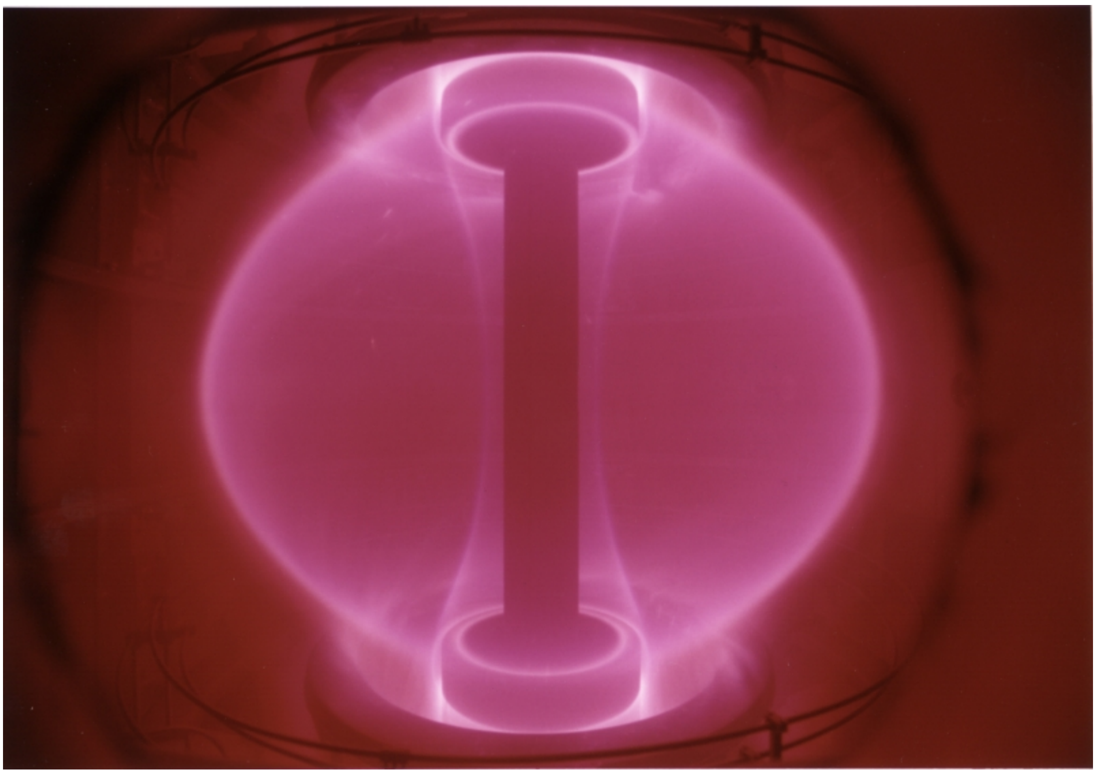
AST2000 Lecture Notes

Part 3C
Nuclear reactions in stellar cores

Questions to ponder before the lecture

1. Protons repel each other due to the electric force between equal charges. How can they possibly come together in a nuclear fusion reactions to create heavier atomic nuclei?
2. What do you think are the necessary conditions for a gas to start nuclear fusions reactions? Or: if you have a tank of hydrogren gas, what would you need to do to that gas in order to start fusion reactions?
3. We have quite detailed theories for which nuclear reactions are going on in the solar core. How would you test such a theory?
4. You might have heard that the energy produced in nuclear reactions are due to mass converted to energy through the equation $E = mc^2$. But which mass is converted to energy? When two protons and two neutrons are fused to form helium, there are still two protons and two neutrons in the new atomic nucleus, no particles are lost? Where is the mass loss?



Plasma in the original START tokamak, a nuclear fusion experiment (Image credit: Alan Sykes)

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Part 3C
Nuclear reactions in stellar cores

1 Mass in special relativity

Another topic which we need to discuss before studying nuclear reactions is the notion of mass in the special theory of relativity. We have already seen that the scalar product of the momenergy four-vector equals the mass of a particle,

$$P_\mu P^\mu = E^2 - p^2 = m^2. \tag{1}$$

Imagine we have two particles with mass m_1 and m_2, total energy E_1 and E_2 and momenta p_1 and p_2. Assume that they have opposite momenta $p_1 = -p_2 = p$,

$$P^1_\mu = (E_1, p), \quad P^2_\mu = (E_2, -p)$$

with $E_1 = \sqrt{m_1^2 + p^2}$ and $E_2 = \sqrt{m_2^2 + p^2}$. These two particles could for instance constitute the proton and the neutron in a deuterium nucleus. The question now is, what is the total mass of the two-particle system (deuterium nucleus)? Let us form the momenergy four-vector for the nucleus

$$P_\mu = P^1_\mu + P^2_\mu = (E_1 + E_2, 0).$$

Using equation 1 we can now find the total mass of the two-particle system (the nucleus),

$$\begin{aligned} M^2 &= P_\mu P^\mu = (E_1 + E_2)^2 \\ &= E_1^2 + E_2^2 + 2E_1E_2 \\ &= m_1^2 + m_2^2 + 2p^2 + \sqrt{(m_1^2 + p_1^2)(m_2^2 + p_2^2)} \end{aligned}$$

where M is the total mass of the nucleus. We have two important observations: (1) Mass is *not* an additive quantity. The total mass of a system of particles is *not* the sum of the mass of the individual particles. (2) The mass of a system of particles depends on the total energy of the particles in the system. The energy of particles in an atomic nucleus includes the potential energy between the particles due to electromagnetic and nuclear forces.

Consider an atomic nucleus with mass M. This nucleus can be split into two smaller nuclei with masses m_1 and m_2. If total mass of the two nuclei m_1 and m_2 is smaller than the total mass of the nucleus, the rest energy is radiated away when the nucleus is divided. This is a nuclear fission process creating energy. Similarly if the total mass of m_1 and m_2 is larger than the total mass of the nucleus, then energy must be provided in order to split the nucleus. The same argument goes for nuclear fusion processes: Consider two nuclei with masses m_1 and m_2 which combine to form a larger nucleus of mass M. If M is smaller than the total mass of the nuclei m_1 and m_2 then the rest mass is radiated away and energy is 'created' in the fusion process. In some cases (particularly for large nuclei), the mass M is larger than the total mass of m_1 and m_2. In this case energy must be provided in order to combine the two nuclei to a larger nucleus. We will soon see that in order to produce atomic nuclei larger than iron, energy must always be provided.

2 Penetrating the Coloumb barrier

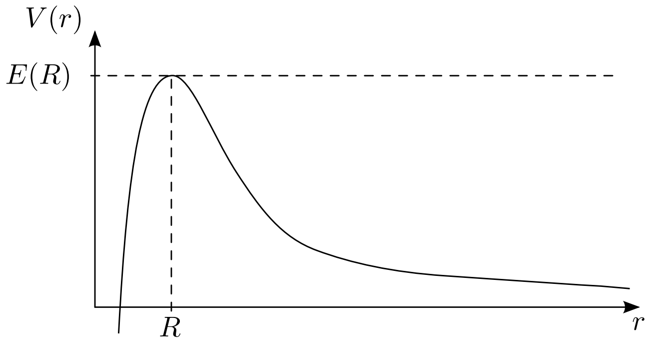


Figure 1: The repulsive Coloumb potential $V(r)$ as a function of distance between nuclei r. At small distances r we see the potential well from the attractive strong forces.

The strong nuclear force (usually referred to as the strong force) is active over much smaller distances than the electromagnetic force. The strong force makes protons attract protons and protons attract neutrons (and vice versa). For two atomic nuclei to combine to form a larger nucleus, the two nuclei need to be close enough to feel the attractive nuclear forces from each other. Atomic nuclei have positive charge and therefore repulse each other at larger distances due to the electromagnetic force. Thus for a fusion reaction to take place, the two nuclei need to penetrate the Coloumb barrier, the repulsive electromagnetic force between two equally charged particles. They need to get so close that the attractive strong force is stronger than the repulsive electromagnetic force. In figure 1 we show the combined potential from electromagnetic and nuclear forces of a nucleus. We clearly see the potential barrier at $r = R$. For a particle to get close enough to feel the attractive strong force it needs to have an energy of at least $E > E(R)$. We can make an estimate of the minimal temperature a gas needs in order to make a fusion reaction happen: The mean kinetic energy of a particle in a gas of temperature T is $E_K = (3/2)kT$ (see the exercises). The potential energy between two nuclei A and B can be written as

$$U = -\frac{1}{4\pi\epsilon_0}\frac{Z_A Z_B e^2}{r},$$

where ϵ_0 is the vacuum permittivity, Z_1 and Z_2 is the number of protons in each nucleus, e is the electric charge of a proton and r is the distance between the two nuclei. For nucleus A to reach the distance R (see figure 1) from nucleus B where the strong force starts to dominate, the kinetic energy must at least equal the potential energy at this point

$$\frac{3}{2}kT = \frac{1}{4\pi\epsilon_0}\frac{Z_A Z_B e^2}{R}.$$

The distance R is typically $R \sim 10^{-15}$ m. Considering the case of two hydrogen nuclei $Z = 1$ fusing to make helium $Z = 2$, we can solve this equation for the temperature and obtain $T \sim 10^{10}$ K. This temperature is much higher than the core temperature of the Sun $T_C \sim 15 \times 10^6$ K. Still this reaction is the main source of energy of the Sun. How can this be?

The secret is hidden in the world of quantum physics. Due to the Heisenberg uncertainty relation, nucleus A can borrow energy ΔE from vacuum for a short period Δt. If nucleus A is close enough to nucleus B, the time Δt might just be enough to use the borrowed energy to penetrate the Coloumb barrier and be captured by the potential well of the strong force. This phenomenon is called *tunneling*. Thus, there is a certain probability that nucleus A spontaneously borrows energy to get close enough to nucleus B in order for the fusion reaction to take place.

3 Nuclear reaction probabilities and cross sections

Quantum physics is based on probability and statistics. Nothing can be predicted with 100% certainty, only statistical probabilities for events to happen can be calculated. When nucleus A is at a certain distance from nucleus B we cannot tell whether it will borrow energy to penetrate the Coloumb barrier or not, we can only calculate the probability for the tunneling to take place. These probabilities are fundamental for understanding nuclear reactions in stellar cores. These probabilities are usually represented as *cross sections* σ.

The exact calculations of cross sections and thereby nuclear reaction rates is outside the scope of this course. Below we show how this can be done (optional) for those who are interested. What we need here is the final expression which can give the energy production rate from a gas with a certain density and temperature. In the calculations below, we arrive at an integral (equation 5) giving the total energy production rate. We will not do the integral here but note that the solution can be Taylor expanded around given temperatures T as

$$\varepsilon_{AB} = \varepsilon_{0,\text{reac}} X_A X_B \rho^{\alpha} T^{\beta},$$

which is the total energy produced per second and per kg of gas. Here ρ is the density, α and β are indices which depend on the temperature T around which the expansion is made and X_A and X_B are the mass fractions of the two nuclei defined as

$$X_A = \frac{n_A m_A}{nm} = \frac{\text{total mass in type A nuclei}}{\text{total mass}},$$

Here n_A is the number density of A particles, n is the total number density of particles, m_A is the mass of A particles and m is the mean mass of a particle in the gas.

Here, $\varepsilon_{0,\text{reac}}$, α and β will depend on the nuclear reaction and can in principle be calculated from the integral in equation 5 for each case, although in this course these numbers will always be given. If we have $\varepsilon_{0,\text{reac}}$, α and β for different nuclear reactions, we can use this expression to find the nuclear reactions which are important for a given temperature T in a stellar core.

The quantity ε_{AB}, or simply ε, given above is the energy release per mass per time. Sinde energy release per time is luminosity, we can therefore write this as

$$\frac{dL}{dm} = \varepsilon$$

The luminosity produced in a shell at a distance r from the center of a star can therefore be written as (NB! check that you can arrive at this expression: how can you write the mass dm for an infinitesimally thin shell of thickness dr?)

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r) \varepsilon(r), \tag{2}$$

which is another of the equations used together with the equation of hydrostatic equilibrium in what is called *stellar model building* which will be described more in part 3D: Solving these two equations combined with some equations from thermodynamics and fluid dynamics one can obtain temperature and density profiles $T(r)$ and $\rho(r)$ as well as detailed knowledge of the different molecules and atoms present at different distances from the center of a star. These models have been used to obtain the understanding we have today of how stars evolve.

3.1 OPTIONAL: Deducing the integral for the nuclear energy production rate

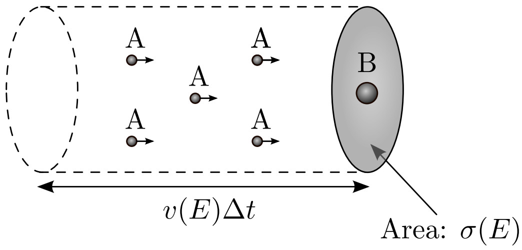


Figure 2: A particles streaming towards the disk with cross section $\sigma(E)$ around the B nucleus. A particles of energy E within the volume $v(E)\Delta t\sigma(E)$ will react with the nucleus B within time Δt.

Before calculating the nuclear reaction rate, we need to understand the definition of cross section in physics. The definition of the cross section is based on an imaginary situation which is a bit different from the real situation but gives an intuitive picture of the reaction probabilities and, most importantly, makes the calculations easier. It can be proven that the calculations made for this imaginary picture gives exact results for the real situation. Instead of the real situation where we have one nucleus A and one nucleus B passing each other at a certain distance (and we want to know the probability that they react), one imagines the nucleus B to be at rest and a number of nuclei of type A approaching it. One imagines nucleus B to have a finite two dimensional extension, like a disk, with area σ. Towards this disk there is a one dimensional flow of A particles (see figure 2). If a nucleus A comes within this disk, it is captured and fusion takes place, if not the nuclei do not fuse. It is important to understand that this is not really what happens: fusion can take place with any distance r between the nuclei. It might also well be that A is within the disk and the fusion reaction is not taking place. But in order to make calculations easier one makes this imaginary disk with an effective cross section σ saying that any nucleus A coming within this disk will fuse. It can be shown that calculations made with this representation gives correct reaction rates even though the model does not give a 100% correct representation of the physical situation. Because of the simplified mathematics, the cross section σ is the most common way of representing a probability for a reaction or collision process to take place.

You will now see how this imaginary picture is used to calculate reaction rates.

The disk cross section (tunneling probability) $\sigma(E)$ depends on the energy E of the incoming nucleus A. Thus the size of the immaginary disk (for the nucleus B at rest) depends on the energy E of the incoming particle A. We will now make calculations in the center of mass system. In exercise 1B.4 in part 1B, you showed that the total kinetic energy of a two-body system can be written as (ignoring gravitational forces)

$$E = \frac{1}{2}\hat{\mu}v^2,$$

where $\hat{\mu}$ is the reduced mass $\hat{\mu} = (m_1 m_2)/(m_1 + m_2)$. We showed that the two-body problem is equivalent to a system where a particle with mass $M = m_1 + m_2$ is at rest and a particle with the reduced mass $\hat{\mu}$ is moving with velocity v. In this case we imagine the nucleus B to be at rest and the particle A is approaching with velocity v.

We will now consider a gas with a total number density of particles n per volume, a number density n_A per volume of A nuclei and a number density n_B per volume of B nuclei. We will try to find how many A nuclei with a given energy E will react with one B nucleus per time interval Δt. The answer is simple: All the A particles with energy E which are in such a distance from B that they will hit the disk with cross section $\sigma(E)$ around nucleus B within the time interval Δt (do you really understand this?). This means that all the A nuclei with energy E at a distance $v(E)\Delta t$ from B moving towards B will react with B. In figure 2 we illustrate the situation. All A nuclei within a volume $V = v(E)\Delta t\sigma(E)$ will react (make sure you get this before continuing!). (again, this is an imaginary situation: only one nucleus A can really react with B, the numbers we obtain are in reality probabilities). Let $n_A(E)$ be the number density of A nuclei with energy E such that $n_A(E)dE$ is the number of A nuclei with energies between E and $E+dE$. Then, the total number of nuclear reactions per nucleus B from A nuclei with energies in the interval E to $E + dE$ is given by

$$dN(E) = v(E)\,dt\,\sigma(E)n_A(E)\,dE. \tag{3}$$

Before continuing we need to know the number density of A nuclei with energy E, $n_A(E)$. Recall from lectures 1A and 1G that we can use the Maxwell-Boltzmann distribution for energy:

$$n(E)dE = \frac{2n}{\sqrt{\pi}(kT)^{3/2}}E^{1/2}e^{-\frac{E}{kT}}\,dE,$$

which is the number of particles in the gas with energy E expressed in terms of the total number of particles in the gas n. Returning to equation 3 we see that what we need is not the total number of particles at energy E, but the total number of A particles at energy E. This can be written as

$$n_A(E)dE = \frac{n_A}{n}n(E)dE,$$

where n_A/n is the fraction of A particles in the gas (over all energies). From equation 3 we thus have

$$\frac{dN(E)}{dt} = \sigma(E)v(E)\frac{n_A}{n}n(E)dE,$$

which is the reaction rate per B nucleus, i.e. the number of reactions taking place for each B nucleus present (independent of the energy of the B nucleus, remember that the B nucleus is at rest). To obtain the total reaction rate r_{AB} between A and B nuclei we thus need to multiply with the total density of B nuclei n_B and integrate over all energies E

$$r_{AB} = \frac{dN}{dt} = \int_0^\infty dE n_A n_B \sigma(E)v(E)\frac{n(E)}{n}.$$

This is the total number of reactions per time and volume. Now we insert the Maxwell-Boltzmann distribution to get

$$r_{AB} = \left(\frac{2}{kT}\right)^{3/2}\frac{n_A n_B}{\sqrt{\hat{\mu}\pi}}\int_0^E dE E e^{-E/kt}\sigma(E).$$

Advanced quantum field theory is needed to calculate $\sigma(E)$. Here we will give the answer

$$\sigma(E) = \frac{S(E)}{E}e^{-b/\sqrt{E}},$$

where

$$b = \frac{\pi\sqrt{\hat{\mu}}Z_A Z_B e^2}{\sqrt{2}\epsilon_0 h}$$

and $S(E)$ is a slowly varying function in E depending on the nuclei involved. The constant b involves the masses and the number of protons in the nuclei. We can thus write the reaction rate as

$$r_{AB} = \left(\frac{2}{kT}\right)^{3/2}\frac{n_A n_B}{\sqrt{\hat{\mu}\pi}}\int_0^\infty dE S(E)e^{-b/\sqrt{E}}e^{-E/(kT)}. \tag{4}$$

We usually express the reaction rate as the energy ε_{AB} which is released per kilogram matter per second. We can write this as

$$\varepsilon_{AB} = \frac{\varepsilon_0}{\rho}r_{AB},$$

where ε_0 (which is **not** the vacuum permittivity ϵ_0) is the energy released per nuclear reaction (why did we include the density ρ here?). Combined with the integral for r_{AB} above we therefore get

$$\varepsilon_{AB} = \frac{\varepsilon_0}{\rho}\left(\frac{2}{kT}\right)^{3/2}\frac{n_A n_B}{\sqrt{\hat{\mu}\pi}}\int_0^\infty dE E e^{-E/kT}\sigma(E), \tag{5}$$

where ε_0 is the energy released in each nuclear reaction between an A and a B nucleus, ρ is the total density of the gas and n_A and n_B are number densities of A and B nuclei.

4 Stellar nuclear reactions

For main sequence stars the most important fusion reaction fuses four ${}^1_1$H atoms to ${}^4_2$He. When writing nuclei, ${}^A_Z$X, A is the total number of nucleons (protons and neutrons), Z is the total number of protons and X is the chemical symbol. There are mainly two chains of reaction responsible for this process. One is the pp-chain,

$$\begin{aligned}
{}^1_1\mathrm{H} + {}^1_1\mathrm{H} &\rightarrow {}^2_1\mathrm{H} + {}^0_0\bar{\mathrm{e}} + {}^0_0\nu_\mathrm{e} \\
{}^2_1\mathrm{H} + {}^1_1\mathrm{H} &\rightarrow {}^3_2\mathrm{He} + {}^0_0\gamma \\
{}^3_2\mathrm{He} + {}^3_2\mathrm{He} &\rightarrow {}^4_2\mathrm{He} + 2\times {}^1_1\mathrm{H}
\end{aligned}$$

Here ${}^0_0\nu_\mathrm{e}$ is the electron associated neutrino, ${}^0_0\gamma$ is a photon and the bar represents antiparticles:

${}^0_0\bar{e}$ is the antiparticle of the electron called the positron. This is the pp-I chain, the most important chain reactions in the solar core. There are also other branches of the pp-chain (with the first two reactions equal) but these are less frequent. The pp-chain is most effective for temperatures around 15 millions Kelvin for which we can write the reaction rate for the full pp-chain as

$$\boxed{\varepsilon_{\rm pp} \approx \varepsilon_{0,\rm pp} X_H^2 \rho T_6^4,}$$

where $T = 10^6 \mathrm{K} T_6$ with T_6 being the temperature in millions of Kelvin. This expression is valid for temperatures close to $T_6 = 15$. For this reaction $\varepsilon_{0,\rm pp} = 1.08 \times 10^{-12}$ Wm^3/kg^2. The efficiency of the pp-chain is 0.007, that is only 0.7% of the mass in each reaction is converted to energy.

The other reaction converting four ${}^1_1$H to ${}^4_2$He is the CNO-cycle,

$$\begin{aligned}
{}^{12}_{6}\mathrm{C} + {}^1_1\,\mathrm{H} &\rightarrow {}^{13}_{7}\mathrm{N} + {}^0_0\,\gamma \\
{}^{13}_{7}\mathrm{N} &\rightarrow {}^{13}_{6}\mathrm{C} + {}^0_0\,\bar{\mathrm{e}} + {}^0_0\,\nu_{\rm e} \\
{}^{13}_{6}\mathrm{C} + {}^1_1\,\mathrm{H} &\rightarrow {}^{14}_{7}\mathrm{N} + {}^0_0\,\gamma \\
{}^{14}_{7}\mathrm{N} + {}^1_1\,\mathrm{H} &\rightarrow {}^{15}_{8}\mathrm{O} + {}^0_0\,\gamma \\
{}^{15}_{8}\mathrm{O} &\rightarrow {}^{15}_{7}\mathrm{N} + {}^0_0\,\bar{\mathrm{e}} + {}^0_0\,\nu_{\rm e} \\
{}^{15}_{7}\mathrm{N} + {}^1_1\,\mathrm{H} &\rightarrow {}^{12}_{6}\mathrm{C} + {}^4_2\,\mathrm{He}
\end{aligned}$$

with a total reaction rate

$$\boxed{\varepsilon_{\rm CNO} = \varepsilon_{0,\rm CNO} X_H X_{\rm CNO} \rho T_6^{20},}$$

where $\varepsilon_{0,\rm CNO} = 8.24 \times 10^{-31}$ Wm^3/kg^2 and

$$X_{\rm CNO} = \frac{M_{\rm CNO}}{M}$$

is the total mass fraction in C, N and O. These three elements are only catalysts in the reaction, the number of C, N and O molecules do not change in the reaction. This expression is valid for $T_6 \approx 20$. We see that when the temperature increases a little, the CNO cycle becomes much more effective because of the power 20 in temperature. In the exercises you will find how much. Thus, the CNO cycle is very sensitive to the temperature. Small changes in the temperature may have large influences on the energy production rate by the CNO cycle.

For stars with an even hotter core, also ${}^4_2$He may fuse to heavier elements. In the triple-alpha process three ${}^4_2$He nuclei are fused to form ${}^{12}_{6}$C.

$${}^4_2\mathrm{He} + {}^4_2\mathrm{He} \rightarrow {}^8_4\mathrm{Be} + {}^0_0\,\gamma$$

$${}^8_4\mathrm{Be} + {}^4_2\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C}^* + {}^0_0\,\gamma$$

Here the reaction rate can be written as

$$\boxed{\varepsilon_{3\alpha} = \varepsilon_{0,3\alpha} \rho^2 X_{He}^3 T_8^{41}.}$$

Here $T = 10^8 \mathrm{K} T_8$, T_8 is the temperature in hundred millions of Kelvin and $\varepsilon_{0,3\alpha} = 3.86 \times 10^{-18}$ Wm^6/kg^3. This expression is valid near $T_8 = 1$. We see an extreme temperature dependence. When the temperature is high enough, this process will produce much more than the other processes.

For higher temperatures, even heavier elements will be produced for instance with the reactions

$${}^4_2\mathrm{He} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{16}_{8}\mathrm{O} + {}^0_0\,\gamma \tag{6}$$

$${}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{24}_{12}\mathrm{Mg} + {}^0_0\,\gamma \tag{7}$$

There is a limit to which nuclear reactions can actually take place: The mass of the resulting nucleus must be lower than the total mass of the nuclei being fused. Only in this way energy is produced. This is not always the case. For instance the reactions

$${}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{16}_{8}\mathrm{O} + 2{}^4_2\mathrm{He} \tag{8}$$

and

$${}^{16}_{8}\mathrm{O} + {}^{16}_{8}\mathrm{O} \rightarrow {}^{24}_{12}\mathrm{Mg} + 2{}^4_2\mathrm{He} \tag{9}$$

require energy *input*, that is the total mass of the resulting nucleus is larger than the total mass of the input nuclei. It is extremely difficult to make such reactions happen: Only in extreme environments with very high temperatures is the probability for such reactions large enough to make the processes take place.

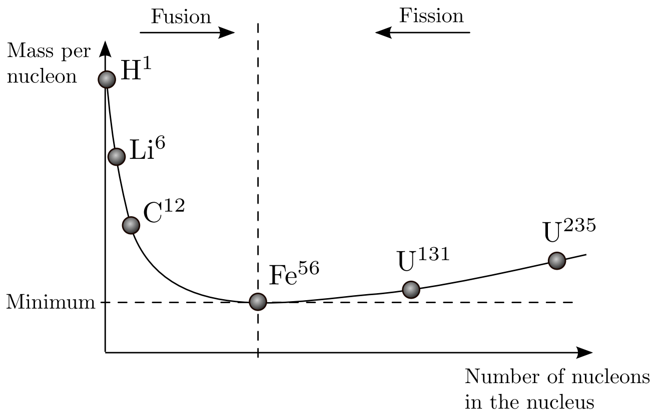


Figure 3: Schematic diagram of mass per nucleon as a function of the number of nucleons in the nucleus. Note that we are only illustrating the general trends. There are for instance a few light elements for which the mass per nucleon increases with increasing number of nucleons in the nucleus.

In figure 3 we show the mass per nucleon for the different elements. We see that we have a minimum for $^{56}_{26}$Fe. This means that for lighter elements (with less than 56 nucleons), the mass per nucleon decreases when combining nuclei to form more heavier elements. Thus, for lighter elements, energy is usually released in a fusion reaction (with some exceptions, see equation 8 and 9). For elements heavier than iron however, the mass per nucleus increases with increasing number of nucleons. Thus, energy input is required in order to make nuclei combine to heavier nuclei. The latter processes are very improbable and require very high temperatures.

We see that we can easily produce elements up to iron in stellar cores. But the Earth and human beings consist of many elements much heavier than iron. How were these produced? In the Big Bang only hydrogen and helium were produced so the heavier elements must have been created in nuclear reactions at a later stage in the history of the universe. We need situations were huge amounts of energy are available to produce these elements. The only place we know about where such high temperatures can be reached are supernova explosions. We will come back to this later.

5 The solar neutrino problem

If you look back at the chain reactions above you will see that neutrinos are produced in the pp-chain and the CNO cycle. We have learned in earlier lectures that neutrinos are particles which hardly react with matter. Unlike the photons which are continuously scattered on charged particles on they way from the core to the stellar surface, the neutrinos can travel directly from the core of the Sun to the Earth without being scattered even once. Thus, the neutrinos carry important information about the solar core, information which would have otherwise been impossible to obtain without being at the solar core. Using the chain reactions above combined with the theoretical reaction rates, we can calculate the number of neutrinos with a given energy we should observe here at Earth. This would be an excellent test of the theories for the composition of the stellar interiors as well as of our understanding of the nuclear reactions in the stellar cores. The procedure is as follows

1. Stellar model building: Solve the coupled set of equations consisting of the equation of hydrostatic equilibrium, equation 2 as well as several equations from thermodynamics describing the transport of energy within the Sun. The solutions to these equations will give you the density $\rho(r)$ and temperature $T(r)$ of the Sun as a function of distance r from the center.

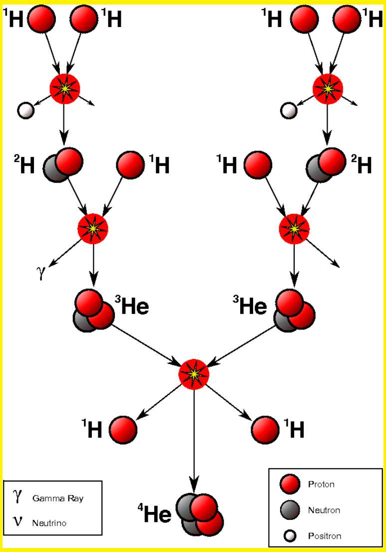
2. The temperature $T(r)$ at a given distance r combined with the above expressions for stellar reaction rates gives the number of neutrinos produced in the different kinds of chain reactions and what energies E these neutrinos should have.

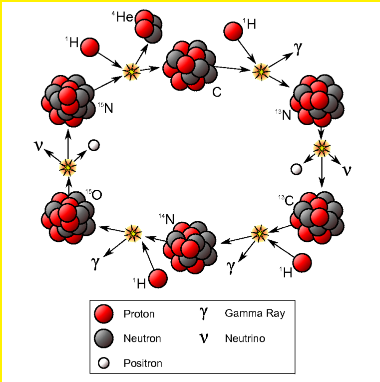
3. Measure the flux of neutrinos for different energy ranges E that we receive on Earth and compare to theoretical predictions.

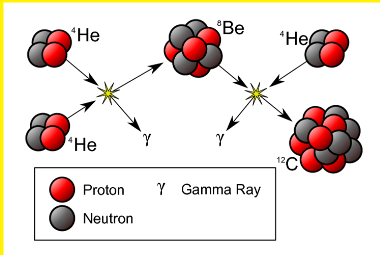
4. If there is agreement, it means we have obtained the correct model for the Sun. If the agreement is not satisfactory, we need to go back to the first step and make the stellar model building with different assumptions and different parameters.

For many years, there was a strong disagreement between the neutrino flux observed at Earth and the solar models. The observed number of neutrinos was much lower than predicted. Now the dis-

Fact sheet: The proton–proton (pp) chain reaction, The carbon-nitrogen-oxygen (CNO) cycle (the helium nucleus is released at the top-left step) and the triple-alpha process.(Figure:Wikipedia)







crepancy is resolved and the solution led to an important discovery in elementary particle physics: It was discovered that the neutrinos have mass. It was previously thought that neutrions were massless like the photons. Elementary particle physics predicted that if the neutrinos have mass, they may oscillate between the three different types of neutrino. If neutrinos have mass, then an electron neutrino could spontaneously convert into a muon or tau neutrino. The first neutrino experiments were only able to detect electron neutrinos. The reason they didn't detect enough solar neutrinos was that they had converted to different types of neutrinos on the way from the solar core to the Earth. Today neutrino detectors may also detect other kinds of neutrino and the observed flux is in much better agreement with the models. But it does not mean that the solar interior and solar nuclear reactions are completely understood. Modern neutrino detectors are now used to measure the flux of different kinds of neutrinos in different energy ranges in order to understand better the processes being the source of energy in the Sun as well as other stars.

But the neutrinos hardly react with matter, how are they detected? This is not an easy task and a very small fractions of all the neutrinos passing through the Earth are detected. One kind of neutrino detector consists of a tank of cleaning fluid C_2Cl_4, by the reaction

$$^{37}_{17}\mathrm{Cl} + ^{0}_{0}\nu_{\mathrm{e}} \rightarrow ^{37}_{18}\mathrm{Ar} + ^{0}_{-1}\mathrm{e}.$$

The argon produced is chemically separated from the system. Left to itself the argon can react with an electron (in this case with its own inner shell electron) by the converse process

$$^{37}_{18}\mathrm{Ar} + ^{0}_{-1}\mathrm{e} \rightarrow ^{37}_{17}\mathrm{Cl} + ^{0}_{0}\nu_{\mathrm{e}}.$$

The chlorine atom is in an excited electronic state which will spontaneously decay with the emission of a photon. The detection of such photons by a photomultiplier then is an indirect measurement of the solar neutrino flux.

6 Exercises

Exercise 3C.1

One of the solar standard models predict the following numbers for the solar core: $\rho = 1.5 \times 10^5$ kgm^{-3}, $T = 1.57 \times 10^7$ K, $X_{\rm H} = 0.33$, $X_{\rm He} = 0.65$ and $X_{\rm CNO} = 0.01$. We will assume that the expressions for energy production per kilogram given in the text are valid at the core temperature of the Sun. We will make this approximation even for the expression for the triple-alpha reaction which is supposed to be correct only for higher temperatures.

1. Calculate the total energy produced per kilogram in the Sun by the pp-chain, CNO-cycle and the triple-alpha process.
2. Find the ratio between the energy production of the pp-chain and the CNO-cycle and between the pp-chain and the triple-alpha process. The energy produced by the CNO cycle is only about 1% of the total energy production of the Sun. If you got a very different number in your ratio between the pp-chain and the CNO-cycle, can you find an explanation for this difference? What would you need to change in order to obtain a more correct answer?
3. Now repeat the previous question using a mean core temperature of about $T = 13 \times 10^6$ K. Use this temperature in the rest of this exercise.
4. At which temperature T does the CNO cycle start to dominate?
5. Assume for a moment that only the pp-chain is responsible for the total energy production in the Sun. Assume that all the energy production in the Sun takes place wihin a radius $R < R_E$ inside the solar core.Assume also that the density, temperature and mass fractions of the elements are constant within the radius R_E. So all the energy produced by the Sun is produced in a sphere of radius R_E in the center of the solar core. Use the above numbers and the solar luminosity $L_\odot = 3.8 \times 10^{26}$ W to find the size of this radius R_E within which all the energy production takes place. Express the result in solar radii $R_\odot \approx 7 \times 10^8$ m. The solar core extends to about $0.2R_\odot$. How well did your estimate of R_E agree with the radius of the solar core?
6. If the CNO-cycle alone had been responsible for the total energy production of the Sun, what would the radius R_E had been? (again express the result in solar radii)

Exercise 3C.2

1. Go through all the nuclear reactions in the pp-chain and CNO cycle. For each line in the chain, check that total charge and total lepton number is conserved. (there might be some printing errors here, if you spot one where is it?)
2. After having checked all these reactions you should have gained some intuition about these reactions and the principles behind them. So much that you should be able to guess the missing numbers and particles in the following reactions

$$\begin{aligned}
{}^{27}_{14}\mathrm{Si} &\rightarrow {}^{27}_{?}\mathrm{Al} + {}^{0}_{0}\,\bar{\mathrm{e}} + ? \\
{}^{27}_{?}\mathrm{Al} + {}^{1}_{1}\mathrm{H} &\rightarrow {}^{24}_{12}\mathrm{Mg} + {}^{?}_{2}? \\
{}^{35}_{17}\mathrm{Cl} + {}^{1}_{1}\mathrm{H} &\rightarrow {}^{36}_{18}\mathrm{Ar} + ?
\end{aligned}$$