Exercise 1: Field from a hemisphere shell

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In this exercise we are going to calculate the field from a hemisphere shell (see figure 1). The shell has constant charge density ρ and radius R.

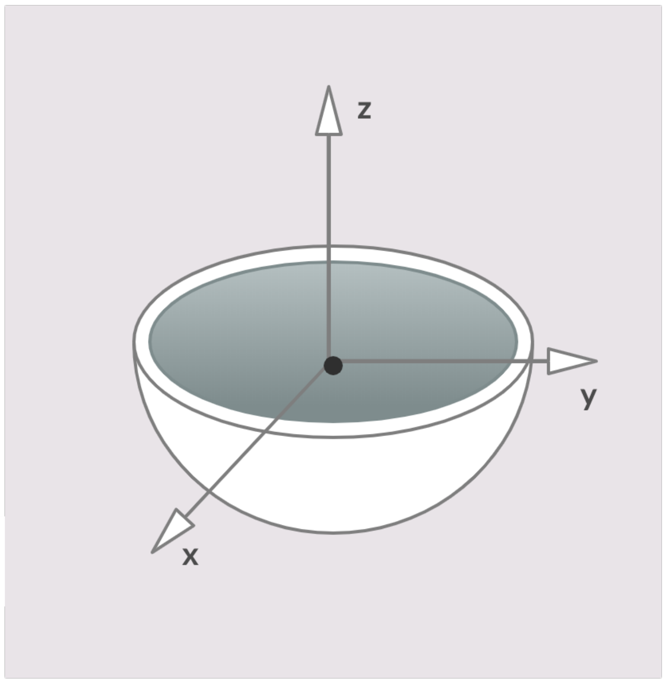

Figure 1: Here you see the hemisphere shell. The origin is equidistant to the hemisphere and the hemisphere is oriented such that it has rotational symmetry along the z-axis.

a) Calculate the field in the origin.

Solution. Electric field from a small surface element is given by

$$d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{\rho}{R^2}\hat{r} \tag{1}$$

Where $\hat{r}$ is a unit vector pointing from the surface element to the origin. Notice that because the origin is equidistant to the hemisphere, the distance from any surface element to the origin is the same. Because of symmetry we only need to look at the component in z-direction (any x- and y-component cancel out).

Therefore we get the following expression:

$$dE_z = d\mathbf{E} \cdot \hat{z} = \frac{1}{4\pi\epsilon_0}\frac{\rho}{R^2}\hat{r} \cdot \hat{z} = \frac{1}{4\pi\epsilon_0}\frac{\rho}{R^2}cos(\theta) \tag{2}$$

Make sure you understand the last transition, if not; express $\hat{r}$ in spherical coordinates and then carry out the dot product. Because we are working with a spherical shape it can be a good idea to transition to spherical coordinates when we do the surface integral. Substituting $dS = R^2 sin(\theta)d\phi d\theta$ we get the following integral:

$$E_z = \frac{\rho}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{\pi/2} sin(\theta)cos(\theta)d\theta d\phi = \frac{\rho}{4\epsilon_0} \tag{3}$$

Therefore we get the Field

$$\mathbf{E}_z = \frac{\rho}{4\epsilon_0}\hat{z} \tag{4}$$