Electromagnetic braking

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Jul 28, 2020

Many of you may be familiar with the experiment of dropping a magnet through a non-magnetic, conductive metal pipe. In this exercise we are going to study the unexpected motion of the falling magnet.

1 Equipment list

Make sure you have all the equipment needed to do the experiment.

- Aluminum foil tube
- Magnets
- Scale
- Stopwatch (Smartphone)
- Tape-measure or ruler

2 Understanding the experiment

Imagine dropping a cylindrical magnet of mass m, radius r and height l vertically through the center of a non-magnetic, conductive metal ring. In Figure 1, the motion of the magnet is depicted at three different positions.

(a) The magnet is above the ring.
(b) The magnet is at the center of the ring.
(c) The magnet is below the ring.

For each of the positions a, b and c answer the following questions:

- Is current being induced in the ring? Why/why not?
- What is the direction of any induced current? (Use Lenz' law)
- How is the movement of the magnet affected and why?

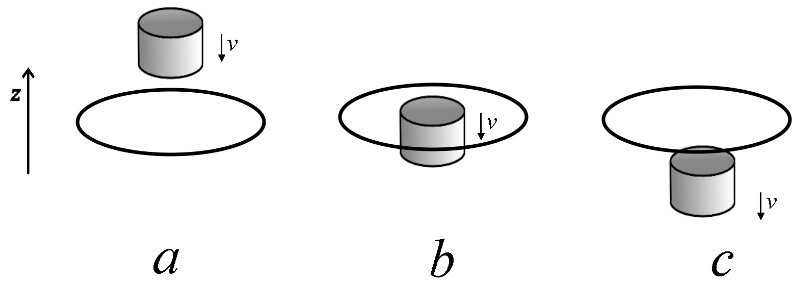

Figure 1: Three different positions of the magnet falling through the ring.

(Solution) As we release the magnet, the magnetic flux in the ring begins to change, and by Faraday's law we know that this induces an electric current in the ring. By Biot-Savart's law this current must itself induce a magnetic field along the z-axis, and by Lenz' law the magnetic field must be aligned in such a way as to oppose the motion of the magnet. When the magnet is moving towards the ring (a), the induced field repels the magnet. If the magnet is in the center of the ring (b), the momentary flux change is zero and there is no effect on the magnet. When the magnet is moving away from the ring (c), the induced field must attract the magnet. This tells us that throughout the entire fall of the magnet (point b is just momentarily and is effectively irrelevant), the ring acts as a brake on the magnet, de-accelerating its motion.

3 Expanding the model

Now, a single metallic ring is obviously different from a lengthy metal pipe. If we wish to study the experiment at hand, we must therefore extend our model. In order to keep the theory simple while also building on our previous discussions, we will model the pipe as a series of N stacked rings, as shown in Figure 2. The length of the pipe is L, and you may assume $l \ll L$.

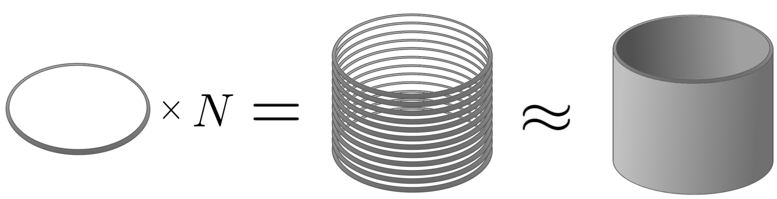

Figure 2: Approximation of solid metal cylinder as stack of metal rings. The number of rings N is arbitrarily large.

Ignore any mutual induction between the "rings". Knowing what you know from the last section, qualitatively answer the following exercises:

- Describe the motion of the magnet inside the pipe
- Letting $l \ll L$ is, to some extent, effectively the same as letting $L \to \infty$. Explain why this leads to a symmetry along the z-axis that means that the force F_M cannot depend on the vertical position of the magnet.
- Why may it be a reasonable approximation to assume the magnetic force on the magnet to be proportional to its velocity? In other words argue that a magnetic force on the form $\mathbf{F_M} = k|\mathbf{v}|\hat{\mathbf{z}}$, for some constant $k > 0$, is not completely unreasonable.
- Show that this means that the velocity *must* terminate in some terminal velocity v_T. Like any sane physicist; neglect air resistance. (you may use some math here).

(Solution) From the study of the magnet falling through one ring, we know that, at any point inside the pipe, the magnet will be attracted to all the "rings" above it, and repelled by all the "rings" below. This means that the magnet will be decelerated by the pipe. In reality F_M depends on the distance from each "ring" to the magnet, but if we make sure that the height of the magnet l is much smaller than the length L of the pipe, $l \ll L$, we may pretend that $L \to \infty$, which grants symmetry along the z-axis. By "symmetry along the z-axis" we mean that if the pipe is infinitely long, any position on the z-axis must result in the same F_M as any other position (there is nothing distinguishing the two situations). By this, F_M can't depend on the distance to any "ring", as it would not be symmetric.
An increase in velocity would mean a greater flux change through the rings, which by Lenz' law means the ring would induce a stronger magnetic field to fight the stronger flux change. Meanwhile a lower velocity would mean the rings wouldn't have to fight as hard against the flux change. At 0 velocity, there would of course not be any flux change, and no induced magnetic field, ergo $F_M = 0$. Due to this behavior, and the above reasoning that F_M shouldn't depend on vertical position, it is not unreasonable to model as a proportionality $F_M = kv$. Because $\mathbf{F_M}$ must point towards positive z, we must have $k > 0$.
The forces acting on the magnet in the pipe are only the magnetic force, $\mathbf{F_M} = k|\mathbf{v}|\hat{\mathbf{z}}$, and gravity, $\mathbf{G} = -mg\hat{\mathbf{z}}$. As the velocity changes, eventually the magnetic- and gravitational force will cancel each other out:

$$\Sigma F = 0 = F_M + G = kv_T - mg \Rightarrow v_T = \frac{mg}{k} \tag{1}$$

In our experiments we may therefore expect the velocity of the magnet to be constant when falling through the pipe.

4 Experimenting

Hold the pipe vertically, centering the magnet above and as close to the center of the pipe as possible. Carefully drop the magnet (make sure the magnet doesn't hit the floor, as they are rather brittle!). What do you observe? Are your results as expected from the predictions of the last section? Compare the motion of the magnet to a control-test where you drop the magnet without the pipe present.

We will now attempt to measure the terminal velocity v_T. We may assume that the magnet reaches its terminal velocity rather quickly (this can qualitatively be observed as well), and that the velocity is constantly v_T through the entire pipe. Using a stopwatch (or, favorably, come up with another way of measuring v_T on your own!), do (at least) 5 measurements of the time t it takes the magnet to fall through the entire length of the pipe L. Take the average of the measured times, and calculate v_T. This process is shown in Figure 3.

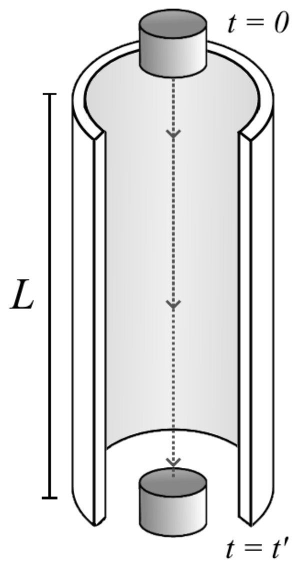

Figure 3: Calculating v_T by dropping the magnet through the pipe.

Measure the mass of the magnet[1] m. Using (1), calculate k. Briefly comment on its value. Do you think your value for k is very accurate? What can be done to increase its accuracy?

5 Measuring the resistivity of aluminum (optional)

Provided as extra material for those of you that are curious you may find that k is approximately given by

$$k \approx \frac{45\pi^2 B_0^2 r^4 w \left(l^2 + r^2\right)}{256 \rho R^4},$$

where B_0 is the magnetic field measured at the center of the magnet surface, R and w are, respectively, the radius and width of the metal pipe. (To measure

[1]In some cases the magnet will attract metallic components within the scale, affecting the measurement. To avoid his, place something between the magnet and the scale.

R take the average of the inner and outer radii of the pipe). Lastly, ρ is the conductivity of the pipe (aluminum), which is what we wish to estimate.

There are several phone apps that lets you use your phones built-in magnetic field sensor. For both iOS and Android we recommend using "Science Journal", by Google. Use your phone to measure B_0,[2] and use this along with your experimental value of v_T to estimate ρ.

Using the internet, look up the actual value of ρ for aluminum. Are your values close? You should be *very* happy if your answer is even within the correct power of ten, as we have done many approximations and estimations to get here. If you wanted a more accurate measurement of ρ, how would you tune the experiment? What do you think is our biggest source of error?

[2]It may be smart to research a little where within your phone the magnetic sensor is located so that you get an accurate reading on the top of the magnet. This may be done by googling something akin to "[phone model] magnetic sensor location", but on most phones it is located on the upper part of the phone, closer to the corners. You may have to spend some time tinkering with the position of the magnet relative to the phone, but ultimately B_0 will be the largest value you manage to read. The magnet is not strong enough to damage the electronics within your phone. However, as a precaution, please distance any magnetic cards like student ID's or bank cards while measuring B_0.