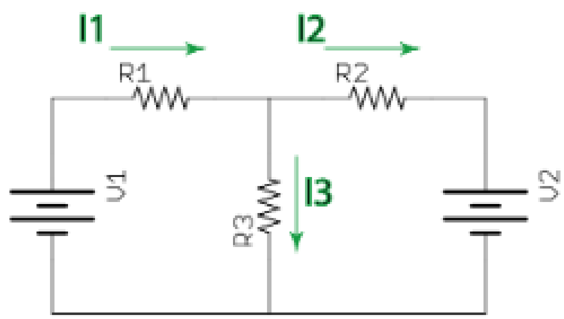
Exercise 1: Kirchoff's circuit laws

In this exercise we are going to look at Kirchoff's laws, and solve problems using linear algebra.

a) Write down Kirchoff's laws.

Solution. 1. The sum of voltages around any loop is zero.
2. The sum of currents at any junction is zero.

b) The voltages are $U_1 = 10\text{V}$ and $U_2 = 5\text{V}$. The resistances are $R_1 = 10\Omega$, $R_2 = 20\Omega$ and $R_3 = 30\Omega$. Using Kirchoff's laws, find the equations needed to solve for the currents. Write it as a matrix equation and use Python to inverse the matrix and find the solution.



Solution. Using the first law on the left-hand loop we get

$$U_1 - I_1 R_1 - I_3 R_3 = 0 \tag{1}$$

Right-hand loop gives

$$U_2 + I_2 R_2 - I_3 R_3 = 0 \tag{2}$$

Using the second law on the top center junction, we get

$$I_1 - I_2 - I_3 = 0 \tag{3}$$

We can write the equations as a matrix equation on the form

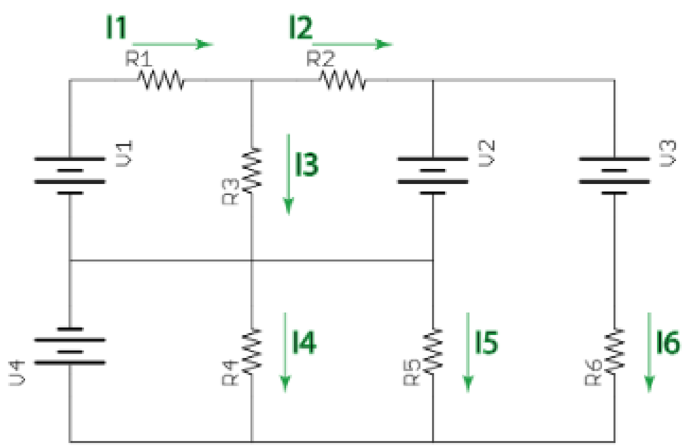
$$\begin{pmatrix} R_1 & 0 & R_3 \\ 0 & -R_2 & R_3 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} U_1 \\ U_2 \\ 0 \end{pmatrix} \tag{4}$$

Using the following code

```
import numpy as np
R1 = 10  # [ohm]
R2 = 20  # [ohm]
R3 = 30  # [ohm]
U1 = 10  # [V]
U2 = 5   # [V]
A = np.array([[R1, 0, R3],[0, -R2, R3],[1, -1, -1]])
#inverting the matrix A
Ainv = np.linalg.inv(A)
B = np.array([10, 5, 0])
#finding the solution
C = np.dot(Ainv,B)
print(C)
```

We found the solutions $I_1 \approx 0.318$A, $I_2 \approx 0.091$A and $I_3 \approx 0.227$A.

c) Do the same for the circuit under as you did for the previous exersice. Use $R_4 = 40\Omega$, $R_5 = 50\Omega$, $R_6 = 60\Omega$, $U_3 = 12$V and $U_4 = 24$V.



Solution. Notice that we can reuse the equations over, so we only need to find three more equations. Using the first law on the bottom left loop we get

$$U_4 + I_4 R_4 = 0 \tag{5}$$

Bottom center loop

$$I_4 R_4 - I_5 R_5 = 0 \tag{6}$$

Right loop

$$U_2 - U_3 - I_6 R_6 + I_5 R_5 = 0 \tag{7}$$

We therefore get the matrix equation

$$\begin{pmatrix} R_1 & 0 & R_3 & 0 & 0 & 0 \\ 0 & -R_2 & R_3 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -R_4 & 0 & 0 \\ 0 & 0 & 0 & R_4 & -R_5 & 0 \\ 0 & 0 & 0 & 0 & -R_5 & R_6 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix} = \begin{pmatrix} U_1 \\ U_2 \\ 0 \\ U_4 \\ 0 \\ U_2 - U_3 \end{pmatrix} \tag{8}$$

Using the code

```
import numpy as np
R1 = 10  # [ohm]
R2 = 20  # [ohm]
R3 = 30  # [ohm]
R4 = 40  # [ohm]
R5 = 50  # [ohm]
R6 = 60  # [ohm]
U1 = 10  # [V]
U2 = 5   # [V]
U3 = 12  # [V]
U4 = 24   # [V]
A = np.array([[R1, 0, R3, 0, 0, 0],
              [0, -R2, R3, 0, 0, 0],
              [1, -1, -1, 0, 0, 0],
              [0, 0, 0,-R4, 0, 0],
              [0, 0, 0, R4, -R5, 0],
              [0, 0, 0, 0, -R5, R6]])
#inverting the matrix A
Ainv = np.linalg.inv(A)
B = np.array([U1, U2, 0, U4, 0, U2-U3])
#finding the solution
C = np.dot(Ainv, B)
print(C)
```

We get the solutions $I_1 \approx 0.318\text{A}$, $I_2 \approx 0.091\text{A}$, $I_3 \approx 0.227\text{A}$, $I_4 = -0.6\text{A}$, $I_5 = -0.48\text{A}$ and $I_6 \approx -0.517\text{A}$.