Questions to discuss during the lectures for part 1A

Important: You also need the lecture notes for part 1A to read together with this guide.
Also note: The lecture guides are made for use during the lectures. They will not make much sense if you are not present at the lecture, in this case, go directly to the lecture notes.

1. Read the question in the second paragraph of exercise 1A.6 (only until before the enumerated questions begin) as well as figure 5 in part 1A. Then discuss the following questions (they are all related so they should be discussed together):

    - How does this rocket engine work, i.e. exactly what is it that makes it move forward?
    - Exactly which physical properties of the particles in the engine does the engine's acceleration depend on?
    - Very roughly, how would you go about to calculate the acceleration of this engine? (step-by-step)

2. What/which properties of the gas determines the speed of the particles in the gas?

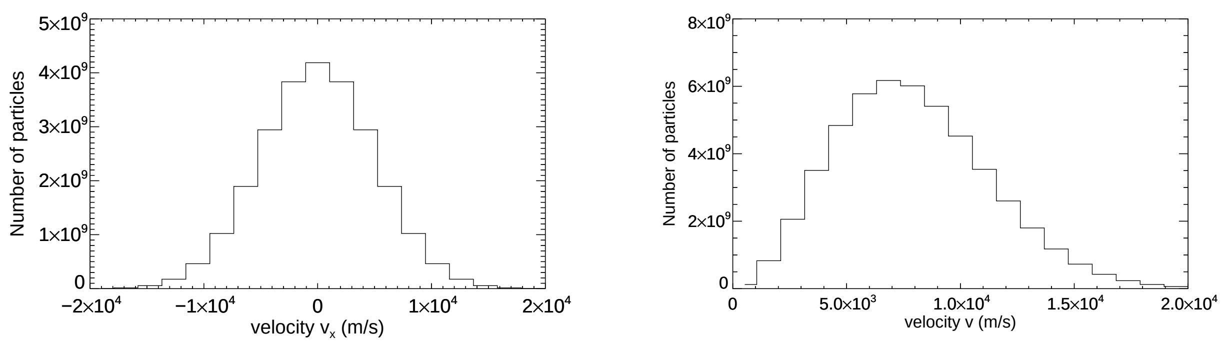

Figure 1: Histogram for the x-component of the velocities (left plot) and the full velocities (right plot) of gas particles

3. In figure 1 you can see a histogram of the particle velocities, the x-component v_x of the velocity and the full velocity v. These are taken from the Maxwell-Boltzmann distribution function for the velocitiy component along an axis and the Maxwell-Boltzmann distribution function for the full velocity.

    - How do you think the histograms for the y- and z-components of the velocity, v_y and v_z, would look like? Justify you answer.
    - What are the main differences between these two histograms?
    - Why is one symmetric while the other one is not?

- Discuss how you think the histograms for a gas at higher temperature and a gas at lower temperature will look like and draw these on top of the figures.

4. The figures show the histograms with limited bin sizes Δv which means that you can only find the number of particles which have a certain range of velocities within Δv, not one exact velocity. If you shrink these bins, would it be possible to find a distribution of velocities $P(v)$ which gives the number of particles with an exact given velocity v? (is this possible, yes or no? why/why not?)

5. Do the Maxwell-Boltzmann distribution for v_x,

$$P(v_x) = \sqrt{\frac{m}{2\pi kT}} e^{-\frac{1}{2}\frac{mv_x^2}{kT}}$$

seem to represent the left plot in figure 1, and does the Maxwell-Boltzmann distribution for v on page 7 in the lecture notes seem to represent the right plot correctly? (remember that the histograms have been integrated over bins Δv) Give arguments: why/why not?

6. How would you use these expressions to solve the first question in exercise 1A.4? (you are not supposed to solve the exercise now, but suggest how to solve it). (Project students have a similar question in challenge A2 in part 1).

7. Given a gas with N particles where you know the full velocity v_i for all particles $i = [1, N]$. How would you go about to find the mean velocity?

8. Given that you only know $P(v)$, how could you go about to find the mean velocity? Use your answer to the previous question combined with the histograms to see if you can find a way.

9. Discuss how you can solve exercise 1A.5, question 1 and 3. (the project student has a similar question in challenge A3, part 1)

10. In figure 2 we show the Maxwell-Boltmann distribution $P(v_x)$ for v_x. What is the mean value of v_x? Can you give an interpretation of the width of the distribution? What would it mean if the distribution was wider/narrower?

11. The *variance* of a distribution $P(v)$ is given as

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(v_i - \langle v \rangle)^2,$$

and the *standard deviation* σ is the square root of the variance. Looking at the expression, can you give an interpretation of the physical meaning of σ?

12. Discuss:

    - do you know what a Gaussian distribution (or normal distribution) is and how it looks like?
    - Without looking it up, make a guess for how the function looks like? Suppose it is a function of a variable v and that the mean of v is μ and the standard deviation is given by σ. Your function should contain all these.

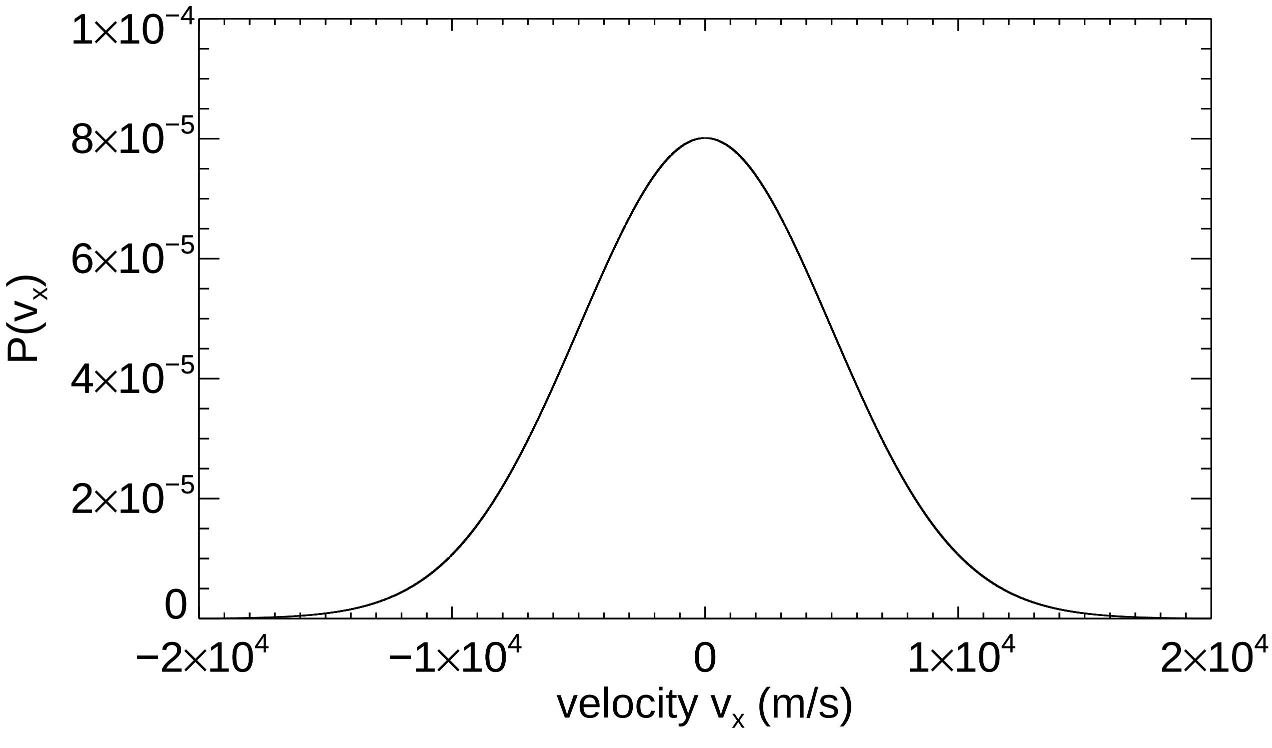

Figure 2: The Maxwell-Boltzmann distribution for the x-component of the velocities of gas particles

- Is $P(v_x)$ and/or $P(v)$ a Gaussian?

13. How would you test the 68-95-99.7 rule?

14. Discuss how you would solve the first two questions of exercise 1A.1. (only discuss!)

15. Look at exercise 1A.2. Discuss how you would implement the code to make a histogram.

16. Assume that $P(h)$ is the probability distribution for the height of a person and $P(IQ)$ is the probability distribution for the IQ of a person. Suppose that these are Gaussian distributed. Using these distributions, how would you calculate the probability that a given person has both height **and** IQ above the average (assuming there is no correlation between being tall and being intelligent). If you know some statistical rule to solve this, forget this rule for the moment, just use pure reasoning.

17. Use the multiplication rules for probability to

    - find an expression for the probability $P(\vec{v})$ for a particle in the gas to have a given velocity vector $\vec{v}$ (or a velocity in the vicinity of $\vec{v}$).
    - discuss how to solve the last question of exercise 1A.4 (only discuss!)
    - discuss how you would solve the last two questions of exercise 1A.1

18. Discuss exercise 1A.6, question 2 (in the project you will need to solve this as well): how would you generate the random position and velocity of a particle in a gas with a given temperature T and particle mass m.

19. Discuss again how you would simulate the gas box.