NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY NTNU
FACULTY OF ENGINEERING
DEPARTMENT OF STRUCTURAL ENGINEERING

TKT4142 Finite Element Methods in Structural Engineering

CASE STUDY 3

SOLUTION PROPOSAL

Problem 1 – Spherical top housing

a) Model the spherical top sensor housing assuming axisymmetry. Mesh the part using linear reduced integration quadrilateral elements (CAX4R). Ensure that there are at least five elements across the thinnest part of the housing wall.

It is advisable to partition the part such that a structured mesh (i.e., when the part turns green) can be achieved and so that edge seeds can easily be applied. Perform a static linear analysis (Static, general). The time period can be arbitrarily chosen as 1.

Solution:

Figure 1 shows the distribution of von Mises stresses in the spherical top housing. The maximum stress is 482.6 MPa and occurs in the intersection between the 4 mm and 10 mm thick sections. The cylindrical shape is generally weaker against external pressures than the spherical lid, so it is reasonable that the highest stresses occur there. We can also see that the stresses are moderate in all other sections of the component, particularly near the mounting the pipe. The stress in the spherical part appears relatively which, ideal for such a pressure vessel.

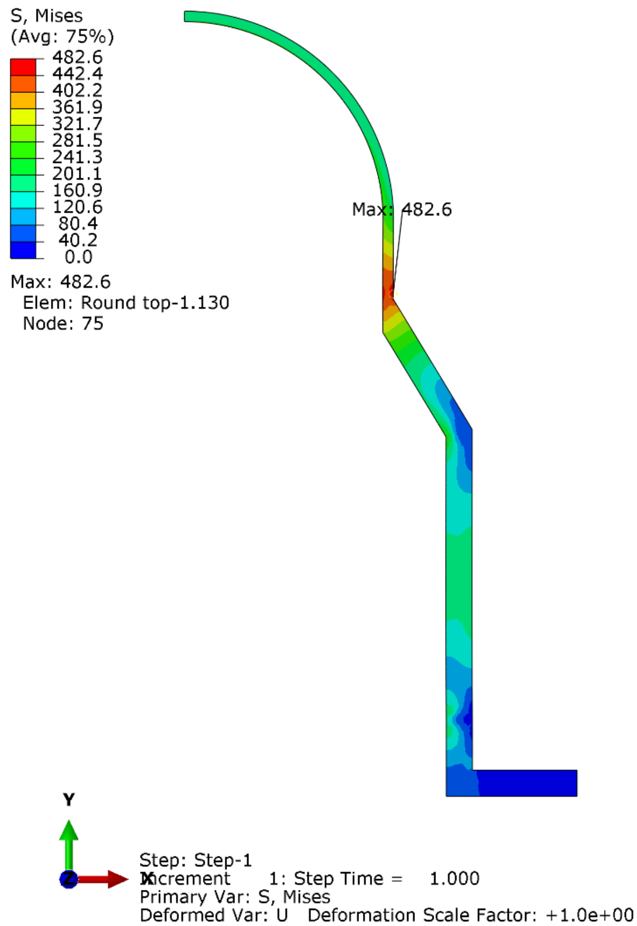


Figure 1: Von Mises stress distribution for the spherical top housing.

b) Evaluate the stresses in the housing:

Max von Mises stress (this is often used to evaluate the capacity of metal parts). Answer with figure and value.

Radial stress (S11), axial stress (S22) and Hoop stress (S33) in the cylindrical part of the housing denoted “Check stresses here” in Figure 2.

Note that the stresses are not neccesarily uniform in this region, but try to make an average across the thickness of the wall at the centre of the “Check stresses here” box.

Compare the two latter to the analytical functions for *Hoop stress in closed cylinder* and *Axial stress in closed cylinder.* Is there a good correspondence?

Compare the von Mises stress in the spherical part of the lid to analytical expression for *Uniform stress in closed sphere.* Is there a good correspondence?

Solution:

Table 1 shows the stresses extracted from the simulation and calculated from the analytical formulas. The stress components are not strictly uniform across neither the length nor the thickness of the cylindrical part. So, any number here is slightly dependent on mesh and where the values are extracted.

There is a good correspondence between the FEA and the analytical formulas, with at most a 5% difference of the stresses in the spherical section. This means that the housing, which is a combination of spheres and cylinders, can to some extent be designed using such analytical formulas.

Table 1: Problem 1b

σ_{vm}	σ_{r}	σ_{z}	σ_{θ}
483 MPa	-9.8 MPa	-201 MPa	-398 MPa
$\sigma_{\mathrm{axial}}^{\mathrm{analytical}}$	$\sigma_{\mathrm{hoop}}^{\mathrm{analytical}}$	$\sigma_{\mathrm{sphere,vm}}$	$\sigma_{\mathrm{sphere}}^{\mathrm{analytical}}$
-191.3 MPa	-382.6 MPa	183 MPa	-191.3 MPa

c) Find the maximum displacement of the part. Answer with figure and value.

Solution:

The largest displacement of the housing is 0.338 mm and occurs close to where the peak stress also was found. Although we did not have a strict displacement requirement, it is promising that they are less than tenth of the wall thickness.

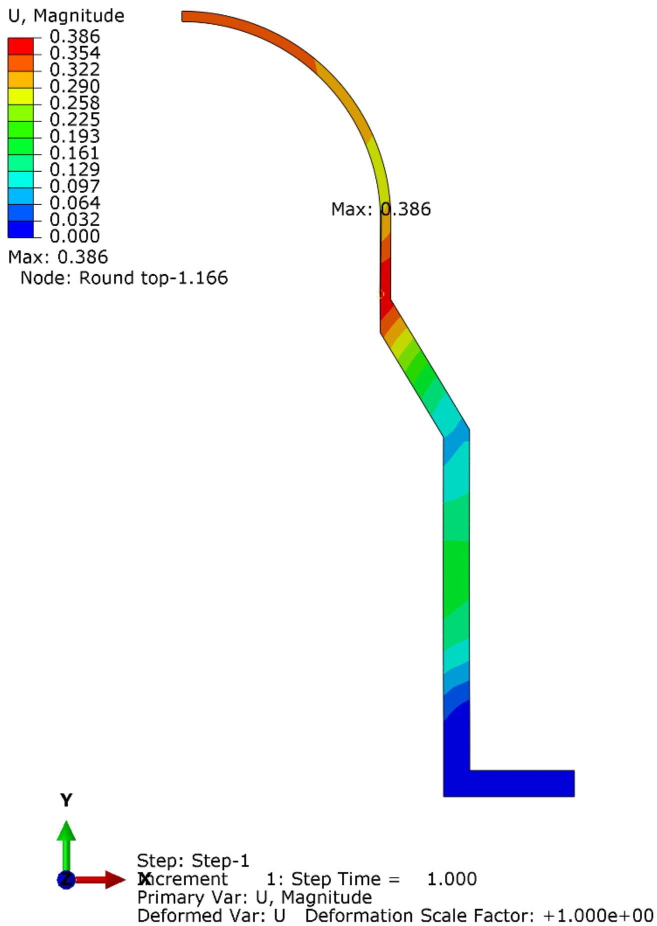


Figure 2: Displacements in the spherical top housing.

d) What is the total reaction force from the axial boundary condition that represents the flange? Compare this force to the external axial force form the pressure.

The external axial force is found as the pressure times the projected area of the housing (i.e., as seen from the top in Figure 5 in the Problem Description).

The sum of forces in the axial direction from the simulation can be found as 1386 kN from the simulation.

The external forces acting in the axial direction can be found as: $F_{\text{axial}} = p \cdot A_{\text{projected}} = p \cdot \pi(150\text{ mm})^2 = 1386\text{ kN}$.

The equality between these numbers is a direct consequence of force equilibrium, $\sum F_{\text{axial}} = 0$. Furthermore, this number is not particularly sensitive to mesh or other assumptions since the finite element model must necessarily balance all external forces and reaction forces.

e) Perform a mesh study using 2, 5 (i.e., from problem 1a), 8 and 12 elements over the thickness. Write down the peak stress for each case.

Solution:

Table 2 shows the change in von Mises stress as a function of elements across the thickness. Note that the limited scale of the axis exaggerates the effect somewhat. We can see that with 5 elements the stresses are noticeably lower than when using a more refined mesh. If we were close to the stress limit of the material it would be advisable to refine the mesh further, and perhaps change the design to avoid such a stress concentration.

Table 2: Mesh convergence study

Elements across the thickness	2	5	8	12
Max Von Mises stress [MPa]	468	482	491.7	497.9
Change relative to one mesh coarser		3.1%	1.9%	1.3%

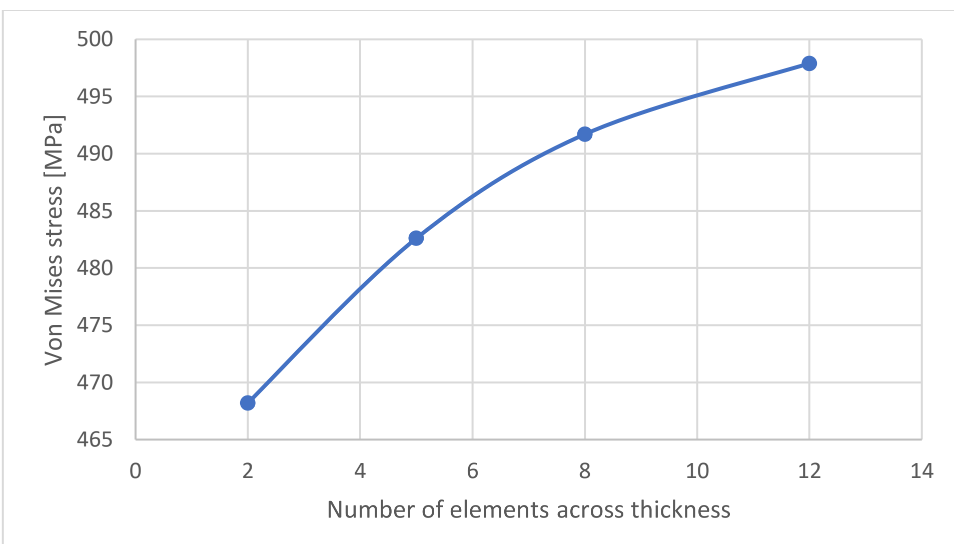
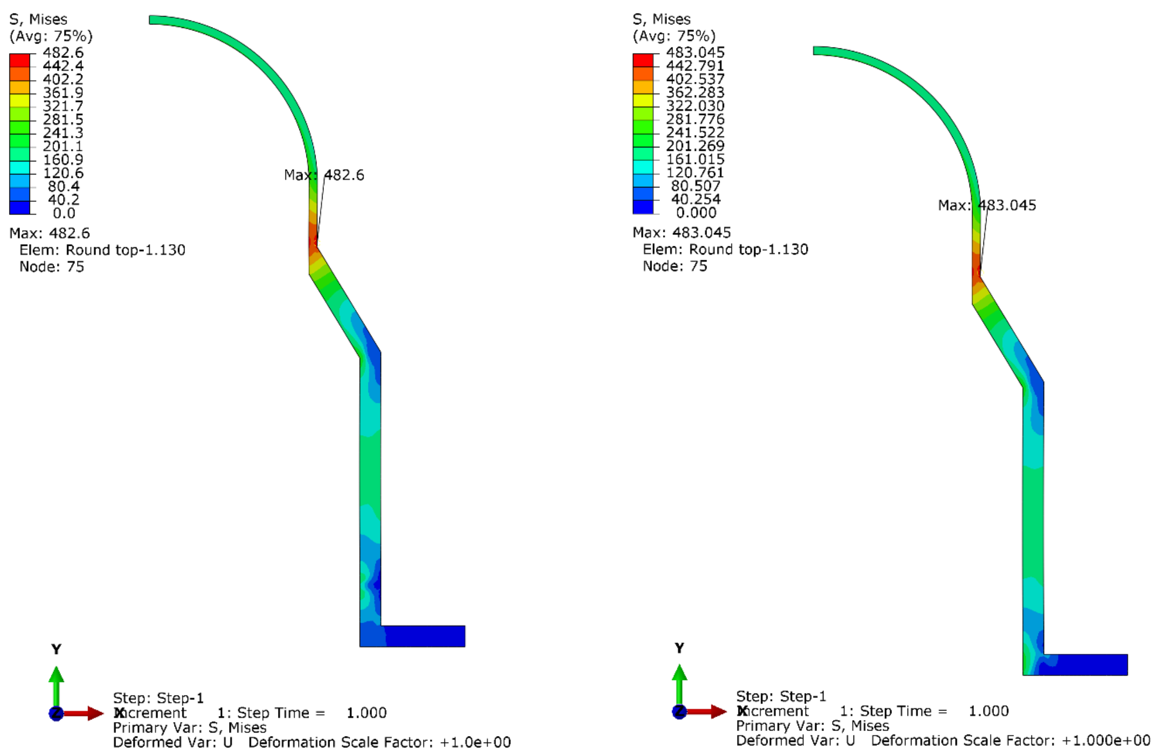


Figure 3: Mesh convergence study

f) Try to remove the radial boundary conditions close to the O-ring grooves in Figure 2. Does it matter in terms of capacity of the housing?

Figure 3 shows a comparison between the stresses in the housing with and without the radial support. We can see that the choice of boundary condition has a large effect on the stresses (and displacements) in close proximity, but not in the region where we saw the highest stresses (i.e., the thin cylindrical part).

This task visualises two extreme choices for how the boundary can be modelled, where the actual behaviour lies somewhere in between. In terms of capacity of the housing, either assumption is valid, but if we were concerned about the stresses closer to the joint we would have to increase the complexity of our model.



(a) *(b)*

Figure 4: Difference in von Mises stress (a) with and (b) without radial support from the pipe flange.

Problem 2 – Flat top housing

a) Model the flat top sensor housing assuming axisymmetry. Mesh the part using linear reduced integration quadrilateral elements (CAX4R). Ensure that there are at least four elements across the thinnest part of the housing wall.

It is advisable to partition the part such that a structured mesh (green) can be achieved and so that edge seeds can easily be applied. Perform a static linear analysis (Static, general). The time period can be arbitrarily chosen as 1.

Solution:

Figure 5 shows the von Mises stress of the flat top housing. The stress in the lid far exceeds the yield stress (with a factor of 4.2) and this housing would certainly cause plastic deformation and very likely a collapse. Since we have used an elastic model the stresses which exceed yield are not realistic at all, but they give us an indication of how much the design must be improved relative to the external loads.

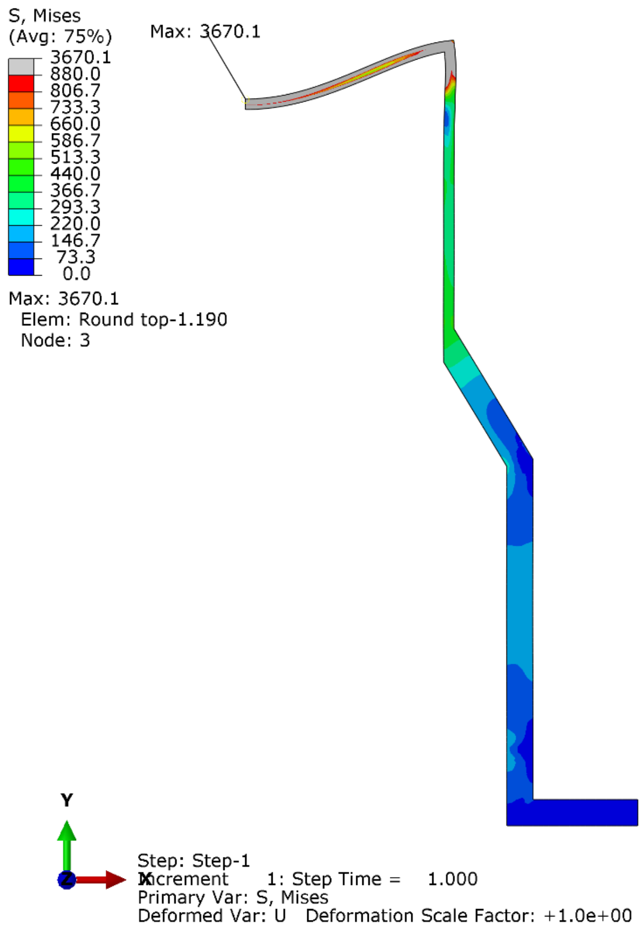


Figure 5: Von Mises stress for the flat top housing

b) Evaluate the stresses in the housing:

Max von Mises stress (this is often used to evaluate the capacity of metal parts). Answer with figure and value.

Radial stress (S11), axial stress (S22) and Hoop stress (S33) in the cylindrical part of the housing denoted "Check stresses here" in Figure 2. Are these different from the spherical top housing?

Compare the displacement of the centre of the the lid to analytical expressions for a pressure loaded circular plate.

The first formula is for a simply supported plate while the second is for a clamped plate. Our case is likely somewhere between these two standard cases since the plate gets some support from the cylindrical part, but not enough to be fully clamped! Which expression fits the displacement from the simulation best?

Solution:

Table 3 shows the stresses from the simulation and the analytical formulas. As we saw for the stress plot, the maximum von Mises stress and the displacements are much higher than for the spherical lid. However, the stresses in the cylindrical part are fairly similar.

Note that these relationships might be sensitive to the slightly non-uniform stress distribution in the regions where we extracted the stresses. This means that much of the stress is carried by the hoop of the part, so the difference in configuration in the axial direction matters less.

Table 3: Problem 2b

	σ_{vm}	σ_{r}	σ_{z}	σ_{θ}	u_{mid}	$u_{\mathrm{mid}}^{\mathrm{supported}}$	$u_{\mathrm{mid}}^{\mathrm{clamped}}$
	3670 MPa	-11 MPa	-202 MPa	-407 MPa	-22.6 mm	-69.4 mm	-17.5 mm
Difference relative to spherical	660%	13.9 %	0.3%	4.5%	6520%	-	-

c) Find the maximum displacement of the whole part. Answer with figure and value.

Solution:

Figure 6 shows the displacement of the flat top housing. In contrast to the spherical housing the displacements are primarily related to bending of the lid. Note that these displacements are so large that one should consider the validity of a linear analysis.

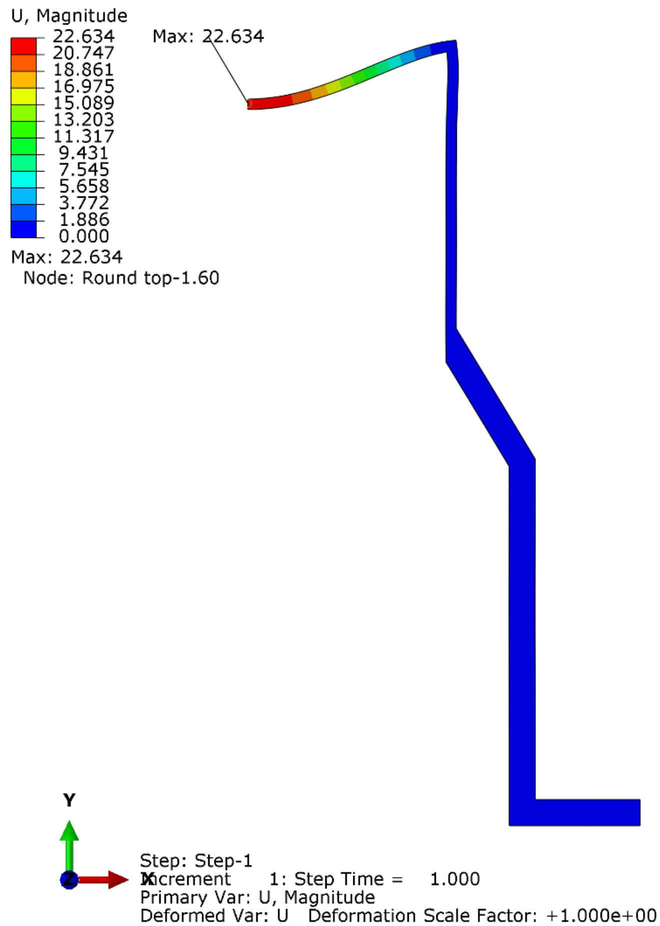


Figure 6: Displacement for the flat top housing

d) Without doing more simulations – at what depth will the maximum stress in the flat head housing start to yield? **Hint**: We are doing a linear analysis.

Solution:

The housing would reach its capacity approximately at 476 m. Max stress is 4.2 times higher than yield, which means that the load and thus the water depth must be $\frac{1}{4.2}$ of the depth the housing was intended for. The realisation that the stress and displacements are proportional to the external load in a linear analysis is very handy.

Problem 3 - Substantiate

a) What causes the large difference in stresses/displacement between the spherical and flat lids?

Solution:

For the flat lid the pressure creates a bending load on the flat lid/plate, while the load for the spherical lid is practically axial. Generally bending leads to higher stresses than axial deformation, just as it does for a simple beam.

b) What simplifications have we made to the boundary conditions how might it affect the results? Will it change our conclusion on whether the part will yield or not?

Solution:

We have modelled the bolted connection as a boundary condition with radial and axial restrictions. This is likely accurate as long as the radial reaction force does not exceed the frictional limit ($N \cdot \mu_{\text{friction}}$).

The radial boundary condition is a slight simplification as the central part of the pipe flange is not completely stiff like we have modelled it (by assigning a boundary condition). However, since it is almost solid it is relatively stiff compared to the housing and will contribute significantly to its radial stiffness. The most accurate way to simulate it would be to model the central part of the pipe flange too.

Neither of these boundaries are in proximity to the regions where we see large stresses or displacements, so we do not need to model these very accurately. They only need to be modelled accurately enough to carry the global forces (i.e., the external pressure).

c) Are there any limitations to the axisymmetric assumption used here? What are they and how could they affect our results.

Solution:

The main non-axisymmetric feature of the component are the bolts which are spatially distributed around the housing and thus not fully symmetric around the central axis. However, because the external load pushes the housing towards the bolted flange so this is a relatively accurate assumption. Such bolts are often tensioned which caused stress to the flange that we have not included.

d) Can other loads be added using the axisymmetric model be included (e.g., weight of internal components, drag). Why/why not?

Solution:

Non-axisymmetric loads cannot simply be included in an axisymmetric simulation so we must make sure that these forces are neglectable in comparison to the external pressure. One argument is that the resulting displacement from the simulation must be axisymmetric since we only have axisymmetric degrees of freedom, even if we somehow were able to apply a directional load. Note that there are options for twist and anti-symmetry available in Abaqus.

e) What could potentially limit the validity of a linear analysis in this case? The spherical lid housing does not deform plastically or with very large displacements so non-linearity is likely not a large source of concern.

Solution:

The flat lid sees very larges stresses and would probably collapse. The simulation is sufficient to show that using the flat lid is not feasible, but the stresses and deformations are not accurate. Note that the analytical formulas also do not account for these non-linearities so they should be equally wrong in that sense.

This container is fairly thin and is exposed to compressive loads so buckling is a clear concern. A separate buckling analysis must be performed, but that is not within the scope of this case. It should be noted that the axisymmetric model would not be usable for a buckling analysis since it would force axisymmetric buckling modes which are generally not most critical.

f) Do we need to account for the internal pressure of the vessel? Why/why not?

Solution:

No.
First, the atmospheric pressure is very small compared to the hydrostatic pressure and would have a neglectable effect. Second and more importantly, the exterior pressure should really be hydrostatic pressure + atmospheric pressure, so one would have to add the atmospheric pressure to both the inside and outside. But that is equivalent to only applying the hydrostatic pressure to the outside like we did already.