POLITECNICO DI MILANO

Finite Element Simulation For Mechanical Design

Convergence analysis

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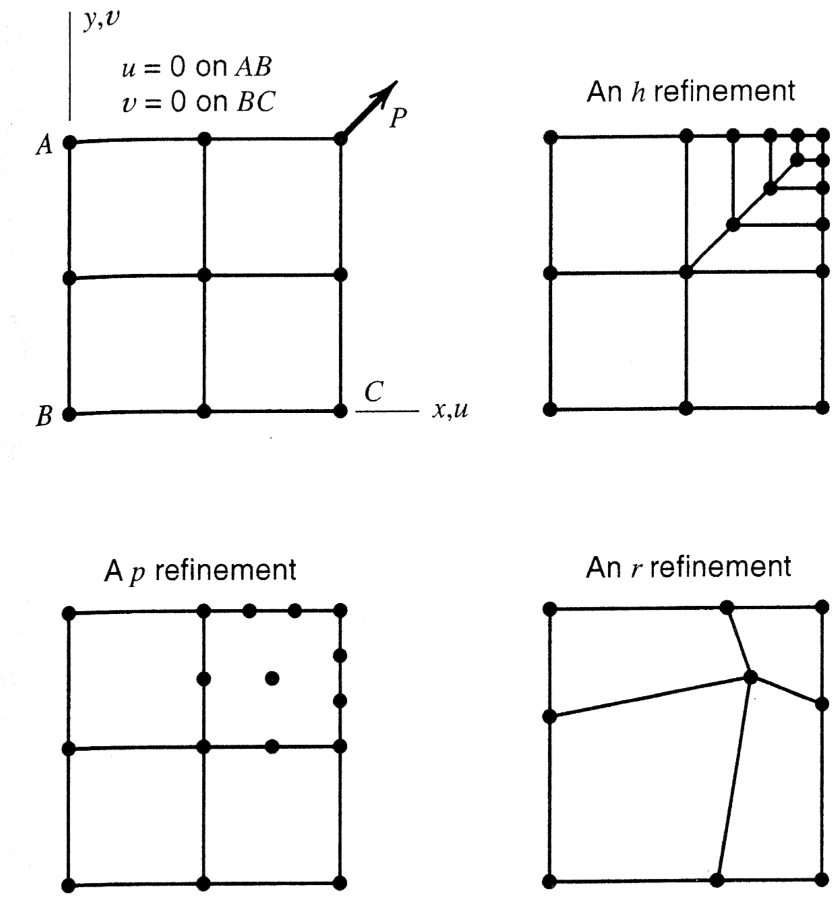
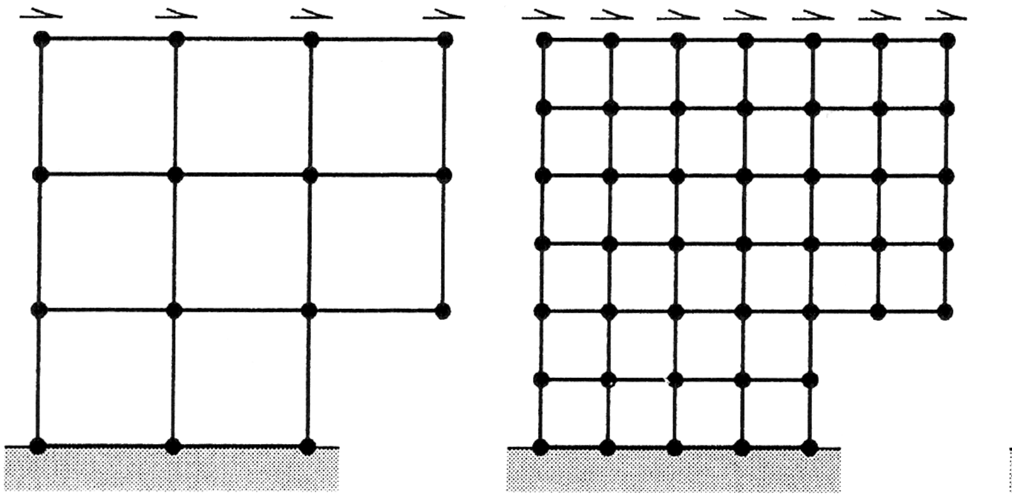
Discretization error: convergence analysis

- To assess the accuracy of results, it is necessary to apply convergence analysis techniques
- The method presented herein is defined as “zero size method”, and it is based on a suitable mesh refinement method and an **estimate of the error** based on the order of the polynomials associated to the element used. The polynomials are used to express the displacement field.
- There are several methods for mesh refinement.



H refinement



h, characteristic dimension

- length, for beam or bar elements

- $A^{1/2}$, for solid 2D or shell

- $V^{1/3}$, for 3D solid

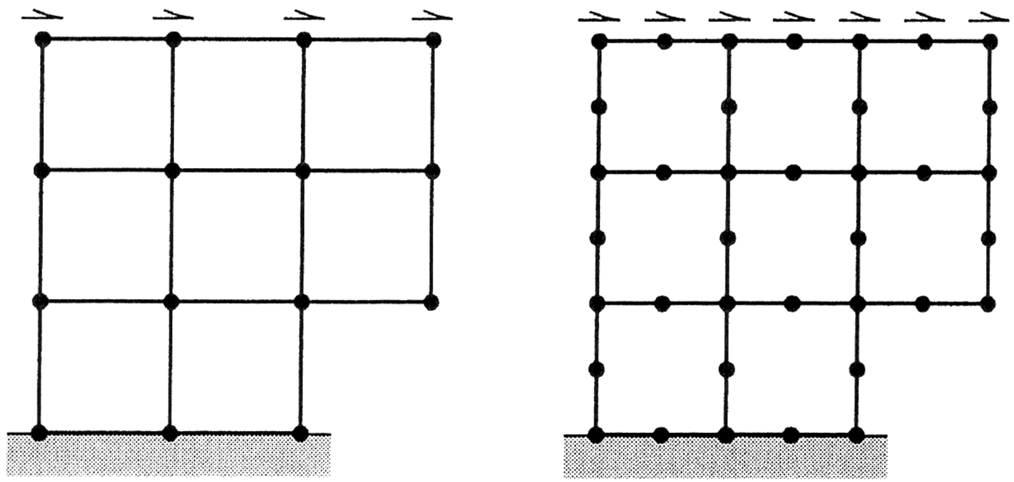
The number of nodes is increased, not the order of the polynomials interpolating displacements inside the element

2 possible refinement methods:

- uniform

- non-uniform

P refinement



The number of elements remain unchanged

Mid nodes are added to the existing elements

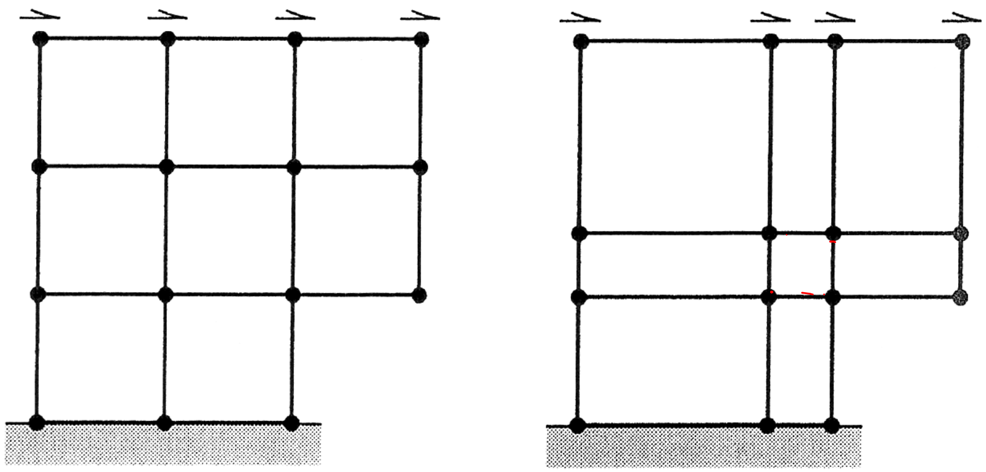
p is the order of the full polynomial that can be identified in the expression of the displacements

Example:

p =1 for Q4 and T3

p = 2 for Q8

R refinement



r = rearrange

The number of elements and nodes is left unchanged, their position is modified

Order of the error

Given:

h, element characteristic length

p, order of the **full** polynomial included in {u}

The discretization error, that measures the speed of convergence, is proportional to:

- h^{p+1} for the description of displacements {u} – with a polynomial of order p it is reasonable to assume that the error is related to the lowest-order (p+1) terms omitted
- h^p for the field obtained by differentiation $\{\varepsilon\}$ e $\{\sigma\}$ – this is an upper bound estimate, because the error depends on the position of the point where stresses or strains are evaluated

Example:

Q4 -> err{u} is proportional to h^2

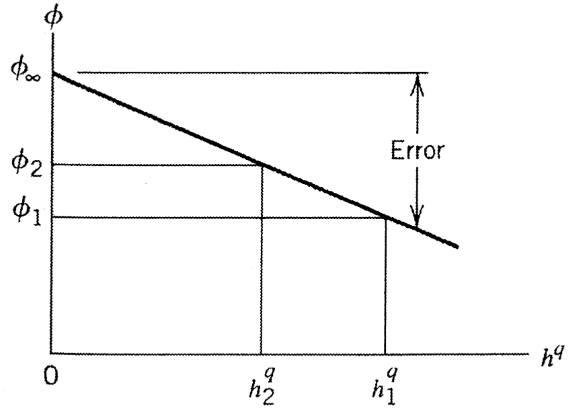
Q8 -> err{u} is proportional to h^3

Convergence analysis

Provided that mesh refinement is regular, i.e.:

- the nodes and the edges of the coarser mesh are preserved
- existing nodes at the vertices and on the edges of the model are preserved
- the point where the quantity Φ of interest is evaluated is not varied
- the type of element is not varied

The error, being function of h^q, being *q* the order of the error for the quantity Φ, varies linearly with h^q



$$\Phi_\infty = \frac{\Phi_1 h_2^q - \Phi_2 h_1^q}{h_2^q - h_1^q}$$

Convergence analysis

q cannot easily be determined a priori:

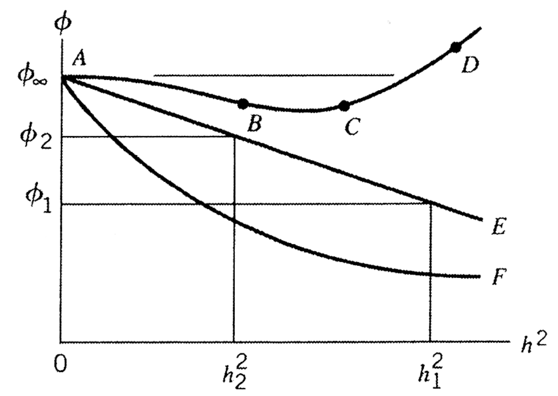
- Depends on the distortion of the elements
- Convergence may be non-monotonic
- The first meshes may be still too coarse

Thus, q is evaluated:

- Based on at least 3 meshes
- Searching for the value of q that makes linear the relationship between Φ and h^q

If the monotonicity is missing, the risks are:

- segment C-D: bad extrapolation
- segment B-C: error apparently null



Discretization error estimate

At the i-th mesh refinement iteration, the percent error can be estimated as

$$e = \frac{\phi_i - \phi_\infty}{\phi_\infty} \cdot 100\,\%$$

For an irregular mesh refinement, a **dimensionless** *h* can be defined as

$$h = \frac{1}{\sqrt[n]{N_{elementi}}}$$

$$\left(\frac{1}{\sqrt{N_{elementi}}}\right)^{-1}$$ It is an estimate of the number of edge elements

with

n = 2 for 2D

n = 3 for 3D

and a linear extrapolation over

3 points should be performed