NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY NTNU
FACULTY OF ENGINEERING
DEPARTMENT OF STRUCTURAL ENGINEERING

TKT4142 Finite Element Methods in Structural Engineering

CASE STUDY 3

Introduction

This case study aims to understand how axisymmetric elements can be used to simplify a component. Utilising symmetry allows for a much more refined mesh at a lower computational cost than using a full 3D model. In addition to the general understanding necessary for using these elements, some of the specific features used to treat axisymmetric models in Abaqus/CAE will be highlighted in this case study.

Learning outcome

- Simplify a component to an axisymmetric problem.
- Draw slightly complex parts in Abaqus (learning how to assign dimensions and constraints)
- Use axisymmetric elements (2D).
- Understand how to work with cylindrical coordinates in Abaqus.
- Learn how to extract loads from a 2D representation.
- Limitations of axisymmetry (buckling, non-axisymmetric loads).
- Simplify bolted connection using boundary conditions.
- Evaluate the difference in performance between the two designs.
- Visualise the full part (revolve Odb).

Problem description

Figure 1 shows a subsea housing for electronic components intended to operate at 2000 m depth. The electronics could for instance be a battery pack or a sensor package. As highlighted in Figure 2, the housing is mounted to a pipe via two flanges which are bolted together. The inside of the housing is at atmospheric pressure, while the outside is subjected to hydrostatic pressure. To avoid leakage, two O-rings will be placed between the housing and the pipe flange. We will not model the O-rings, but the housing will get some radial support from the pipe.

Since the housing is exposed to seawater and high pressures, it is made with Titanium Ti-6Al-4V (Grade 5), which is both strong and resists corrosion. The material properties can be found in Table 1.

In *Problem 1* a housing with a spherical top will be analysed, and its dimensions can be found in the sketch in Figure 5. However, a previous design had been implemented with a flat top, shown in Figure 6. Luckily, some clever engineers advised against it and suggested the spherical design instead. In *Problem 2* we will look at why that might have been a good idea.

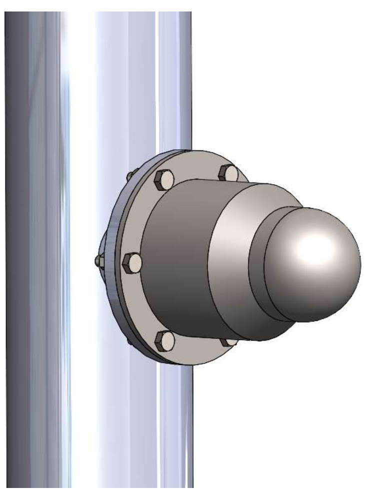


Figure 1: Mounted sensor housing

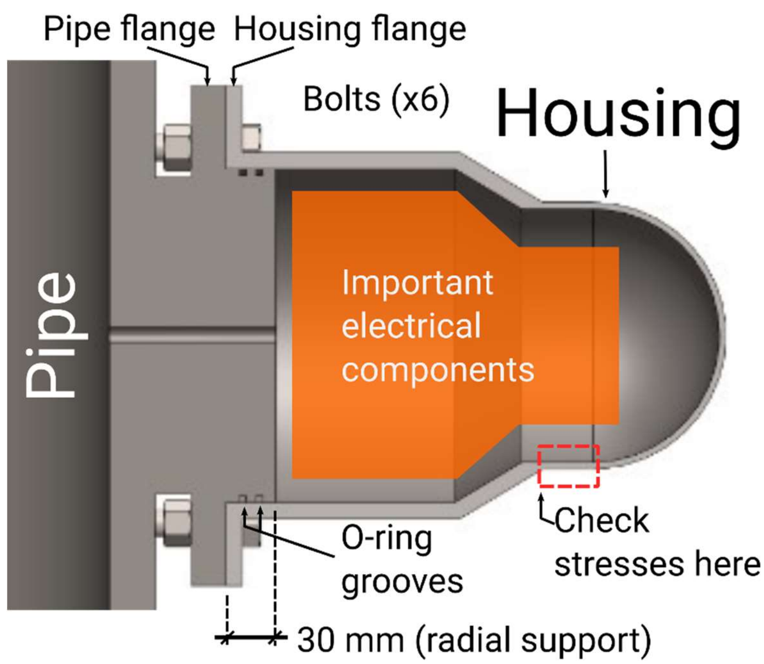


Figure 2: Mounted sensor housing

Table 1: Material parameters for Titanium Ti-6Al-4V

E [MPa]	ν [$-$]	ρ [kg/m^3]	σ_y [MPa]
113800	0.342	4430	880

A bit about cylindrical coordinates and axisymmetric elements

Axisymmetric elements can be used to save computational time for problems that are symmetric around a single axis of rotation. Cylindrical coordinates are used for such elements instead of the more common cartesian coordinates. When discussing an axisymmetric problem, it is common to denote loads, displacements and stresses related to the relevant coordinate, for instance: Radial direction (R) -> radial stress, tangential direction (theta) -> hoop stress or tangential stress, axial direction (z) -> axial stress.

Note that in the 2D axisymmetric space Abaqus numbers the coordinate stresses as radial (S11), axial (S22) and hoop (S33), which is <u>not</u> the same order as the coordinates are typically listed (which is radial, tangential and axial).

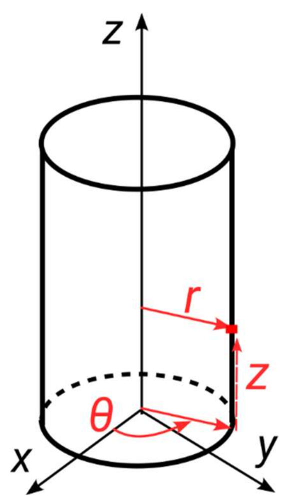


Figure 3: Cylindrical coordinate system

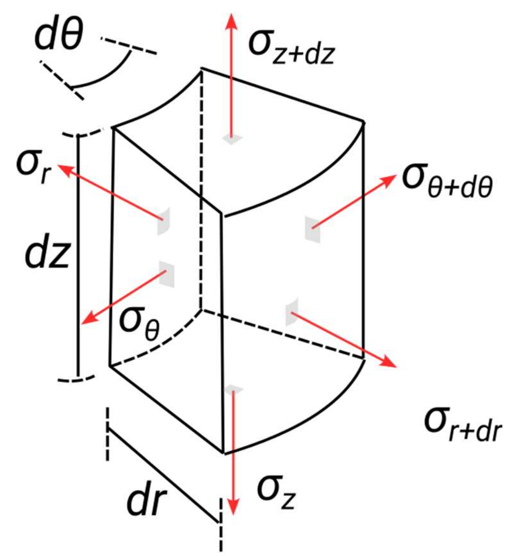


Figure 4: Stresses in cylindrical coordinates

Modelling assumptions

Loads

The external pressure follows the relationship,

$$p = \rho g h,$$

where $\rho = 1000$ kg m^{-3} is the density of water, $g = 9.81$ ms^{-2} is the gravitational constant and h is the water depth.

The internal pressure can be neglected for now.

Boundary conditions

- Modelling the bolts accurately would increase the complexity of the model and disqualify the use of axisymmetry (since they are spaced around the circumference of the part). Because the housing is pushed into the flange by the external pressure, the bolts essentially just carry any bolt pre-tension load that might have been applied to make sure the component stays sealed.
  Therefore, we will represent the bolted connection by assigning a boundary condition in the axial directions along the flange (i.e., at the bottom of Figure 5).

  It is probably a good approximation to fix the radial direction too since there will be substantial friction between the two flanges due to the bolt pre-load and the external pressure.

- There is no need to fix the edges on the symmetry axis radially, Abaqus takes care of that.

- The housing is constrained radially around the O-ring grooves marked in Figure 2 as radial support ensures that the component remains sealed. Fix this edge radially to emulate this boundary. We will investigate this assumption in *Problem 1*.

Problem 1 – Spherical top housing

a) Model the spherical top sensor housing assuming axisymmetry (Figure 5). Mesh the part using linear reduced integration quadrilateral elements (CAX4R). Ensure that there are at least five elements across the thinnest part of the housing wall.

It is advisable to partition the part such that a structured mesh (i.e., when the part turns green) can be achieved and so that edge seeds can easily be applied. Perform a static linear analysis (Static, general). The time period can be arbitrarily chosen as 1.

b) Evaluate the stresses in the housing:

Maximum von Mises stress (this is often used to evaluate the capacity of metal parts). Answer with figure and value.

Radial stress (S11), axial stress (S22) and hoop stress (S33) in the cylindrical part of the housing denoted “Check stresses here” in Figure 2.

Note that the stresses are not neccesarily uniform in this region, but try to make an average across the thickness of the wall at the centre of the "Check stresses here" box.

Compare the two latter to the analytical functions for *Hoop stress in closed cylinder* and *Axial stress in closed cylinder.* Is there a good correspondence?

Compare the von Mises stress in the spherical part of the lid to analytical expression for *Uniform stress in closed sphere.* Is there a good correspondence?

c) Find the maximum displacement of the part. Answer with figure and value.

d) What is the total reaction force from the axial boundary condition that represents the flange? Compare this force to the external axial force form the pressure. See **Instructions/Tips** for a way to extract the forces.

The external axial force is found as the pressure times the projected area of the housing (i.e., as seen from the top in Figure 5).

e) Perform a mesh study using 2, 5 (i.e., from problem 1a), 8 and 12 elements over the thickness. Write down the peak stress for each case.

f) Try to remove the radial boundary conditions close to the O-ring grooves in Figure 2. Does it matter in terms of capacity of the housing?

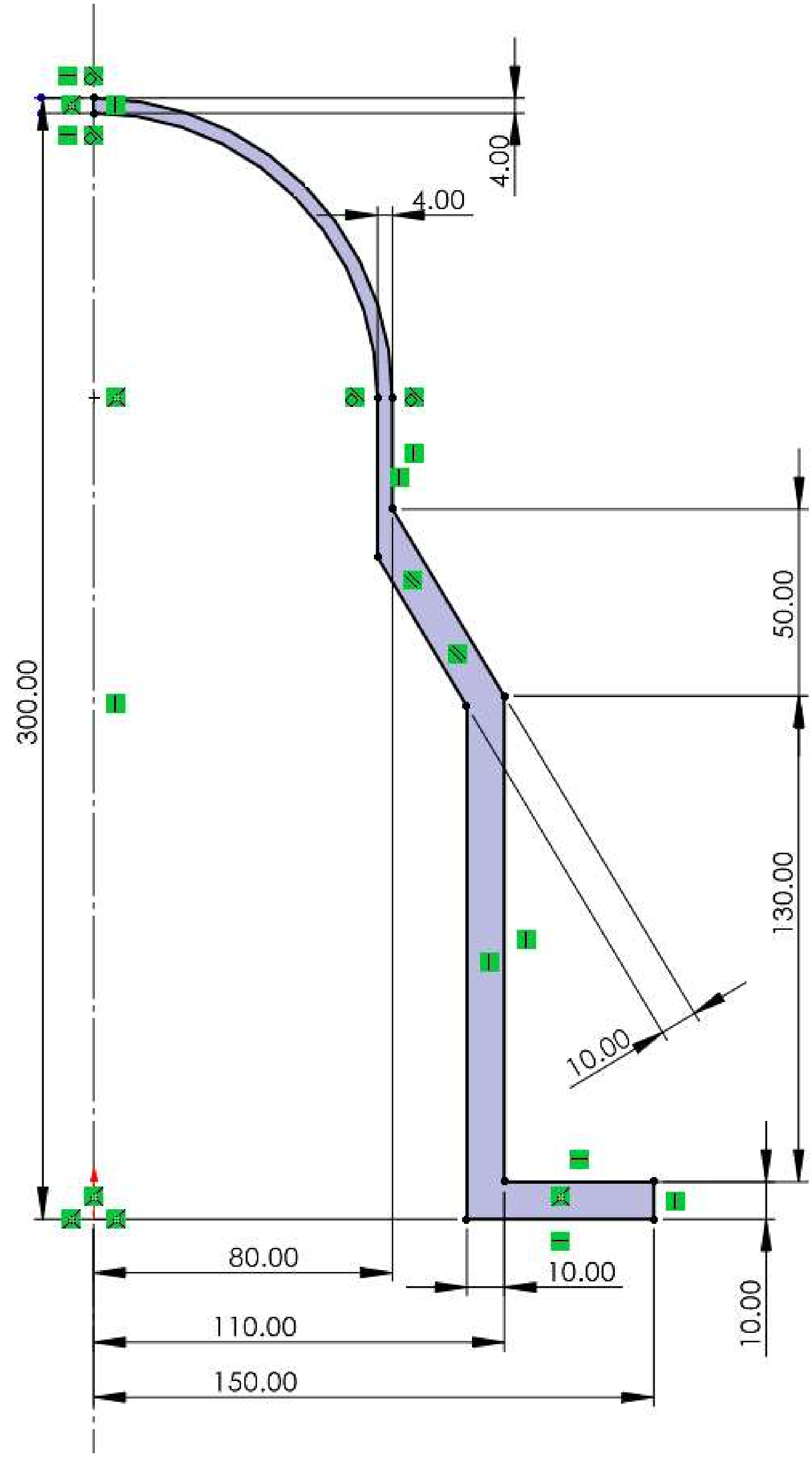


Figure 5: Cross-section of spherical top housing

Problem 2 – Flat top housing

a) Model the flat top sensor housing assuming axisymmetry (Figure 6). Mesh the part using linear reduced integration quadrilateral elements (CAX4R). Ensure that there are at least four elements across the thinnest part of the housing wall.

It is advisable to partition the part such that a structured mesh (green) can be achieved and so that edge seeds can easily be applied. Perform a static linear analysis (Static, general). The time period can be arbitrarily chosen as 1.

b) Evaluate the stresses in the housing:

Maximum von Mises stress (this is often used to evaluate the capacity of metal parts). Answer with figure and value.

Radial stress (S11), axial stress (S22) and hoop stress (S33) in the cylindrical part of the housing denoted "Check stresses here" in Figure 2. Are these different from the spherical top housing?

Compare the displacement of the centre of the the lid to analytical expressions for a pressure loaded circular plate.

The first formula is for a simply supported plate while the second is for a clamped plate. Our case is likely somewhere between these two standard cases since the plate gets some support from the cylindrical part, but not enough to be fully clamped! Which expression fits the displacement from the simulation best?

c) Find the maximum displacement of the whole part. Answer with figure and value.

d) Without doing more simulations – at what depth will the maximum stress in the flat head housing start to yield? **Hint**: We are doing a linear analysis.

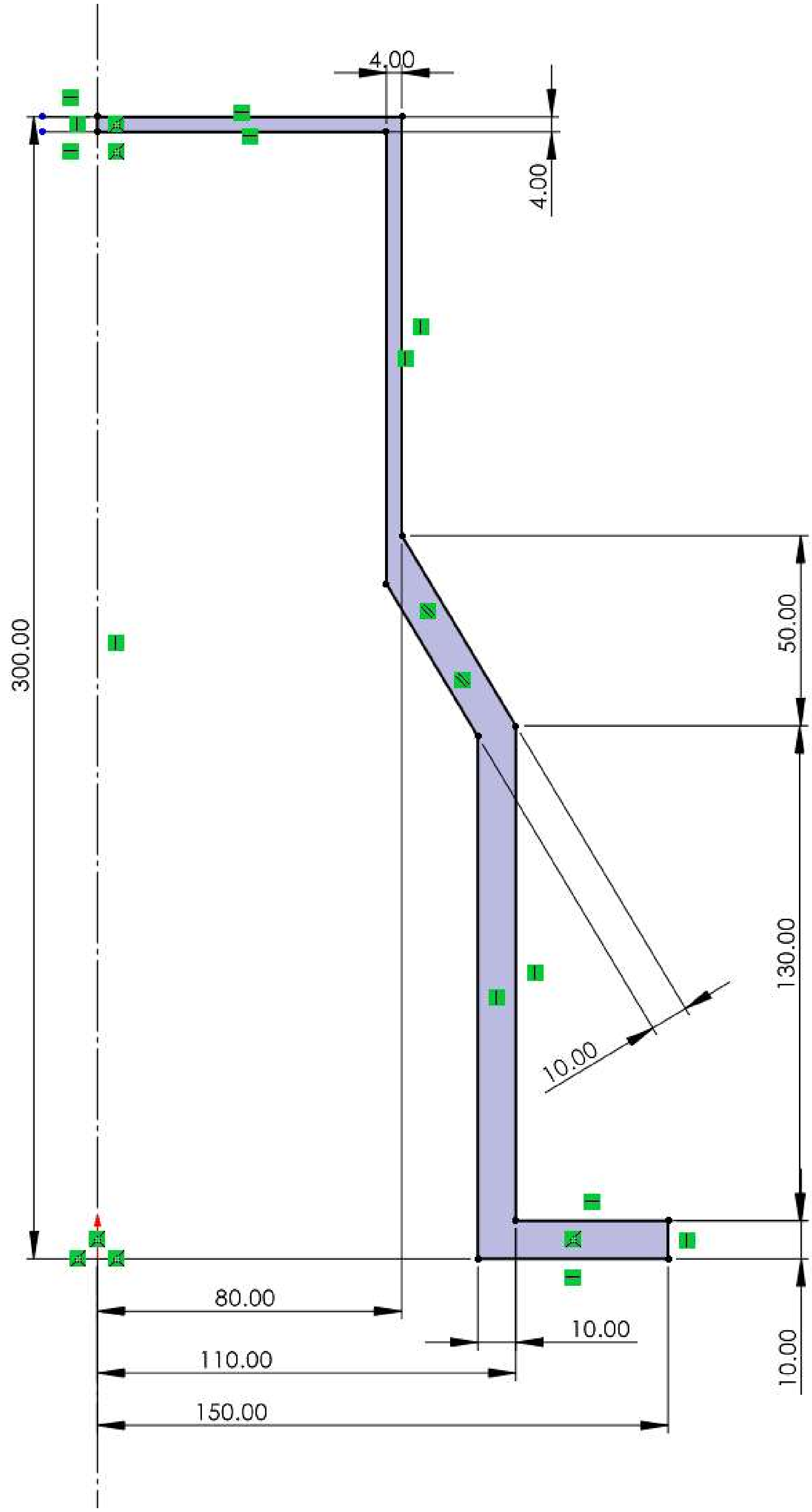


Figure 6: Cross-section of flat top housing

Problem 3 – Substantiate or make a qualified guess

a) What causes the large difference in stress/displacement between the spherical and flat lids?

b) What simplifications have we made to the boundary conditions and how might these affect the results? Could these simplifications change our conclusion on whether the part will yield or not?

c) Are there any limitations to the axisymmetric assumption used here? What are they and how could they affect our results?

d) Can other loads be added using the axisymmetric model be included (e.g., the weight of internal components, drag). Why/why not?

e) What could limit the validity of a linear analysis in this case?

f) Do we need to account for the internal pressure of the vessel? Why/why not?

Analyical formulas

For the following formulas: p is the pressure, r_m is the mean radius of the wall, t is the wall thickness and ν is Poisson's ratio. Note that thin-walled assumptions have been used for these equations.

Table 2: Analytical formulas for cylinders and spheres

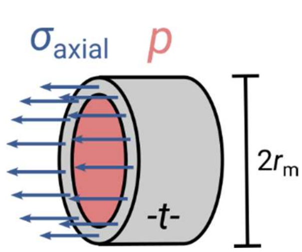
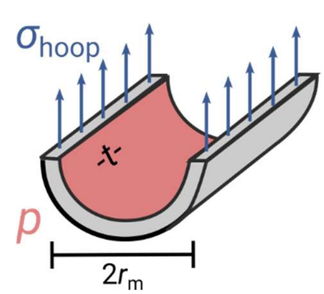
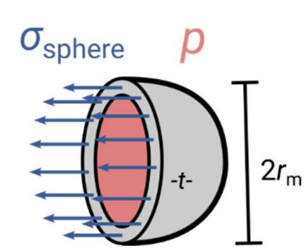
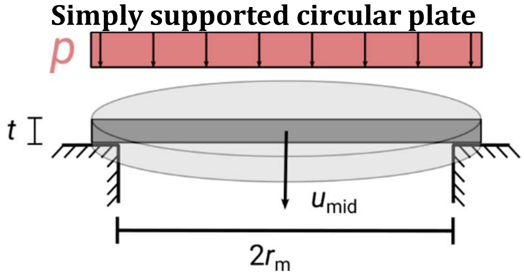
Axial stress in closed cylinder	**Hoop stress in closed cylinder**	**Uniform stress in closed sphere**
		
$\sigma_{\text{axial}} = \frac{pr_m}{2t}$	$\sigma_{\text{hoop}} = \frac{pr_m}{t}$	$\sigma_{\text{sphere}} = \frac{pr_m}{2t}$

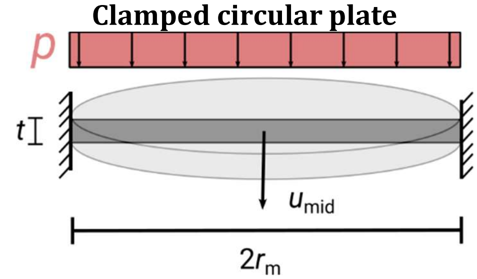
Table 3: Analytical formulas for pressure loaded cylindrical plates

Simply supported circular plate



$$u_{\text{mid}} = 0.696\frac{pr_m^4}{Et^3}$$

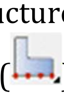
Clamped circular plate



$$u_{\text{mid}} = 0.171\frac{pr_m^4}{Et^3}$$

Instructions/Hints:

- **Sketching**: It is often a good idea to draw in lines and curves at approximate lengths, and then use the dimensioning tools and constraints to set final shape. Constraints are very useful when drawing a complex part, e.g., make sure that parallel lines are actually parallel. The tangential constraint is useful for connecting straight and curved lines. The V and H make lines either vertical or horizontal. These are often auto assigned to lines that are drawn vertical.

- **Partitioning** the part close to the transition from wall thickness of 10 mm to 4 mm is useful for creating a structured mesh. This can be done using the partition face tool () Use edge seeds () to refine the mesh in the thin parts.

- **Boundary conditions**

  Since we sketch in 2D the displacements are numbered as U1, U2, and UR3. These are displacements in radial and axial directions and rotations about the tangential direction.

  Since the part is bolted to a separate flange the contacting surface on the housing will practically not move axially, so the lower edge can be fixed in that direction.

  The housing has been fitted onto the flange connected to the pipe so the interior of the housing will be restricted from moving radially (inwards). A boundary condition can be assigned to an edge by isolating it in the sketch. The edge can be created by the partition edge or the partition face tool.

- **Pressure** in 2D is essentially assigned to an edge with the same value as if it was assigned to a regular 3D a surface.

- Abaqus returns reaction forces as the total force around the circumference of the structure. So, there is no need to integrate them across the circumference of the part.

- To **visualise** the 3D geometry: **View**, **Odb display options, Sweep/Extrude, Sweep elements**.

- **Measuring the sum reaction forces along an edge/surface**
  Method 1: Create a set of the edge/surface. Create a history output to extract the reaction force for that specific set (i.e., not the whole model). Run simulation. **XY-Data, Create, ODB History output,** Select all forces (for all nodes) that should be summed**, Save as, sum((XY,XY,…)).**
  Read result from graph: **Tools, Query, Probe values,** Select the last data point which will be the sum of reaction forces.

  Method 2 (more elegant but complex): Instead of assigning the boundary condition to the edge/surface – a reference point can be constrained (equation or coupling constraint) to the edge/surface in the direction of the boundary. Assign the boundary condition to that node (other direction must also be locked to prevent a singularity). Read the reaction force from only that node.