Questions to discuss during the lectures for part 1C

Important: You also need the lecture notes for part 1C to read together with this guide.
Also note: The lecture guides are made for use during the lectures. They will not make much sense if you are not present at the lecture, in this case, go directly to the lecture notes. You may then use the lecture guide after reading the lecture notes to test your understanding.

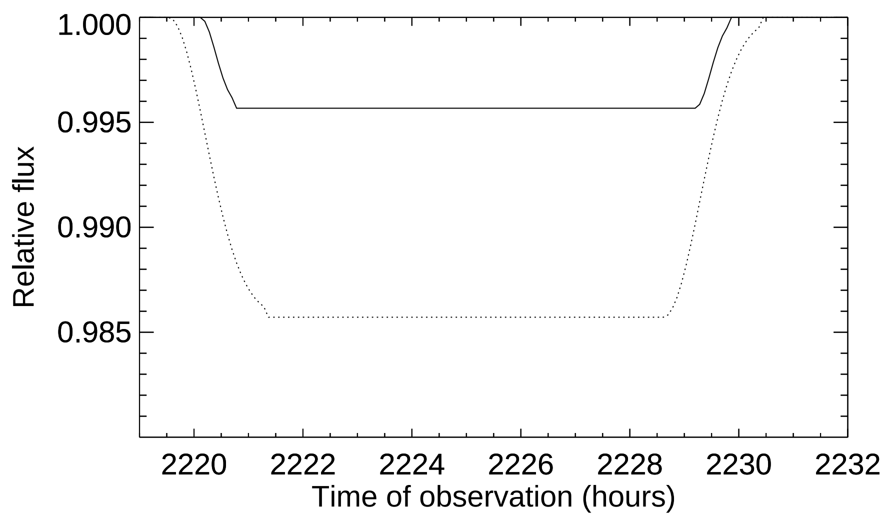

Figure 1: Light curves from a star with eclipsing planet. Solid line shows the light curve of a wavelength without a spectral line, the dotted line shows the light received on the wavelength of a particular spectral line of water vapour.

1. In figures 1 you see the light curves from a star, i.e., the flux of received light as a function of time. The observations shown by a solid line are made on a wavelength of light where no spectral line is expected. The observations shown by a dotted line is made on a wavelength where water vapour absorbs light. The observations are made when a planet is passing in front of the star. What do these plots tell you about the planet orbiting the star? Try to imagine the disc of the planet passing in front of the disc of the star, step by step.

2. Roughly how would you go about to find the radius of this planet using the curves? (what more info would you need?)

3. How can you conclude that this planet has an atmosphere and roughly how could you go about to find the thickness of the atmosphere?

4. Is there a way you could find the orbital velocity of the planet from this plot?
5. Are there other ways you could get information about this planet?
6. Why is it so difficult to see the planets directly with the telescope?
7. The small angle formula $d = r\Delta\theta$ shown in figure 2 is valid for small angles $\Delta\theta$. Try to deduce the small angle formula.

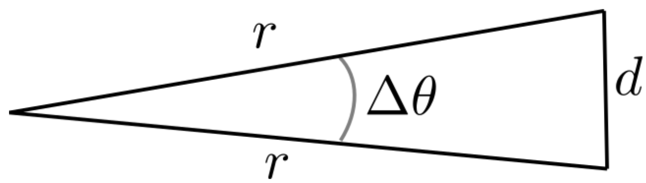

Figure 2: Figure for deriving the small-angle formula.

8. Assume that d in the figure corresponds to the distance between a star and a planet and r is the distance to the observer. If Sirius (at a distance of 8.6 ly) has a planet with a circular orbit, what is the minimum radius of the orbit in order to be able to resolve the planet from Earth with resolution $0.4''$? Give the answer i units of AU. And from the Hubble Space Telescope with resolution $0.1''$?
9. In figure 3 the observed wavelength of the $H\alpha$ spectral line from a star as a function of time is shown. In the laboratory, this spectral line appears at the wavelength 656.28nm. The sinusoidal variation of the wavelength of the spectral line (seen in the light of the star) is caused by the presence of a planet. How?
10. Why is the curve not oscillating about 0? (which would correspond to no line shift)
11. Try to draw by hand the radial velocity curve (the velocity component along your line of sight) of the star as a function of time with numbers and units on the axes.
12. In figure 4 the orbit of the start around the centre of mass with the planet is shown. Where are the points A, B, C and D on your velocity plot?
13. How could such a curve be used to find the mass of the planet? Could you use Newton's version of Kepler's 3rd law?
$$P^2 = \frac{4\pi^2 a^3}{G(m_* + m_p)}$$
14. How can you find P in this equation? Assuming circular orbits, can you eliminate a from this equation and instead use the velocity inferred from your velocity curve? How?
15. Show that
$$m_p + m_* = \frac{P}{2\pi G}(v_* + v_p)^3$$
What is v_* and v_p here?

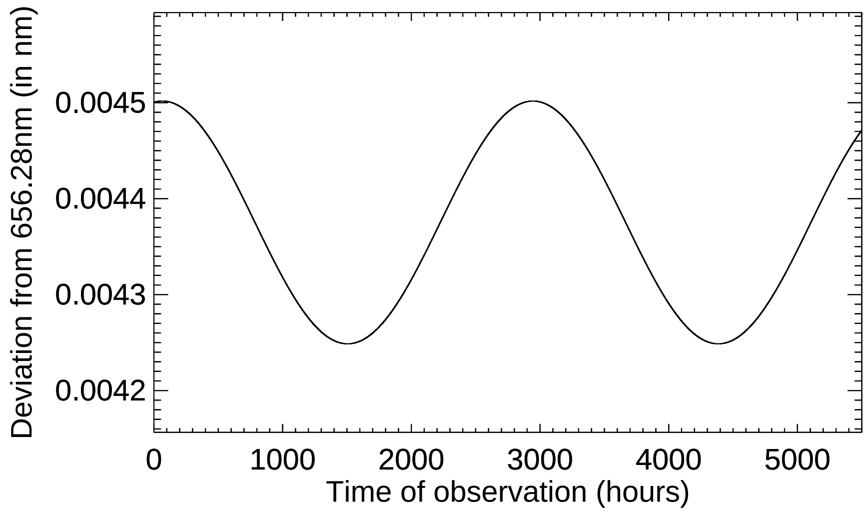

Figure 3: The observed wavelength of the $H\alpha$ spectral line minus the wavelength 656.28nm (the wavelength at which the spectral line appears in the laboratory) from a star as a function of time

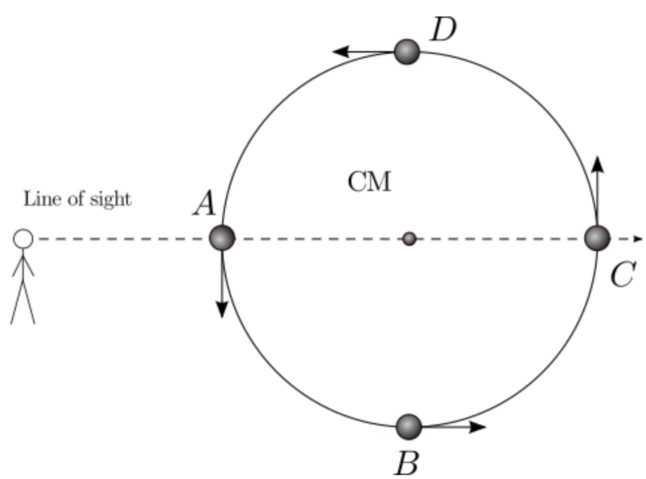

Figure 4: The movement of the star in orbit around the center of mass

16. Do you measure v, v_* or v_p from the velocity curve? How do you obtain the velocity to use from your curve?

17. Which one is larger, v_p or v_*?

18. Using the relation between $\vec{r}$ and $\vec{r}_*$ and $\vec{r}_p$ (which relation is this? Look at the discussion on center of mass from the previous lecture notes), and taking the derivative of this relation, try to find a relation between v_p and v_*.

19. Use this to find m_p expressed in terms of v_*, m_* and P.

20. How can you find v_* from observations? Can you always find v_* from observations? (figure 2 in the lecture notes may help here)

21. If the inclination is unknown, can you still say something about m_P?

22. Discuss how you would solve exercise 1C.1, questions 1, 2 and 3.

23. Discuss how you would solve exercise 1C.2, questions 1 and 3

24. Discuss how you would solve 1C.3

25. The velocity curve which you used in exercise 1C.3 is noisy. In order to find correct values from a noisy curve, we will need to fit a model of the velocity curve to the noisy data. We will look at this in detail later, but let's first find an equation for the velocity curve which will serve as our model. We will in the following assume an inclination of 90° and circular orbits. The situation is shown in figure 5. The observed velocity is the velocity along the line of sight, v_{*r} whereas the full velocity v_* is constant due to the circular orbit. Convince yourself that we can write the observed velocity curve as

$$v_{*r}(t) = v_* \cos\theta(t)$$

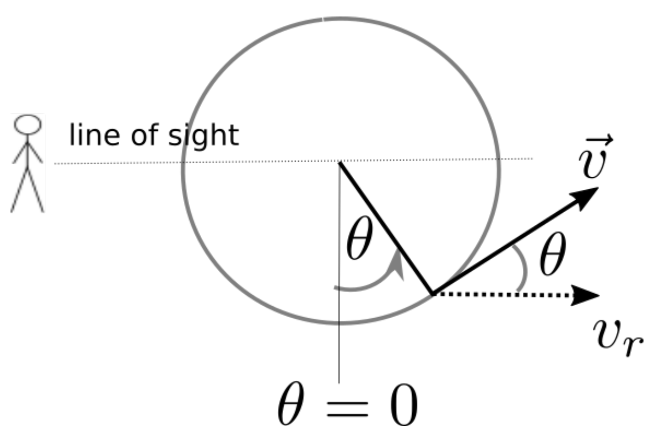

Figure 5: Figure used to obtain the model for the observed velocity.

26. Show that the time variation of the angle θ is given by

$$\theta(t) = \theta_0 + \frac{2\pi}{P}t$$

where θ_0 is the angle when $t = 0$. Hint: Since the orbit is circular, the full velocity equals the tangential velocity which we found to be $v_* = r\dot{\theta}$ in the lectures on celectial mechanics (remember how we arrived at this? If not, look back, you should be able to deduce this for the exam). Write $\dot{\theta} = d\theta/dt$.

27. We rewrite θ_0 in terms of another variable t_0:

$$v_*(t) = v_{*r}^B \cos\frac{2\pi}{P}(t - t_0)$$

where v_{*r}^B is the observed velocity at point B in figure 3 in the lecture notes (why at this point???) What is the interpretation of t_0?

28. Figure 5 and 9 in the lecture notes are examples of how observed velocity curves look like. We have learned that in order to estimate the planet mass. we need to find the velocity v_{*r}^B as well as the period. But the noise in these curves makes it difficult to find exact values for v_{*r}^B and P from the data. Having obtained a model for the observed curve $v_{*r}(t)$, and having access to observed data $v_{*r}^{\text{obs}}(t_i)$ for all N observations $i = [1, 2, 3, ..., N]$, how would you find the best possible values for v_{*r}^B and P numerically?

29. The noise in the curves can often be approximated as Gaussian noise. At one given observation i taken at time t_i, we can write the observed velocity as

$$v_{*i}^{\text{obs}} = v_{*i}^{\text{real}} + \delta v_i$$

That the noise is Gaussian would mean that the number δv_i is a random number drawn from a Gaussian distribution. The mean value of this Gaussian distribution is normally 0. We denote the standard deviation of the noise by σ_n. Write an expression for the distribution function $P(\delta v_i)$, i.e. the probability that the noise δv_i for observation i has a given value δv_i.

30. Figure 6 shows a velocity curve of a star observed with two different telescopes with different quality of the detectors. Roughly what is σ_n for each of these two sets of observations?

31. The second part of the data set has considerably worse quality than the first part. When making a least squares fit to this curve, it does not take into account the fact that different parts of the data has different quality. Could you suggest some small modification to the method of least squares in order to take this into account?

32. Assume now that every single observation i has a different standard deviation of the noise denoted by σ_i. Write an expression for the probability distribution $P(\delta v_0, \delta v_1, ..., \delta v_N)$ for the noise in all observations to have the values $\delta v_0, \delta v_1, ..., \delta v_N$. Hint: law of multiplication of probabilities.

33. Rewrite δv_i in terms of v_{i*}^{obs} and v_{i*}^{real}. Hint: there is an equation a few lines above...

34. Can you see where the method of least squares and the method of χ^2 is coming from? How can you interpret the least squares or least χ^2 in statistics? (what does it actually mean to find the least square).

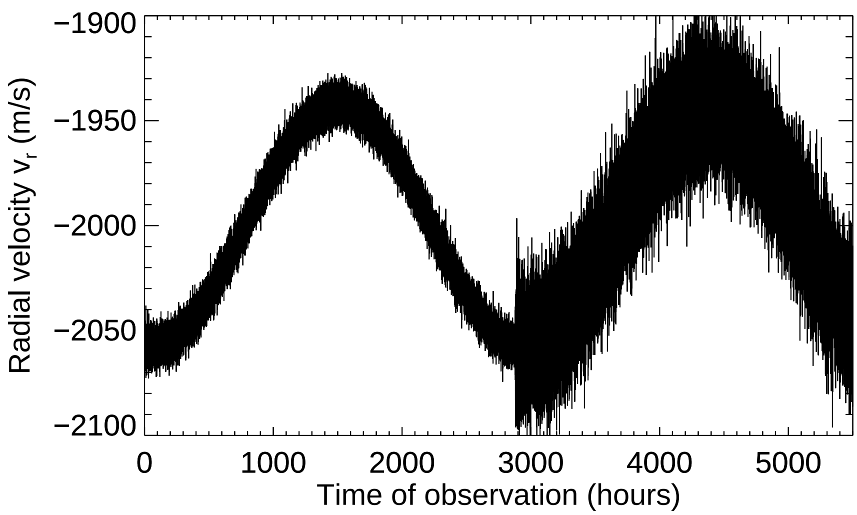

Figure 6: Velocity observations of a star with time-varying noise properties.