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Performance analysis and optimisation of stochastic flow lines with limited material supply

Julia Mindlina and Horst Tempelmeier

Supply Chain Management Area, University of Cologne, Cologne, Germany

ABSTRACT
We consider stochastic flow lines with limited buffer sizes and limited material supply. In these systems, the configuration of the flow line parameters and the configuration of the material supply determine the system output. Shortages of material supply can limit the performance of the production system. We use flexible (mixed-integer) linear programming approaches to evaluate and optimise the performance of long stochastic flow lines with limited material supply in discrete and continuous time. The approaches are used to quantify the impact of material shortages on the system output. Further, they are applied to determine the minimum material levels that are required to prevent material shortages of a given flow line configuration. The results of the numerical study reveal insights on the approximation accuracy of the linear programs as well as on the dependence of optimal material levels on flow line characteristics such as the presence of bottleneck machines and the system variability. The contribution of this paper consists of both, integrated models for stochastic flow lines with limited material supply and new insights on the optimal material supply of stochastic flow lines.

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Limited material supply; milkrun system; stochastic flow line; linear programming; manufacturing system design

1. Introduction

In the last years, numerous researchers have studied problems related to the configuration of asynchronous flow lines with random processing times and/or failures. Thereby, the impact of system parameters on the output such as the number of stations, processing times, breakdown characteristics and buffer sizes has been studied analytically. While the focus lies on the movement of the workpieces through the system, the supply of the stations with material to be mounted on the workpieces is almost completely neglected. However, in many cases, the material supply of flow lines in industrial applications represents a decisive driver of the system output. The flow line and the component supply imply a mutual interdependence since the material supply impedes the flow of workpieces through the flow line in case of material shortages. Further, the configuration of the flow line determines the demand for components. Consequently, integrated evaluation approaches are required that can be used for the optimisation of the material supply configuration of a specific flow line.

In the following, we focus on line-side inventory which describes the storage of material at the stations in the flow line. This part feeding policy is suitable for flow lines with a high volume and a low variety of components (see Hua and Johnson 2010 as well as Sali, Ahin, and Patchong 2015). To reduce material handling efforts, the transport of material to several material stocks along a flow line can be consolidated. Thereby, the transport vehicles circulate in a fixed time interval and supply the material stocks at the machines in pre-determined sequences with machine-specific replenishment quantities. This part-feeding policy is also known as milkrun material supply.

Milkrun material supply systems are applied in the automotive as well as in the electronics and mechanics industrial sectors. In these industrial applications, flow lines with equal deterministic machine processing times and random machine failures as well as flow lines with random continuous processing times caused by human workers are supplied with milkruns.

We propose integrated linear programming approaches to analyse and optimise longer milkrun supplied stochastic flow lines in discrete and continuous time. These approaches are used to quantify the impact of material shortages on the system output and to calculate the minimal material levels that are required to feed the respective flow line preventing material shortages.

CONTACT Julia Mindlina mindlina@wiso.uni-koeln.de

The results of the numerical study reveal a high approximation accuracy of the linear programming approaches for flow lines in discrete and continuous time. The optimisation results indicate a strong dependence of the material levels on the configuration of the flow line as, for example, the heterogeneity of the machines and the variability in the production system.

The contributions of this paper are both, integrated models for the evaluation and optimisation of milkrun supplied flow lines as well as new insights on the optimal material supply of stochastic flow lines.

The remainder of the paper is structured as follows. Section 2 discusses the relevant literature associated with stochastic flow lines with limited material supply and provides a description of the modelled production systems. Section 3 presents the respective linear programming models. Section 4 incorporates the analysis of the approximation accuracy and the system behaviour as well as insights on the optimal material levels. Further, the optimisation results of the integrated approach are compared to a sequential approach. The conclusion and further research are described in Section 5.

2. Modelling stochastic flow lines with limited material supply

2.1. Literature review

Performance evaluation problems of stochastic flow lines, on the one hand, and material supply problems, on the other hand, are almost completely treated in isolation in the literature. The existing papers considering asynchronous stochastic flow lines with limited material supply treat other optimisation problems and make use of other modelling approaches.

The models presented in this paper are based on Chapter 6 of the Ph.D. thesis by Mindlina (2019) and incorporate limited material supply into the linear programming models proposed by Helber, Schimmelpfeng, and Stolletz (2011) and Helber et al. (2011). The discrete-time modelling approach is suitable for milkrun supplied systems due to the discrete material replenishment process. Further, it allows for modelling the material levels as decision variables. For further mathematical programming approaches, we refer to Schruben (2000), Matta and Chefson (2005), Chan and Schruben (2008) and Pedrielli et al. (2018).

Further, there are related literature streams to flow lines with limited material supply in the context of the performance analysis of assembly networks. However, assembly networks imply that workpieces are merged and passed as a single unit to the downstream station after the processing completion of two different machines with stochastic processing times or stochastic failures. In contrast, we model deterministic and stochastic batch deliveries of material parts that are used for stochastic processing operations.

Integrated models for flow lines with a synchronous flow of workpieces and line-side material components are presented in Bukchin and Meller (2005), Emde, Fliedner, and Boysen (2012), Klenk, Galka, and Günther (2015), Alnahhal and Noche (2015), Sternatz (2015) and Baller et al. (2020).

The following publications refer to asynchronous stochastic flow lines with limited material supply but do not focus on material shortages and the optimisation of the material levels. Yan et al. (2010) propose a solution algorithm for the so-called line-side buffer assignment problem which represents the assignment of line-side buffers to a limited capacity of transport vehicles for part delivery so that material shortages are prevented. Weiss, Matta, and Stolletz (2017) use a sample-based mixed-integer linear program to solve the buffer allocation problem of a stochastic flow line with limited material supply only at the first station. Further, Kiesmüller and Zimmermann (2018) propose an exact Markov-chain approach for the evaluation of a stochastic flow line with a limited amount of line-side spare parts and demonstrate the impact of the simultaneous optimisation of the buffer sizes between the machines of the flow line and the spare part inventory levels.

Further publications address material shortages of flow lines using other modelling approaches and solve different optimisation problems than this article. Mindlina and Tempelmeier (2017) model the same systems but they propose an exact Markov chain approach to evaluate the performance of milkrun supplied two-machine lines in discrete time. Chang et al. (2013) develop a modified max-plus linear system to approximate the performance measures of a flow line with random machine failures and limited material supply. Additionally, they calculate the utilisation of the component transport system. Bozer and Ciemnoczolowski (2013) develop an approximate analytical approach to calculate the probability to exceed the physical capacity of a tugger train on a random milkrun tour or to exceed the target cycle time with the required tour completion time. Ciemnoczolowski and Bozer (2013) propose closed-form analytical models to approximate the material starving probability of the machines in a milkrun-supplied manufacturing system and Korytkowski and Karkoszka (2016) analyse the interactions between the milkrun material supply and a stochastic flow line with 10 machines using a simulation model.

In summary, to the best of our knowledge, there is no publication where the system output is formulated as a function of the configuration of a stochastic flow line and the associated material supply system. Further, there are no studies that refer to the optimal material levels in dependence of the flow line configuration.

2.2. System characteristics

We consider a stochastic flow line which is composed of serially arranged stations (machines) and finite buffer sizes. Further, there is limited material supply at each machine. Figure 1 shows such a flow line modelled as a queueing network. Workpieces move through the system where at each station material supplied from the associated line-side stocks is attached to the workpiece. Thereby, each processing task on a workpiece requires one unit of material. The line-side material stocks are replenished periodically after r time units. Thereby, the inventory mr_i is refilled up to a station-specific order up-to level S_i $(i = 1, \ldots, M)$.

The first machine never starves for workpieces and the last machine is never blocked. The processing times of the tasks can either be random or deterministic with random failures. Adjacent machines are decoupled by buffers with a limited capacity b_i. If material components are not available, the workpieces wait in the upstream buffers until the next material arrival. If a finished workpiece cannot be passed to the downstream buffer, the machine is blocked until the end of the current processing operation of the downstream machine or the arrival of a required material unit at this machine. Hence, we assume blocking-after-service. As a consequence, the flow of workpieces can be interrupted by starving for material in addition to the conventional starving and blocking effects. The shortage of material may also increase the proportion of blocking effects.

2.3. Application in industry

The main targets of the implementation of milkrun material supply in industry refer to the standardisation of the material supply process, the reduction of material inventories and the required storage area at the flow line as well as the consolidation of transportation effort and the prevention of material shortages. In order to achieve these aims, the system characteristics and assumptions presented in Section 2.2 are sometimes extended.

In practice, several mechanisms may secure an adequate supply of material. As the example of a German car manufacturer demonstrates, either sufficient line-side material is replenished on the milkrun tours or prioritised emergency deliveries are provided in the case that line-side material impedes the processing operations of the flow line. Thereby, the responsible workers may send alerts manually or automatic orders are carried out.

Further, in multi-product flow lines, workpieces waiting for components can be ejected from the flow line in order not to propagate the disturbance through the whole flow line. These workpieces are sent to a queue in an additional storage area from which they are retrieved when sufficient components have been replenished.

Further, the direct supply of machines by automated guided vehicles may become a competitive solution in the future. However, the modelling approach of this paper implies strong flexibility allowing for individual adaptations according to practitioners' needs.

Apart from the model, the accuracy of the determined optimal material levels is strongly affected by accurate data serving as the input parameters for the model.

3. Linear programming models of flow lines with limited material supply

In this paper, we use linear programming (LP) models to mimic the dynamic development of the multi-station production processes in a stochastic flow line. The LP model is constructed with a large number of realisations of the relevant random variables which are generated in a preprocessing step and which are then input as deterministic data into the LP model. The processing times are modelled by means of deterministic samples. Thereby we use descriptive sampling which goes back to Saliby (1990). The samples are transformed into processing

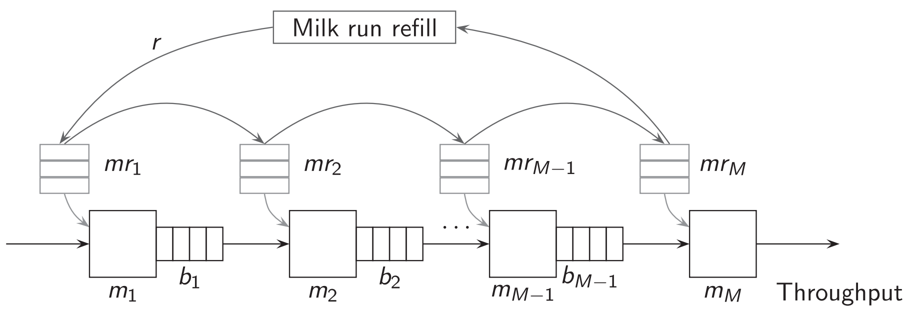


Figure 1. Flow line with limited buffers and material supply.

capacities for each production stage and each discrete time period. Thereby, we count the number of finished workpieces at a production stage within a time period. For flow lines with generally distributed processing times, we make use of the gamma distribution. To depict the failures in discrete Bernoulli lines, we generate pseudo-random numbers. Thereby, Bernoulli lines are modelled with discrete time slots that equal the cycle times of the machines. Machine m_i is up during a time slot with a probability of p_i with $i = 1, \ldots, M$. For Bernoulli lines, the number of samples equals the number of modelled time periods. For gamma-distributed processing times, the number of samples and covered time periods may deviate.

The flow of the workpieces through the line is depicted by linear equations. The output of the linear program is an estimate of the production rate as well as optimised material order up-to levels.

Using these models for discrete-time systems can result in simulation errors whereas the approximation of production systems in continuous time may incorporate simulation and discretisation errors. However, the considered flow lines imply a discrete time replenishment process of discrete material components which fits the discrete time modelling and strongly reduces the discretisation error in comparison to models of stochastic flow lines without material supply.

The advantage of the linear programming approach is the flexibility regarding the configuration of the flow line and the material supply as well as the probability distribution of the processing times. In the following, we present a linear program for open stochastic flow lines with milkrun material supply. However, it can be adapted to a closed flow line as in Helber, Schimmelpfeng, and Stolletz (2011). Further, the material supply mode as well as the flow line configuration and features as, for example, the blocking condition can be easily adjusted.

3.1. Basic model formulation

In the following, we present the linear program for open stochastic flow lines with limited material supply and the associated notation.

Input Parameters

i	$= 1, \ldots, M$ machines
t	$= 1, \ldots, T$ periods
T_0	number of warm-up periods
$c_{i,t}$	processing capacity of machine i in period t
b_i	number of buffer spaces available at machine i
rep_t	binary period-specific replenishment parameter
S_i	material order up-to level at machine i

Decision Variables

$q_{i,t}$	production quantity of machine i in period t
$bl_{i,t}$	end-of-period inventory behind machine i in period t
$mr_{i,t}$	end-of-period material inventory at machine i in period t
$R_{i,t}$	material replenishment quantity at machine i in period t

$$\max \quad PR = \frac{1}{T - T_0} \cdot \sum_{t=T_0+1}^{T} q_{Mt} \tag{1}$$

s.t.

$$bl_{i,t-1} + q_{i,t} = bl_{i,t} + q_{i+1,t+1} \qquad i = 1, \ldots, M-1 \quad t = 2, \ldots, T-1 \tag{2}$$

$$q_{i,t} \leq c_{i,t} \qquad i = 1, \ldots, M \quad t = 1, \ldots, T \tag{3}$$

$$bl_{i,t} \leq b_i \qquad i = 1, \ldots, M-1 \quad t = 1, \ldots, T \tag{4}$$

$$bl_{i,T-1} + q_{i,T} = bl_{i,T} \qquad i = 1, \ldots, M-1 \tag{5}$$

$$q_{i,1} = bl_{i,1} + q_{i+1,2} \qquad i = 1, \ldots, M-1 \tag{6}$$

$$q_{M,1} = bl_{M,1} \tag{7}$$

$$R_{i,t} = (S_i - mr_{i,t-1}) \cdot rep_t \qquad i = 1, \ldots, M \quad t = 2, \ldots, T \tag{8}$$

$$R_{i,t} + mr_{i,t-1} = mr_{i,t} + q_{i,t+1} \qquad i = 1, \ldots, M \quad t = 2, \ldots, T-1 \tag{9}$$

$$R_{i,T} + mr_{i,T-1} = mr_{i,T} \qquad i = 1, \ldots, M \tag{10}$$

Objective function (1) aims at maximising the production rate PR which is the average production quantity of the last machine. Equations (2) model the flow of workpieces through the flow line and account for the corresponding amount of workpieces in the buffers and the servers. Equations (3) restrict the production quantities to the processing capacities of a machine. Thereby, the processing capacities are a result of the sampling procedure. Equations (4) restrict the end-of-period inventories to the buffer spaces behind the machines. Equations (8) imply that the material replenishment quantity equals the difference between the material order up-to level of the machine and the end-of-period inventory in case of replenishment. Thereby, the binary replenishment variable is an input variable that is calculated in dependence on the replenishment interval. The variable is represented by $rep_t = 1$ for $t \mod r = 0$ and $rep_t = 0$ for $t \mod r > 0$. Consequently, $rep_t = 1$ holds for every rth time period enforcing a material supply with a replenishment interval of r. If $rep_t = 0$, no material replenishment is triggered. Equations (9) ensure that the production of workpieces only takes place if material is available either from stock or due to replenishment. Further equations of the linear program refer to the special cases of the last machine in the flow line or the last and the first period.

The flexibility of the linear program allows us to additionally model stochastic material supply. Therefore, we modify the generation of the binary replenishment variable rep_t for $t = 1, \ldots, T$. For each period with a planned replenishment where $t \mod r = 0$, we draw a pseudorandom number representing a potentially failing replenishment.

3.2. Optimisation of the material order up-to levels

On the basis of the models for the evaluation of flow lines with limited material supply, we propose reformulations for the optimisation of the material order up-to levels equivalently to Helber, Schimmelpfeng, and Stolletz (2011). In order to optimise the material order up-to levels of the machines, we define the material order up-to levels S_i^* for $i = 1, \ldots, M$ as decision variables instead of input parameters. Additionally, we incorporate material order up-to levels into the objective function by reformulating it into the profit function (11). Thereby, the linear program is turned into a mixed-integer linear problem that can be solved via branch and bound or branch and cut algorithms. The gross margin gm stands for a value that is much higher than the line-side area costs per unit ca. In this way, it is ensured that the optimal material order up-to levels prevent material shortages and enable the highest possible production rate. However, the line-side area costs ca cause that the optimal material order up-to levels are chosen as low as possible to prevent material shortages. In this case, the resulting costs are not decisive in contrast to the resulting optimal material order up-to levels. However, the weights gm and ca can also be chosen in a way that the optimal solution allows for material shortages and a resulting reduction of the production rate. This is the case for relatively high line-side area costs. In this way, the profit function can be flexibly adapted to industrial needs

$$\max Pf = gm \cdot \frac{1}{T - T_0} \cdot \sum_{t=T_0+1}^{T} q_{Mt} - ca \cdot \sum_{i=1}^{M} S_i^*. \tag{11}$$

The model can be easily extended to simultaneous optimisation of the material levels and the buffer allocation in the flow line by modelling the buffers as decision variables and adding a constraint with the total sum of the buffers as in Helber et al. (2011).

4. Numerical analysis

4.1. Outline of the numerical study

In the numerical study, we aim to derive insights on the modelling accuracy of the approaches as well as on the material supply configurations at optimality and the merit of the integrated modelling of stochastic milkrun supplied flow lines.

First, to decide on a suitable number of samples for further computations, we investigate the impact of the number of samples on the approximation accuracy and on the run time performance in Section 4.2. Using these results, we compare the approximated production rates of the evaluation approaches to the simulated throughput in Section 4.3. Afterwards, we analyse the characteristic system behaviour of stochastic flow lines with limited material supply in Section 4.4. In Section 4.5, we analyse the impact of the flow line characteristics and the replenishment interval on the optimal material levels. Section 4.6 compares the results of the integrated approach to a sequential optimisation approach of the material levels.

Although the evaluation and the optimisation-oriented models imply different objective functions, the approximation accuracy is particularly influenced by the constraints modelling the inventory of workpieces in the flow line.

All the results of the numerical study are based on three sample paths per scenario. Especially the evaluation-oriented linear programs lead to very robust production rate estimates. Hence, we report the exemplary results of one sample path, respectively, which is supposed to represent the overall study. Concerning the optimised configurations, the specific material levels show minor differences throughout the three sample paths. However, we are interested in the general relations between the input variables and the output. The presented hypothesis is tested with statistical tests for all sample paths. The numerical study is performed on an Intel Core i7-4800MQ with 2.7 GHz and 8 GB RAM.

Table 1. Parameter sets for the numerical study in discrete and continuous time.

Parameter	Variation
Number of machines M	$\{3, 7, 15\}$
Buffer capacity b_i	$\{3, 5, 10\}$
Bottleneck material level $\lceil 0.6 \cdot S_i \rceil$	$\{\text{none}, S_1, S_{\lceil M/2 \rceil}, S_M\}$
Material ratio $\frac{S}{r}$	$\{0.7, 0.9\}$
Replenishment interval r	$\{30, 60\}$
Machine reliability probability p	$\{0.7, 0.9\}$
Bottleneck machine $p_i - 0.2$	$\{\text{none}, p_1, p_{\lceil M/2 \rceil}, p_M\}$
Expected value of processing rate μ	$\{1\}$
Coefficient of variation of processing time ζ	$\{0.3, 0.5, 0.7\}$
Bottleneck machine $\mu_i - 0.5$	$\{\text{none}, \mu_1, \mu_{\lceil M/2 \rceil}, \mu_M\}$

The approximation accuracy is investigated for flow lines in discrete and continuous time separately. Thereby, Bernoulli lines and flow lines with gamma distributed processing times are modelled. The test set for the evaluation-oriented numerical study of the linear programming approach is shown in Table 1. It comprises 1260 scenarios.

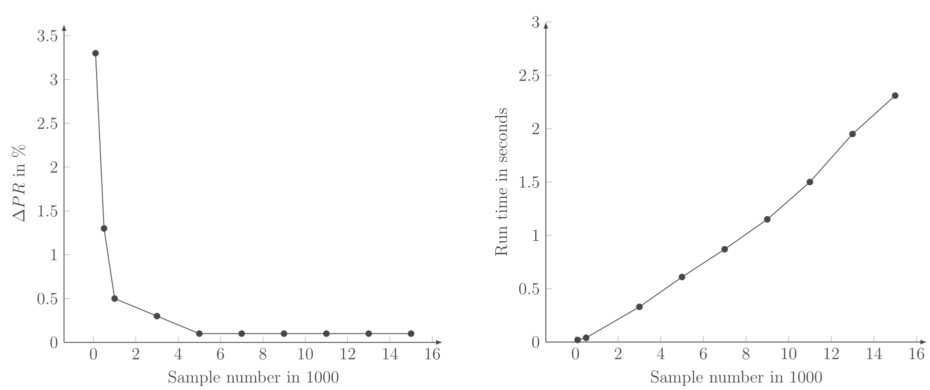
Further, we study the optimal material order up-to levels of different flow line configurations in discrete and continuous time. The optimisation study refers to 752 configurations which represent a subset of the configurations generated from Table 1. The mixed-integer linear programming approach is applied to flow lines with up to 15 machines and 150 buffers and all reported variations of the parameters are covered in the optimisation study. Sometimes additional parameter variations are incorporated into the optimisation study as, for example, higher coefficients of variation of the processing times or a longer replenishment interval in order to emphasise the effects. Details on these cases are provided in the corresponding sections.

Further, we study the impact of deterministic and stochastic replenishment on the optimal material order up-to levels. Thereby, replenishment disruption probabilities up to 5% are modelled.

In our optimisation study, we focus on insights on the optimal material levels from a practical perspective. Since in most cases, the gross margin is much higher than the line-side area costs, we set the weights in the profit function such that material shortages are prevented. We choose the relation between the parameters gm and ca to be 100000:1 which almost certainly ensures availability of line-side material while minimising the inventory. Nevertheless, it might be profitable to accept a loss of gross margin in case of very high line-side area costs. However, to accept a lower gross margin, very accurate information on the cost parameters is required. Since reliable data on costs can be a challenge in practice and the aim of a conservative practitioner is presumably to prevent material shortages, we decide on these parameters although the model is capable of modelling the cost trade-off.

4.2. Impact of the number of samples on approximation accuracy and run time performance

Figure 2 demonstrates the approximation quality of the production rate and the computation time in dependence of the sample number for a homogenous three-machine Bernoulli line.[1] Figure 2(a) shows that the accuracy of the production rate estimate, $\Delta PR = |\frac{PR_{Approx} - PR_{Sim}}{PR_{Sim}}|$, rises with an increasing sample number of processing times until it reaches the point where the sample size does not change the approximation quality anymore. In the underlying example, the deviation between the approximation and the simulation can be reduced to 0.1% with a sample size of 5000. However, this figure shows an exemplary development of the production rate estimate and the number of machines as well as the configuration of the flow line exert an influence on the development of the approximation accuracy with the sample size. In order to secure an adequate sample size for a large range of production systems, we consider 10,000 time periods



(a) Samples versus accuracy

(b) Samples versus run time

Figure 2. Impact of the sample number.

with a warm-up time of 500 time units throughout the numerical study.

Figure 2(b) illustrates the linear development of the computation time with an increase of the sample size for the 3-machine line example. Though, the computation time strongly depends on the number of machines and implies a stronger increase for longer flow lines.

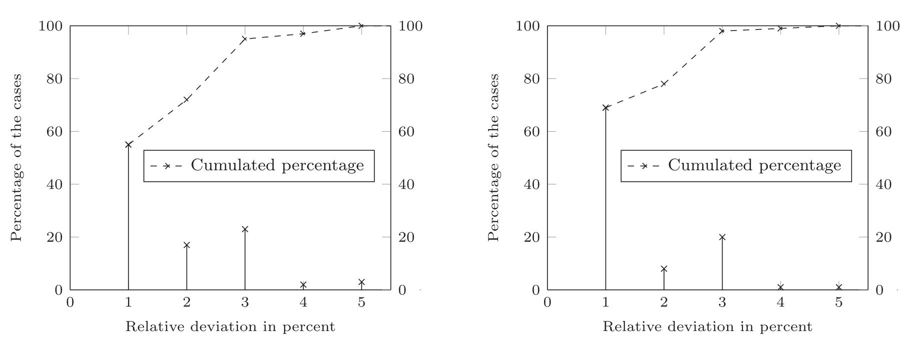
4.3. Approximation accuracy and runtime performance of the linear program

In order to assess the approximation accuracy of the production rate, we compare the results of the evaluation-oriented linear program with simulation results. Figure 3 depicts the distributions of the absolute values of the relative deviation between the production rate estimates and the simulation. Both approaches imply a very high approximation accuracy whereby the results in continuous time show a slightly higher accuracy. Both studies, in continuous and discrete time, lead to a maximum relative deviation of the production rate of 5%. However, this is only true for 3% (discrete time) and 1% (continuous time) of the cases. The mean deviations of the approximated from the simulated production rates are 1.00% for the discrete-time and 0.6% for the continuous-time flow lines. Figure 3(a,b) also shows that 95% (discrete time) and 98% (continuous time) of the scenarios lead to an approximation error that is equal to or below 3%. The results of the different sample paths are not identical but the differences between the percentages of cases lie within a range that can be neglected. Hence, Figure 3(a,b) is representative of all sample paths.

Table 2 shows that the high approximation accuracy holds throughout the whole test bed. Conducting a six-way ANOVA and calculating Spearman's rank correlation coefficient to search for a significant influence of the input parameters on the relative deviation between the production rate estimate and the simulation, we observe that there is a significant influence of the machine number, the machine reliability probability and the coefficient of variation of the processing times on the approximation accuracy. However, the approximation errors lie within a very small range as depicted in Table 2. Therefore, we do not go into detail on the significance of the relation between flow line parameters and the approximation accuracy.

Besides, the study demonstrates that the approximation accuracy of the linear program is very high for different material ratios. Consequently, the approximation accuracy of the production rate estimates associated with the optimal material levels in Section 4.5 does not differ from the approximation accuracy of exogenously chosen material levels.

Further, for instance, the buffer sizes cover a range of 3–10. For flow lines in continuous time, the coefficients of variation of the processing times range from 0.3 to 0.7.



(a) Discrete-time flow lines (b) Continuous-time flow lines

Figure 3. Distributions of the relative throughput deviation.

Table 2. Approximation quality and computation time of the linear program for flow lines in discrete and continuous time.

Measures	Total	Impact of						
		$M=3(7,15)$	$b=3(5,10)$	$p\backslash\mu=hom(het)$	$r=30(60)$	$S=hom(het)$	$\frac{S}{r}=0.7(0.9)$	$\zeta=0.3(0.5,0.7)$
Discrete time: ΔPR in %	1.00	0.58(1.04,1.36)	1.24(0.79,1.05)	1.09(0.87)	0.91(1.09)	0.92(1.10)	1.19(0.8)	–
Discrete time: Run-time in seconds	7	0(4,18)	5(7,10)	5(9)	7(7)	7(8)	7(8)	–
Continuous time: ΔPR in %	0.64	0.47(0.60,0.84)	0.67(0.45,0.80)	0.63(0.64)	0.64(0.63)	0.70(0.56)	0.65(0.63)	(0.41,0.57,0.93)
Continuous time: Run-time in seconds	7	0(4,18)	6(7,9)	8(7)	7(7)	7(8)	8(7)	(9,7,6)

Note: ΔPR: $|\frac{PR_{Approx}-PR_{Sim}}{PR_{Sim}}|$; b: buffer capacity behind each machine; r: replenishment interval; M: number of stations; p: machine reliability probabilities; $p\backslash\mu = hom$: homogenous machine reliability probabilities or processing rates; $p\backslash\mu = het$: heterogenous machine reliability probabilities or processing rates; $S = hom$: homogenous material order up-to levels; $S = het$: heterogenous material order up-to levels; ζ: coefficient of variation of the processing times in continuous-time flow lines.

Considering buffer sizes of 1 and coefficients of variation of the processing times ≥ 1 would lead to less accurate results implying an increase of the mean deviation by up to 5%. However, these scenarios are not representative of realistic production systems and would distort the results for the relevant configurations.

Besides, the mean computation time amounts to 7 s per evaluation. The results in Table 2 indicate that the number of machines as well as the buffer sizes have the strongest influence on the computation time. However, the mean computation time for a scenario with 15 machines is 18 s and the average run-time for scenarios with 10 buffers between each machine amounts to 10 s.

4.4. Insights on system behaviour

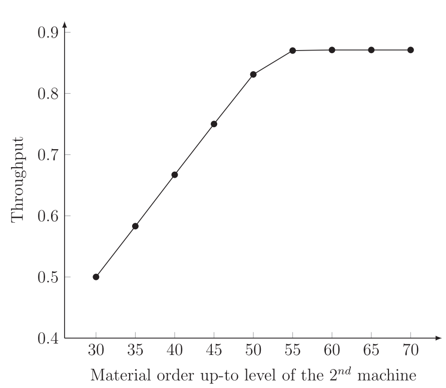
Figure 4(a) shows the throughput of a stochastic two-machine line as a function of the material order up-to level of the second machine.[2] The first machine does not suffer from material shortage. Therefore, the development of the throughput is the result of the limited material supply at station 2. The throughput increases until it reaches an upper bound. This state is reached when material is available for every processing operation of each machine without delay. In this situation, the throughput of the production system is only restricted by a limited buffer size that cannot completely compensate for the randomness of the machine failures. The maximum throughput of the production system in this example is determined with (S_2/r). The production system cannot produce more products during the replenishment time than the number of available material units because one material part is required for every workpiece.

Figure 4(b) illustrates the results for a characteristic development of the throughput in dependence of the replenishment interval.[3] The throughput does not change for a replenishment interval that is short enough to ensure the material supply of the flow line without material shortages. For these configurations, the frequency of the material replenishment does not change the production rate because exclusively disturbances within the flow line due to blocking and starving reduce the output. With a longer replenishment interval, the throughput sinks because of disturbances of the workpiece flow by material shortages. The longer the replenishment interval, the stronger the throughput decreases. Since material replenishment tours are related to costs for the milkrun transport vehicles, the human operators of the tugger trains as well as for the material handling, it is economic to install a replenishment interval that is just as short that it prevents material shortages.

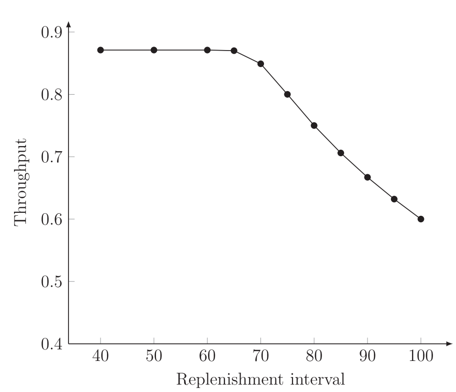
4.5. Insights on optimal line-side material levels

The optimisation results of the material levels are analysed with eight perspectives. A multi-factor ANOVA and Spearman's rank correlation coefficient are calculated in the Stata environment to derive significant effects of the flow line configuration and the replenishment interval on the optimal material levels. Box plots are used to depict the results of the experiments. In comparison to Korytkowski and Karkoszka (2016) that work out the effects of the flow line configuration on the production rate using multi-factor ANOVA, we analyse the material levels at optimality. These findings represent a decision support for material supply design.

The analysis of milkrun supplied flow lines in continuous and discrete time is performed separately. The impact of the variables in both systems on the optimal material levels is similar. To avoid repetitions, we exemplarily present the results of one system type respectively. However, the results are representative of all treated systems.



(a) Material order up-to level versus throughput



(b) Replenishment interval versus throughput

Figure 4. Impact of the material supply on the system throughput.

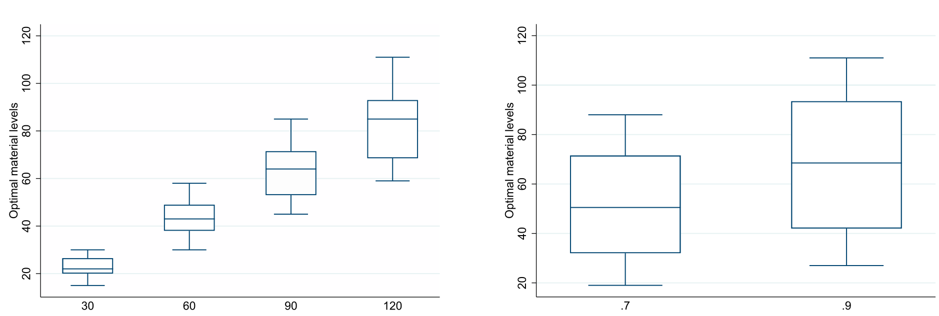
Further, we find minor differences in the individual material levels and the resulting parameters of the statistical analysis throughout the different sample paths. Though, we are interested in general system relationships and present only conclusions of the analysis that hold for all scenarios and all sample paths of the test bed.

The multi-factor ANOVA including the variables replenishment interval, processing time/machine reliability probability, station number, buffers and coefficients of variation (only in continuous time) leads to a minimum R^2 of 0.98 throughout all experiments indicating that 98% of the variation in the material levels is explained by these input variables.

(1) The replenishment interval exerts a significant influence on the optimal material order up-to levels which is confirmed by the results of the multi-factor ANOVA ($F(3) = 736.61$, $p < .001$) and Spearman's rank correlation coefficient ($rho = 0.95$, $p < .001$). Figure 5(a) shows an exemplary box plot of the results in discrete time. We observe that the material order up-to levels rise with a higher replenishment interval. This effect corresponds to the results of classical inventory management and is due to a longer risk period of material demand that has to be covered till the next replenishment. The growth of the material levels is increased if unbalanced flow lines are considered as is the case in Figure 5(a). For homogenous flow lines, the increase of the optimal material levels is less than proportional to the length of the replenishment interval. This result indicates that pooling effects of the material components have to be considered for a longer replenishment interval. The box plot in Figure 5(a) also indicates that the spread of the optimal material order up-to levels in our test set increases with a higher replenishment interval. Our analysis reveals that this effect is strengthened by the consideration of unbalanced flow lines implying a higher system variability than balanced lines.

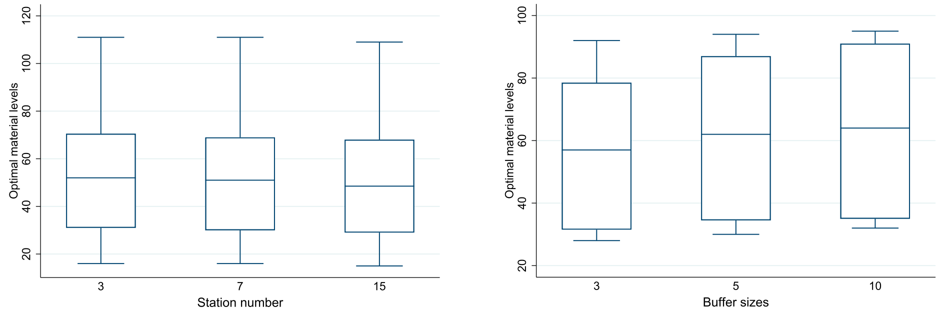
(2) The increase of the machine reliability probability in Bernoulli lines leads to a significant growth of the associated material order up-to levels according to the multi-factor ANOVA ($F(1) = 225.91$, $p < .001$) and Spearman's rank correlation coefficient ($rho = 0.27$, $p = .021$). Figure 5(b) illustrates the growth of material order up-to levels whereby the box plot only comprises balanced flow lines in discrete time. In this way, the impact of the machine reliability probability is not disturbed by bottleneck machines. The explanation for this effect is that a higher machine reliability probability implies less machine failures leading to less blocking and starving effects within the flow line. Consequently, the throughput of the flow line is increased and the demand for material components rises.

(3) The number of stations exerts a significant influence on the material levels according to the multi-factor ANOVA ($F(2) = 7.18$, $p = .002$). However, Spearman's rank correlation coefficient does not reveal a significant relation between the station number and the material levels throughout all samples. The impact of the station number, if it is proved to be significant for a sample path, is very low in comparison to the impact of the replenishment interval or the machine reliability probabilities. Figure 6(a) illustrates the impact of the station number for the underlying scenarios of the discrete test set. We observe that an increase of the machine number at least does not change the material order up-to levels and on average rather reduces them. Additional machines increase the blocking and starving effects in the flow line and therefore may cause a reduction of the throughput of the flow line. Consequently, the demand for material components is decreased. However, this effect is rather marginal



(a) Material levels versus replenishment interval (b) Material levels versus machine reliability

Figure 5. Optimal material order up-to levels versus r and p.



(a) Material levels versus station number (b) Material levels versus buffer sizes

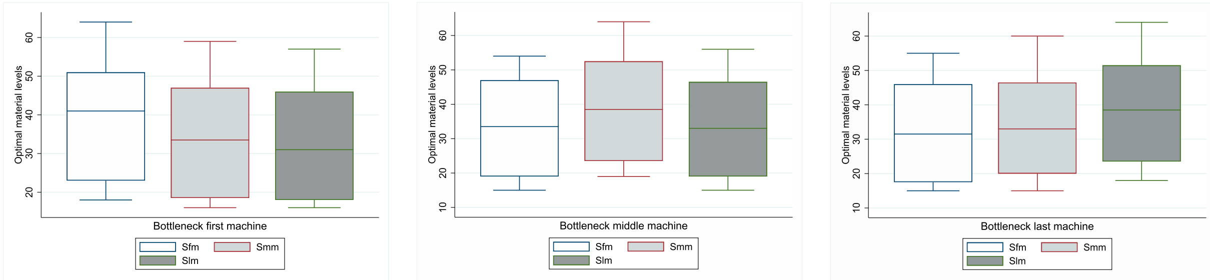
Figure 6. Optimal material order up-to levels versus M and b.

and an adaptation of the material order up-to levels should only be initiated if the machine number is changed considerably.

(4) The multi-factor ANOVA shows a significant influence of the buffer sizes on the optimal material order up-to levels ($F(2) = 38.68$, $p < .001$). As in the previous case, Spearman's rank correlation coefficient does not show a significant relationship between the buffer sizes and the material levels in all cases. Though, the p-values of Spearman's rank correlation coefficient are, for instance, $p = .059$ if they exceed the defined 5%-probability level and the associated exemplary $rho = 0.22$. From these results, we conclude that there can be a relationship between the buffer sizes and the optimal material levels, where the material levels grow with the buffer sizes. This relationship is stronger than the impact of the number of stations. However, the impact of the buffer increase strongly depends on the associated flow line configuration. Buffering against blocking and starving effects helps to reduce the variability in the system and increases the throughput. This effect can lead to a higher demand for material components. On average this effect can be observed in our test set as depicted in Figure 6(b).

(5) Flow lines with homogenous machines imply homogenous material order up-to levels in most of the cases. However, this result is caused by the assumption of equal buffer capacities. Optimising the buffer allocation and the material levels simultaneously, we allow for heterogenous buffer capacities. The result is that, equivalently to the inverted bowl shape of buffer capacities in an optimised buffer allocation, the optimal material levels of the internal machines of the flow line are higher than those of the other machines in all tested cases. Hence, an inverted bowl shape of material levels can be observed for homogenous machines with an optimised buffer allocation where increasing the material levels for the internal machines is beneficial due to higher associated buffer capacities. Heterogenous flow lines incorporate a significant pattern of optimal material order up-to levels where the machine with the lowest processing rate or the highest machine failure probability is supplied with an at least equal or in most cases higher material order up-to level than the others. This pattern holds for each scenario of the discrete and continuous data set. Thereby, it is independent of the location of the bottleneck machine in the flow line. The adjacent machines of the bottleneck machine also incorporate increased material order up-to levels. Spearman's rank correlation coefficient supports this effect with $rho = -0.20$ and $p = .034$ for the bottleneck machine, respectively. In the numerical study, the bottleneck machines imply material order up-to levels that are up to 15% higher than the smallest order up-to levels of the other machines. Figure 7(a–c) illustrates the material supply pattern for heterogenous machines. Thereby, the optimal material order up-to levels are denoted with Sfm, Smm and Slm which stands for the material order up-to levels of the first machine, of the machine in the middle of the flow line and of the last machine in the flow line. The explanation for this observation is that the bottleneck machine has to be supplied sufficiently in order not to lose production potential. Since the bottleneck machine is the limiting factor for the system output, it has to be ensured that material shortages do not further limit the bottleneck machine. Hence, it is important that the material supply of the bottleneck machine covers the system variability as much as possible.

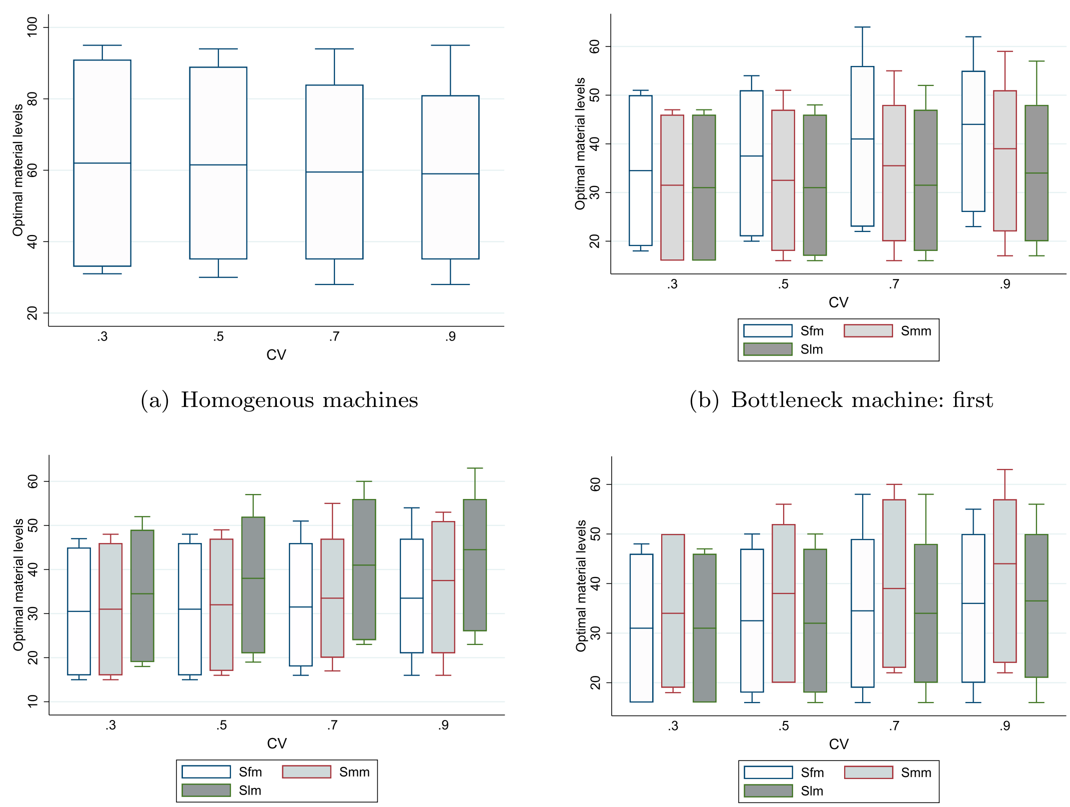
(6) Increasing the coefficient of variation of the continuous processing times, we observe two opposed effects. While the increase of the coefficients of variation leads to a slight decrease of the optimal material order up-to levels of flow lines with homogenous machines, we learn that heterogenous flow



(a) (b) (c)

Figure 7. Optimal material order up-to levels of heterogenous flow lines. (a) Bottleneck machine: first (b) Bottleneck machine: middle (c) Bottleneck machine: last.

lines imply an increase for both, the material of the bottleneck machine and the material of the other machines. Thereby, the optimal pattern for heterogenous flow lines described in the previous paragraph still holds. The effect is independent of the location of the bottleneck machine. This effect is significant according to Spearman's rank correlation coefficient with $rho = 0.20$ and $p = .003$ for all machines. The development of the material levels of homogenous machines is not significant according to Spearman's rank correlation coefficient. However, we conclude that there is at least no increase of the material levels with the coefficient of variation of the processing times in these cases. Figure 8(a,b,d) depicts these results. An increase in the variability in the system leads to the requirement to supply the bottleneck machine with even more material than it is the case for a lower variability in the system. The same holds for the adjacent machines of the bottleneck machine. Assuming that it is essential to prevent material shortages of the bottleneck machine, the material levels have to be increased to cope with growing stochasticity of the bottleneck machine. In contrast, for homogenous flow lines blocking and



(a) Homogenous machines (b) Bottleneck machine: first

(c) Bottleneck machine: last (d) Bottleneck machine: middle

Figure 8. Material levels versus coefficients of variation of the processing times.

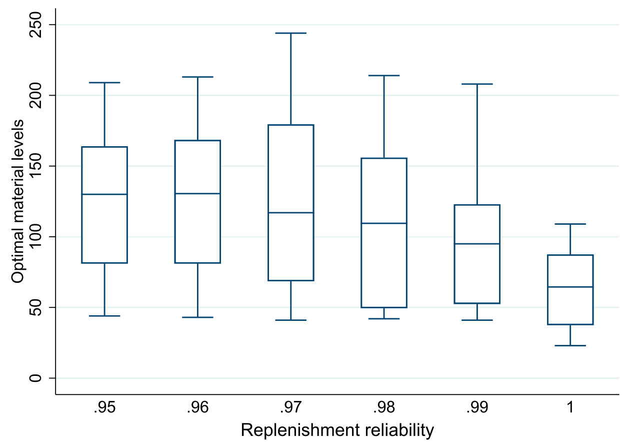


Figure 9. Material levels versus replenishment probability.

starving effects in the flow line are increased and the result can be a decreased demand for material components. Consequently, the reaction to an increase of the variability of the processing times should be adapted to the characteristics of the flow line.

(7) Concerning stochastic material replenishment, the reliability of material replenishment significantly affects the material order up-to levels. This is confirmed by Spearman's rank correlation coefficient ($rho = -0.34,\ p < .001$). The higher the replenishment probability, the lower the optimal material order up-to levels as depicted in Figure 9. Thereby, a stronger decrease of the optimal material order up-to levels can be observed for a higher replenishment reliability probability. The longer the replenishment interval, the stronger the growth of the optimal material order up-to levels since longer time intervals without replenishment have to be covered potentially.

The average computation time for the optimisation of the material order up-to levels grows with the machine number from milliseconds to 96 s for an increase from 3 to 15 machines. Further, a growth of the buffer sizes between each machine from 3 to 10 buffers results in an increase of the computation time from 7 to 82 s per scenario. The longer the replenishment interval, the shorter the required computation time since less replenishment events have to be accounted for by the linear program.

4.6. Comparison between integrated and sequential approach

In order to demonstrate the necessity for integrated evaluation and optimisation approaches for milkrun-supplied flow lines, we compare the optimisation results of the

Table 3. Optimal results of integrated versus sequential approach for 3-machine line.

System configuration	Material levels using LP	Material levels using sequential approach
$r = 60, b_i = 3, p_i = 0.9\ \forall i$	57, 57, 57	60, 60, 60
$r = 60, b_i = 1, p_i = 0.9\ \forall i$	55, 55, 55	60, 60, 60
$r = 60, b_i = 3\ \forall i, p_1 = 0.9, p_2 = 0.9, p_3 = 0.6$	39, 42, 45	60, 60, 47

linear program to a sequential approach where the demand for material components is derived from the machine reliability probability of the Bernoulli lines. In the sequential approach, we first calculate the demand parameters that result from an r-times convoluted machine reliability probability. A classical inventory optimisation approach for an (r, S) -policy is used to calculate the optimal material order up-to levels associated with these demand parameters. Thereby, we aim at a service level close to 100%.

We consider a three-machine Bernoulli line with $r = 60,\ b_i = 3,\ p_i = 0.9\ \forall i$. The linear programming approach leads to optimal material order up-to levels of 57 for each machine, whereas we calculate optimal material order up-to levels of 60 for each machine using the sequential approach. Consequently, we observe an overestimation of the sequential approach by 3 material parts per machine. If we assume a 15-machine production line, the result is an overestimation of 45 material parts. Assuming that we supply valuable components, we can achieve considerable cost savings with an integrated approach.

Further, assuming that we have a flow line configuration with $r = 60, b_i = 1, p_i = 0.9\ \forall i$, the linear programming approach leads to optimal material order up-to levels of 55 material parts at each machine. Using the sequential approach, we do not have the possibility to consider the impact of the reduced buffer sizes. In this case, we have an overestimation of the required material parts of 5 components per machine (respectively, 75 components for a 15-machine line). Hence, in this example, we observe that the overestimation of the sequential approach is higher the smaller the buffers are.

Additionally, treating heterogenous flow lines as, for example, $r = 60,\ b_i = 3\ \forall i$ and $p_1 = 0.9, p_2 = 0.9, p_3 = 0.6$, the linear program leads to optimal material order up-to levels of $S_1 = 39,\ S_2 = 42,\ S_3 = 45$. Using the sequential approach, we can only treat the machines separately which results in optimal material order up-to levels of $S_1 = 60, S_2 = 60, S_3 = 47$. Thereby, the interaction between the machines in a flow line cannot be captured, and the result is that the bottleneck is supplied with less material than the other machines. In this example, material shortages are prevented. However, the interaction between the machines is totally ignored. Table 3 summarises the optimisation results.

Consequently, we learn that a sequential optimisation approach of the material levels leads to an overestimation of the required material for the illustrated examples resulting in costs that can be saved with the linear programming approach. Further and even more important, there is no possibility to capture and react to the interaction between flow line parameters as, for example, buffers and the existence of bottleneck machines using the sequential approach. Using the production rate of the flow line as the input for the sequential approach would allow to incorporate the influence of the buffers. However, the differences between the machines for unbalanced lines cannot be captured since the production rate is identical for all machines in steady state.

5. Conclusion and further research

We present integrated approaches for the evaluation and optimisation of the performance of stochastic flow lines with limited material supply. We quantify the impact of material shortages on the production rate and optimise the material supply of given flow line configurations. We present (mixed-integer) linear programming approaches for longer flow lines in discrete and in continuous time which incorporate high modelling flexibility and can easily be adapted to further flow line configurations as, for example, closed or assembly flow lines and further optimisation targets as the simultaneous optimisation of material levels and the buffer allocation in the flow line.

The approximation accuracy of the approaches is very good implying a reasonable computation time. On the basis of the mixed-integer linear programs, we derive insights on the optimal material order up-to levels. The material supply has to be adapted to the flow line configuration since the flow of workpieces through the flow line represents the demand for material components. Additionally, the bottleneck machine and adjacent machines in heterogenous flow lines have to be supplied with the highest amount of material components. Considering system variability, a different treatment of balanced and unbalanced lines is essential for an adequate material supply of the flow line. Thereby, a higher system variability requires an increase of the material level of the bottleneck machine to prevent a reduction of the output of the bottleneck and to cope with the stochasticity in the system.

Further research could address the development of analytic solutions and efficient optimisation approaches for stochastic flow lines with limited material supply. Additionally, realistic system characteristics as emergency deliveries could be integrated into the models. A focus could also be laid on the simultaneous optimisation of the material levels and the buffers in the flow line since material shortages change the optimal buffer allocation in the flow line.

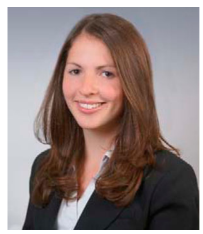
Notes

1. $p_i = 0.9, b_i = 3, r = 30, S_i = 24\ \forall i$.
2. $p_i = 0.9, b_1 = 3, r = 60, S_1 = 60\ \forall i$.
3. $p_i = 0.9, b_1 = 3, S_i = 60\ \forall i$.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributors



Julia Mindlina works as a research assistant at the University of Cologne. She was awarded a Ph.D. from the University of Cologne in 2019. Her main research interest lies in the analysis and optimisation of stochastic manufacturing systems.

Horst Tempelmeier has been Full Professor for Supply Chain Management and Production at the University of Cologne from 1993 to 2020. He has published articles in the *International Journal of Production Research*, *European Journal of Operational Research* and *Management Science*, among others. He is the (co-)author of textbooks on Flexible Manufacturing Systems and on Inventory Analytics. His research interests are Production Management, Inventory Management, and Stochastic Models in Supply Chains Management.

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