Exercise 1: Force between two moving particles

Here we are going to visualize the magnetic force between two moving particles, in 3D. First we need to find the magnetic force between the two particles.

a) Write down the equation the for magnetic force between two moving particles.

Solution.

$$\mathbf{F}_{\text{from1on2}} = \frac{\mu_0}{4\pi}\frac{Q_2\mathbf{v}_2 \times \left(Q_1\mathbf{v}_1 \times \mathbf{R}\right)}{R^3} \tag{1}$$

b) Write a function (in Python or MatLab) that takes in the positions of the particles, their charges and their velocities, and outputs the magnetic force from one of the particles on the other.

Solution.

```
def Magnetic_force(r1, r2, v1, v2, Q1, Q2):
    #Returns the magnetic force from between two moving particles
    r = r2 - r1
    R = np.linalg.norm(r)
    part1 = mu0/(4*np.pi*R**2)*Q2*v2
    part2 = np.cross(Q1*v1,r/R)
    F = np.cross(part1,part2)
    return F
```

Note that you have to import NumPy and assign the value of the permeability of vacuum in order for this to work.

c) Plot the velocity vectors for both particles and the force applied on one of the particles. Try for different initial conditions. Do you get what you expected?

Hint 1. To get reasonable sizes use a scaled version for the marmeability of vacuum.

Hint 2. Reasonable initial conditions are: $\mu_0 = 1$, $Q_1 = 1$, $Q_2 = 2$, $\mathbf{v}_1 = [0, -4, 0]$, $\mathbf{v}_2 = [0, 4, 0]$, $\mathbf{r}_1 = [0, 0, 0]$ and $\mathbf{r}_2 = [1, 0, 0]$.

Solution.

```
#Complicated initial conditions
# v1_0 = np.array([1,2,3]) #initial velocity of particle 1
# v2_0 = np.array([-1,-5,5]) #initial velocity of particle 2
# r1 = np.array([0,0,0]) #initial position of particle 1
# r2 = np.array([1,0,0]) #initial position of particle 2
#Easy initial conditions
v1_0 = np.array([0, -4, 0])  # initial velocity of particle 1
v2_0 = np.array([0, 4, 0]) #initial velocity of particle 2
```

```
r1 = np.array([0, 0, 0]) #initial position of particle 1
r2 = np.array([1, 0, 0]) #initial position of particle 2
Q1 = 1 #charge of particle 1
Q2 = 2 #charge of particle 2
#calculate the magnetic force between particle 1 and 2
F12 = Magnetic_force(r1, r2, v1_0, v2_0, Q1, Q2)
#The data we are going to plot
positions = np.array([r1,r2,r2])
vectors = np.array([v1_0, v2_0, F12])
#\labels and colors we are going to use
\labels = ['Velocity particle 1',
          'Velocity particle 2',
          'Magnetic force on particle 2']
colors = ['r', 'g', 'b']
#creating the figure we are going to use
fig = plt.figure()
ax = fig.add_subplot(111, projection = '3d')
#Plotting velocity and force vectors
for i in range(3):
    ax.quiver(positions[i, 0], positions[i, 1], positions[i, 2],
              vectors[i, 0], vectors[i, 1], vectors[i, 2],
              color=colors[i], \label=\labels[i])
#Plotting the postions of the particles
ax.scatter(r1[0], r1[1], r1[2], color='r', \label='Position particle 1')
ax.scatter(r2[0], r2[1], r2[2], color='g', \label='Position particle 2')
#setting the axis
ax.set_xlim([-5, 5])
ax.set_ylim([-5, 5])
ax.set_zlim([-5, 5])
#\labeling the axis
ax.set_x\label('x-axis')
ax.set_y\label('y-axis')
ax.set_z\label('z-axis')
#showing the plot
ax.legend()
plt.show()
```