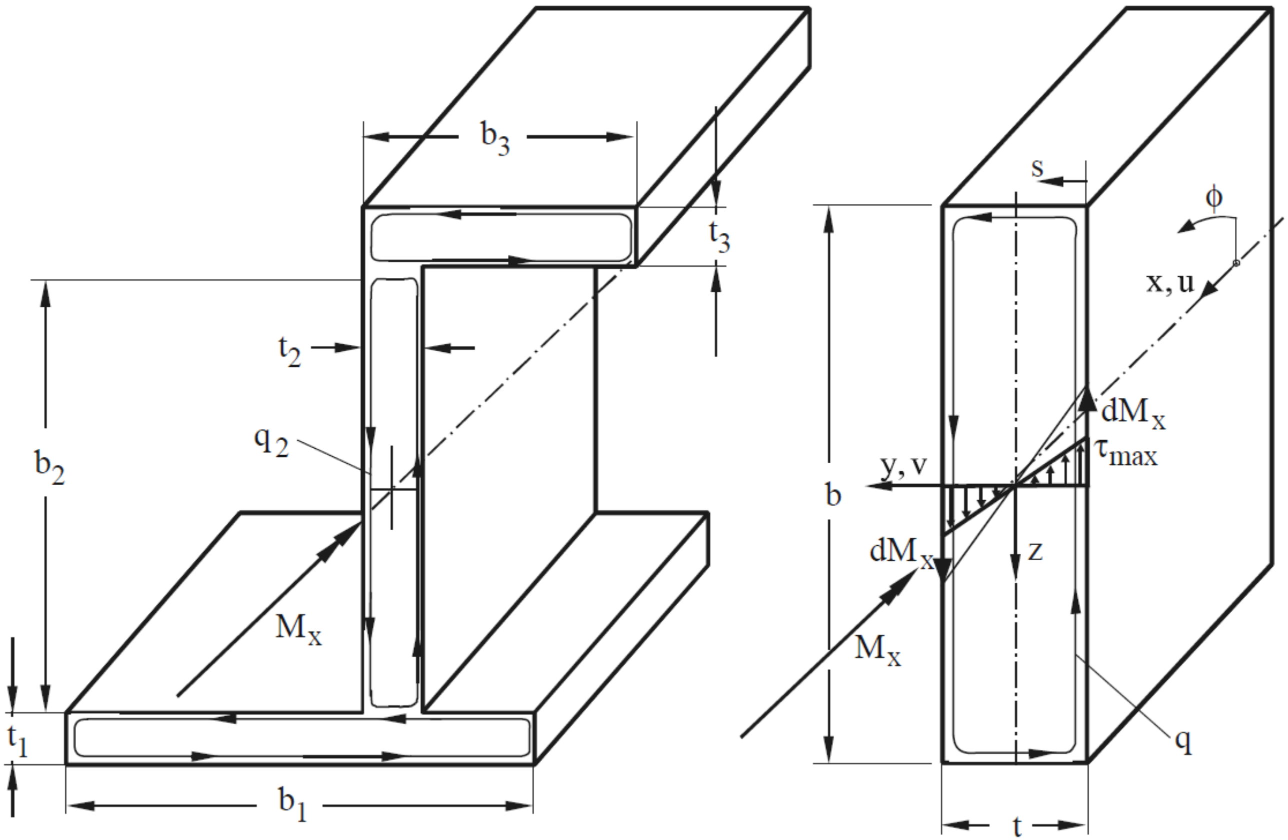
$$\frac{1}{2} M_x \cdot \phi = \frac{1}{2} \int_0^x \int_A \frac{\tau^2}{G}\, dA\, dx = \frac{1}{2} \int_0^x \left(\oint_s \frac{q^2}{G \cdot t(s)}\, ds \right) dx = \frac{1}{2} \cdot \frac{M_x{}^2 \cdot x}{4 \dot{A}^2 \cdot G} \cdot \oint_s \frac{ds}{t(s)}$$

$$\phi' = \frac{M_x}{4 \dot{A}^2 \cdot G} \oint \frac{ds}{t(s)}\,.$$

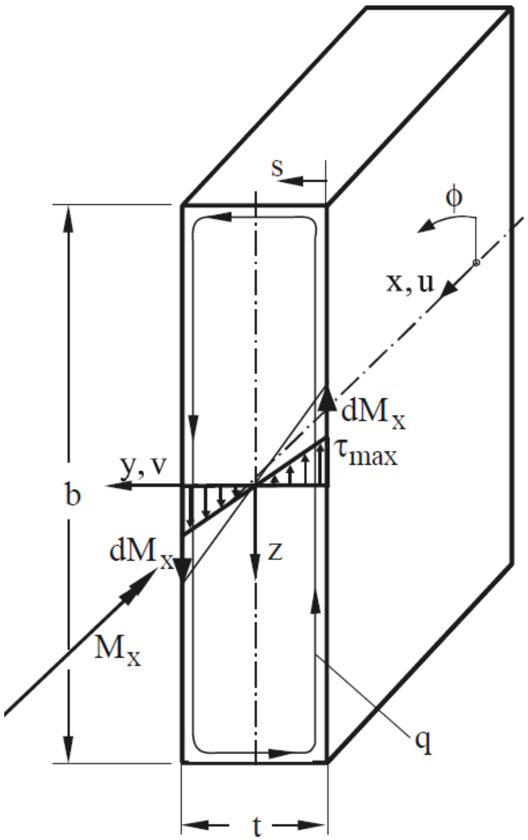
$$J_t = \frac{4\, \dot{A}^2}{\oint \frac{ds}{t(s)}}\,.$$

Torsion constant

Open, thin walled cross sections



Open, thin walled cross sections



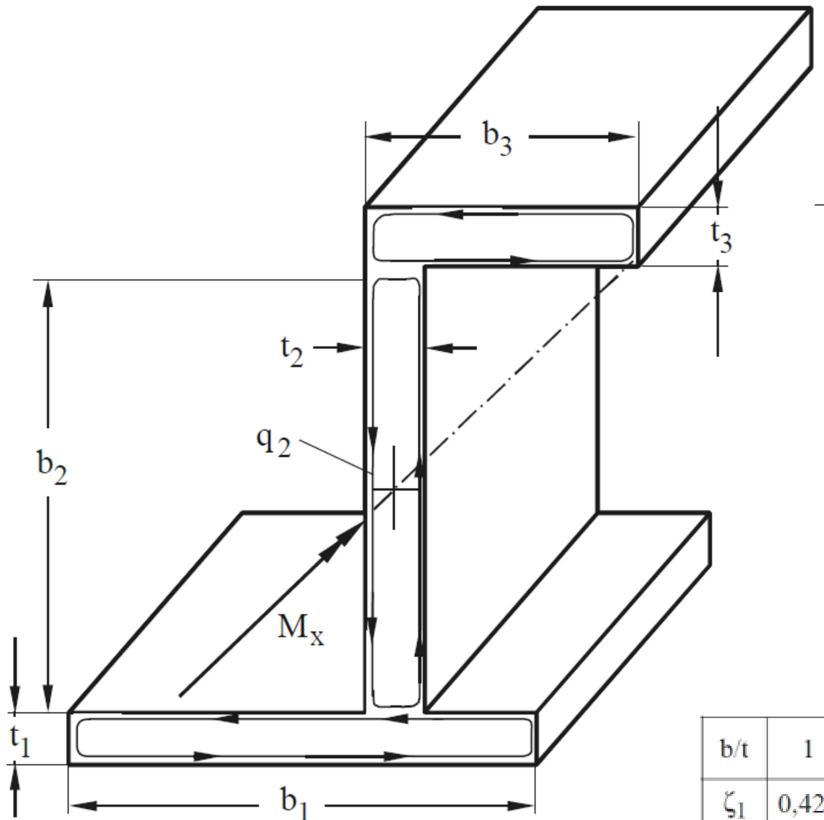
$$\phi' = \frac{M_x}{G \cdot J_t}$$

$$J_t = \frac{1}{3} t^3 \cdot b \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t}{b} \left[\tanh\left(\frac{\pi \cdot b}{2\ t}\right) + \frac{1}{243} \tanh\left(\frac{3\pi \cdot b}{2\ t}\right) \right] \right\} \approx \frac{1}{3} \zeta_1 \cdot t^3 \cdot b$$

b/t	1	1,5	2	3	4	5	6	8	10	∞
ζ_1	0,424	0,588	0,687	0,789	0,843	0,875	0,897	0,920	0,938	1
ζ_2	0,625	0,664	0,737	0,801	0,845	0,873	0,894	0,919	0,936	1

$$J_t \approx \frac{1}{3} \cdot t^3 \cdot b$$

Open, thin walled cross sections



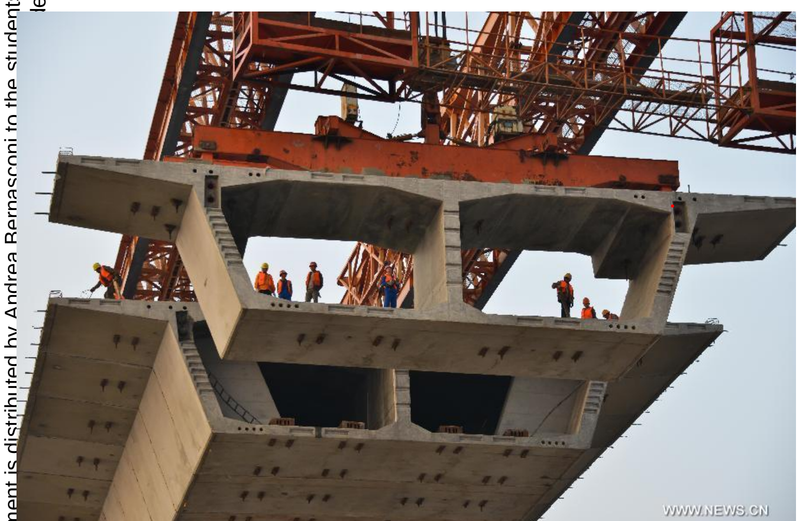
$$\tau_{max} = \frac{3 t_{max} M_x}{\sum_i^n t_i^3 b_i}$$

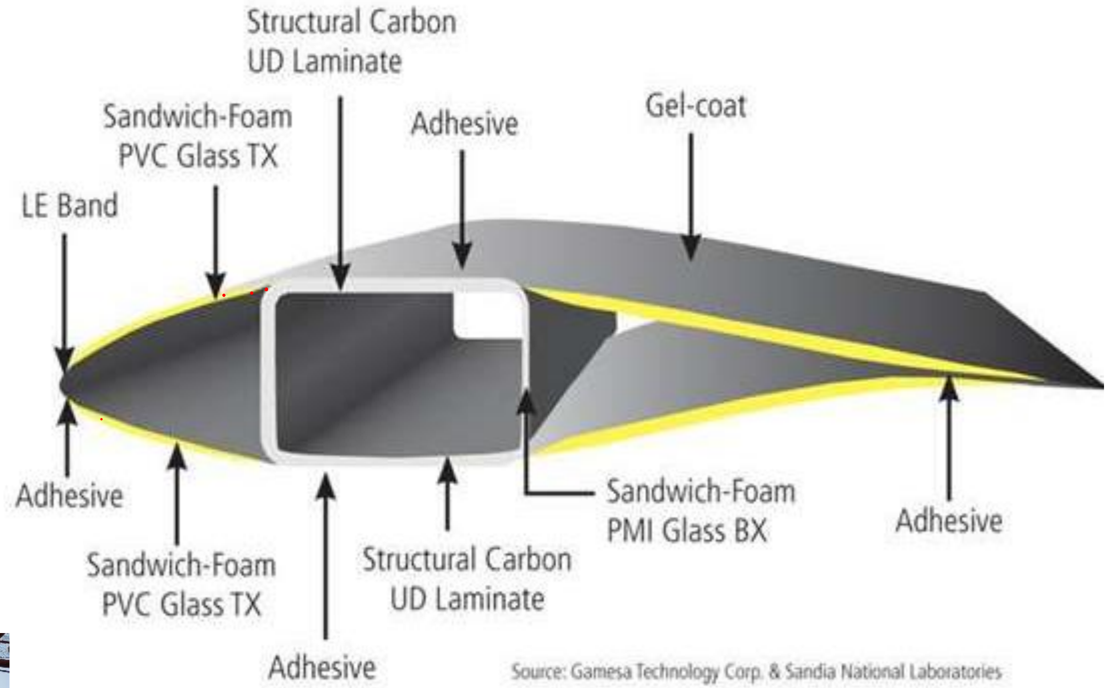
$$J_t \approx \sum_i^n \frac{1}{3} \zeta_{1_i} \cdot t_i^{\,3} \cdot b_i$$

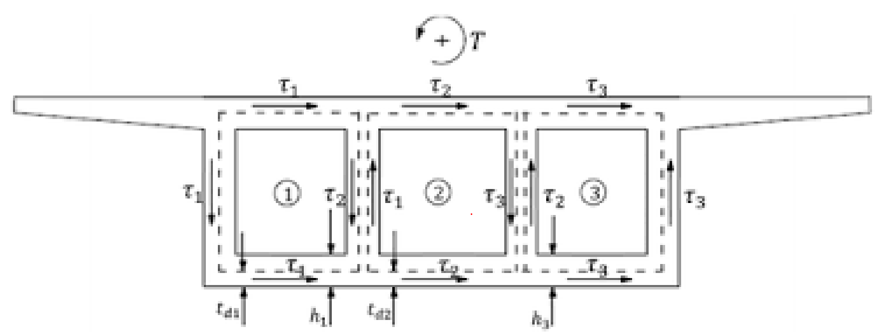
b/t	1	1,5	2	3	4	5	6	8	10	∞
ζ_1	0,424	0,588	0,687	0,789	0,843	0,875	0,897	0,920	0,938	1
ζ_2	0,625	0,664	0,737	0,801	0,845	0,873	0,894	0,919	0,936	1

Shape does not matter, only the number and the length of each member

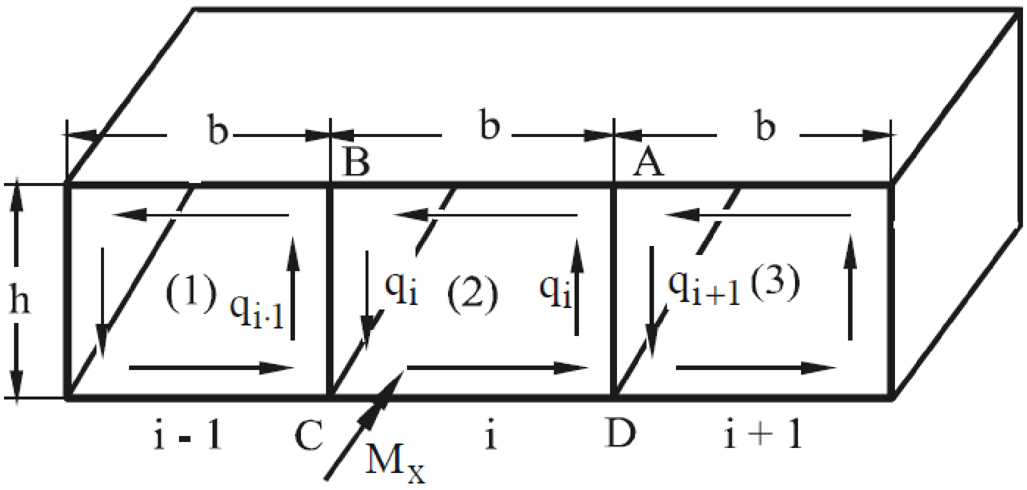
Multi cell cross sections





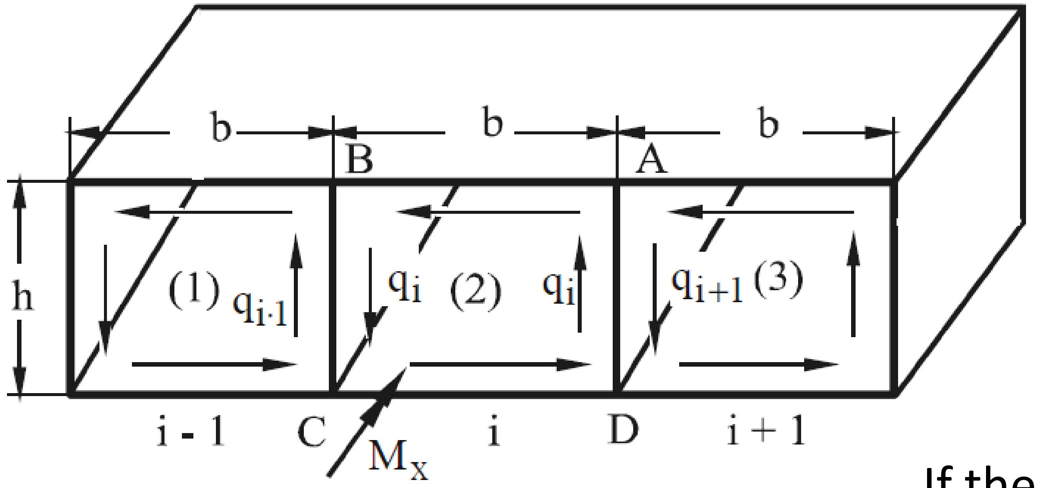


Multi cell cross sections



$$M_x = \sum_{i=1}^{3} M_{xi} = \sum_{i=1}^{3} q_i \cdot 2\,\dot{A}_i \;.$$ Equilibrium

$$\phi_1{}' = \phi_2{}' = \phi_3{}' = \phi'$$ Compatibility



If there were only q_i

$$M_{xi} = G \cdot J_{t_i} \cdot \phi' = G \frac{4\,\dot{A}_i^{\,2}}{\left(\oint \frac{ds}{t(s)}\right)_i} \phi' = q_i \cdot 2\,\dot{A}_i$$

$$q_i \cdot \left(\oint \frac{ds}{t(s)}\right)_i = 2\dot{A}_i \cdot G \cdot \phi' \ .$$

$$q_i \int_A^B \frac{ds}{t} + \left(q_i - q_{i-1}\right) \int_B^C \frac{ds}{t} + q_i \int_C^D \frac{ds}{t} + \left(q_i - q_{i+1}\right) \int_D^A \frac{ds}{t} = 2\,\dot{A}_i \cdot G \cdot \phi' \ .$$

$$-\frac{q_{i-1}}{G \cdot \phi'} \int_B^C \frac{ds}{t} + \frac{q_i}{G \cdot \phi'} \left(\oint \frac{ds}{t}\right)_i - \frac{q_{i+1}}{G \cdot \phi'} \int_D^A \frac{ds}{t} = 2\,\dot{A}_i \ .$$

$$u_i = \frac{q_i}{G \cdot \phi'}$$

$$a_{i,L} = \int_B^C \frac{ds}{t}$$

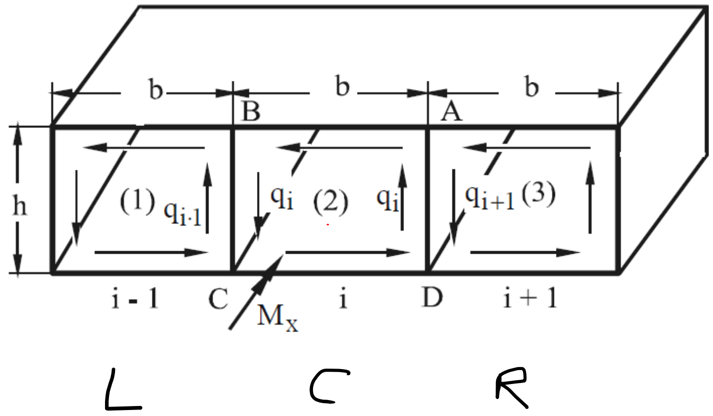
$$a_{i,Z} = \left(\oint \frac{ds}{t} \right)_i$$

$$a_{i,R} = \int_D^A \frac{ds}{t}$$

For symmetric sections:

$$a_{i,R} = a_{i+1,L}$$

$$a_{i,L} = a_{i-1,R}$$

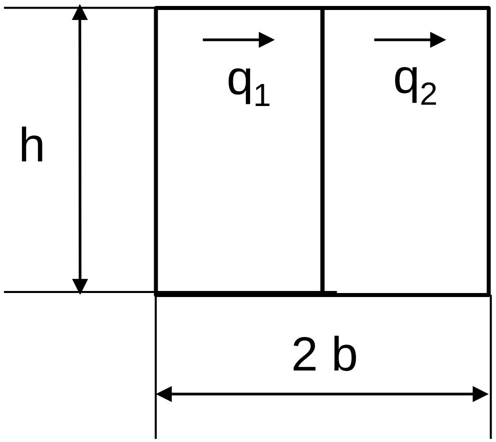


$$-a_{i,L} \cdot u_{i-1} + a_{i,Z} \cdot u_i - a_{i,R} \cdot u_{i+1} = 2\,\dot{A}_i \qquad (i = 1, 2, 3).$$

$$\begin{aligned}
a_{1,Z} \cdot u_1 - a_{1,R} \cdot u_2 &= 2\,\dot{A}_1 \\
-a_{2,L} \cdot u_1 + a_{2,Z} \cdot u_2 - a_{2,R} \cdot u_3 &= 2\,\dot{A}_2 \\
-a_{3,L} \cdot u_2 + a_{3,Z} \cdot u_3 &= 2\,\dot{A}_3
\end{aligned}$$

$$q_i = (G \cdot \phi') \cdot u_i .$$

Example: two cells section



Wall thickness t is uniform

$$T = M_1 + M_2 = GJ_t\phi'$$

$$T = q_1 2bh + q_2 2bh$$

$$q_1\left(\frac{2b+h}{t}\right) + q_1\frac{h}{t} - q_2\frac{h}{t} = 2\,bh\,G\,\phi'$$

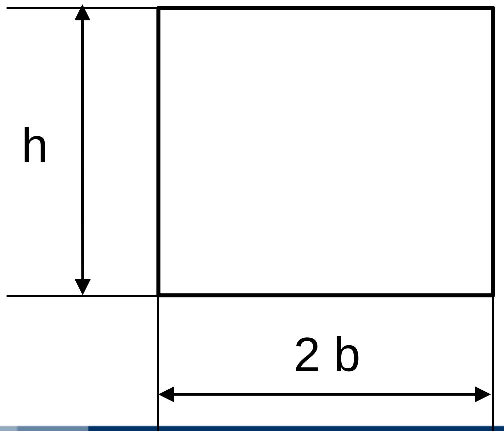
$$-q_1\frac{h}{t} + q_2\left(\frac{2b+h}{t}\right) + q_2\frac{h}{t} = 2\,bh\,G\phi'$$

$$q_1 = q_2$$

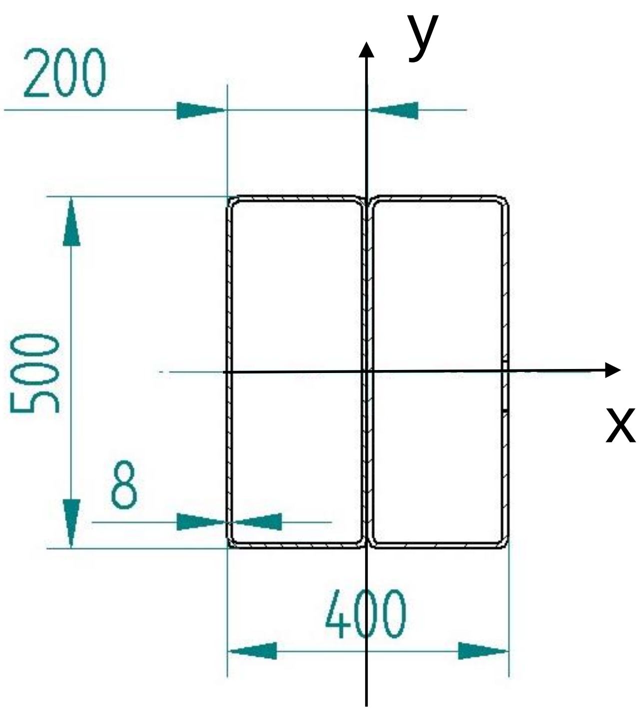
$$q_1 = \frac{2\,bh\,G\phi'}{\left(\frac{2b+h}{t}\right)}$$

$$T = 2\frac{2\,bh\,G\phi'}{\left(\frac{2b+h}{t}\right)}2bh \qquad J_t = \frac{8\,b^2h^2t}{2b+h}$$

Same as

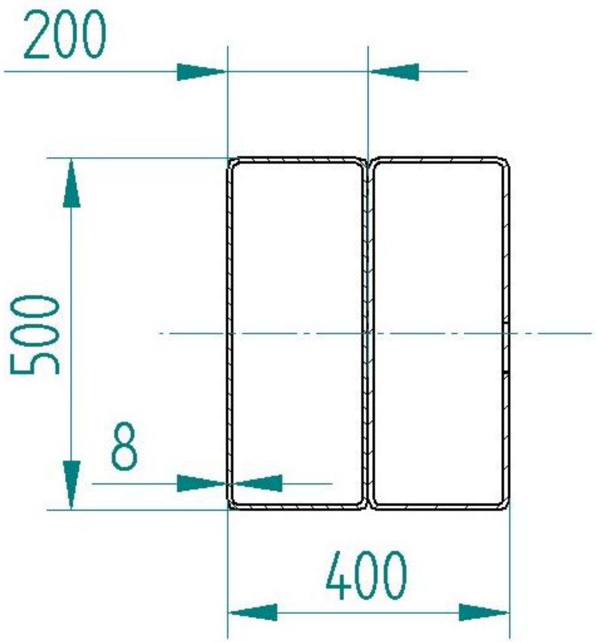


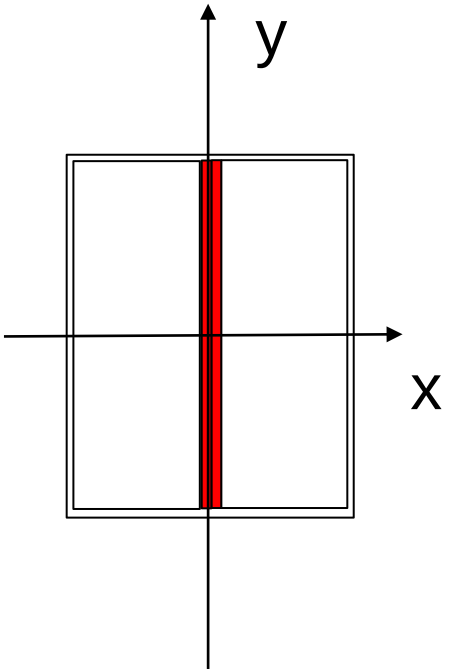
In this case, due to the double thickness of the central web, there is a slight change in J_t

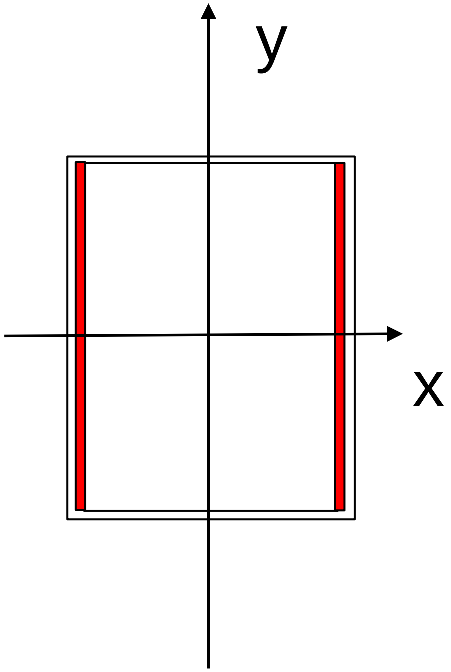


$$J_t = \frac{16\, b^2 h^2 t}{4b + h}$$

An equivalence can be applied to section B only if in-plane vertical loads are applied







Same J_x

Same J_t

Different J_y

$$J_t = \frac{16\, b^2 h^2 t}{4b + h}$$