### IT UNIVERSITY OF COPENHAGEN

# Clustering Player Behaviors in Data Streams using K-means in MapReduce

by

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in the Computer Science Software Development and Technology

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"Any intelligent fool can make things bigger, more complex, and more violent. It takes a touch of genius – and a lot of courage – to move in the opposite direction."

Albert Einstein

## Abstract

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# Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor...

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# Abbreviations

LAH List Abbreviations Here

## Chapter 1

## Introduction

### 1.1 Motivation

We live in a world where data is being generated at an amazing rate everywhere around us. Data can describe characteristics of e.g. Internet activities, social interactions, user behaviors in games, mobile phone activities, scientific experiments and measurements from different devices and sensor equipments. Amount of data that is being registered and stored around us is growing massively in volume and complexity. Organizations are storing more and more historic data than before, moving from large databases storage towards more commonly online distributed file systems called cloud storage where petabytes ( $10^{15} = 1.000 \ terabytes$ ) of information can be stored. We live in a world of  $Big\ Data$ , a journey exploring unknown patterns and structures without knowing where it will lead us in the future.

"Information is the oil of the 21st century, and analytics is the combustion engine."

Peter Sondergaard, senior VP at Gartner

In digital games, data about in-game user interactions have been logged and behaviors analyzed since the first game came out. Analyzing the user experience and behaviors of players have mostly been done in laboratories in the past, both during game development and after game launch to see if the game was played as designed. Game designs

have becoming increasingly complex in the recent years offering much more freedom to the players by increasing the number of actions available, items to interact with and online massive multi-player persistent worlds that continue to exist after a player exits a game [1, 2]. This complexity generates much more user-centric data than before and is increasingly challenging when evaluating game designs [3, 4]. The user interactions being registered is called *user telemetry* and is translated to *game metrics* as referred in game development, providing detailed and objective numbers, e.g. total playtime, monsters killed, puzzles solved.

Collecting user's telemetry can give very detailed quantitative information on player behavior and using data mining techniques can supplement traditional qualitative approaches with large-scale behavioral analysis [5], for example show where users are getting stuck and finding actionable behavioral profiles [1, 2, 6]. In the recent years user behavior analysis have in part been driven by the emergence of massive multi-player on-line (MMO) games and Free-to-Play (F2P) games which can have millions of users and objects that can form highly complex interactions. These game models, especially of persistent nature, are constantly monitoring users actions and their behaviors by driving their revenue with subscriptions or offer players to buy virtual items via micro transactions [1, 2, 4, 7].

One way of doing a behavioral analysis is use an unsupervised machine learning technique called clustering. Cluster analysis is a popular exploratory data mining technique that groups set of data objects together in a cluster that are more similar to each other than data objects in other groups [8]. Human beings categorizes or classifies a new object or a phenomenon based on similarity or dissimilarity of the object's descriptive features and is one of most primitive activies of humans [9]. Clustering explores the unknown patterns of the data and provide compressed data representation for large-scale data. In computer games cluster analysis or behavioral categorization can find behavioral profiles that are actionable and give high valuable insights into the game development as well as increasing the monetization [10, 11].

Most clustering algorithms are designed for modern sizes of datasets where the whole data can fit into memory or allows few passes into a database (where each data object is read more than once). It can be very expensive analyzing large-scale datasets and to

get answers efficiently then one needs to reduce the set of data to be analyzed, e.g. sample fewer players and have fewer features (dimensions) to be compared. Computations for large-scale data takes time and needs to be distributed to be able to complete in reasonable amount of time. Google's MapReduce programming model was introduced in 2004 [12] and allows automatic parallelization and distribution of computations on large clusters of commodity computers. Allowing programmers and researchers to easily implement highly scalable algorithms to process large amount of data using the MapReduce model without worrying about handling failures and distributing the data with a large amount of complex code.

### 1.2 Problem Statement

How can clustering using incremental k-means find general player behaviors in largescale behavioral game data in reasonable time?

Considering the massive size of user telemetry data being logged and processed, and the complexity of game designs. There is a knowledge gap when it comes to analyzing such large-scale data efficiently. Number of players are increasing and the complexity of player-game and player-player interactions grows exponentially. One of the largest massively multiplayer online role-playing game (MMORPG) World of Warcraft has a population of around 12 million users where players live in a persistent world that can create many millions of different interactions in the game.

User telemetry from games can arrive in daily chunks and need to be processed incrementally (in mini batches). There is a need for algorithms that can process massive amount of data that doesn't fit in a computer memory to extracts knowledge in a reasonable time. The k-means algorithm can find clusters that represent general game behaviors. When implemented in the MapReduce framework, k-means can cluster the data in parallel and is highly scalable with running time increasing linearly with the size of the input.

Our goal with this project is to implement a scalable clustering algorithm to find the general behaviors in a specific real life game dataset in collaboration with GameAnalytics (GA) [13]. The goal is not to implement a complete product but a scalable algorithm that provides information about the general player behavioral profiles for a specific game.

Also the algorithm should allow GA to easily develop a product that can cluster general player behaviors in large-scale games.

The success criteria of this project is:

- A scalable k-means clustering algorithm finding clusters describing the general behaviors of a real life game dataset provided by GameAnalytics.
- The general behaviors found must be intuitively interpretable and actionable to game developers.
- The algorithm must be able to process incrementally cluster daily arriving chunks of game metric data.

GameAnalytics is Software as a Service (SaaS) start-up, a data and analytics engine for game studios with its headquarters located in Copenhagen. Analyzing large quantities of game metric data that needs to be processed efficiently returning actionable results to aid game design and development.

### 1.3 Method

TODO A short description of the methods

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### 1.4 Contributions

TODO Description of the contributions made in the project

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### 1.5 Project Outline

References are cited by index in the bibliography and are in order of appearance, e.g. [2] is a citation number two that is referenced in the thesis. Referring to other sections is by the number of that section, e.g. 4.1.2.

The organization of the thesis is as follows:

- Chapter 2 Background Describes a short background theory about clustering player behaviors and the MapReduce framework for large-scale data parallel processing.
- Chapter 3 Related Work Overview of related and recent work regarding clustering player behaviors and large-scale data with k-means as focus.
- Chapter 4 Methodology Our design and implementation work is described; Description of the real game dataset and selection of features, the k-means algorithm in MapReduce and the experimental set-up.
- Chapter 5 Results Results and observations from experiments are explained.
- Chapter 6 Conclusions Conclusions are drawn from the study including future research.

## Chapter 2

# **Background Theory**

### 2.1 Player Behavior Profiles

TODO Describe player behavior

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#### 2.1.1 Game Metric

TODO Describe User Telemetry and Game Metric. Features and behavioral variables

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## 2.2 Clustering

The goal of clustering is to categories or groups similar objects together into so called clusters (hidden data structures) while different objects belong to other clusters. A cluster are set of objects that are similar to each other, while objects in different clusters are dissimilar to each other. Identifying descriptive features of an object one can compare these features to a known object based on their similarity or dissimilarity based on some criteria. Cluster analysis can be achieved by various algorithms and is a common technique in statistical data analysis that is used in many fields, e.g. machine learning, pattern recognition, image analysis and bioinformatics. In this thesis we focus on a popular clustering algorithm called k-means that is a centroid based clustering algorithm where a cluster is represented by a central vector called centroid.

### 2.2.1 K-means clustering method

Many clustering methods exists but one of the most popular ones is called k-means [14, 15], also known as the Lloyd algorithm [16] which was further generalized for vector quantization [17]. K-means seeks to group similar data points into k partitions or hyperspherical clusters using a distance measure and giving insights into the general distributions in the dataset. A cluster is represented by a centroid that characterize the geometric center of the cluster that is calculated as the mean of all data instances belonging to that cluster. The objective function in k-means is to minimizes the squared error between a cluster's centroid (mean) and its assigned points and over all set of clusters minimizing the Sum of Squared Error (SSE). Let a set of data points  $x_i \in \Re^d$ , i = 1, ..., N, where each point x is a real number d-dimensional vector and we want to partition them into K clusters  $C = \{c_1, ..., c_K\} \in \Re^d$ , then the objective function is defined as

$$SSE = \sum_{k=1}^{K} \sum_{x_i \in c_k} \|x_i - \mu_k\|^2$$

where  $||x_i - \mu_k||$  is a chosen distance measure between a data point  $x_i$  and its cluster centroid  $\mu_k$  (mean). The centroid is calculated as

$$\mu_k = \frac{1}{n_k} \sum_{x_i \in c_k} x_i,$$

where  $c_k$  is a cluster number k and its corresponding  $n_k$  data points  $i=1,...,n_k$ . The algorithm for the k-means starts by initializing K centroids by choosing random data points from the dataset or according some heuristic procedure. In each iteration of the algorithm it assigns data points to it's nearest centroid by calculating the minimum distance to the K centroids for each instance. After assigning all the data points to clusters the centroids are updated so they represent the mean value of all the points in the corresponding cluster. The algorithm stops the iteration when the centroids do not change from the previous iteration or the error (SSE) is below some specific threshold. Stopping can also be done by predefine a maximum number of iterations to be run.

TODO INSERT PICTURE, showing some simple iterations...

One of the weaknesses of k-means is that it is sensitive of the initial selection of the centroids which can lead to local optimum, that is the algorithm converges and fails

to find the global optimum. One solution is to run the algorithm n times and pick the initialization that gave the lowest SSE result. Another weakness of k-means is that a user has to predefine the number of centers k-means need to cluster, this is most often not known in advance. Many methods exist and most popular one is to run the algorithm with by increasing the k number of clusters to some K and pick a good k candidate using the popular Scree plots [18]. Noise in data and outliers can dramatically increase the squared error and centroids shifting from data distribution in question towards outliers far away, thus representing skewed distributions. Solutions involve removing these noise in preprocessing or normalize the data with the zero mean normalization [8, 18].

The solution to the optimal partition can be found by checking all possibilities using a brute force method but that is a NP-hard problem [19] and cannot be solved in a reasonable time. The k-means algorithm is a heuristic approach for the clustering problem with running time of the algorithm O(NKdT) where T is number of iterations. Usually K, d and T is much less than N, k-means is good for clustering large-scale data because of approximately linear time complexity.

The above implementation of k-means is called the *batch* k-means, where the centroids are updated after all the data points have been assigned. The *online* (incremental) mode of the algorithm processes each data point sequentially. For each data point x the nearest cluster centroid  $c_{min}$  is calculated and that centroid is updated right away, defined as

$$c_{min}(old) = \underset{k}{\operatorname{argmin}} ||x - c_k||$$
$$c_{min}(new) = c_{min}(old) + \eta(x - c_{min}(old))$$

where the cluster centroid  $c_{min}$  is updated towards the data point x using the learning rate  $\eta$  which determines the adaptation speed to each data point. The online approach is however highly dependent on the order of which the data points are processed. A variant of this method is used when clustering an endless stream of data where data points arrive one at a time or in chunks.

#### 2.2.2 Player Behaviors

TODO Describe clustering player behaviors with focus using k-means

K-means algorithm have been shown to be very useful in behavioral analysis to give good insights in the general behaviors found in a game [CITE].

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### 2.3 MapReduce and Large-Scale Data

TODO Describe MapReduce and Large-Scale data

MapReduce is a programming model used in the Hadoop project.

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## Chapter 3

# Related Work

Clustering player behaviors and processing large data sets have been researched actively in the recent years. In our work we find the general player behaviors in a the case study using k-means clustering algorithm in the MapReduce framework for high scalability and parallel processing of large-scale data. The behavior of the data can change from day to day like when dealing with an endless stream of data is also discussed. In subsequent sections some of the recent and related work are given a short introduction.

### 3.1 Clustering Player Behaviors

Many researches have been done on clustering and predicting player behaviors over the last years to get a better understanding which kind of user behaviors are to be found when playing a game that can be actionable for game developers [2, 6, 20–22]. User behavior analysis has becoming increasingly popular in the recent years because of rise of the free-to-play (F2P) genre games in *Facebook* and *Google Play* where populations can be in millions creating complex game interactions [1, 2]. Playing these games are free and many are of persistent nature where the world in the game continues when a player exits. To be profitable these games drive their revenue via micro transactions, e.g. players buying upgrades or virtual items in game for real money [1, 2, 4, 7]. Major game publishers have also been collecting and analyzing large scale of behavior telemetry data but details of their methods are kept confidential [5, 23]. Most available research work

is case-based where a specific algorithm is applied to a specific game and commercial game data sets has only become accessible recently for academic researchers [5].

Predicting player behavior in a major commercial game Tomb Raider: Underworld (TRU) was presented in a study of Drachen et al. [10]. Authors classified 1365 players in a moderate data set into four user behavioral groups using six statistical gameplay features based on core game design as inputs of an emergent self-organizing map to identify dissimilar behavior clusters. Behavior profiles covering 90 percent of the users in the dataset were labeled in game terminology usable for game designers. Mahlmann et al. [11] did a follow up on the research using eight gameplay features and classified behavior of 10,000 players. The authors presented also how to predict behavior based on early play analysis, a popular topic which can be used to prevent churn (attrition) [7].

Analyzing social groups in the highly popular Massively Multiplayer Online Role-Playing Game (MMORPG) World of Warcraft was done by Thurau and Bauckhage [24]. They analyzed how groups (guilds) evolve over time from both American and European based guilds. Their paper is the first study analyzing such amount of data in a MMORPG, analyzing large-scale data gathered on-line from 18 million players belonging in 1.4 million groups over a period of 4 years. Convex-Hull Non Negative Matrix Factorization (CH-NMF) [22] technique was applied to the data to find the extremes rather than averages and the results show no significant cultural difference in formation processes of guilds from either the US or the EU. Interpretability of CH-NMF was more distinguishable and representing archetypal guilds than the more conventional clustering method k-means that represent the cluster centroids with similar characteristic.

Drachen et al. [6] did a clustering analysis for two major commercial games applied to large-scale of high-dimensionality player behavior telemetry. K-means and Simplex Volume Maximization (SIVM) clustering were applied to the MMORPG Tera and the multi-player first-person shooter strategy game Battlefield: Bad Company 2. SIVM clustering is an adaption of Archetype Analysis (AA) for large-scale data sets to find extreme player behaviors profiles [22, 25]. The authors show the contribution differences from the two algorithms where k-means gives insights into the general distribution of behaviors vs. SIVM showing players with extreme behaviors. The selection of the most important features from the data set were followed by a method suggested by Drachen

et al. [10], behavioral profiles were extracted and interpreted in terms of design language [10, 23].

In a recent study by Drachen et al. [26] the authors compare four different popular methods with purpose of clustering player behaviors and develop profiles from large-scale game metric data set from the highly popular commercial MMORPG World of Warcraft. The data set was collected from mining the Warcraft Realms site, recordings of on-line time and what level each player reached for each day in the years 2005-2010 for approx. 70 thousands of players. The authors selected playtime and leveling speed as their behavioral variables to show a measure of the overall player engagement in the game, where playtime is one of the most important measure for calculating the churn rate [4, 7]. Interpretable behaviors profiles where only generated by the k-means and the SIVM algorithm. The SIVM Archetype Analysis algorithm produces however significantly different behaviors that result in easier interpretation of behavior profiles compared to the k-means algorithm where the centroids are overall similar.

### 3.2 Clustering Large Data

K-means is one of the most studied clustering algorithm out there and is still actively researched. It's a simple algorithm that partition the data into k partitions by minimizing its objective function sum of squared error. From its appearance in a standard algorithm version by Stuart Lloyd in 1982 [16], it has been one of the most popular clustering algorithm to research because of its simplicity. There are many different research areas regarding k-means e.g. manually set the number of k partitions to cluster, initializing the centers of the partitions, dealing with outliers in data and scalability when dataset increases. In recent years k-means has also been very popular algorithm to study in the Map-Reduce framework where the algorithm can easily be applied to cluster large amount of data sets in parallel [12].

Guha et al. [27] designed an algorithm called STREAM that is based on the divideand-conquer strategy and achieves a constant-factor approximation solving the k-median problem (a k-means variant). The algorithm divides the dataset into m pieces of similar sizes. Each of the pieces are independently clustered sequentially and all the centers from all the pieces are then clustered further. They show a new k-median algorithm called LSEARCH that is used by the stream algorithm and is based on local search algorithm solving the facility location problem [28] to solve the k-median problem. Results show that LSEARCH produced better quality clusters than k-means and the hierarchical algorithm BIRCH [29] but took longer to run. In 2009 Ailon et al. [30] extended the the work of Guha et al. introducing an one pass streaming algorithm for k-means with approximation guarantees. Achieving that they introduced a new algorithm called k-means# that builds on the non-streaming algorithm k-means++ by Arthur and Vassilvitskii [31] that is a combined algorithm of seeding the initial centers and running k-means. k-means# provides a bi-criterion  $(\alpha, \beta)$  approximation algorithm by choosing  $\alpha * k$  centers with approximation factor  $\beta$ . In the divide-and-conquer strategy they run the k-means# independently on each piece of the data to achieve O(k \* log(k)) random centers non-uniformly and use the k-means++ algorithm to find k centers from the intermediate centers from all the pieces of the data set.

Another approach is using a coreset by selecting a weighted subset from the original data set such that by running any k-means algorithm on the subset will give near similar results to running k-means on the original data set. Ackermann et al. [32] introduce a new algorithm called StreamKM++ that uses k-means++ algorithm from Arthur et al. [31] to solve k-means on the subset and the also design a new data structure called coreset tree to speed up the time for the sampling in the center initialization. StreamKM++ is a streaming version of k-means++ to cluster large data sets. Their approach was shown to be on par with LSEARCH algorithm in cluster quality but outperformed BIRCH by factor of 2. A recent work by Shindler et al. [33] proposed an algorithm called Fast streaming k-means based on the online facility location algorithm [34] and extends the work of Braverman et al. [35] proving a faster running time and a better approximation factor. Their algorithm outperformed both work of Ailon et al. [30] and Ackermann et al. [32] mentioned above.

Clustering data streams of an unknown length, evolving over time [36, 37] are challenging where it is not possible to access historic data points because of the amount of data arriving continuously. Aggarwal et al. [38] proposed a well-known stream clustering framework called CluStream for clustering large evolving data streams and is guided by application-centered requirements. CluStream has an online component that maintains snapshots of statistical information about micro-clusters (a.k.a. cluster feature vector [29]) in a pyramidal time window and an offline component that uses the compact

intermediate summary statistics from the micro-clusters to find higher level k clusters using k-means, in a time horizon defined by an analyst. The authors proposed also a new high-dimensional, data stream clustering algorithm called HPStream that is highly scalable [39]. HPStream uses projected clustering [40], which can determine clusters for a subset of dimensions, to data streams and a new data structure called *fading cluster structure* that allows historical and current data to integrate nicely with a user-specified fading factor. Zhou et al. [41] presented a clustering algorithm called SWClustering that clusters evolving data streams over sliding windows to be able to analyze also the evolution of the individual clusters by eliminating influence by historic data points while the new data points arrive. The Authors show that the CluStream algorithm is more sensitive to influences of outdated data and is less efficient.

Processing large of amount of data efficiently using parallel processing is an active research and gain much of popularity when the Google's MapReduce programming model was introduced by Dean and Ghemawat [12] that allows researchers easily to create highly scalable and fault tolerance algorithms by just implementing a Map and a Reduce function. Zhao et al. [42] implemented a parallel version of k-means (PKMeans) in MapReduce framework and show that their algorithm can effectively run on large data sets. They designed the Map function to calculate the closest cluster centroid for each point at a time and output a  $\langle key, value \rangle$  pair where key is the closest cluster centroid id and value is the data point. After each map task they apply a combiner function called Combine that is executed on the same machine as the Map function. The Combine function partly sum the values of the data points assigned to the same cluster and outputs a  $\langle key, value \rangle$  pair where key is the id of the cluster centroid and the value is comprised of the sum values and the number of data points in that cluster. Using a combiner reduces the amount of intermediate information that is processed and sent over the network to the Reduce task [12, 42]. Then the Reduce function sums up all the intermediate sub sum values from all the combiners and calculates the new centers for the next MapReduce iteration. To overcome the sensibility of k-means to outliers in data, Li et al. [43] proposed an algorithm called MBK-means using the ensemble learning method called Bagging [44], generating k new data sets from the original data set with replacement sampling. For each of the new data set the k-means is run using the MapReduce framework until convergence then finally the k sets of k centroids are merged to form the final k centroids.

Many extensions on the traditional MapReduce framework have been proposed to support algorithms running iteratively [45–50] and incrementally [50–52] efficiently. The incremental MapReduce frameworks are interesting and relates to our work since we are incrementally clustering data but the use of these frameworks are not in our study.

### 3.3 Our study

The algorithm in this thesis is most similar to PKMeans [42] described above, a parallel k-means implementation in MapReduce using a Combine function to reduce the intermediate data sent between the mappers and the reducers. We however implement a parallel k-means algorithm in MapReduce so that each Map function efficiently calculates the distance to nearest cluster centers by calculating the distance matrix between all data points and the cluster centers instead of processing each data point separately. We apply the algorithm to incrementally but non-iteratively cluster player behaviors on theoretically large data sets that arrive daily.

In our work we show k-means is a good approach to provide valuable insights into the general of behaviors for a specific real game data provided by GameAnalytics [13]. Work of Drachen et al. [6, 26] relates to ours when selecting and building important behavioral variables from user's telemetry and extracting behavioral profiles by analyzing and interpret the centroids (basis vectors) from the k-means algorithm running in MapReduce. Additionally we perform experiments with a controlled data set where we have different normal distributions of data arriving separately each day with results and future work are discussed.

## Chapter 4

# Methodology

TODO Describe introduction to the methodology

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### 4.1 Data and Preprocessing

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#### 4.1.1 Feature selection and behavioral variables

TODO Describe feature selection and behavioral variables

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### 4.2 K-means algorithm in MapReduce

TODO Describe the k-means algorithm implementation in MapReduce and the relevant pseudo codes and pictures

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### 4.2.1 Map

#### TODO Describe the map function

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#### 4.2.2 Combine

#### TODO Describe the Combine function

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#### **4.2.3** Reduce

### TODO Describe the Reduce function

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#### 4.2.4 Distance measure

TODO Describe the distance measure used in k-means

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### 4.3 Experiment Set-Up

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## Chapter 5

## Results and Discussion

### 5.1 Results

TODO Show results with relevant pictures and what they mean

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### 5.2 Discussion

TODO Describe the results with more detailed explanations

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## Chapter 6

## Conclusions

### 6.1 Conclusions

TODO Write Conclusions. Convince the reader that the research question was answered/solved. Write what is relevant to the research question. Use short statements directly related to the research question.

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### 6.2 Summary of Contributions

TODO Describe the contributions may overlap Conclusions (Note: maybe move into Conclusions section). Short numbered statements.

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#### 6.3 Future Research

TODO What is the future work. What can be done differently, what needs to be addressed?

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# Appendix A

# Appendix Title Here

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