



Step 1: Grid creation with elements, nodes and properties

	A	B	C	D	E	F	G	H
1	P	10000	N					
2								
3	element	1st node	2nd node	L	A	E	beta	k
4	1	2	3	2000	100	206000	1.570796	10300
5	2	1	3	2500	100	206000	2.214297	8240
6	3	1	2	1500	100	206000	3.141593	13733.33

Step 2: Individual stiffness matrices

Step 2.1: Orientation contributions

7	$[K] = \frac{AE}{L} \begin{bmatrix} \cos^2 \beta & \cos \beta \sin \beta & -\cos^2 \beta & -\cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta & -\cos \beta \sin \beta & -\sin^2 \beta \\ -\cos^2 \beta & -\cos \beta \sin \beta & \cos^2 \beta & \cos \beta \sin \beta \\ -\cos \beta \sin \beta & -\sin^2 \beta & \cos \beta \sin \beta & \sin^2 \beta \end{bmatrix}$					
14	[K1]=k1*	0.00	0.00	0.00	0.00	u2
15		0.00	1.00	0.00	-1.00	v2
16		0.00	0.00	0.00	0.00	u3
17		0.00	-1.00	0.00	1.00	v3
19	[K2]=k2*	0.36	-0.48	-0.36	0.48	u1
20		-0.48	0.64	0.48	-0.64	v1
21		-0.36	0.48	0.36	-0.48	u3
22		0.48	-0.64	-0.48	0.64	v3
24	[K3]=k3*	1.00	0.00	-1.00	0.00	u1
25		0.00	0.00	0.00	0.00	v1
26		-1.00	0.00	1.00	0.00	u2
27		0.00	0.00	0.00	0.00	v2

Step 2.2: Matrix expansion

30	[K1]=k1*	0.00	0.00	0.00	0.00	0.00	0.00	u1
31		0.00	0.00	0.00	0.00	0.00	0.00	v1
32		0.00	0.00	0.00	0.00	0.00	0.00	u2
33		0.00	0.00	0.00	1.00	0.00	-1.00	v2
34		0.00	0.00	0.00	0.00	0.00	0.00	u3
35		0.00	0.00	0.00	-1.00	0.00	1.00	v3
37	[K2]=k2*	0.36	-0.48	0.00	0.00	-0.36	0.48	u1
38		-0.48	0.64	0.00	0.00	0.48	-0.64	v1
39		0.00	0.00	0.00	0.00	0.00	0.00	u2
40		0.00	0.00	0.00	0.00	0.00	0.00	v2
41		-0.36	0.48	0.00	0.00	0.36	-0.48	u3
42		0.48	-0.64	0.00	0.00	-0.48	0.64	v3
44	[K3]=k3*	1.00	0.00	-1.00	0.00	0.00	0.00	u1
45		0.00	0.00	0.00	0.00	0.00	0.00	v1
46		-1.00	0.00	1.00	0.00	0.00	0.00	u2
47		0.00	0.00	0.00	0.00	0.00	0.00	v2
48		0.00	0.00	0.00	0.00	0.00	0.00	u3
49		0.00	0.00	0.00	0.00	0.00	0.00	v3

Step 2.3: Multiplying by $k_i = \frac{A_i E_i}{L_i}$

[K1] =	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	10300	0	-10300
	0	0	0	0	0	0
	0	0	0	-10300	0	10300
[K2] =	2966.4	-3955.2	0	0	-2966.4	3955.2
	-3955.2	5273.6	0	0	3955.2	-5273.6
	0	0	0	0	0	0
	0	0	0	0	0	0
	-2966.4	3955.2	0	0	2966.4	-3955.2
	3955.2	-5273.6	0	0	-3955.2	5273.6
[K3] =	13733.333	-1.68254E-12	-13733.333	1.683E-12	0	0
	-1.68E-12	2.06136E-28	1.683E-12	-2.06E-28	0	0
	-13733.333	1.68254E-12	13733.333	-1.68E-12	0	0
	1.683E-12	-2.06136E-28	-1.68E-12	2.061E-28	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0

Step 3: Sum individual matrices to obtain the global stiffness matrix and complete the governing equation

16699.73	-3955.20	-13733.33	0.00	-2966.40	3955.20	*	u1	=	0
-3955.20	5273.60	0.00	0.00	3955.20	-5273.60		v1		-P
-13733.33	0.00	13733.33	0.00	0.00	0.00		u2=0		rh2
0.00	0.00	0.00	10300.00	0.00	-10300.00		v2=0		rv2
-2966.40	3955.20	0.00	0.00	2966.40	-3955.20		u3=0		rh3
3955.20	-5273.60	0.00	-10300.00	-3955.20	15573.60		v3		0

Step 4: Swap rows and columns to isolate the two problems
(unknown displacement/known forces vs unknown forces/known displacements)

swap 3rd and 6th column									
16699.73	-3955.20	3955.20	0.00	-2966.40	-13733.33				
-3955.20	5273.60	-5273.60	0.00	3955.20	0.00				
-13733.33	0.00	0.00	0.00	0.00	13733.33				
0.00	0.00	-10300.00	10300.00	0.00	0.00				
-2966.40	3955.20	-3955.20	0.00	2966.40	0.00				
3955.20	-5273.60	15573.60	-10300.00	-3955.20	0.00				
swap 3rd and 6th row									
16699.73	-3955.20	3955.20	0.00	-2966.40	-13733.33	u1		0	
-3955.20	5273.60	-5273.60	0.00	3955.20	0.00	v1	-P		
3955.20	-5273.60	15573.60	-10300.00	-3955.20	0.00	v3		0	
0.00	0.00	-10300.00	10300.00	0.00	0.00	v2=0		rv2	
-2966.40	3955.20	-3955.20	0.00	2966.40	0.00	u3=0		rh3	
-13733.33	0.00	0.00	0.00	0.00	13733.33	u2=0		rh2	

Step 5: Solve first problem to obtain unknown displacements

16699.733	-3955.2	3955.2	*	u1	=	0
-3955.2	5273.6	-5273.6		v1		-10000
3955.2	-5273.6	15573.6		v3		0
inverse matrix						
7.282E-05	5.46117E-05	1.994E-20				
5.461E-05	0.00032767	9.709E-05				
1.617E-20	9.70874E-05	9.709E-05				
u1	=	-0.546117	mm			
v1	=	-3.276699	mm			
v3	=	-0.970874	mm			

Step 6: Solve second problem to obtain unknown forces

coefficients multiplying the previously unknown displacements		
1.683E-12	-2.06136E-28	-10300
-2966.4	3955.2	-3955.2
-13733.33	1.68254E-12	0
rv2	=	10000
rh3	=	-7500
rh2	=	7500

Step 7: Compute nodal forces

52	Nodal forces								
53									
54	F2x	0.00	0.00	0.00	0.00	u_2	0.00	0	
55	F2y	0.00	1.00	0.00	-1.00	v_2	0.00	10000	
56	F3x	0.00	0.00	0.00	0.00	u_3	0.00	0	
57	F3y	0.00	-1.00	0.00	1.00	v_3	-0.97	-10000	
58									
59	F1x	0.36	-0.48	-0.36	0.48	u_1	-0.55	7500	
60	F1y	-0.48	0.64	0.48	-0.64	v_1	-3.28	-10000	
61	F3x	-0.36	0.48	0.36	-0.48	u_3	0.00	-7500	
62	F3y	0.48	-0.64	-0.48	0.64	v_3	-0.97	10000	
63									
64	T	-0.60	0.80	0	0	F*1	-12500		
65		0	0	-0.60	0.80	F*3	12500		
66									
67									
68									
69	F1x	1.00	0.00	-1.00	0.00	u_1	-0.55	-7500	
70	F1y	0.00	0.00	0.00	0.00	v_1	-3.28	0	
71	F2x	-1.00	0.00	1.00	0.00	u_2	0.00	7500	
72	F2y	0.00	0.00	0.00	0.00	v_2	0.00	0	



Rotate if nodal forces are needed in the local coordinate system