



Learning with Covariance Matrices: Foundations and Applications to Network Neuroscience

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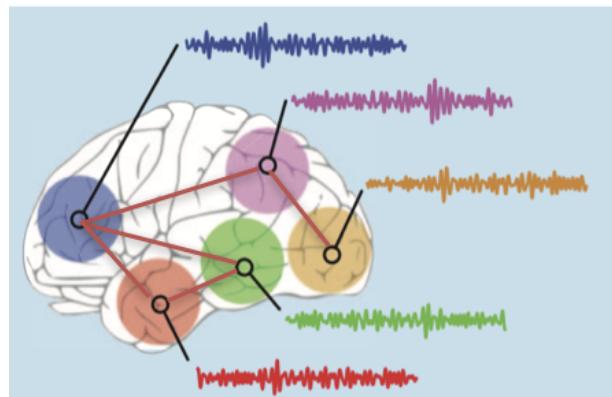
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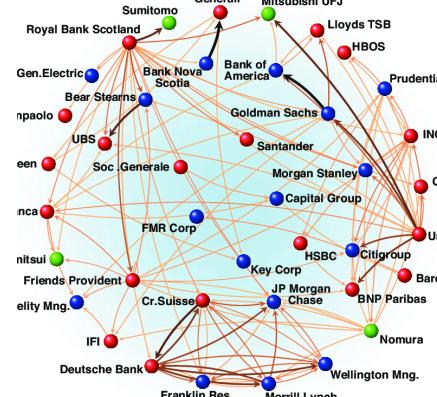
IEEE International Workshop on Machine Learning
for Signal Processing (MLSP), 2025

Covariance Matrix

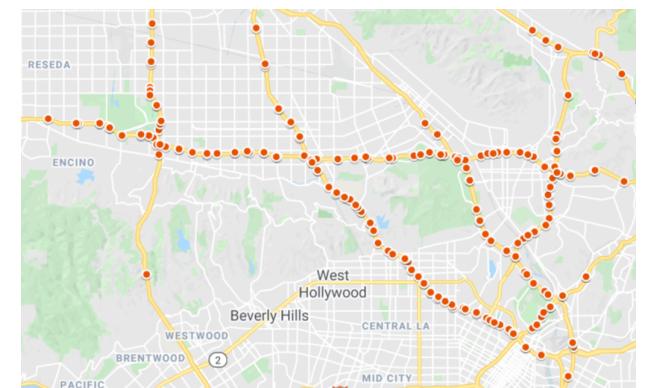
- Covariance matrix captures the **redundancies** between data points (features)
 - **Brain datasets:** some areas of the brain activate together
 - **Financial datasets:** stock prices fluctuate in tandem
 - **Traffic datasets:** traffic volume is correlated across intersections



Brain



Finance



Traffic

Covariance Matrix

➤ Evaluating a covariance matrix

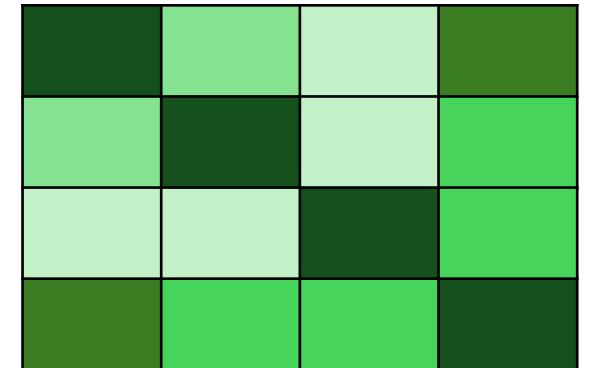
- Consider a random variable $\mathbf{x} \in \mathbb{R}^m$
- The covariance is

$$\mathbf{C} = \mathbb{E}[(\mathbf{x} - \boldsymbol{\mu})(\mathbf{x} - \boldsymbol{\mu})^\top], \text{ where } \boldsymbol{\mu} = \mathbb{E}[\mathbf{x}]$$

- In practice, we have **sample** covariance matrix (an estimate)

$$\hat{\mathbf{C}} = \frac{1}{n-1} \sum_{i=1}^n (\mathbf{x}_i - \hat{\boldsymbol{\mu}})(\mathbf{x}_i - \hat{\boldsymbol{\mu}})^\top, \text{ where } \hat{\boldsymbol{\mu}} = \frac{1}{n} \sum_{i=1}^n \mathbf{x}_i$$

n: number of samples (size of a dataset)



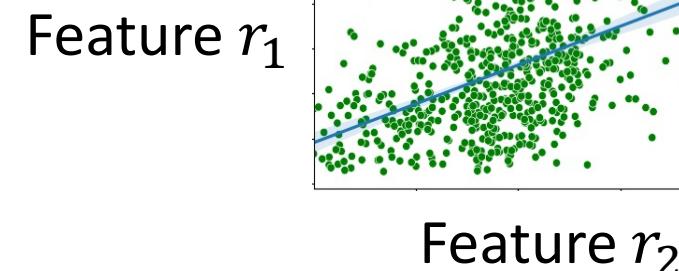
Covariance Matrix

- Covariance matrix encodes **redundancies** between different features in data

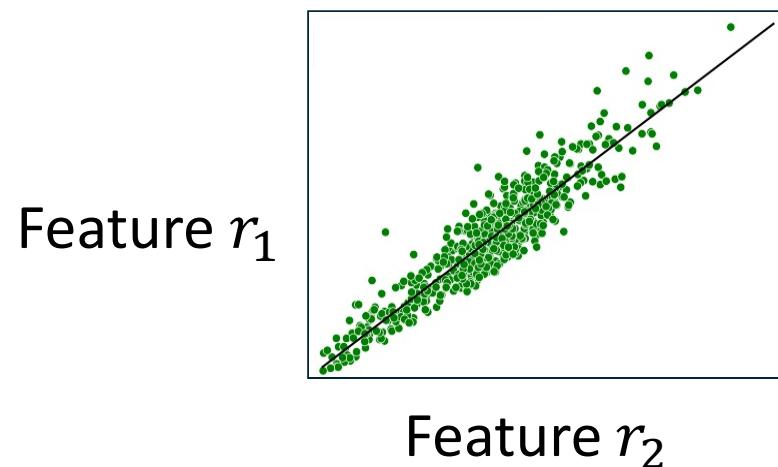
Covariance matrix
(2-feature dataset)

$\sigma^2(r_1)$	$\sigma(r_1, r_2)$
$\sigma(r_1, r_2)$	$\sigma^2(r_2)$

Low redundancy
(smaller $\sigma(r_1, r_2)$)



High redundancy
(higher $\sigma(r_1, r_2)$)



$\sigma(r_1, r_2)$ = how features r_1 and r_2 vary with respect to each other

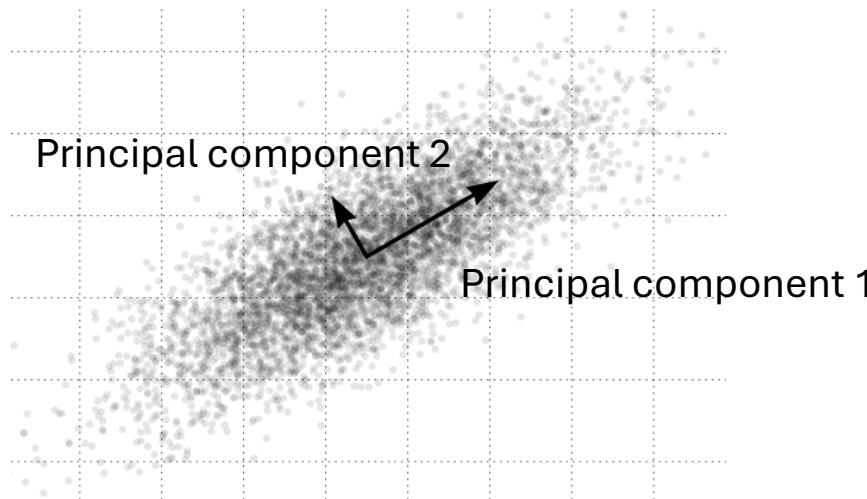
Covariance matrices are widespread in signal processing and machine learning

➤ Principal component analysis (PCA)

- Eigenvectors of the covariance matrix form principal components (PCs)
- PCs inform the shape of a dataset (directions of variance)

Given sample \mathbf{x} and eigendecomposition $\hat{\mathbf{C}} = \hat{\mathbf{V}}\hat{\Lambda}\hat{\mathbf{V}}^T$,

$$\text{PCA transform: } \tilde{\mathbf{x}} = \hat{\mathbf{V}}^T \mathbf{x}$$



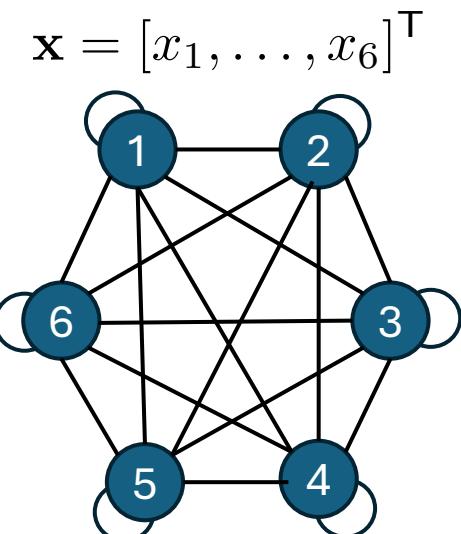
PCA transform in ML

- Unsupervised learning (dim. reduction)
- Supervised learning (regression, classification)

Covariance matrices are widespread in signal processing and machine learning

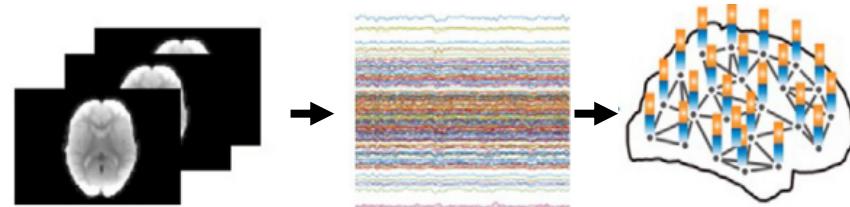
➤ Covariance matrices are leveraged as **graphical** representations of data

- A graph $G = (V, E, W)$
 - Set of nodes V
 - Set of edges E
 - A weight function W
- Covariance matrix is a **fully connected graph**,
 - nodes are the features
 - edges associated with pairwise covariance values

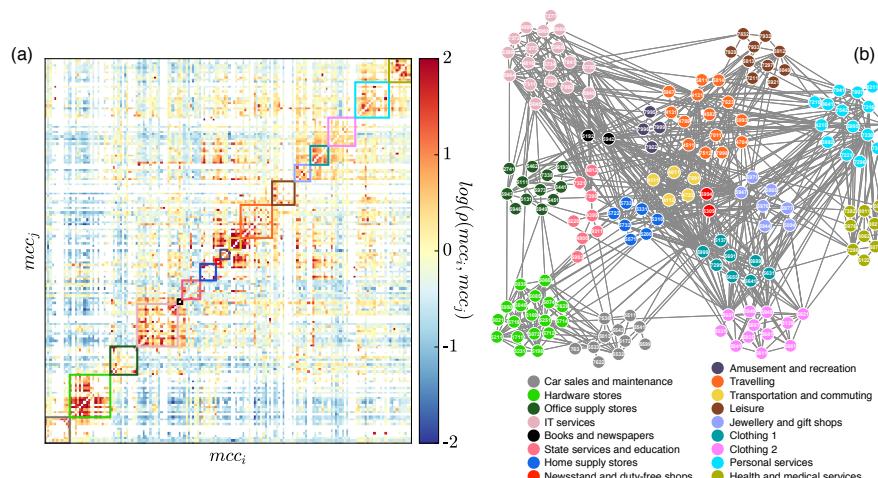


Covariance matrices are widespread in signal processing and machine learning

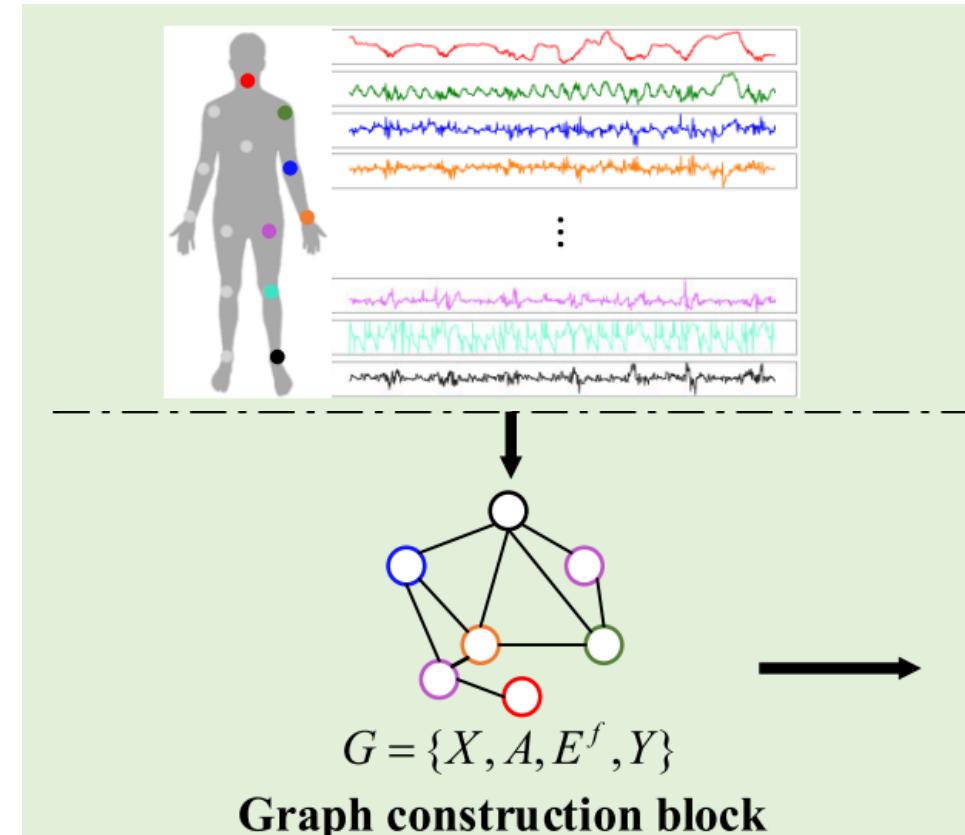
- Covariance matrices as **graphical** representations; used in graph neural nets



Brain connectome [Li, et al. 2021]



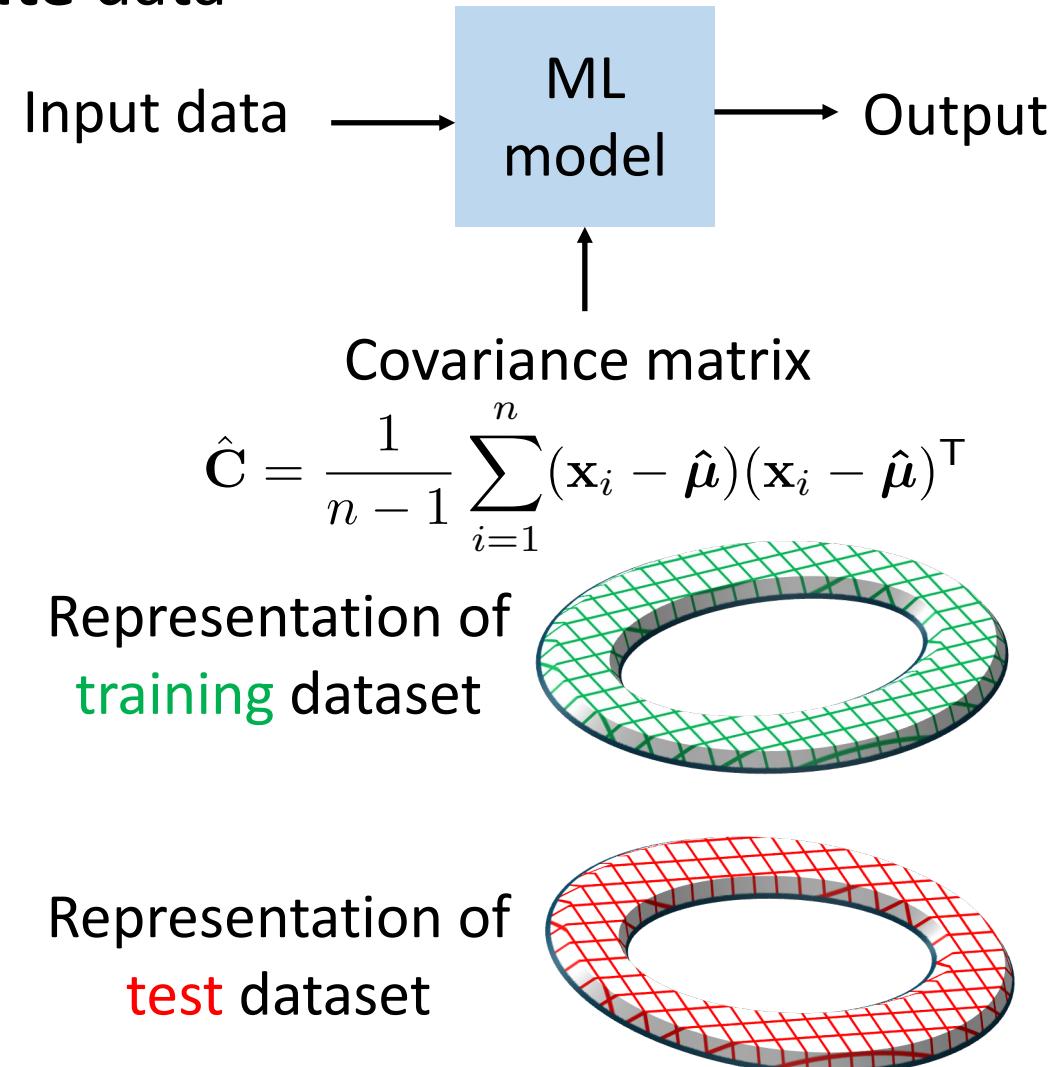
Socio-economic networks [Leo, et al. 2016]



Wearable devices [Wang, et al. 2023]

Learning with covariance matrices: Challenges

- Sample covariance matrix is estimate from **finite** data
- ML model is trained on **training** dataset, deployed on **test** dataset
- Statistical spaces defined by **training** and **test** data may not align perfectly

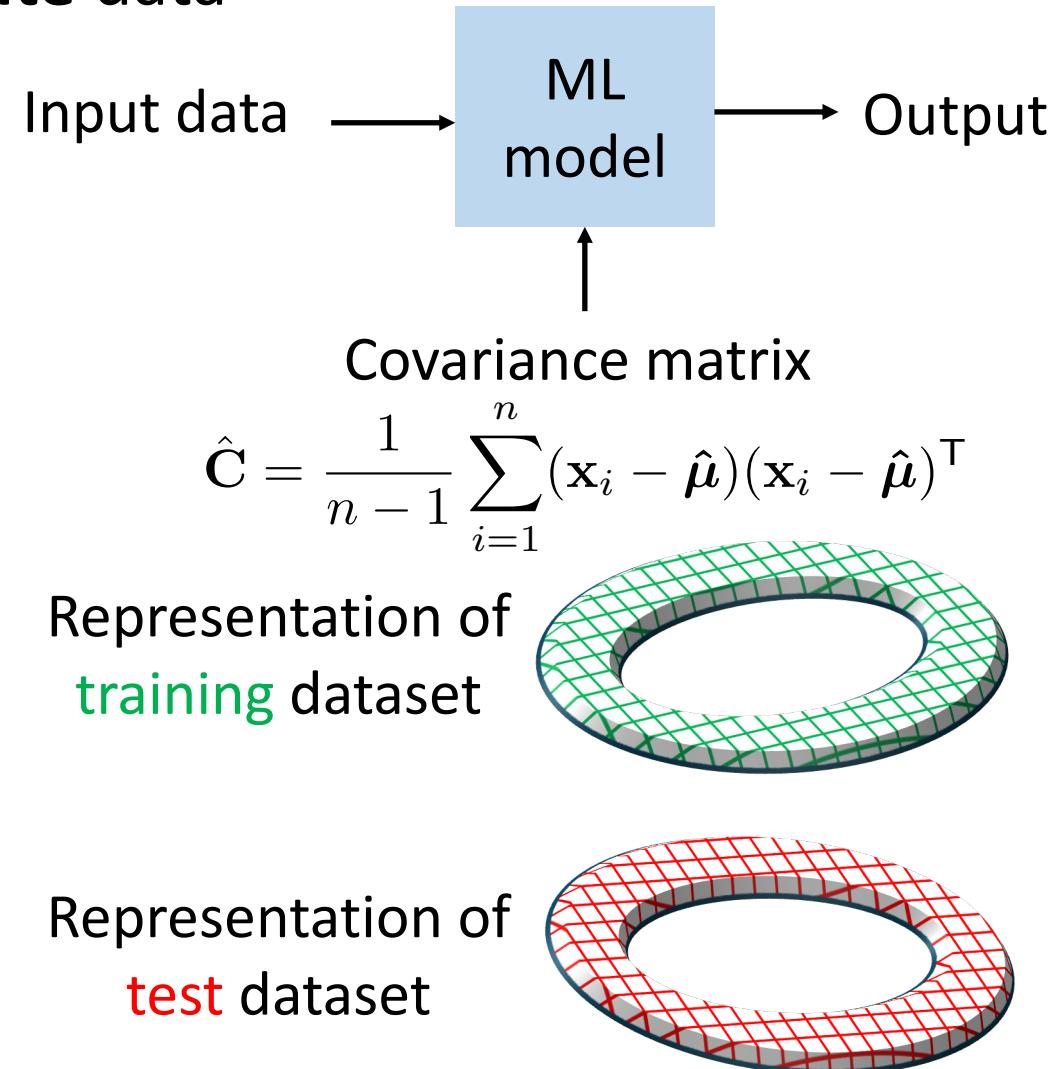


Learning with covariance matrices: Challenges

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Challenge 1 (stability)

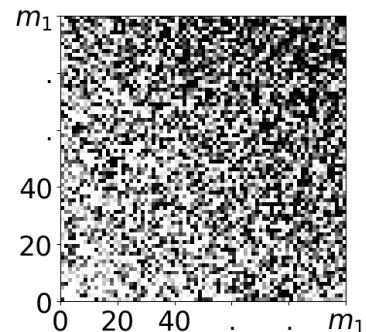
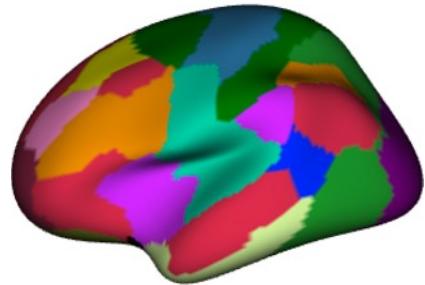
Are inference outcomes **stable** to perturbations in covariance matrix (finite sample effect)?



Learning with covariance matrices: Challenges

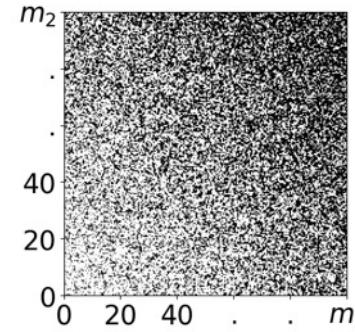
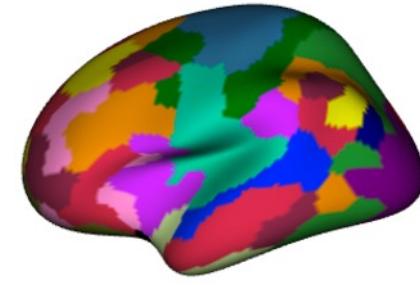
- Datasets capture information about same phenomenon at **different scales**

Dataset with m_1 features



Covariance matrix \mathbf{C}_{m_1}
(size $m_1 \times m_1$)

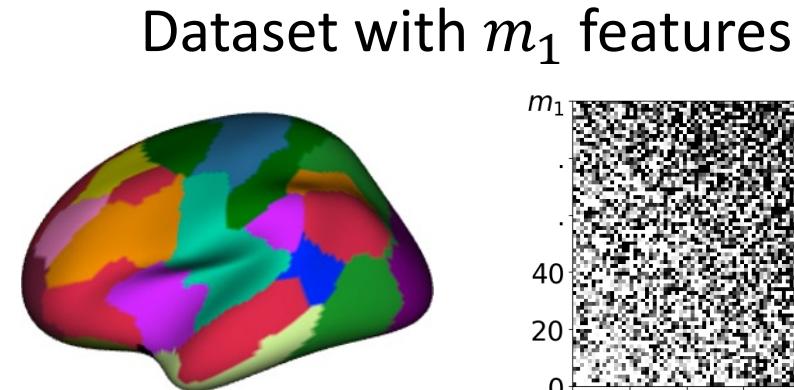
Dataset with m_2 features



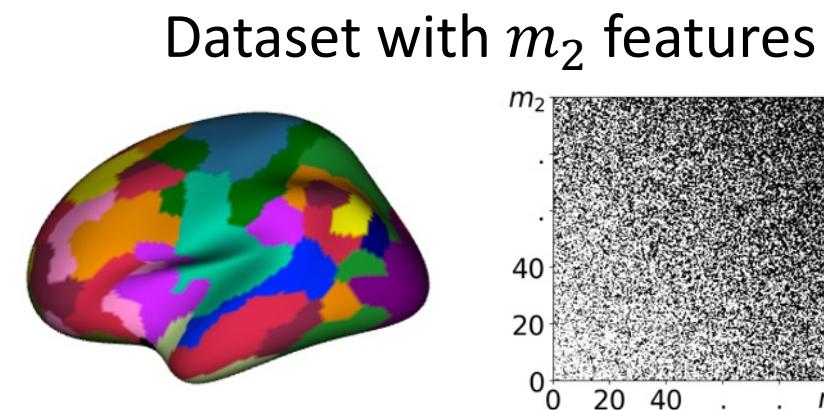
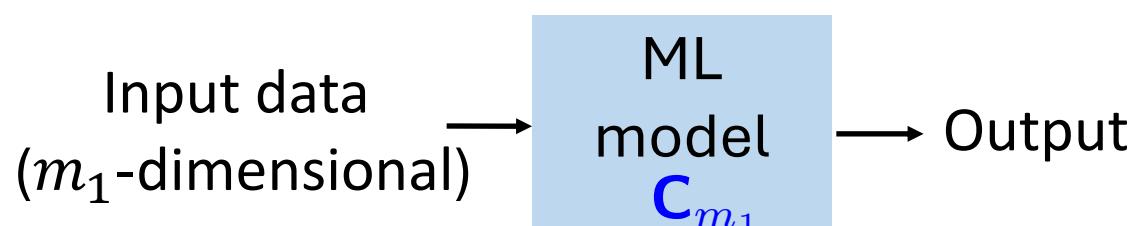
Covariance matrix \mathbf{C}_{m_2}
(size $m_2 \times m_2$)

Learning with covariance matrices: Challenges

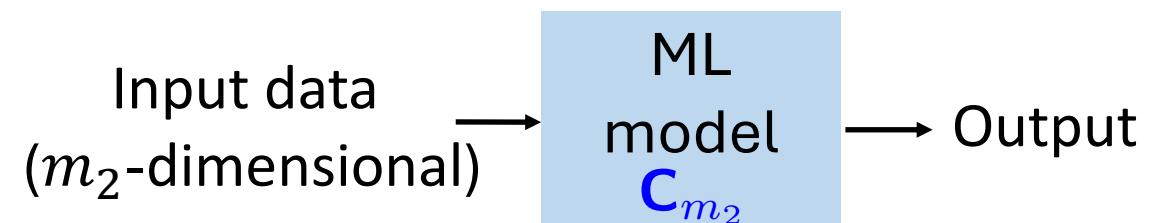
- Datasets capture information about same phenomenon at **different scales**



Covariance matrix \mathbf{C}_{m_1}
(size $m_1 \times m_1$)



Covariance matrix \mathbf{C}_{m_2}
(size $m_2 \times m_2$)



Challenge 2 (transferability)

Can the redundancy in covariance matrices of datasets of different sizes be exploited?

Learning with covariance matrices: A GSP approach

- Signal and information processing is about exploiting signal structure
- Graph signal processing (GSP): broaden classical signal processing to graphs



Graph Signal Processing: Overview, Challenges, and Applications

This article presents methods to process data associated to graphs (graph signals) extending techniques (transforms, sampling, and others) that are used for conventional signals.

By ANTONIO ORTEGA¹, Fellow IEEE, PASCAL FROSSARD, Fellow IEEE, JELENA KOVACIĆ, Fellow IEEE, JOSE M. F. MOURA², Fellow IEEE, and PIERRE VANDERHEYDEN

ABSTRACT | Research in graph signal processing (GSP) aims to develop tools for processing data defined on irregular graph domains. In this paper, we first provide an overview of core ideas in GSP and their connection to conventional digital signal processing, along with a brief historical perspective to highlight how concepts recently developed in GSP build on top of prior research in other areas. We then summarize recent advances in developing basic GSP tools, including methods for sampling, filtering, or graph learning. Next, we review progress in several application areas using GSP, including processing and analysis of sensor network data, biological data, and applications to image processing and machine learning.

KEYWORDS | Graph signal processing (GSP); network science and graphs; sampling; signal processing

I. INTRODUCTION AND MOTIVATION

Data is all around us, and massive amounts of it. Almost every aspect of human life is now being recorded at all levels: from the marking and recording of processes inside the cells starting with the advent of fluorescent markers, to our personal data through health monitoring devices and apps, financial and banking data, our social networks, mobility and traffic patterns, marketing preferences, fads, and many more. The complexity of such networks [1] and interactions mean that the data now reside on irregular and complex structures that do not lend themselves to standard tools.

Typical graphs that are used to represent common real-world data include Erdős-Rényi graphs, ring graphs, random geometric graphs, small-world graphs, power-law graphs, nearest-neighbor graphs, scale-free graphs, and many others. These model networks with random connections (Erdős-Rényi graphs), networks of brain neurons (small-world graphs), social networks (scale-free graphs), and others.

As in classical signal processing, graph signals can have properties that are specific to the graph structure they are defined on. They can also be represented via basis atoms and can have a spectral representation. In particular, the graph Fourier transform allows us to develop the intuition gathered in the classical setting and extend it to graphs: we can talk about the notions of frequency and bandlimitness,



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Graph Signal Processing

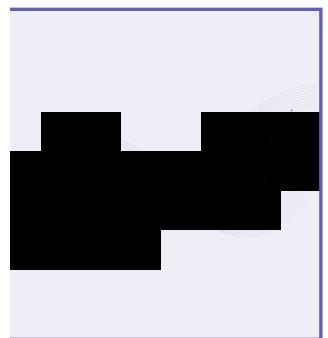
History, development, impact, and outlook

Xiaowen Dong, Dorina Thanou, Laura Toni,
Michael Bronstein, and Pascal Frossard

GRAPH SIGNAL PROCESSING:
FOUNDATIONS AND EMERGING DIRECTIONS

Graph Signal Processing for Machine Learning

A review and new perspectives



The effective representation, processing, analysis, and visualization of large-scale structured data, especially those related to complex domains, such as networks and graphs, are one of the key questions in modern machine learning. Graph signal processing (GSP) is a branch of signal processing models and algorithms that aims at handling data supported on graphs, opens new paths of research to address this challenge. In this article, we review a few important contributions made by GSP concepts and tools, such as graph filters and transforms, to the development of novel machine learning algorithms. In particular, our discussion focuses on the following three aspects: exploiting data structure and relational priors, improving data and computational efficiency, and enhancing model interpretability. Furthermore, we provide new perspectives on the future development of GSP techniques that may serve as a bridge between applied mathematics and signal processing on one side and machine learning and network science on the other. Cross-fertilization across these different disciplines may help unlock the numerous challenges of complex data analysis in the modern age.

Introduction

We live in a connected society. Data collected from large-scale interactive systems, such as biological, social, and financial networks, become largely available. In parallel, the past few decades have seen a significant amount of interest in the machine learning community for network data processing and analysis. Networks have an intrinsic structure that conveys very specific properties to data, e.g., interdependences between data entities in the form of pairwise relationships. These properties are traditionally captured by mathematical representations such as graphs.

In this context, new trends and challenges have been developing fast. Let us consider, for example, a network of interconnected interacting nodes and the evolution of individual genes at every point in time. Some typical tasks in network biology related to this type of data are 1) discovery of key genes (via protein grouping) affected by the infection and 2) prediction of how the host organism reacts (in terms of gene expression)

Manuscript received November 25, 2017; revised March 01, 2018; accepted March 23, 2018. First posted February 24, 2018. Corresponding author: Antonio Ortega.

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Digital Object Identifier 10.1109/MSP.2018.280126

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- **Graph signal processing (GSP):** broaden classical signal processing to graphs



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Graph Signal Processing *History, development, impact, and outlook*



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Graph Signal Processing

History, development, impact, and outlook

Signal processing (SP) excels at analyzing, processing, and inferring information defined over regular (first continuous, later discrete) domains such as time or space. Indeed, the last 75 years have shown how SP has made an impact in

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Graph Signal Processing for Machine Learning

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GRAPH SIGNAL PROCESSING:
FOUNDATIONS AND EMERGING DIRECTIONS

The effective representation, processing, analysis, and visualization of large-scale structured data, especially those related to complex domains, such as networks and graphs, are one of the key questions in modern machine learning. Graph signal

Our view: GSP perspective well-equipped to address challenges to learning with covariance matrices

Manuscript received from the authors on 10 January 2018; revised on 10 April 2018; accepted on 10 May 2018. This work was supported by grants from the Swiss National Science Foundation (SNSF) and the European Union (EU).

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Digital Object Identifier 10.1109/ICASSP.2018.8420126

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ence or similarity across nodes, and the links must be inferred from the data themselves. As a result, GSP is a broad framework that encompasses and extends classical SP methods, tools, and algorithms to application domains of the modern technological world, including social, transportation, communication,

opening new doors to us compared to the classic, a network of individual genes at every point in time. Some typical tasks in network biology related to this type of data are 1) discovery of key genes (via protein grouping) affected by the infection and 2) prediction of how the host organism reacts (in terms of gene expression)

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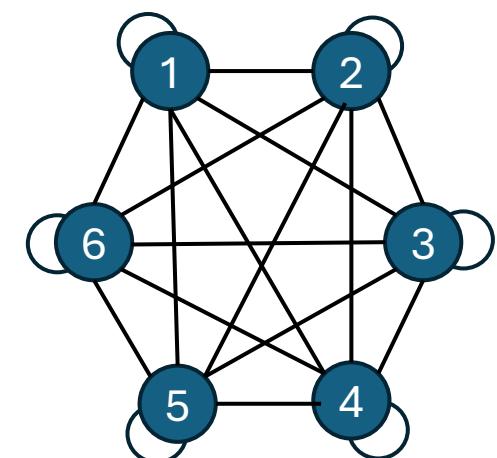
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Learning with covariance matrices: A GSP approach

- Graph neural networks (GNNs) have been shown to be [Ruiz et al., 2023]
 - stable to (**abstract**) perturbations in graph structure
 - generalizable to graph structures of different sizes
(similar to convolutional neural nets for images)

$$\mathbf{x} = [x_1, \dots, x_6]^T$$



- Covariance matrix is a **data-driven** graph
 - interplay between perturbation theory of covariances and ML over them

Outline

- PCA and the graph Fourier transform
- CoVariance neural networks (VNNs)
- Theory of VNNs: Stability and transferability
- Variants of VNNs

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- PCA and the graph Fourier transform
- Covariance neural networks (VNNs)
- Theory of VNNs: Stability and transferability
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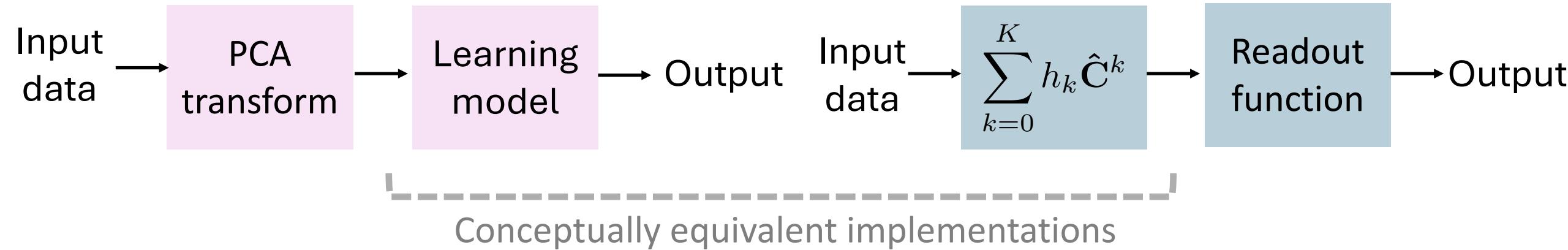
Key takeaways:

- VNNs offer a novel GSP-inspired perspective to PCA
 - ➡ addressing challenges in modern data analysis
- Principled deep learning solution for finite-data regimes

PCA and Graph Fourier Transform

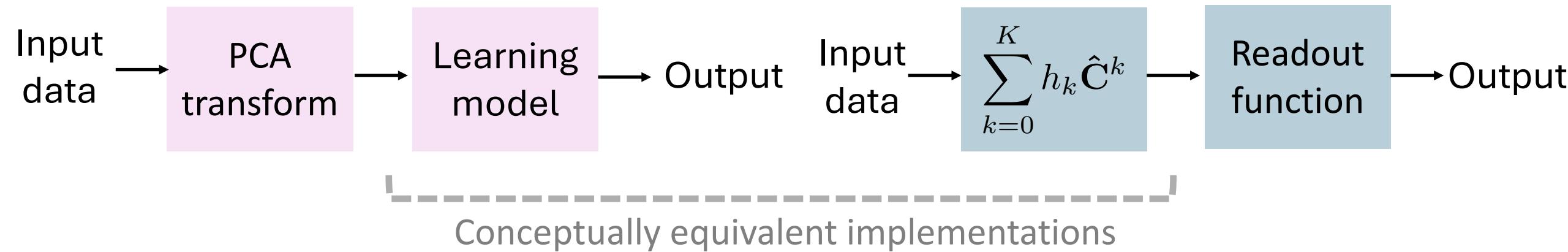
A graph filter implementation of PCA inference

- **To show:** PCA-based inference can be implemented with a polynomial over $\hat{\mathbf{C}}$



A graph filter implementation of PCA inference

- **To show:** PCA-based inference can be implemented with a polynomial over $\hat{\mathbf{C}}$



- **How:** Follows from the graph Fourier transform analysis of $\sum_{k=0}^K h_k \hat{\mathbf{C}}^k$
- **Implications:**
 - Alternative implementation of PCA-based inference using polynomial over $\hat{\mathbf{C}}$
 - But more importantly, polynomial implementation is **stable, transferable**

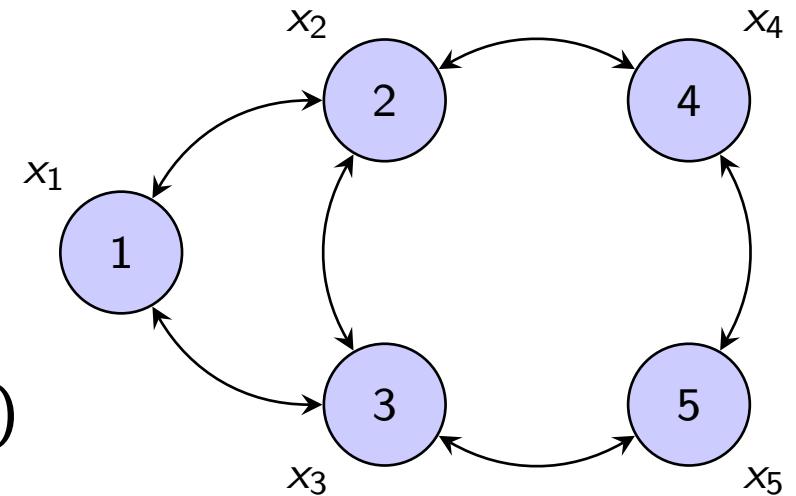
Preliminaries: Graph

➤ **Graph:** a triplet (V, E, W)

- A set of **nodes** $V = \{1, \dots, m\}$
- A set of (undirected) **edges** $E \subseteq V \times V$

Edge between node i and j denoted by (i, j)

- An **edge function** $W: E \mapsto \mathbb{R}$ that maps edge (i, j) to weight $w_{ij} \in \mathbb{R}$



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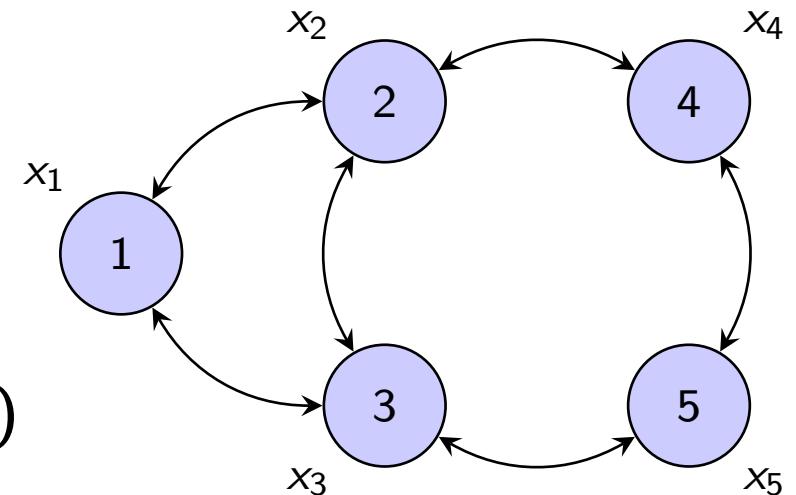
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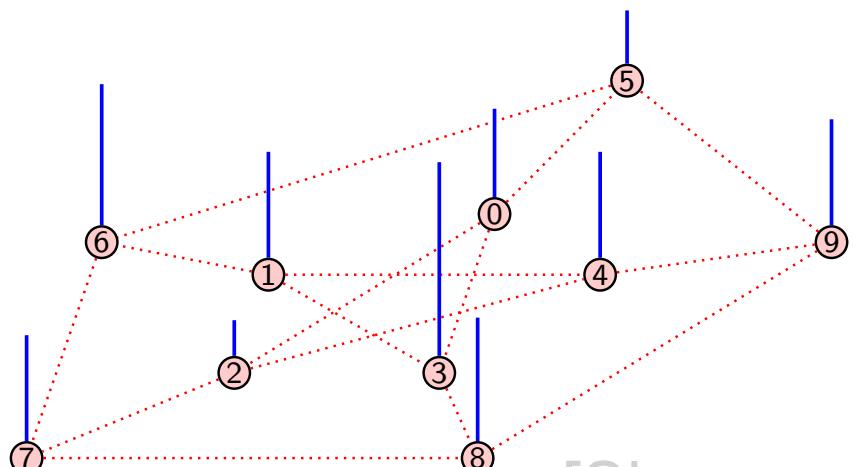
➤ **Adjacency matrix** representation of graph

$$[\mathbf{A}]_{ij} = \begin{cases} w_{ij}, & \text{if } (i, j) \in E, \\ 0, & \text{otherwise} \end{cases}$$



Preliminaries: Graph signal

- **Graph signals** are mappings $x: V \mapsto \mathbb{R}$
 - ➡ graph signal is defined on the vertices of the graph
- **Graph signal** can be represented as a vector $\mathbf{x} \in \mathbb{R}^m$
 - ➡ x_i denotes the graph signal at i -th vertex in V

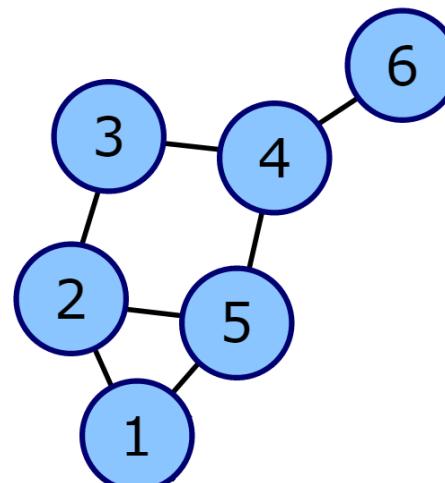


[Shuman, 2013]

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_9 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.7 \\ 0.3 \\ \vdots \\ 0.7 \end{bmatrix}$$

Preliminaries: Graph shift operator (GSO)

- To understand and analyze graph signal \mathbf{x} , GSP accounts for the graph structure
- Graph structure is encoded in a **graph shift operator** $\mathbf{S} \in \mathbb{R}^{m \times m}$
 - ➡ $[\mathbf{S}]_{ij} = 0$ for $i \neq j$ and $(i, j) \notin E$ (\mathbf{S} captures local graph structure)



$$\mathbf{S} = \begin{pmatrix} S_{11} & S_{12} & 0 & 0 & S_{15} & 0 \\ S_{21} & S_{22} & S_{23} & 0 & S_{25} & 0 \\ 0 & S_{23} & S_{33} & S_{34} & 0 & 0 \\ 0 & 0 & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & 0 & S_{54} & S_{55} & 0 \\ 0 & 0 & 0 & S_{64} & 0 & S_{66} \end{pmatrix}$$

- Examples: adjacency matrix, Laplacian

Covariance matrix is a **data-driven** adjacency matrix

Preliminaries: Graph Fourier Transform (GFT)

- Generically, eigendecomposition of GSO $\mathbf{S} = \mathbf{U}\Phi\mathbf{U}^{-1}$
- **GFT** is the projection of graph signal on the eigenvector space \mathbf{U}

$$\tilde{\mathbf{x}} = \mathbf{U}^{-1}\mathbf{x}$$

- **Inverse GFT** is defined as



$$\mathbf{x} = \mathbf{U}\tilde{\mathbf{x}}$$

Eigenvectors $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_m]$ are the frequency basis

When GSO is covariance matrix...

- GFT over covariance matrix

Given eigendecomposition

$$\hat{\mathbf{C}} = \hat{\mathbf{V}} \hat{\Lambda} \hat{\mathbf{V}}^T$$

GFT of \mathbf{x} is

$$\tilde{\mathbf{x}} = \hat{\mathbf{V}}^T \mathbf{x}$$

When GSO is covariance matrix...

- GFT over covariance matrix

Given eigendecomposition

$$\hat{C} = \hat{V} \hat{\Lambda} \hat{V}^T$$

GFT of \mathbf{x} is

$$\tilde{\mathbf{x}} = \hat{V}^T \mathbf{x}$$

- PCA transform

Projection of sample \mathbf{x} on principal components of \hat{C}

PCA transform: $\tilde{\mathbf{x}} = \hat{V}^T \mathbf{x}$

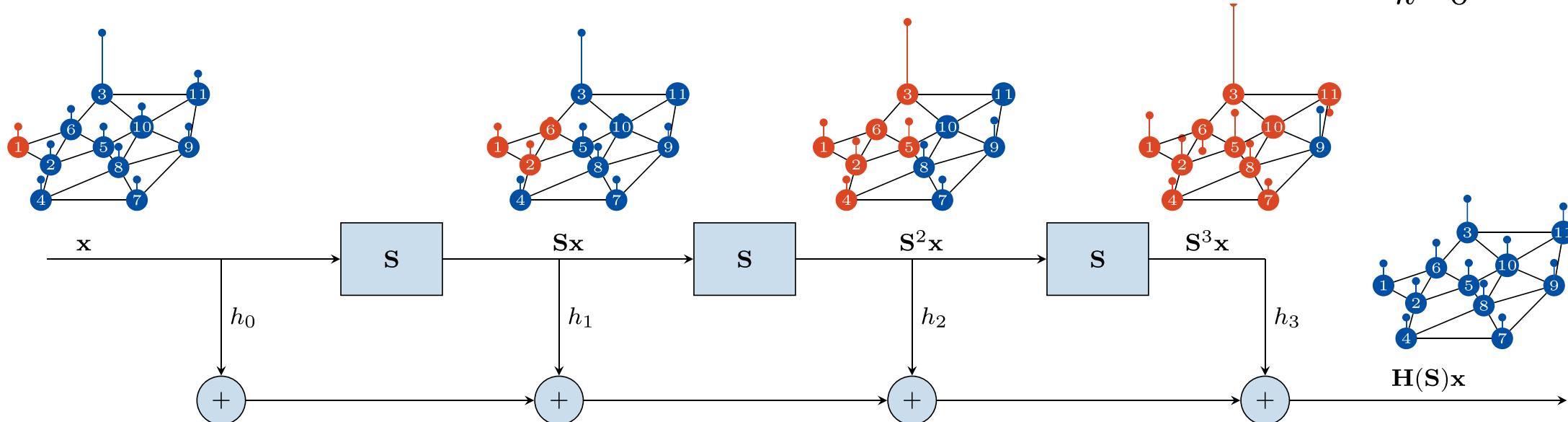
PCA transform is GFT with respect to the covariance graph!

Preliminaries: Graph filter

- **Graph filter \mathbf{H}** maps graph signal \mathbf{x} to another graph signal \mathbf{z} via linear-shift-and-sum operation

$$\mathbf{z} = \mathbf{H}(\mathbf{S})\mathbf{x},$$

$$\text{where } \mathbf{H} := h_0 \mathbf{S}^0 + h_1 \mathbf{S}^1 + h_2 \mathbf{S}^2 + \cdots + h_K \mathbf{S}^K = \sum_{k=0}^K h_k \mathbf{S}^k$$

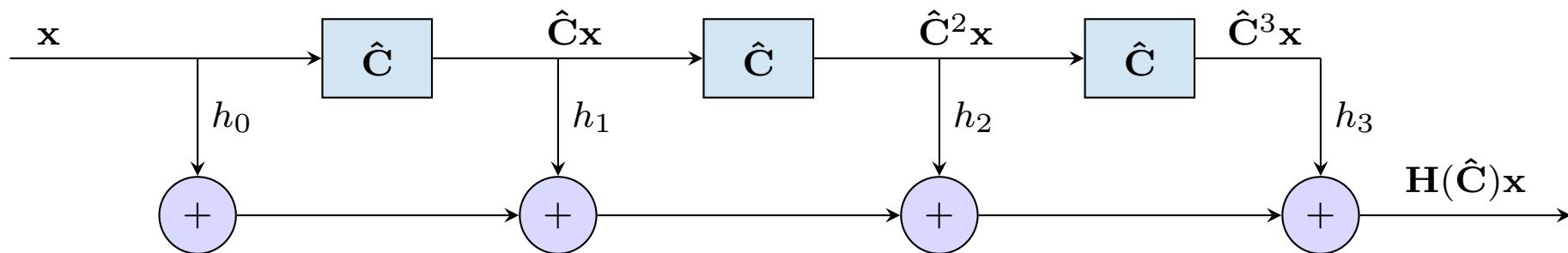
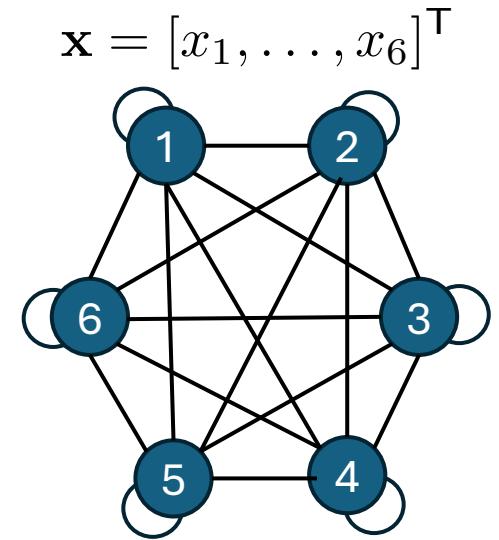


[Isufi et. al, IEEE TSP, 2024]

Graph filter on covariance matrix

- Covariance matrix forms a fully-connected graph where
 - nodes are features
 - edges are covariance values
- Graph filter on covariance matrix $\hat{\mathbf{C}}$ is defined as

$$\mathbf{H}(\hat{\mathbf{C}}) = \sum_{k=0}^K h_k \hat{\mathbf{C}}^k \mathbf{x}$$



CoVariance filter

- Analogy between $\mathbf{H}(\hat{\mathbf{C}})$ and PCA
 - Using eigendecomposition $\hat{\mathbf{C}} = \hat{\mathbf{V}}\hat{\Lambda}\hat{\mathbf{V}}^T$, it follows that

$$\mathbf{z} = \mathbf{H}(\hat{\mathbf{C}})\mathbf{x} = \sum_{k=0}^K h_k \hat{\mathbf{C}}^k \mathbf{x} = \sum_{k=0}^K h_k \hat{\mathbf{V}} \hat{\Lambda}^k \hat{\mathbf{V}}^T \mathbf{x} = \hat{\mathbf{V}} \left(\underbrace{\sum_{k=0}^K h_k \hat{\Lambda}^k}_{\text{Frequency response}} \right) \hat{\mathbf{V}}^T \mathbf{x}$$



CoVariance filter

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PCA

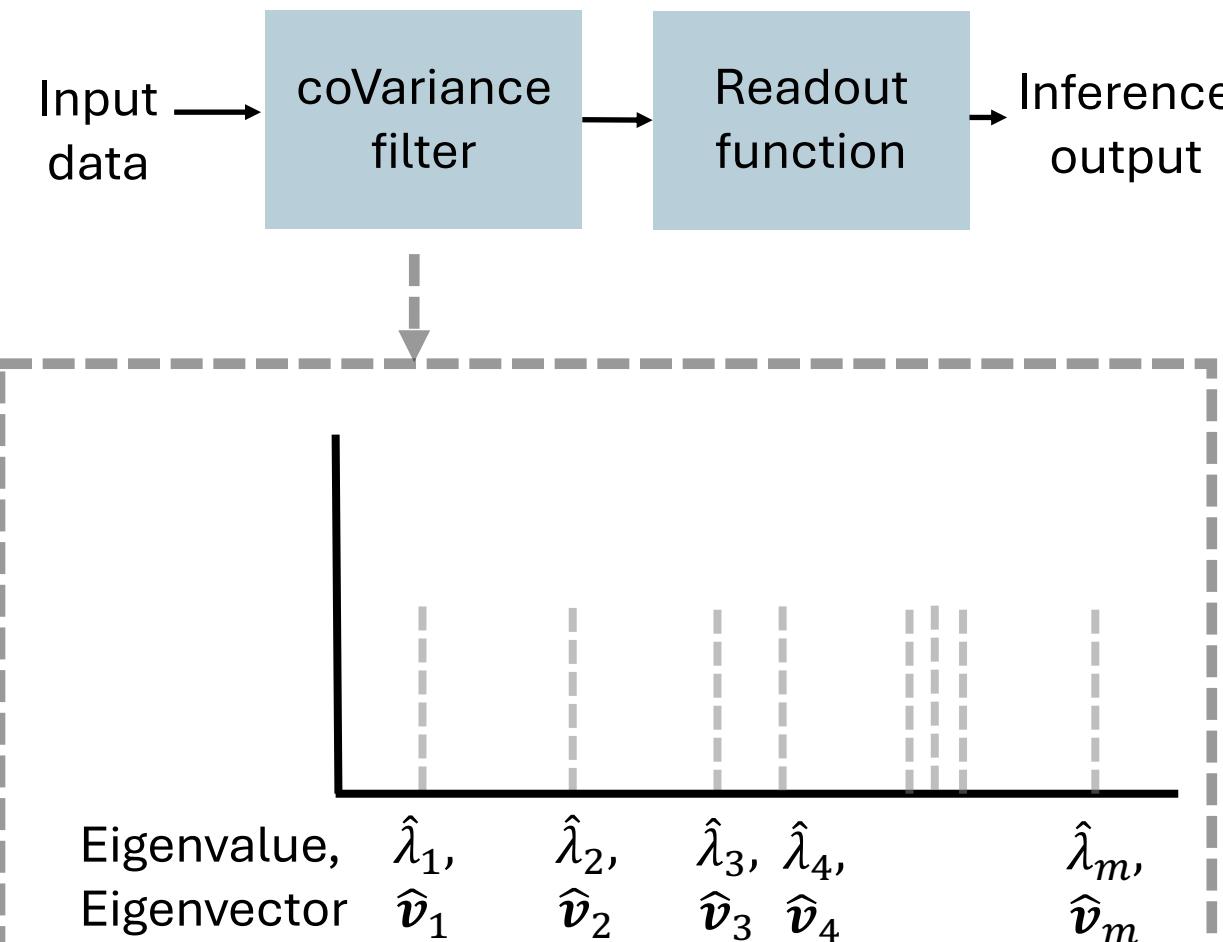
- GFT of coVariance filter output \mathbf{z} and PCA are **equivalent**

$$\tilde{\mathbf{z}} = \left(\sum_{k=0}^K h_k \hat{\Lambda}^k \right) \hat{\mathbf{V}}^T \mathbf{x}$$

i -th component of $\tilde{\mathbf{z}}$ is modulated by $h(\lambda_i) = \sum_{k=0}^K h_k \lambda_i^k$

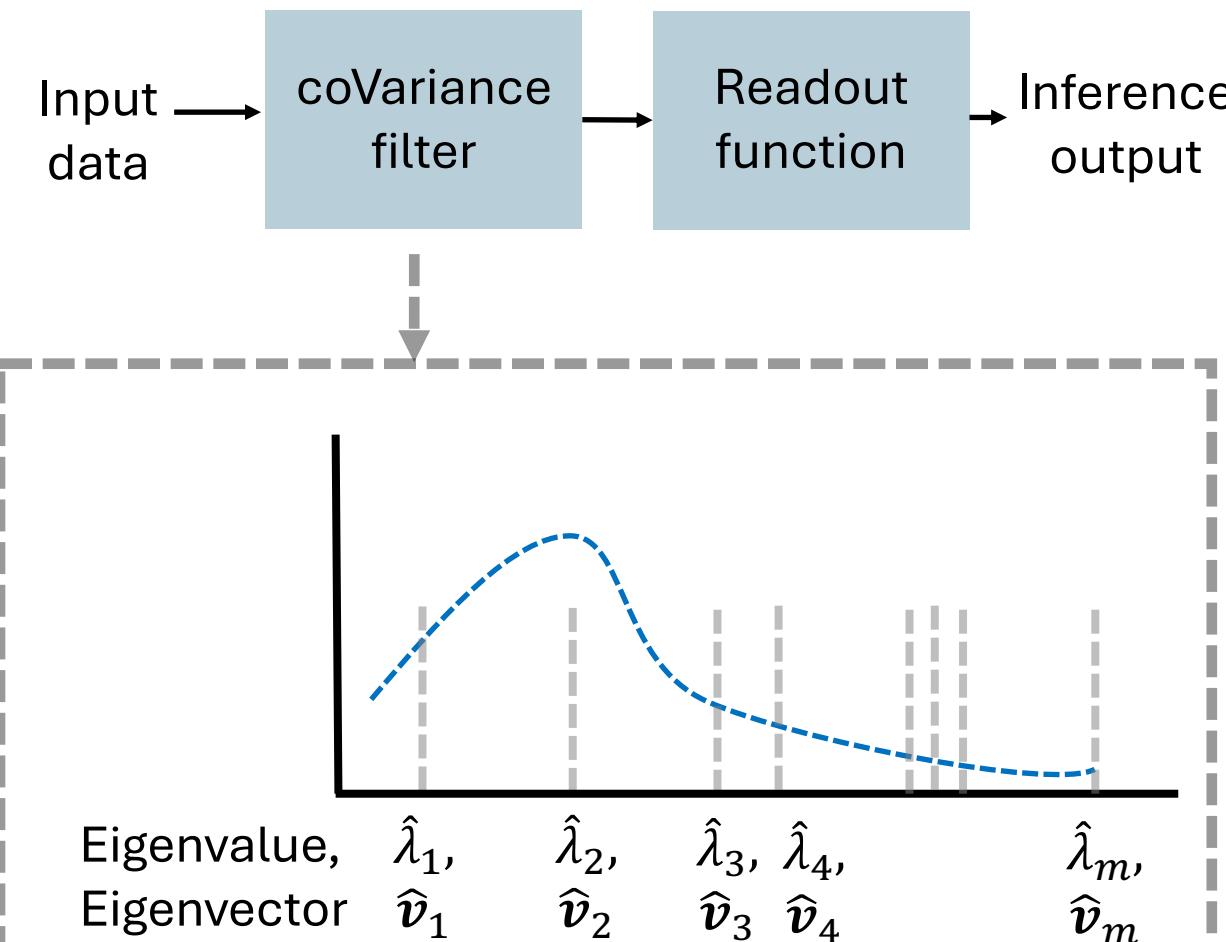
Learning with coVariance filter versus PCA-based learning

- Learning with a **coVariance** filter



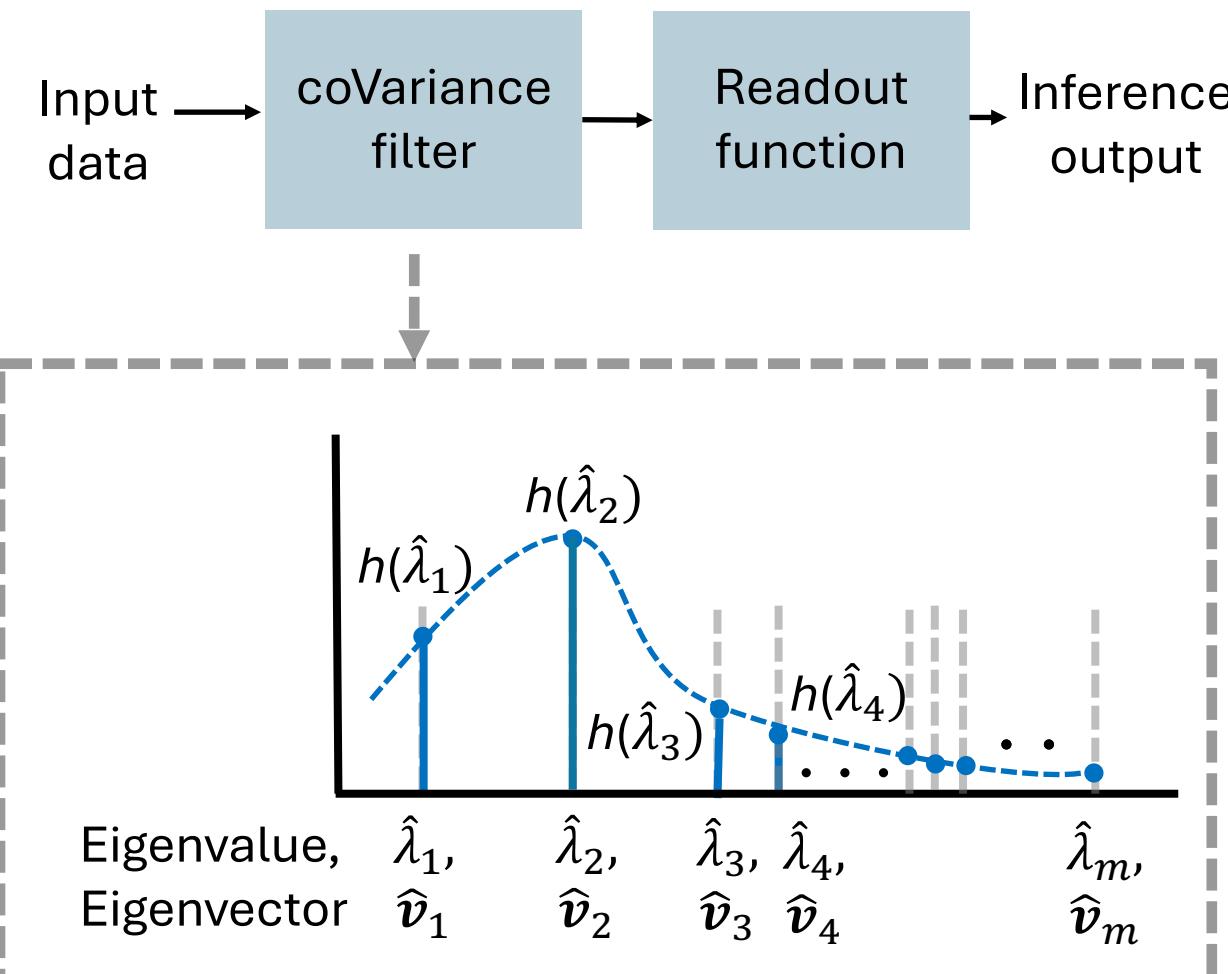
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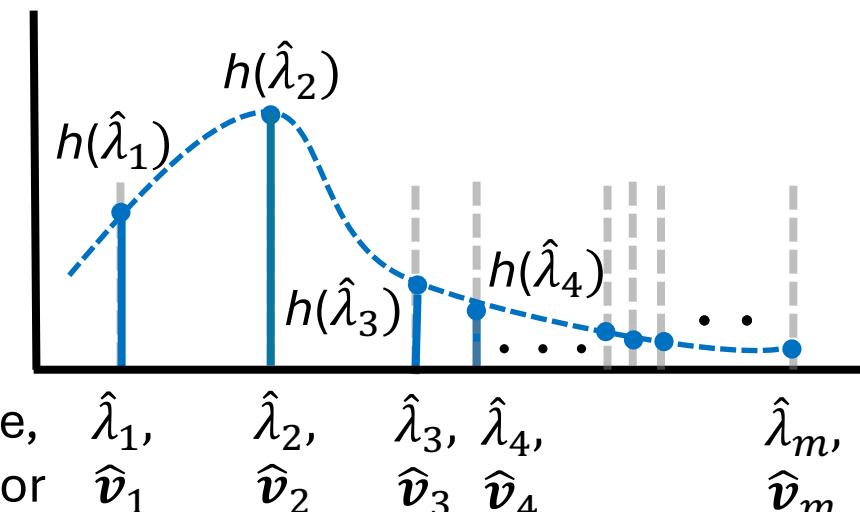
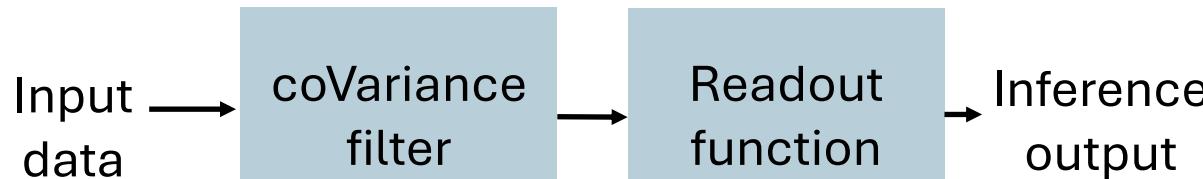
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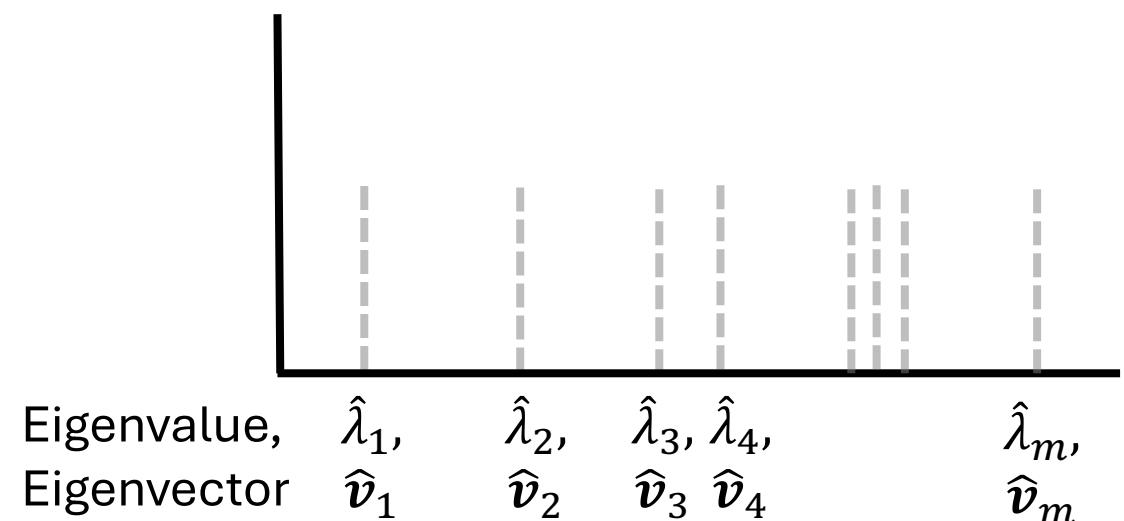
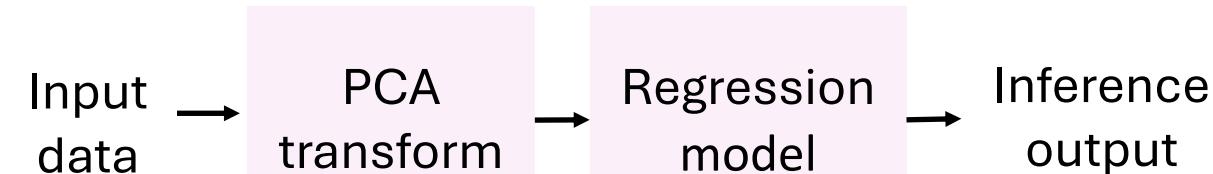


Learning with coVariance filter versus PCA-based learning

- Learning with a **coVariance** filter

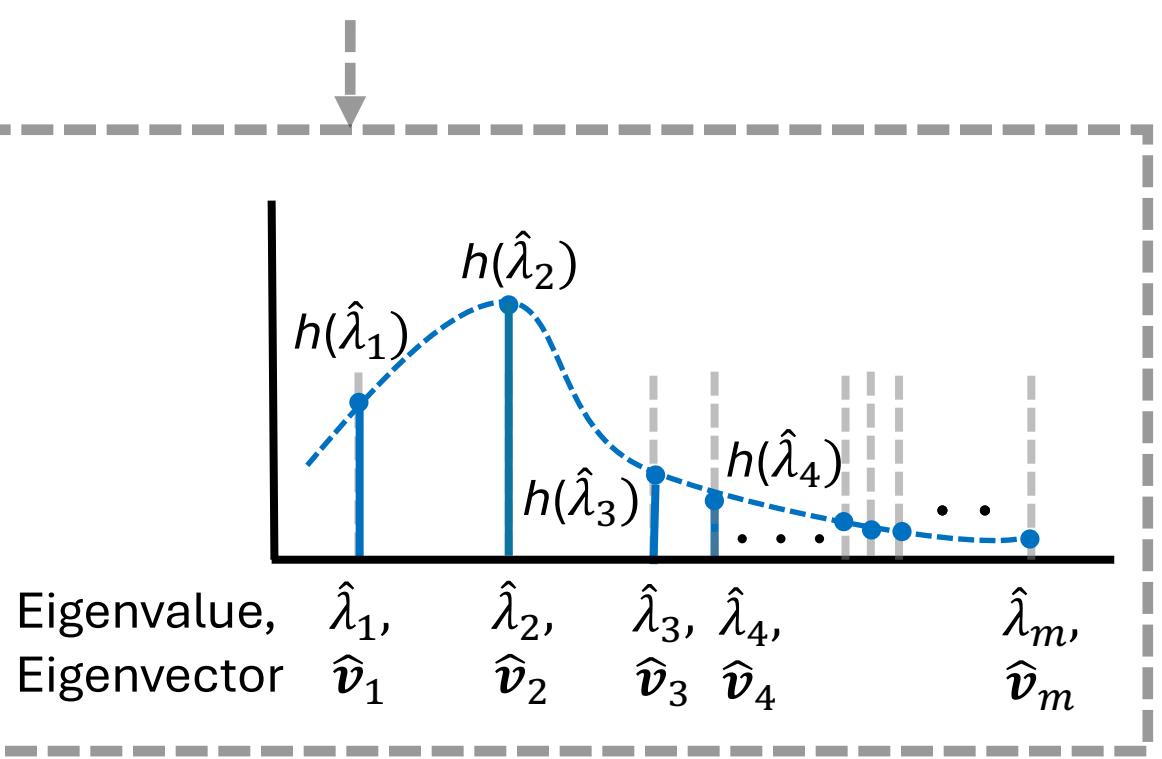
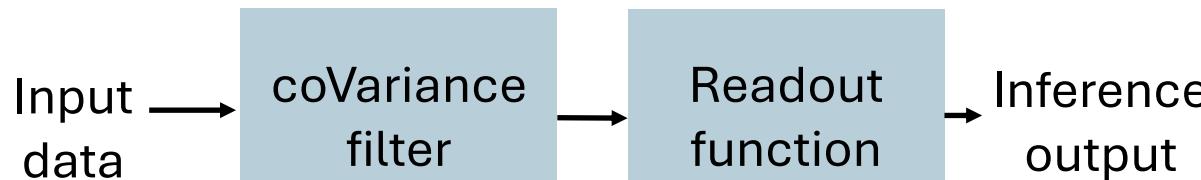


- PCA-based learning

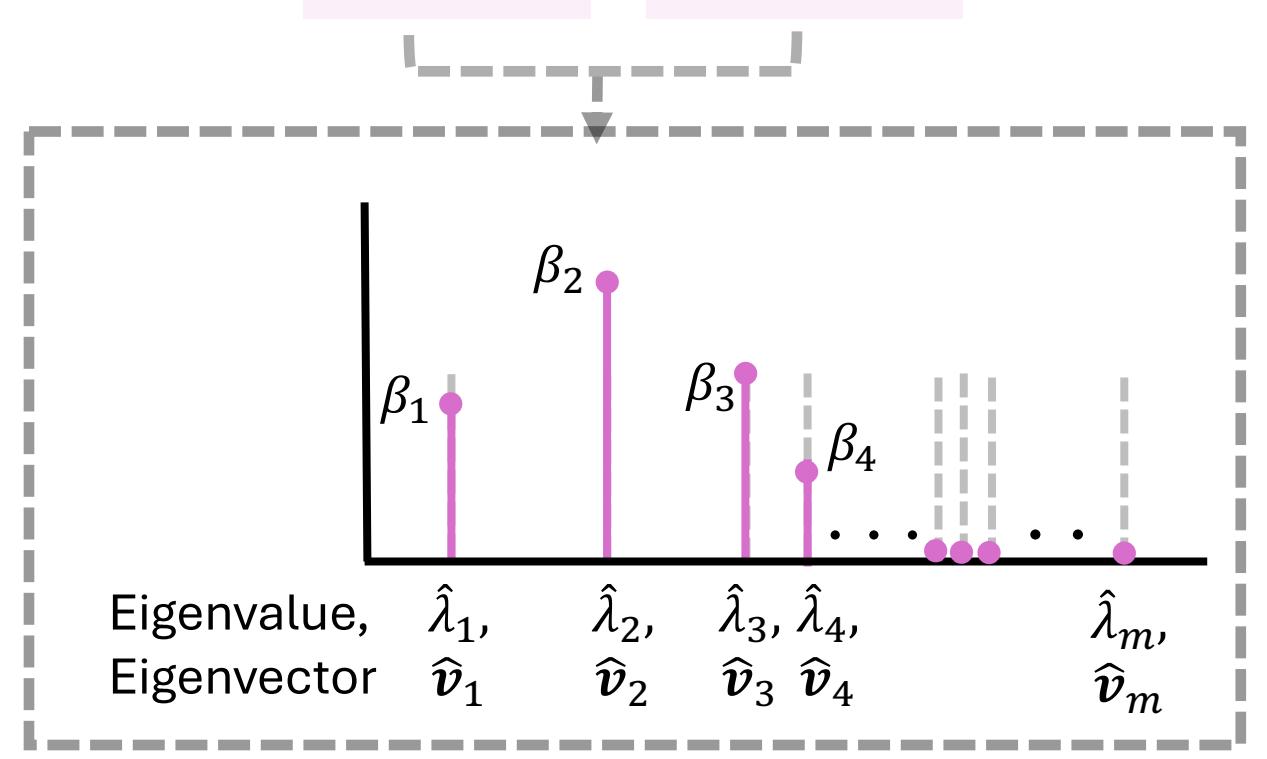
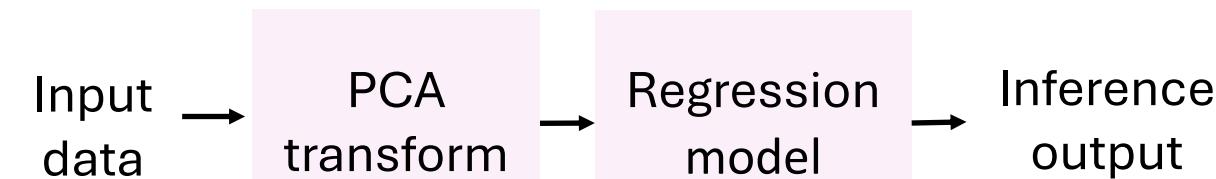


Learning with coVariance filter versus PCA-based learning

- Learning with a **coVariance** filter



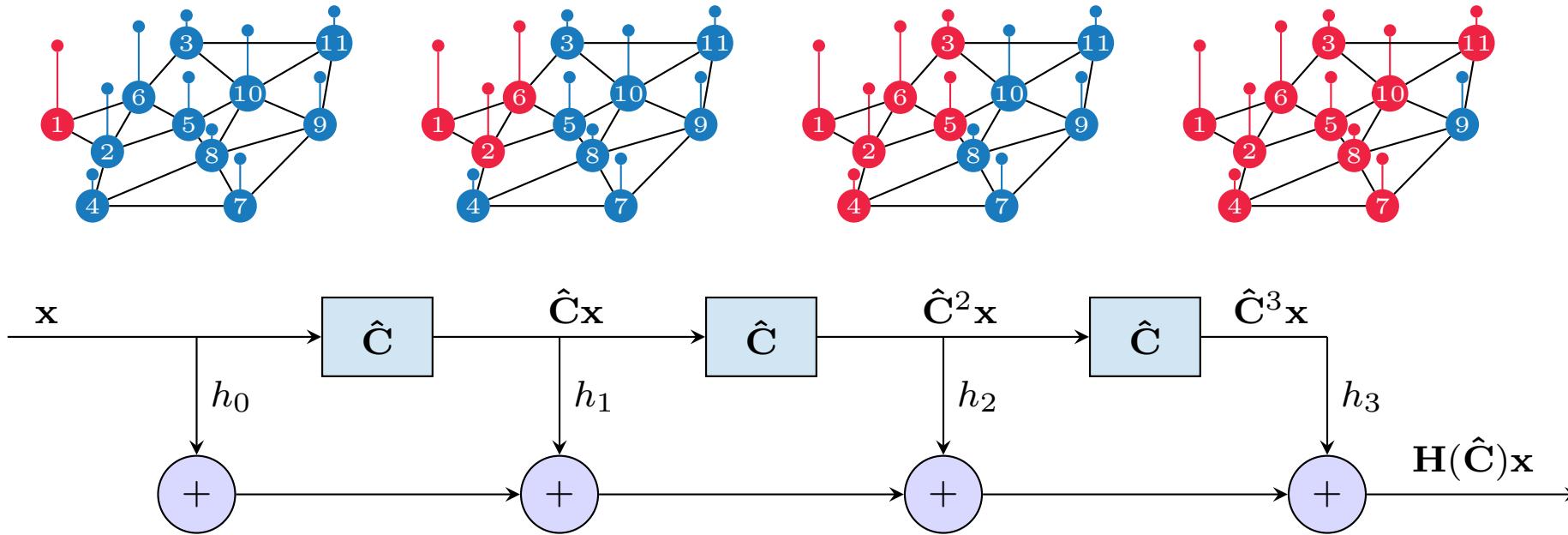
- PCA-based learning



coVariance Neural Networks (VNNs)

coVariance filters as convolutional operators

- Operation $\hat{\mathbf{C}}^k \mathbf{x}$ performs a k -shift of signal \mathbf{x} over graph defined by $\hat{\mathbf{C}}$



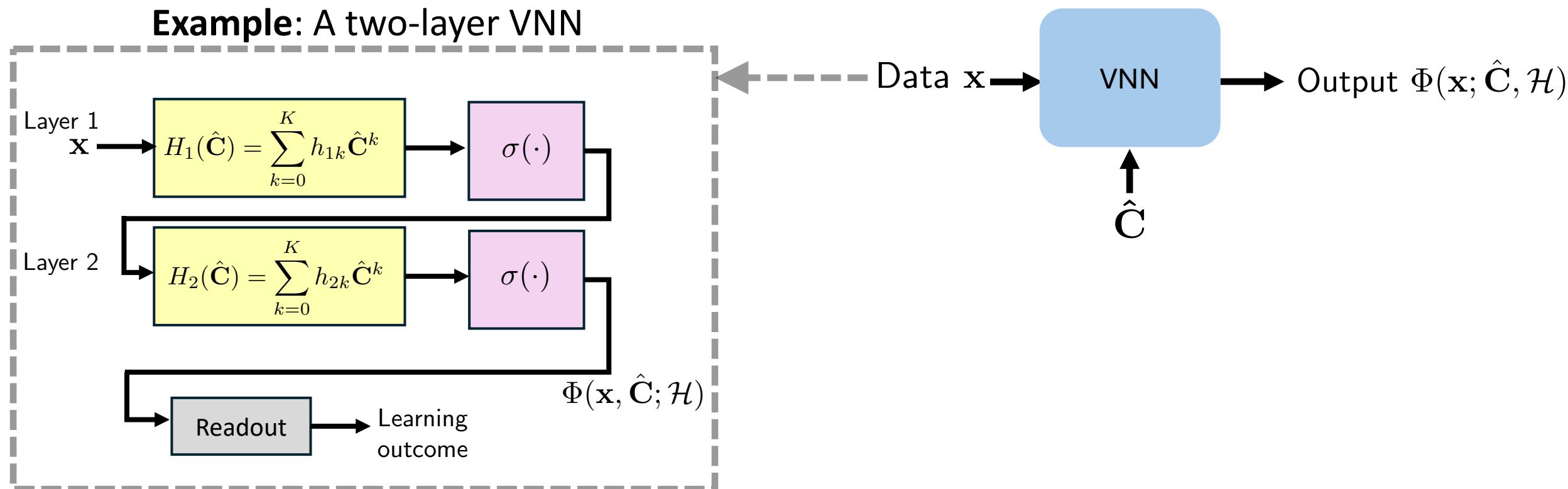
- Parameters $\{h_k\}$ are called **filter taps**, are **scalars** and **learnable** parameters

CoVariance Neural Networks (VNNs)

- coVariance filters can learn only **linear** representations
- To accommodate learn **non-linear** representations, concatenate coVariance filter with pointwise non-linearity σ (for e.g., ReLU, sigmoid, etc.)

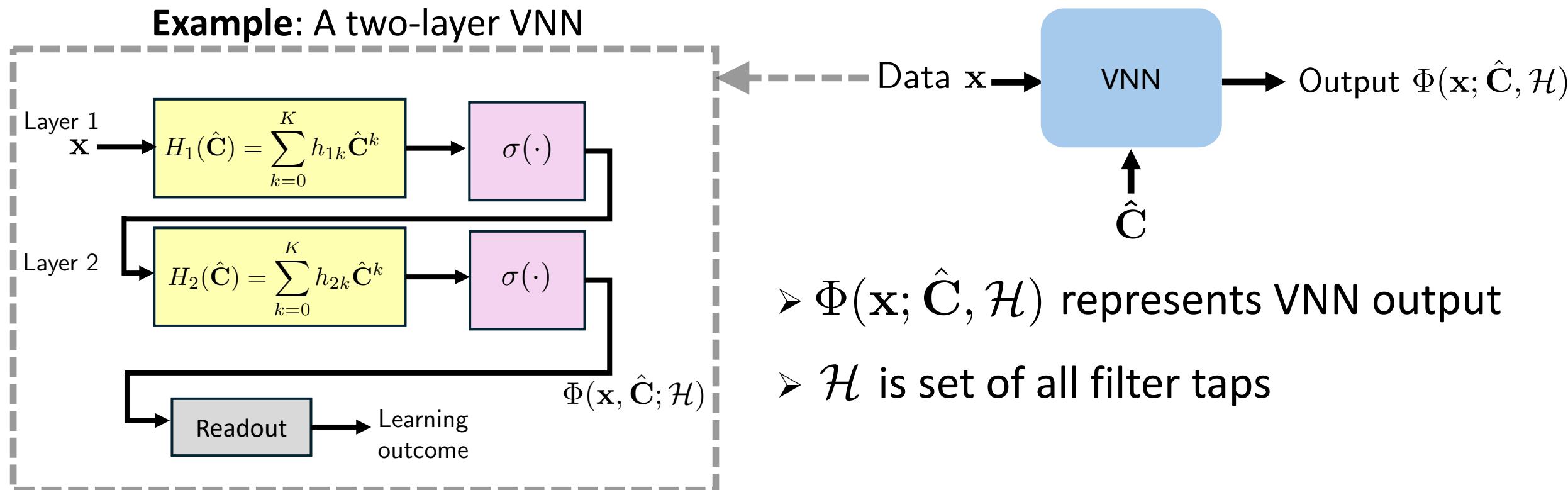
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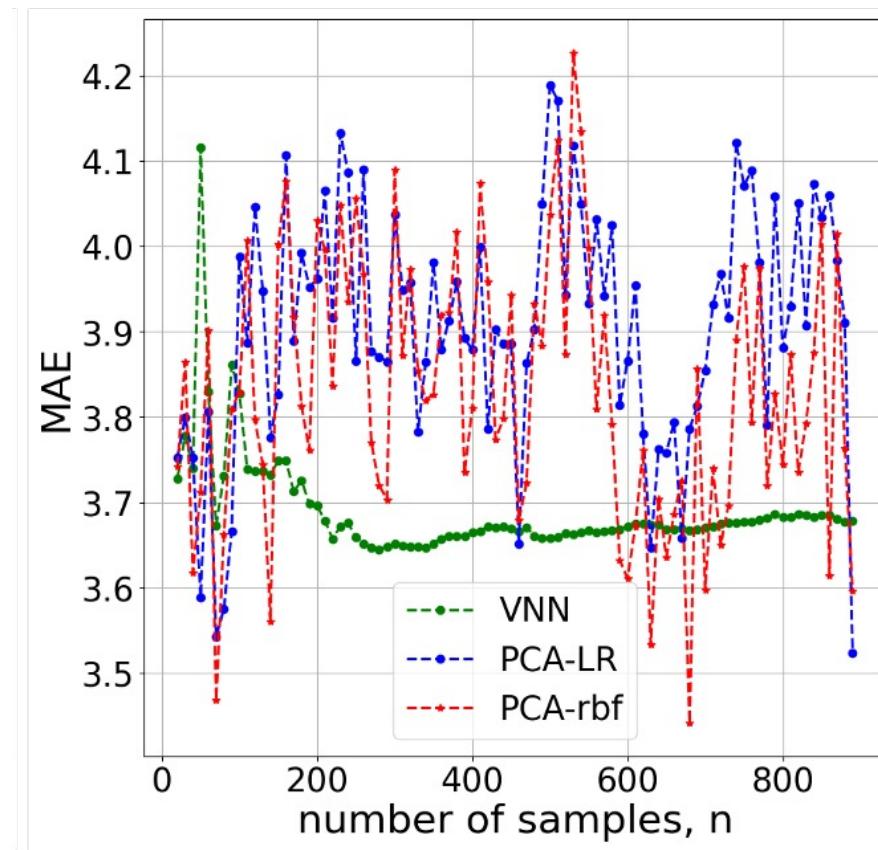
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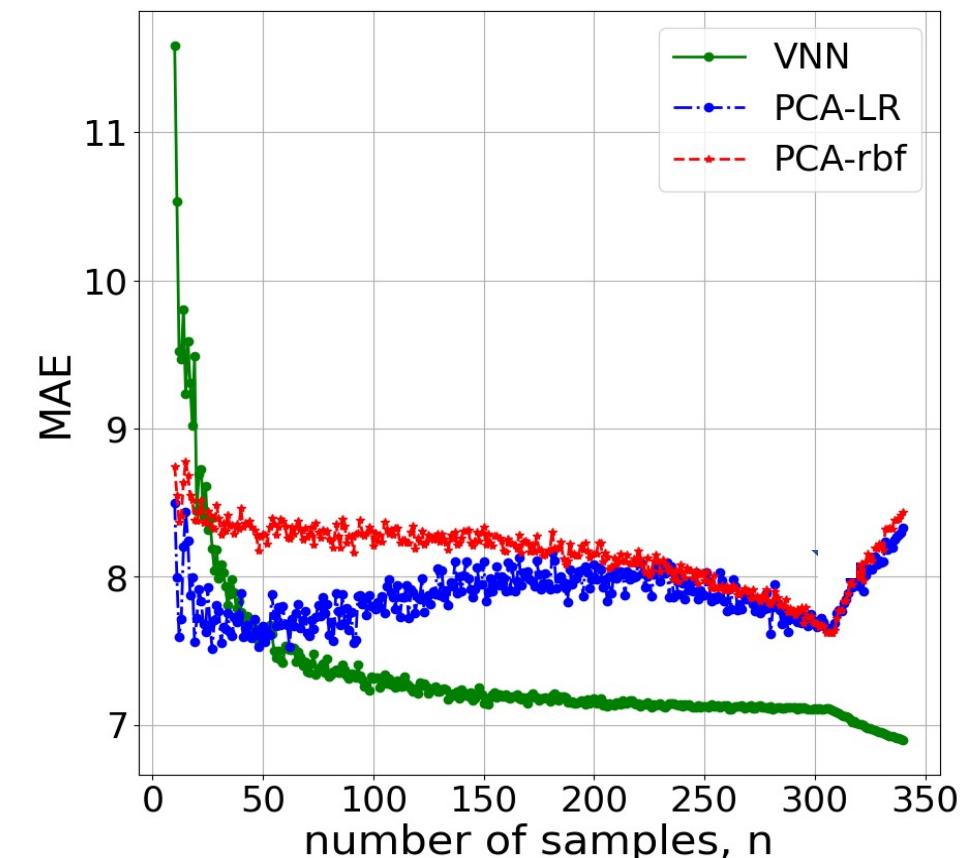


VNNs outperform PCA (regression task)

Synthetic data
(Friedman regression problem)



Neuroimaging data
(age prediction task)

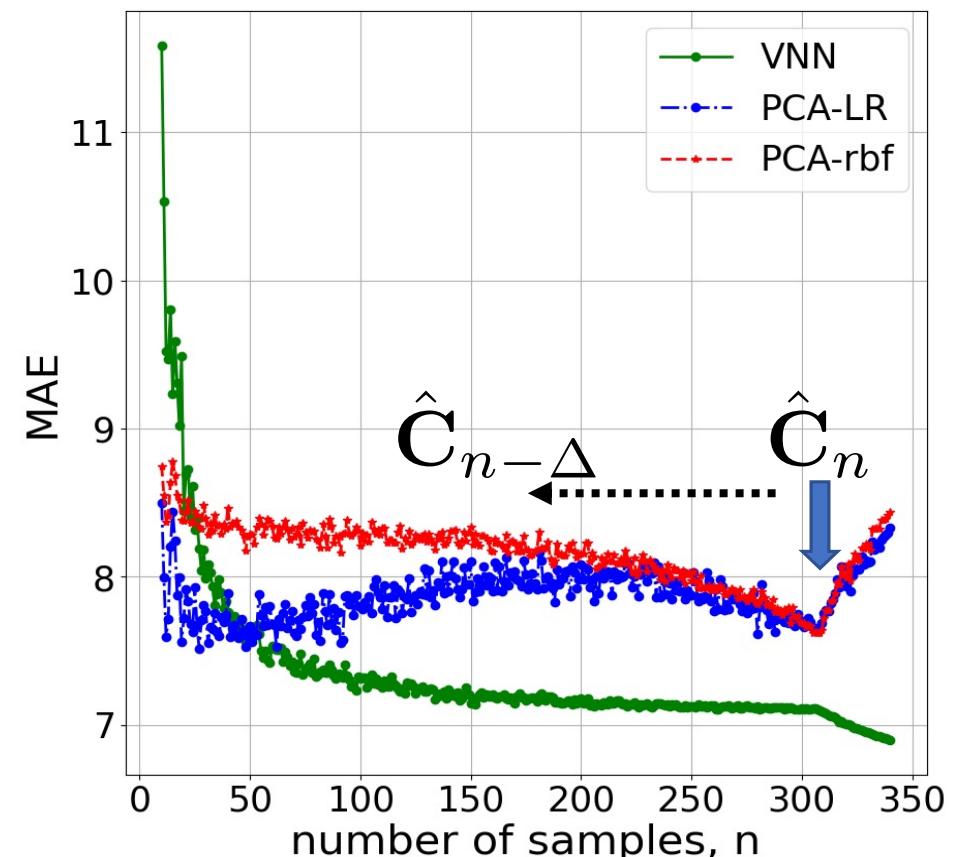


Stable Inference with VNNs

Stability of inference with PCA and VNNs

- PCA-driven inference can be **unstable** to stochastic perturbations in sample covariance matrix ([finite sample effect](#))
- VNNs provide **stable** outcomes
→ enhanced reproducibility

Performance on regression task



\hat{C}_n : estimated from n samples

Stochastic perturbations in sample covariance matrix

- Recall: Sample covariance matrix $\hat{\mathbf{C}}$ is estimate of true covariance matrix \mathbf{C}

$$\hat{\mathbf{C}} = \frac{1}{n-1} \sum_{i=1}^n (\mathbf{x}_i - \hat{\boldsymbol{\mu}})(\mathbf{x}_i - \hat{\boldsymbol{\mu}})^T \quad \mathbf{C} = \mathbb{E}[(\mathbf{x} - \boldsymbol{\mu})(\mathbf{x} - \boldsymbol{\mu})^T]$$

➡ eigenvectors/eigenvalues $\hat{\mathbf{V}}, \hat{\boldsymbol{\Lambda}}$ of $\hat{\mathbf{C}}$ are estimates of $\mathbf{V}, \boldsymbol{\Lambda}$ of \mathbf{C}

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- Convergence between $\hat{\mathbf{V}}, \hat{\boldsymbol{\Lambda}}$ and $\mathbf{V}, \boldsymbol{\Lambda}$ [*]

$$\|\hat{\mathbf{V}}_{\mathbf{x}} - \mathbf{V}_{\mathbf{x}}\| = \mathcal{O}\left(\frac{1}{n^{1/2} \min_{i \neq j} |\lambda_i - \lambda_j|}\right)$$

[*] Loukas, Andreas, 2017

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➡ **Unstable PCA transform when eigenvalues of covariance are close**

[*] Loukas, Andreas, 2017

Stability of coVariance filter

- How to gauge stability?

$$\mathbf{x} \longrightarrow \boxed{\mathbf{H}(\hat{\mathbf{C}})} \longrightarrow \mathbf{z} = \mathbf{H}(\hat{\mathbf{C}})\mathbf{x}$$
$$\hat{\mathbf{C}} = \frac{1}{n-1} \sum_{i=1}^n (\mathbf{x}_i - \hat{\boldsymbol{\mu}})(\mathbf{x}_i - \hat{\boldsymbol{\mu}})^\top$$

➡ Output \mathbf{z} must be robust to number of samples n used to estimate $\hat{\mathbf{C}}$

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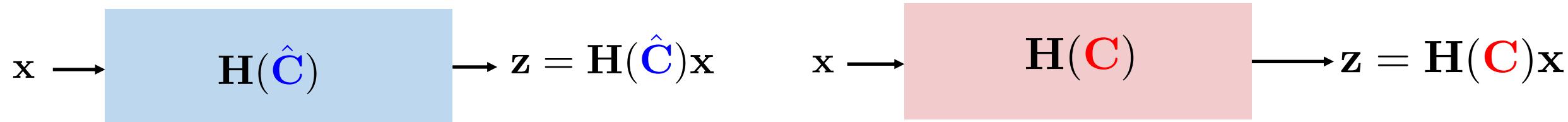
➡ Output \mathbf{z} must be robust to number of samples n used to estimate $\hat{\mathbf{C}}$

- Compare filter outputs for sample and true covariance matrix

$$\mathbf{x} \longrightarrow \boxed{\mathbf{H}(\hat{\mathbf{C}})} \longrightarrow \mathbf{z} = \mathbf{H}(\hat{\mathbf{C}})\mathbf{x} \quad \mathbf{x} \longrightarrow \boxed{\mathbf{H}(\mathbf{C})} \longrightarrow \mathbf{z} = \mathbf{H}(\mathbf{C})\mathbf{x}$$

➡ metric of interest: $\|\mathbf{H}(\hat{\mathbf{C}}) - \mathbf{H}(\mathbf{C})\|$

Stability of coVariance filter

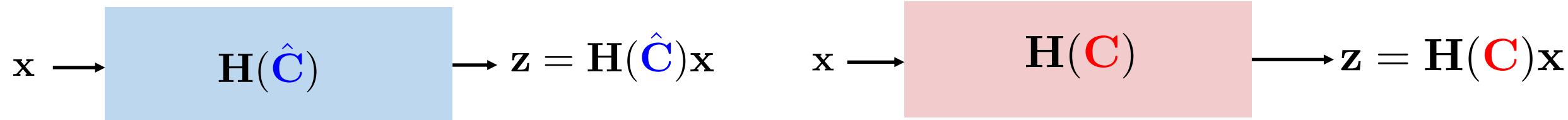


Stability result [Sihag et al., 2022]

$$\left\| H(\hat{C}) - H(C) \right\| = \mathcal{O} \left(\frac{1}{n^{1/2-\varepsilon}} \right)$$

} coVariance filter output is asymptotically consistent

Stability of coVariance filter



Stability result [Sihag et al., 2022]

$$\left\| H(\hat{C}) - H(C) \right\| = \mathcal{O} \left(\frac{1}{n^{1/2-\varepsilon}} \right)$$

Assumption.

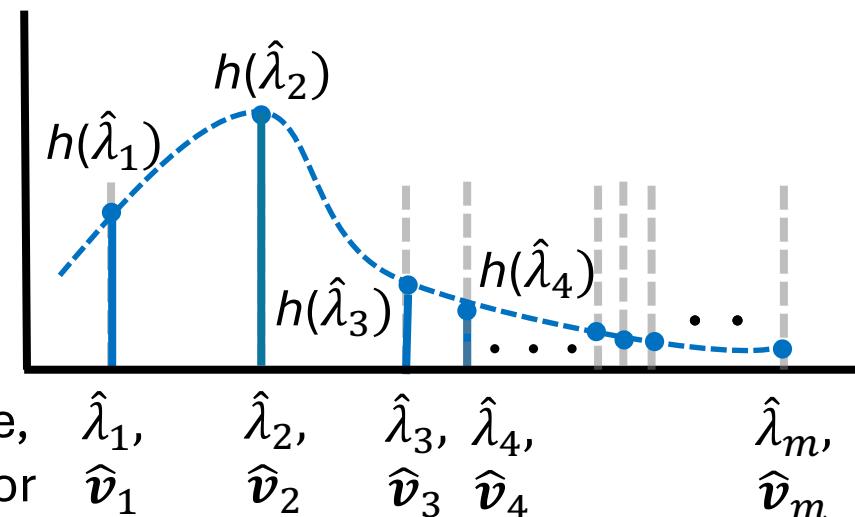
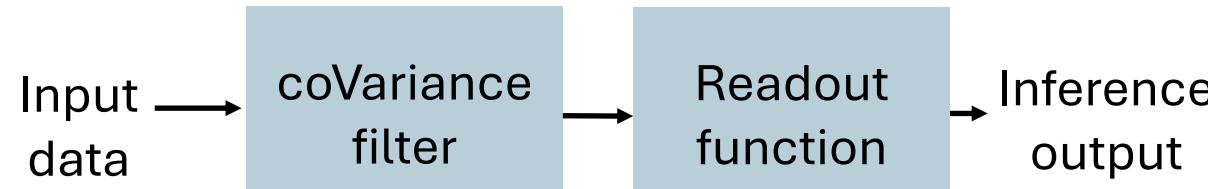
Frequency response of filter $H(C)$ satisfies

$$|h(\lambda_i) - h(\lambda_j)| \leq Q \frac{|\lambda_i - \lambda_j|}{k_i}$$

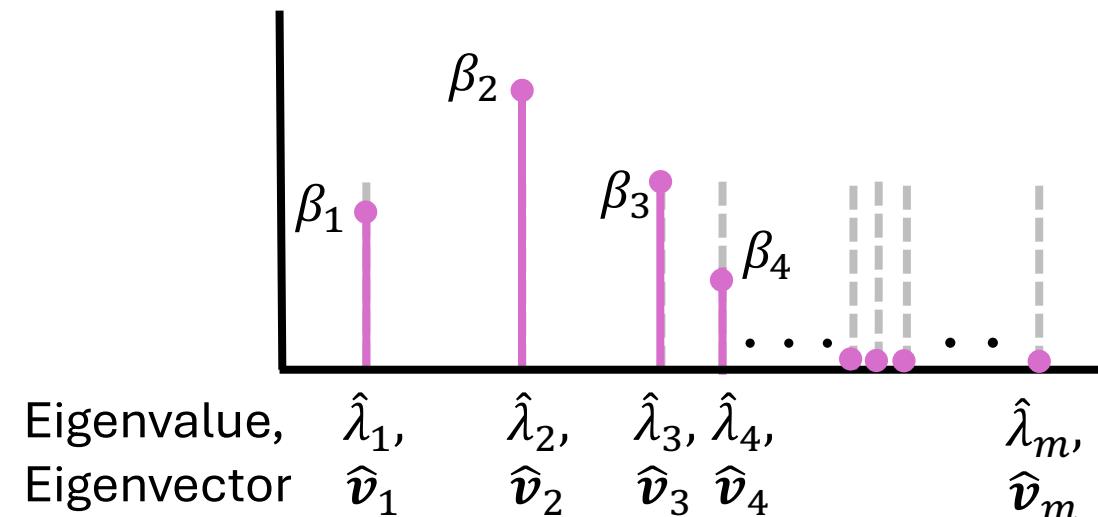
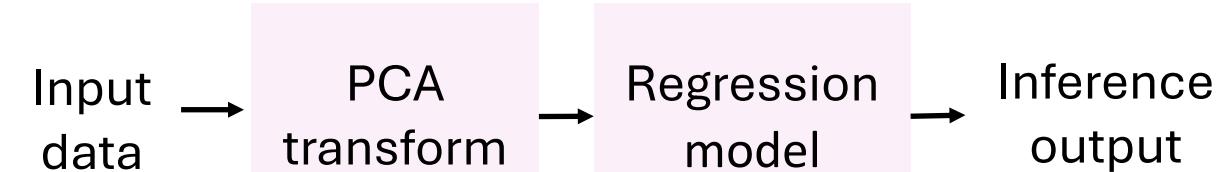
- } coVariance filter output is asymptotically consistent
- } coVariance filter sacrifices discriminability between close eigenvalues for stability

Recall: Learning with coVariance filter versus PCA-based learning

➤ Learning with a **coVariance** filter

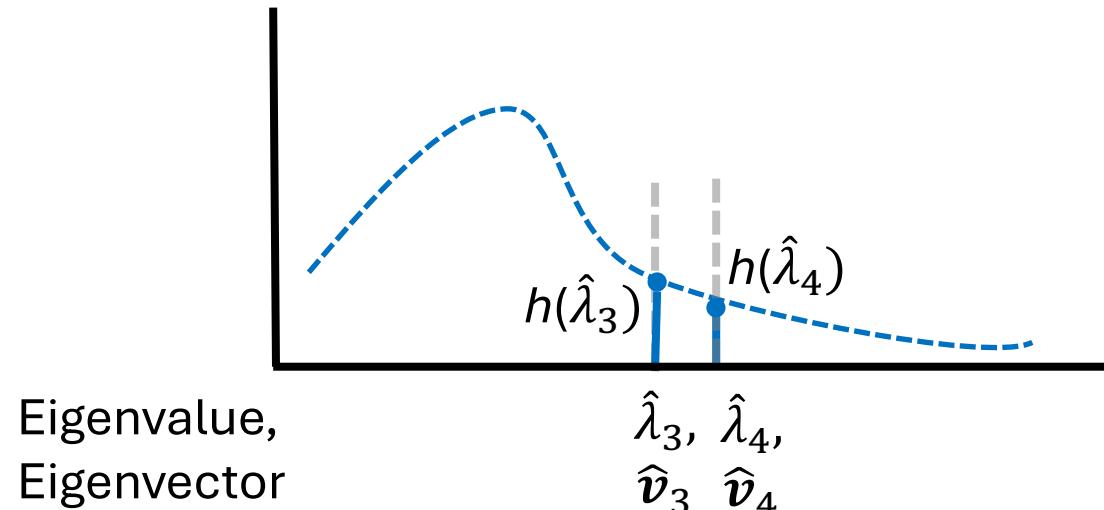
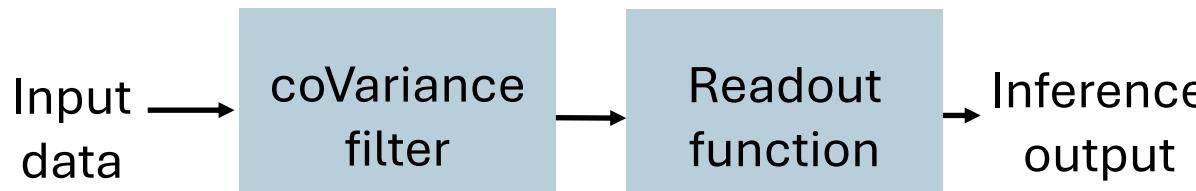


➤ PCA-based learning

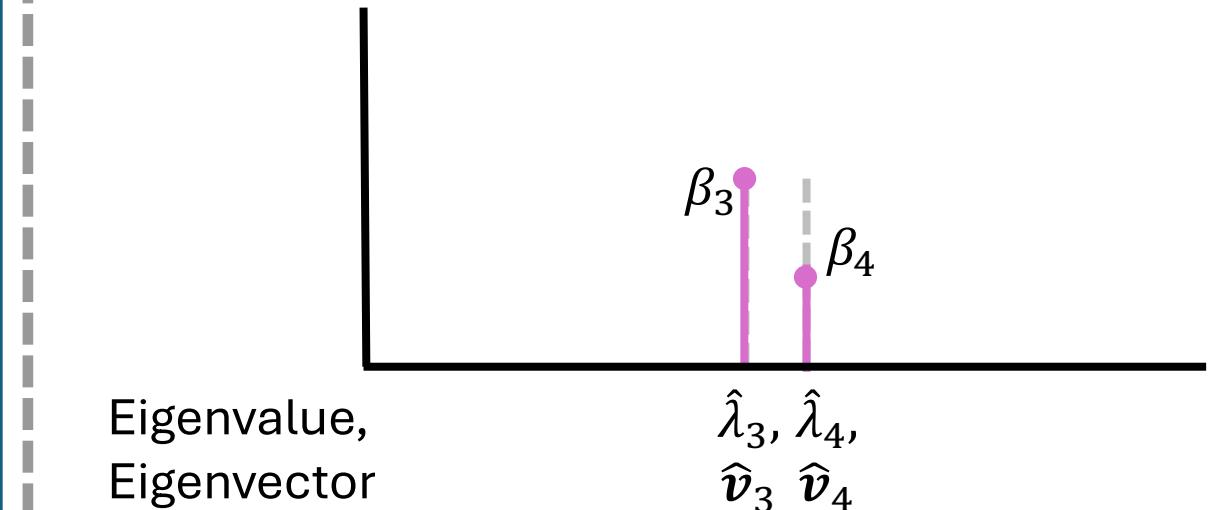
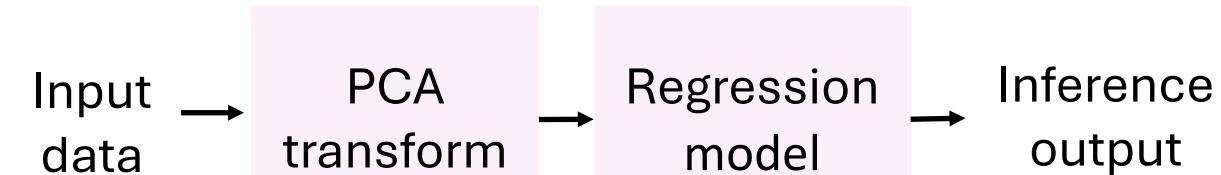


Why is coVariance filter more stable than PCA?

➤ Learning with a **coVariance** filter

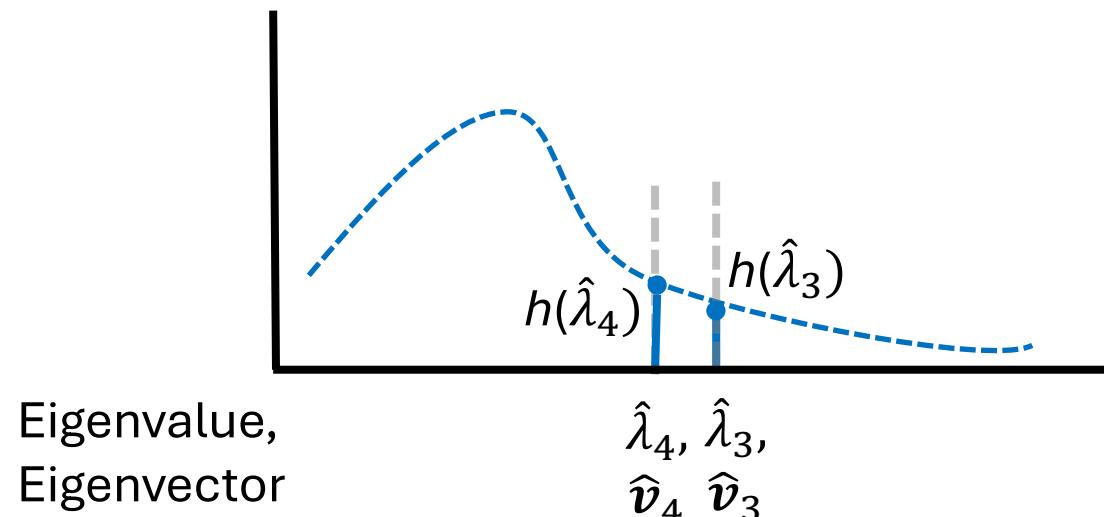
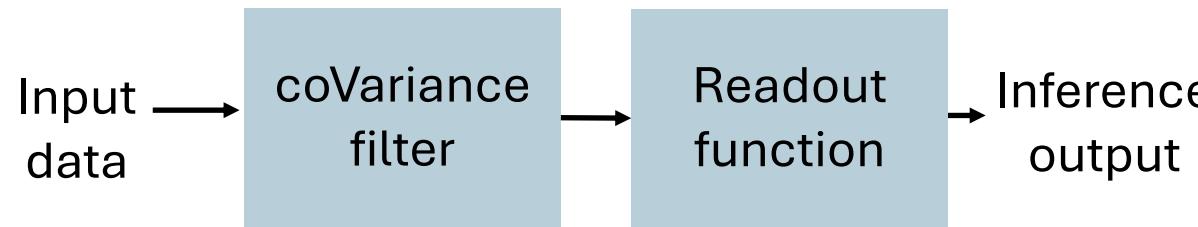


➤ PCA-based learning

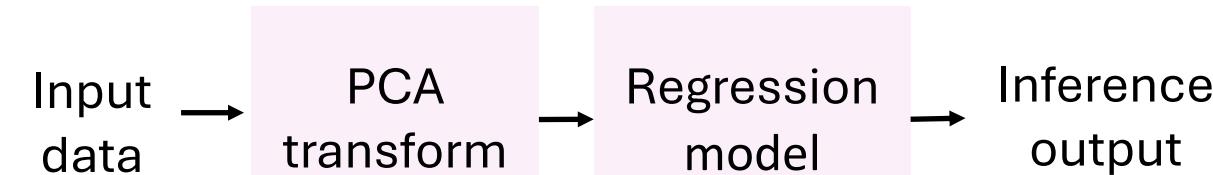


Why is coVariance filter more stable than PCA?

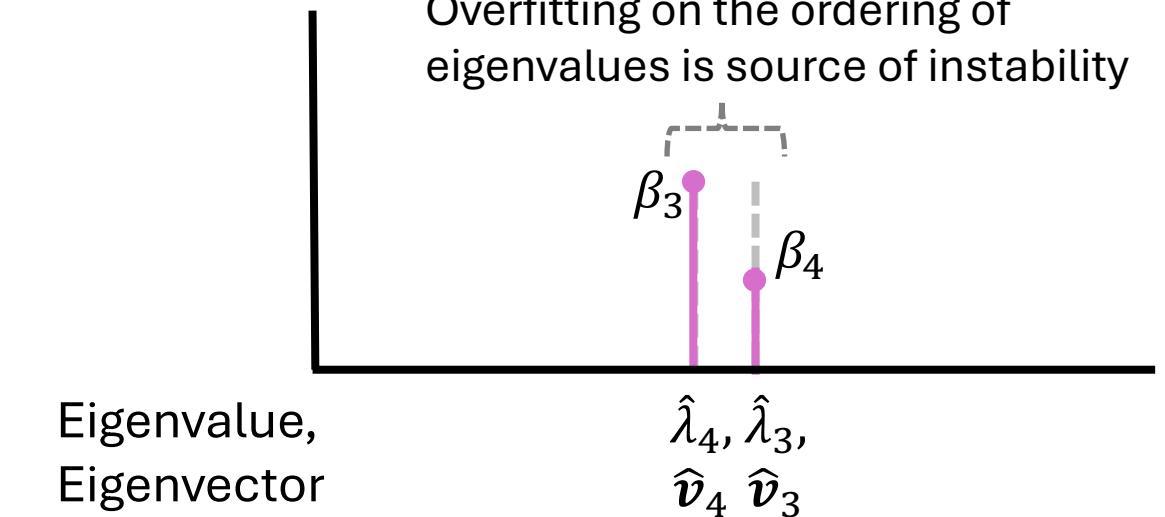
➤ Learning with a **coVariance** filter



➤ PCA-based learning



Overfitting on the ordering of eigenvalues is source of instability



Stability of VNNs

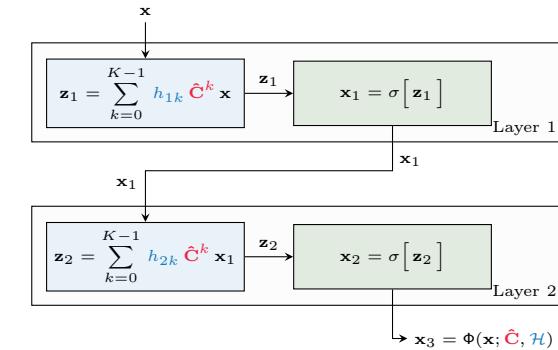
- VNNs inherit the stability from coVariance filters
 - Stability bound depends on the bound for filters

$$\left\| \mathbf{H}(\hat{\mathbf{C}}) - \mathbf{H}(\mathbf{C}) \right\| = \mathcal{O} \left(\frac{1}{n^{\frac{1}{2}} - \varepsilon} \right) = \alpha_n$$

- For a VNN with L layers and F filters in parallel,

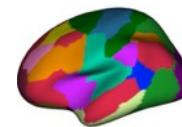
$$\left\| \Phi(\mathbf{x}, \hat{\mathbf{C}}; \mathcal{H}) - \Phi(\mathbf{x}, \mathbf{C}; \mathcal{H}) \right\| \leq LF^{L-1} \alpha_n$$

- Stability bound increases with number of layers and size of filter banks

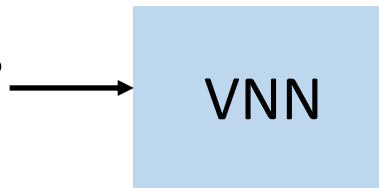


Stability of VNNs: Experiments

- Regression task



Cortical thickness
data

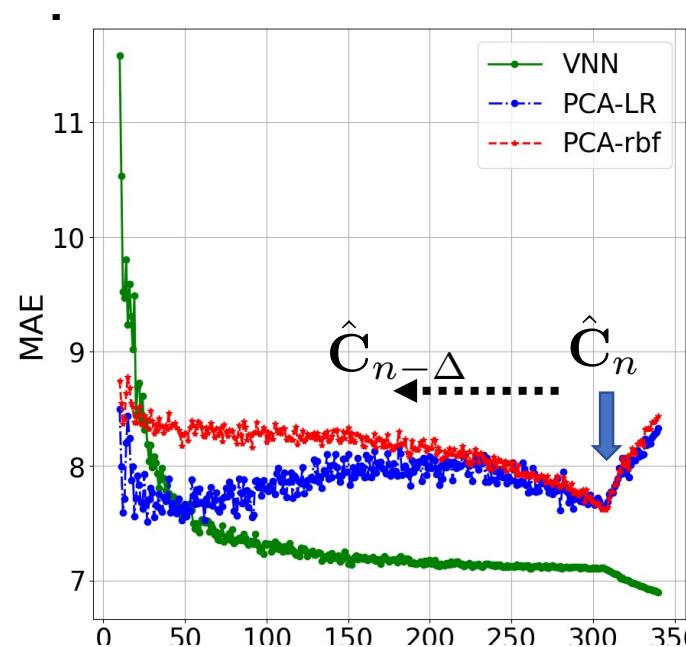


→ Estimate of age

- Comparison against PCA-regression

Data: cortical thickness dataset ($m = 104$) from ($n = 341$) human subjects

- **Metric:** MAE (mean absolute error)



VNN: coVariance Neural Network

PCA-LR: PCA-regression with linear kernel

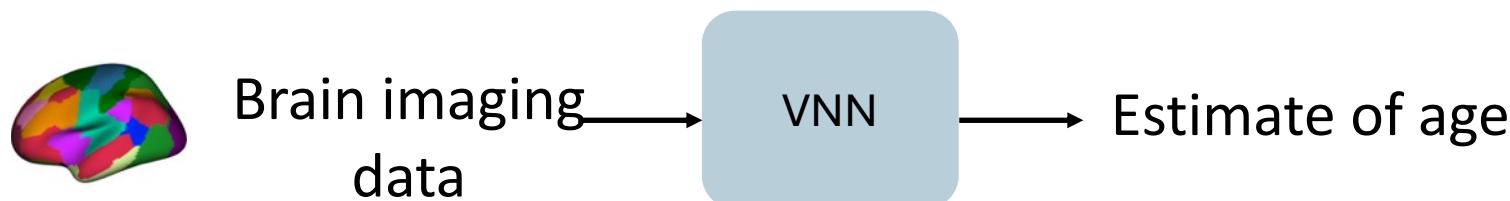
PCA-rbf: PCA regression with rbf kernel

VNN outperforms PCA and is more stable

Transferability of VNNs

Empirical evidence of transferability across multiscale data

- Transferability across multiscale datasets
 - **Multiscale** datasets capture same phenomenon at different scales

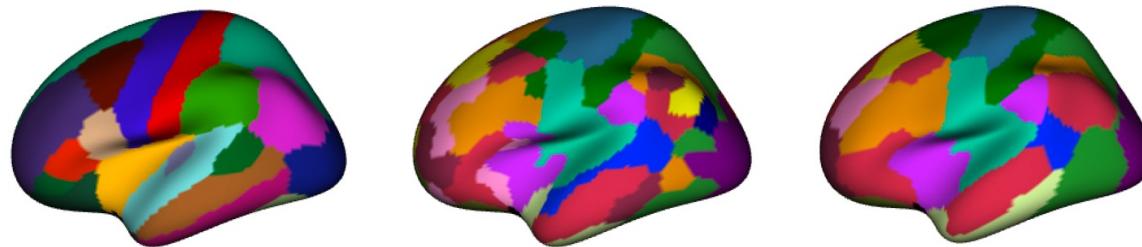


Transferability across datasets with different number of features

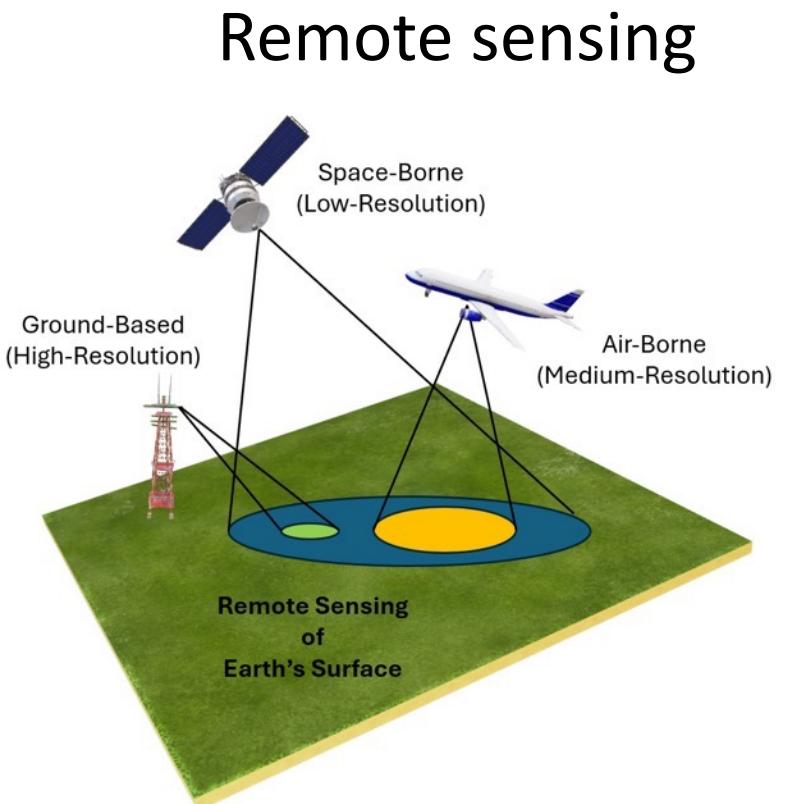
Training	Testing	
	100-feature dataset	300-feature dataset
100-feature dataset	5.39 ± 0.084	5.5 ± 0.101

Transferability

- Learning models could generalize to **compatible** datasets
- **compatible:** different dimensionalities and describing the same domain



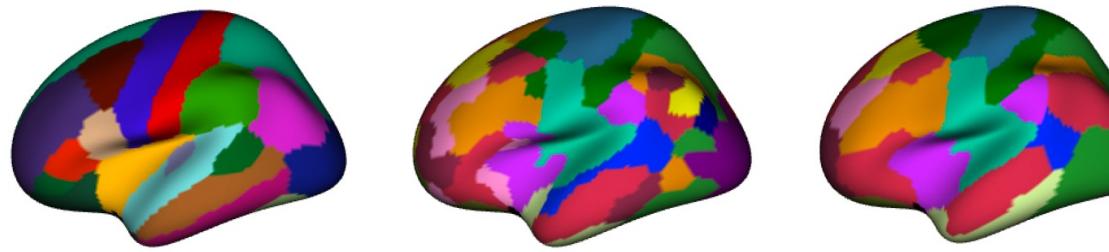
Brain imaging data



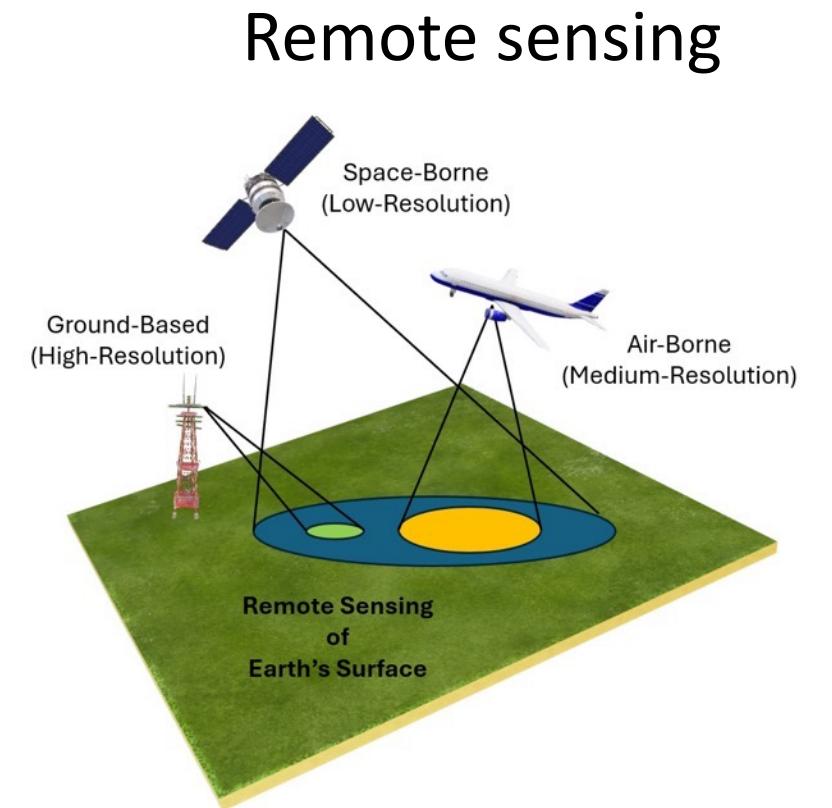
Credit: Mustafa Aksoy, UAlbany

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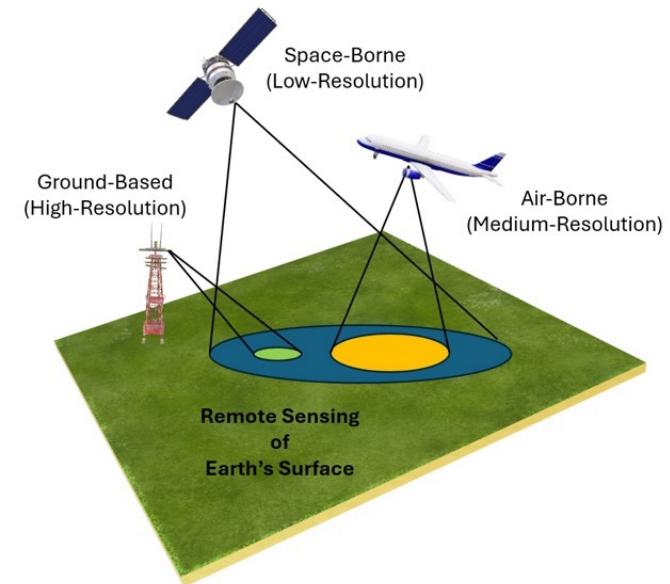
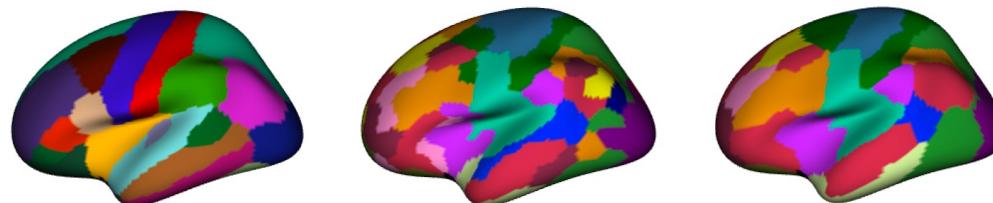
Credit: Mustafa Aksoy, UAlbany

- **Motivation:** novel metric for generalizability, managing high dimensional data...

Transferability

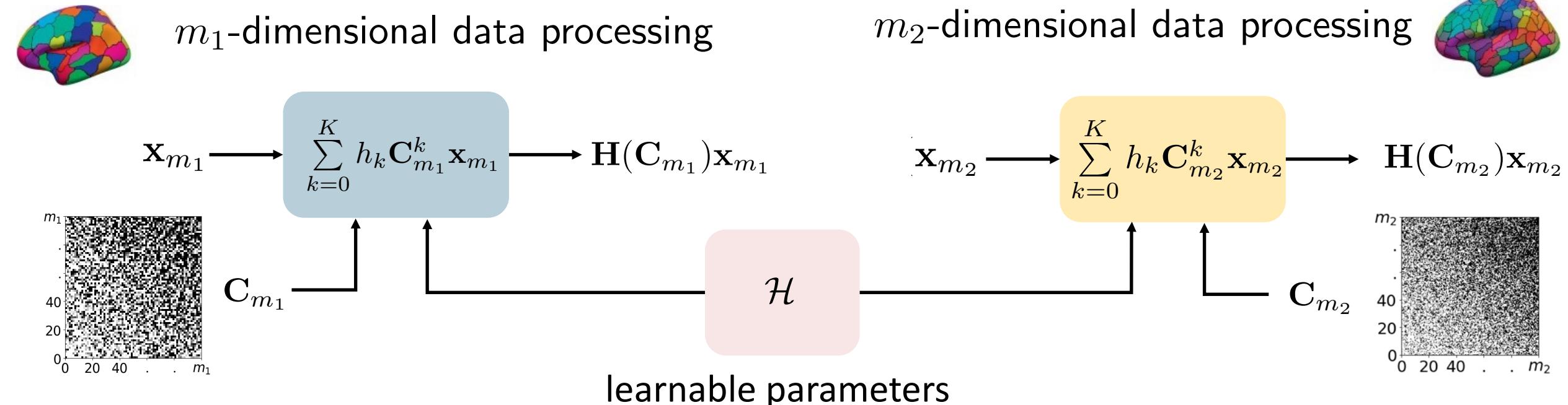
- Most statistical approaches, including PCA, operate within the dimensionality
 - ➡ seamless transference not possible across different dimensionalities
- **This section: How do VNNs transfer?**

When is transference successful?



Credit: Mustafa Aksoy, UAlbany

coVariance filters are scale-free models

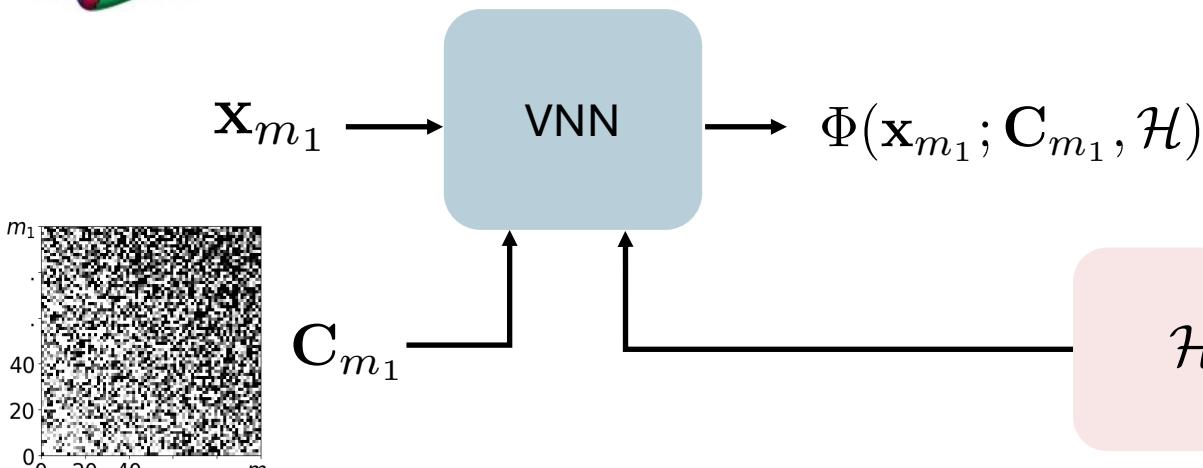


- A coVariance filter $\mathbf{H}(\cdot)$ with scalar filter taps $\{h_k\}$ can process dataset (covariance matrix) of any arbitrary dimensionality: **scale-free model**

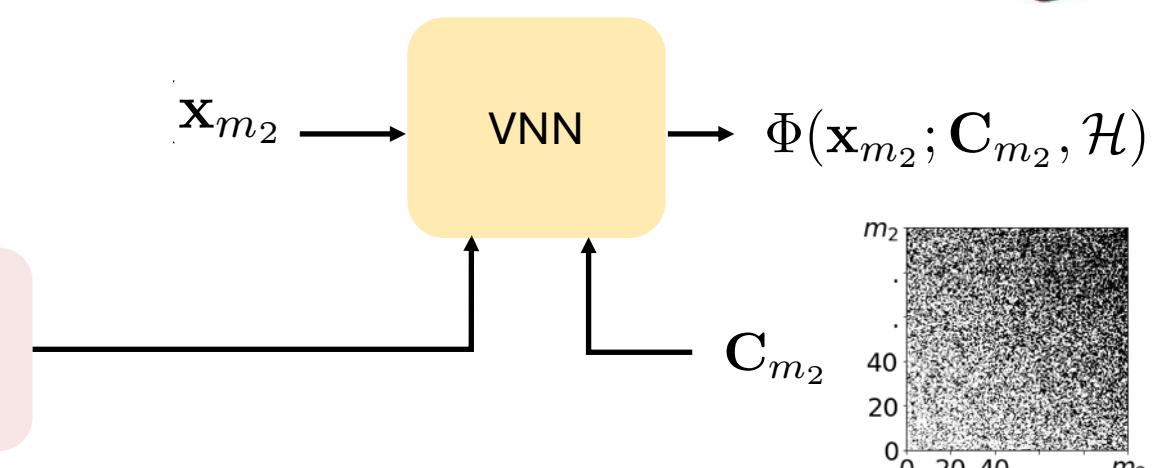
VNNs as scale-free models



m_1 -dimensional data processing



m_2 -dimensional data processing



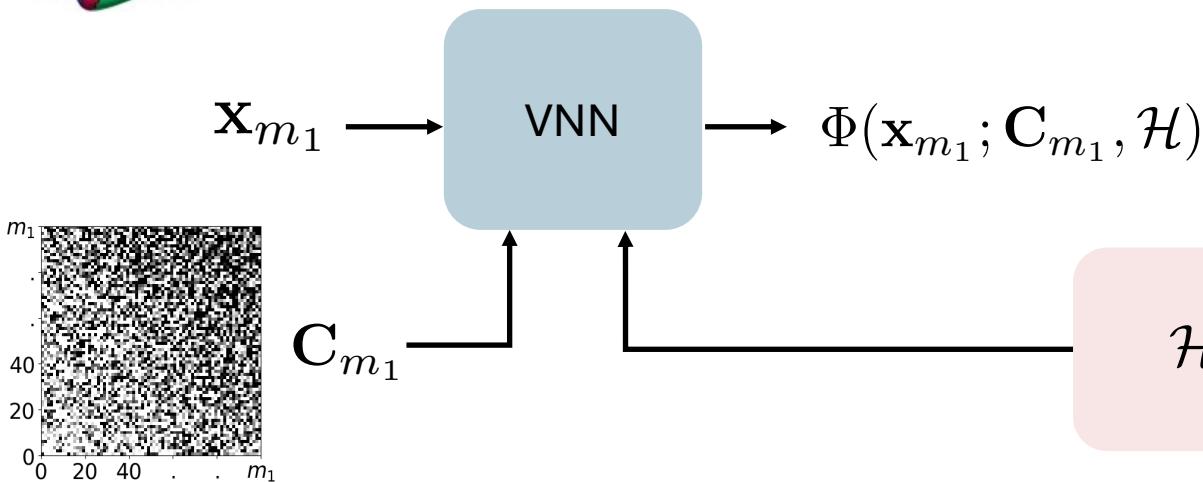
learnable parameters

How to compare $\Phi(\mathbf{x}_{m_1}; \mathbf{C}_{m_1}, \mathcal{H})$ and $\Phi(\mathbf{x}_{m_2}; \mathbf{C}_{m_2}, \mathcal{H})$?

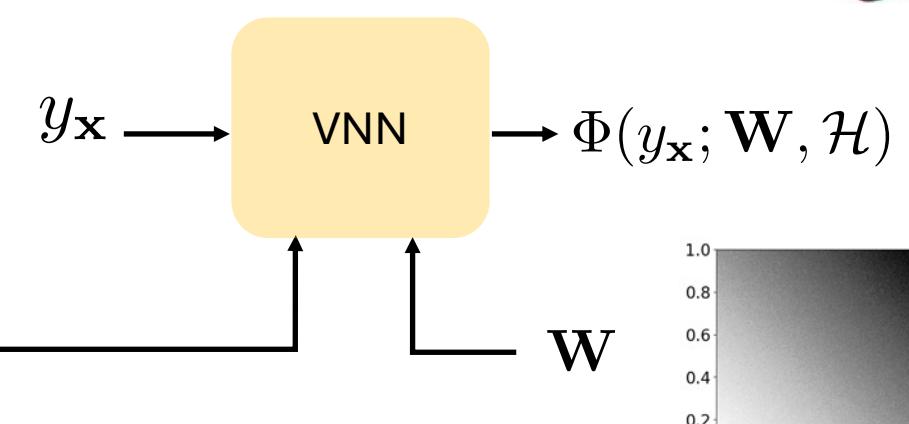
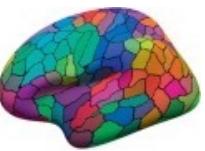
VNNs as scale-free models



m_1 -dimensional data processing



data processing in continuous limit

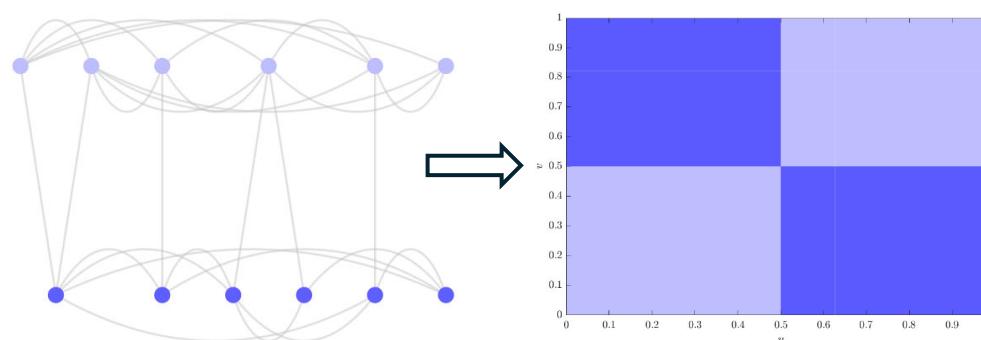


**Continuous limit of covariance matrices
as $m \rightarrow \infty$**

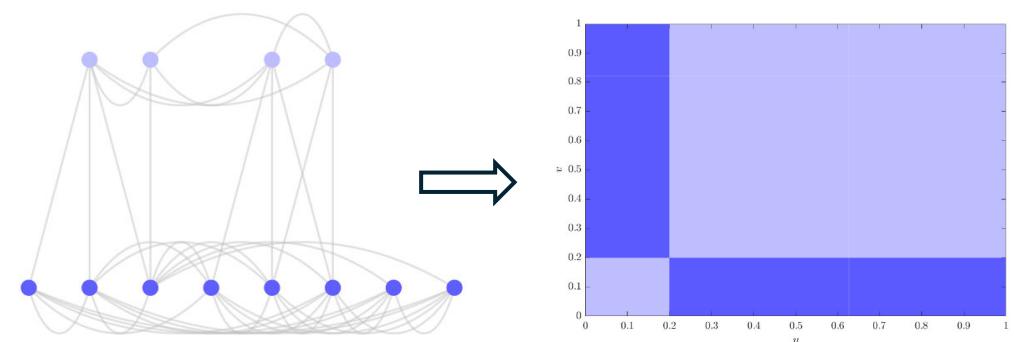
How to compare $\Phi(\mathbf{x}_{m_1}; \mathbf{C}_{m_1}, \mathcal{H})$ and $\Phi(y_{\mathbf{x}}; \mathbf{W}, \mathcal{H})$?

Graphons as continuous limits

- Graphs can have **limit objects** with uncountable number of nodes
- Example: Stochastic block models [Ruiz et al., TSP, 2021]



Balanced SBM



Unbalanced SBM

Graphons as continuous limits

➤ **Graphon:** A graphon is a symmetric, bounded measurable function

- Node labels are graphon arguments $u \in [0,1]$
- edge weights are graphon values $\mathbf{W}(u, v) = \mathbf{W}(v, u)$

$$\mathbf{W} : [0, 1]^2 \mapsto \mathbb{R}$$

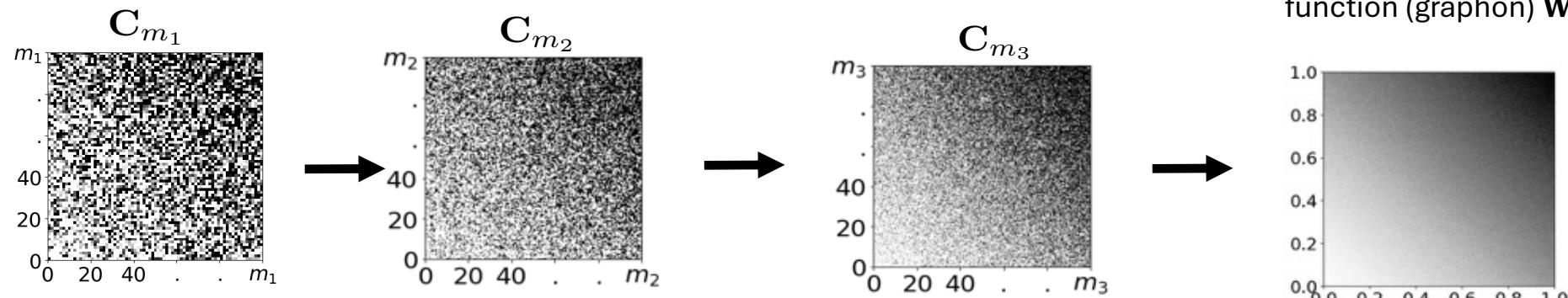
Graphons as continuous limits

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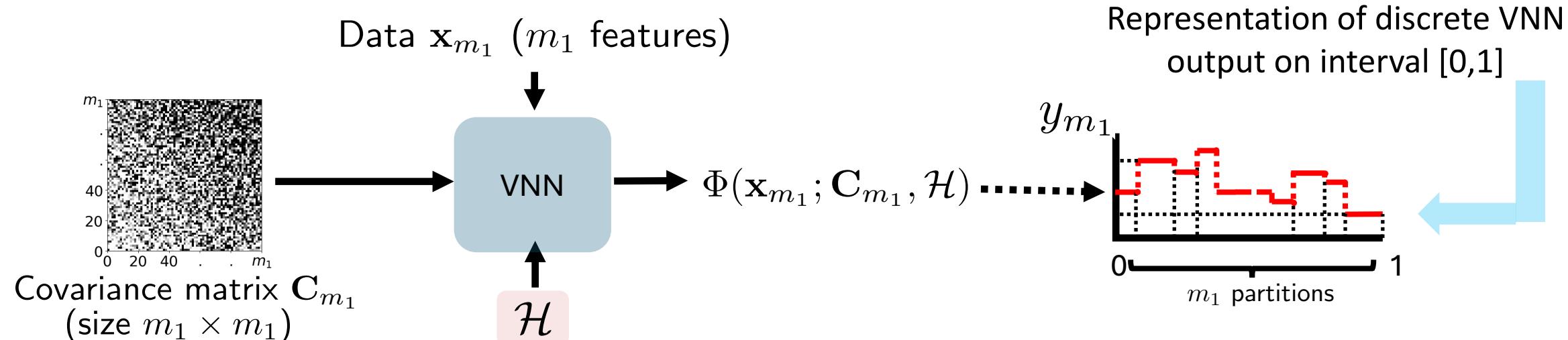
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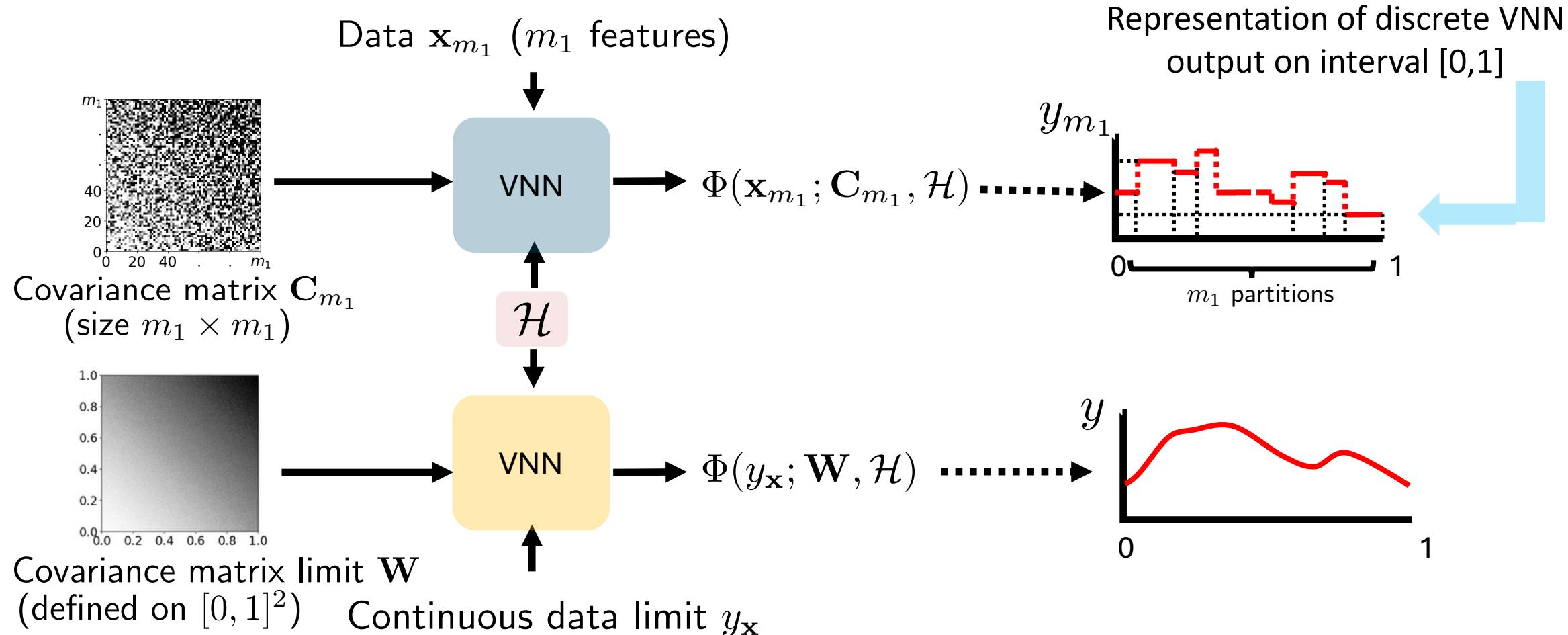
➤ Transferability when covariance matrix is part of some converging sequence



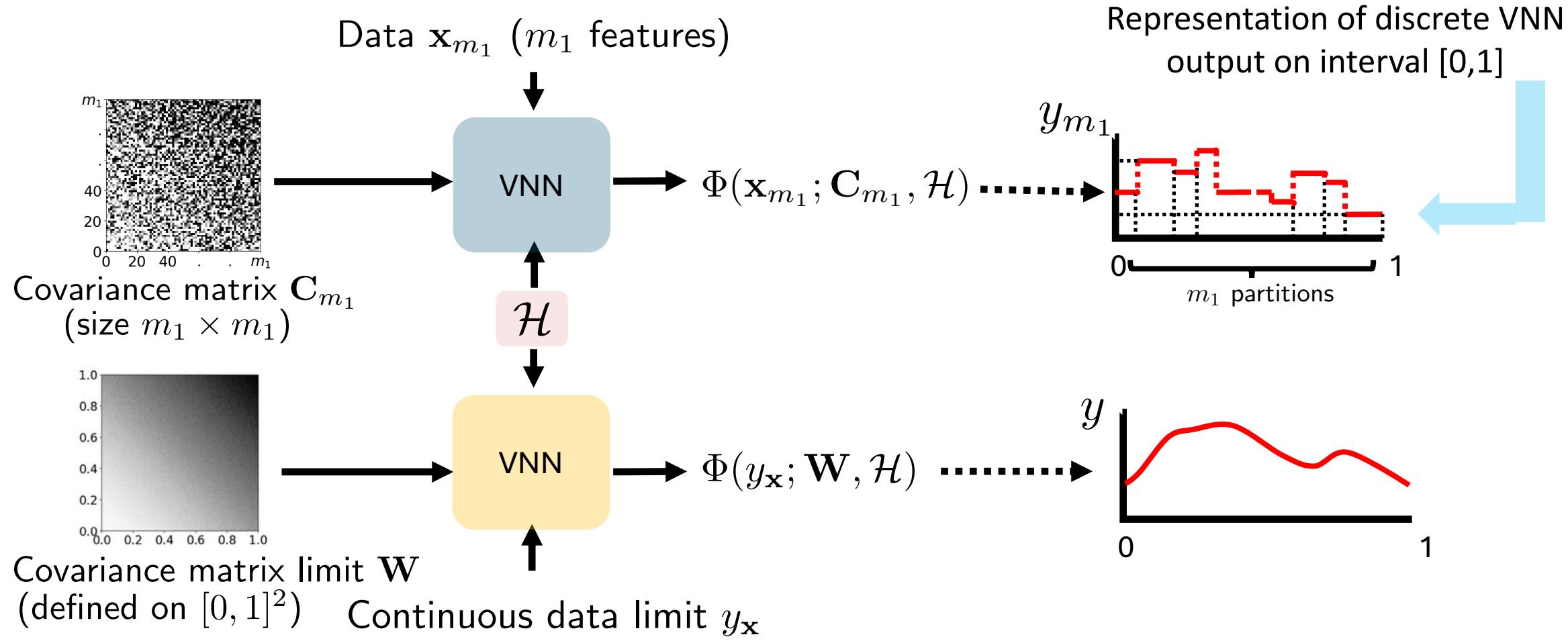
Problem formulation for transferability



Problem formulation for transferability

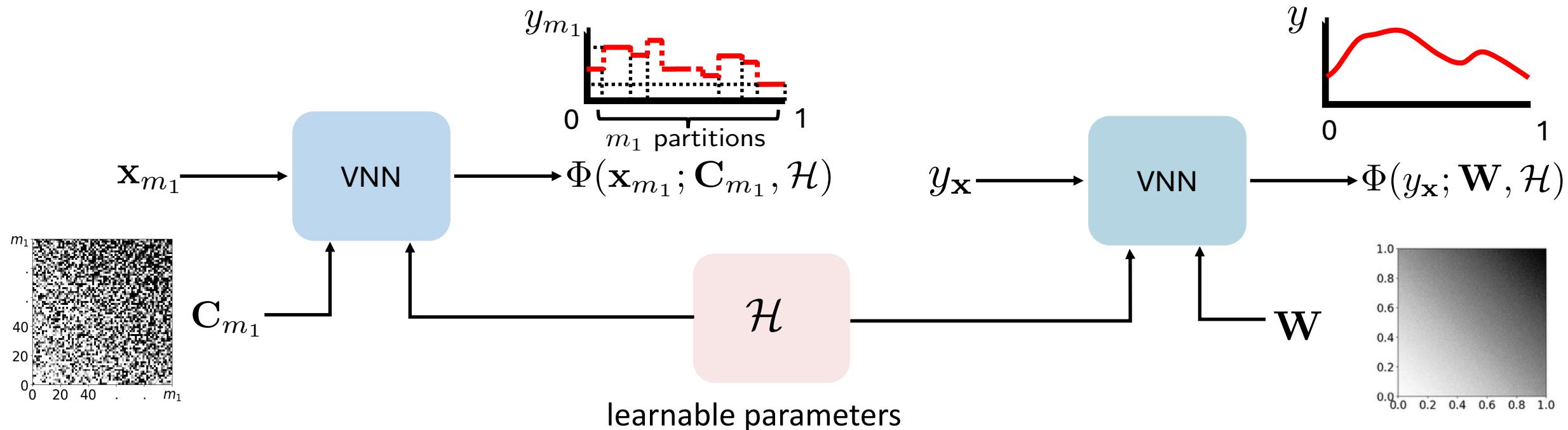


Problem formulation for transferability



Find ϑ , such that, $\|y_{m_1} - y\|_2 \leq \vartheta$

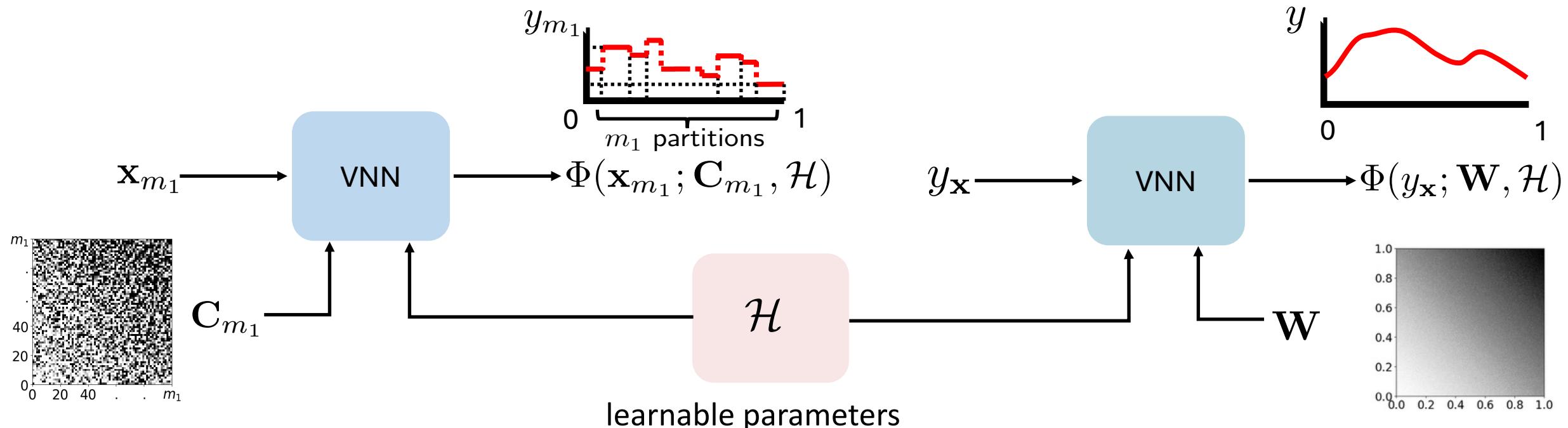
VNNs are provably transferable



Transferability bound* [Sihag et al., 2024]

$$\|y_{m_1} - y\| \propto \mathcal{O}\left(\frac{1}{m_1^{3\zeta/2-1}}\right), \text{ for } \zeta \in (2/3, 1]$$

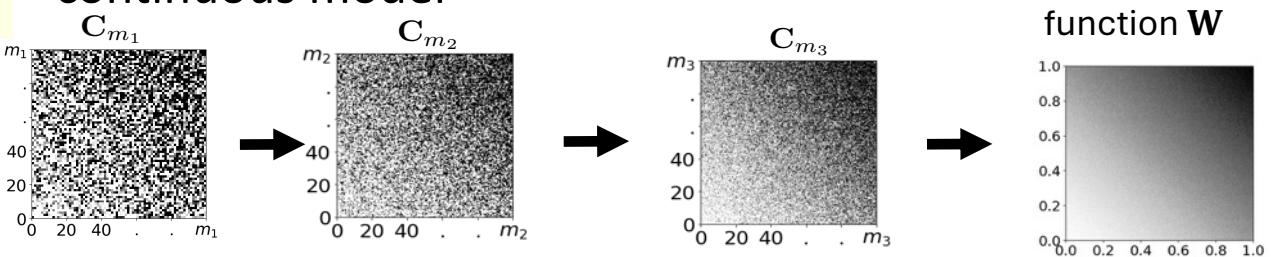
VNNs are provably transferable



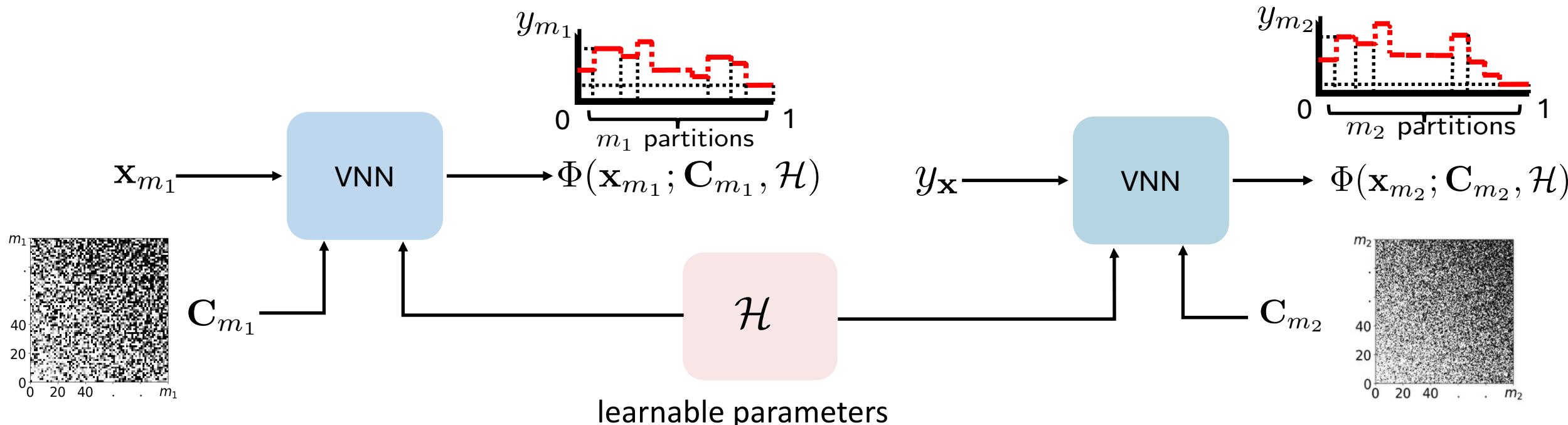
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***Assumption:** data is a discretization of a common continuous model



VNNs are provably transferable

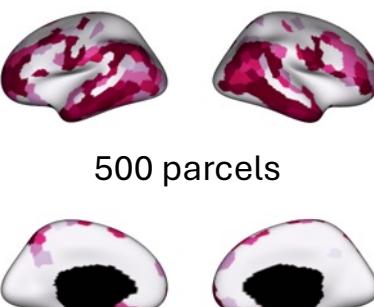
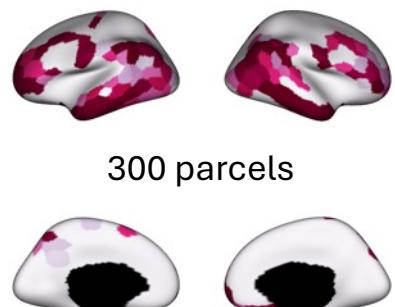
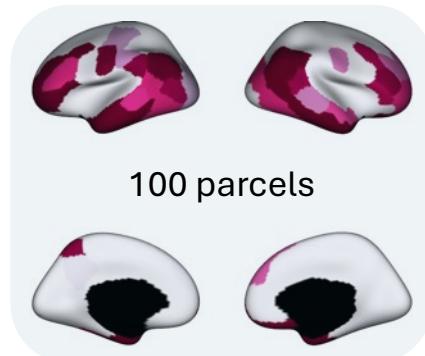


Transferability bound

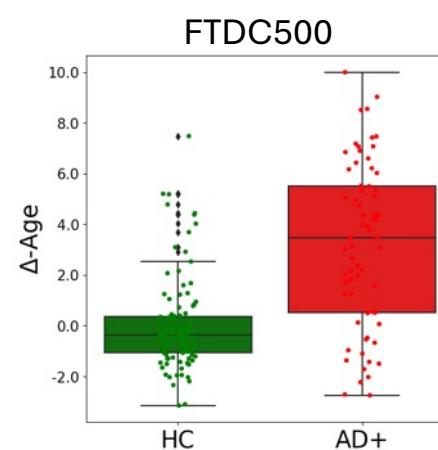
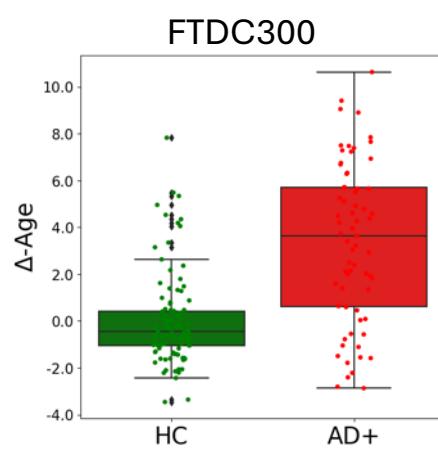
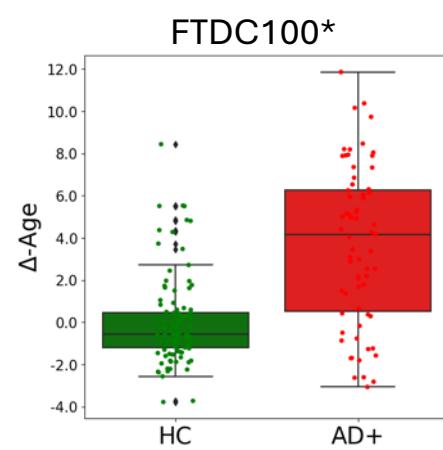
$$\|y_{m_1} - y_{m_2}\| \propto \mathcal{O}\left(\frac{1}{m_1^{3\zeta/2-1}} + \frac{1}{m_2^{3\zeta/2-1}}\right), \text{ for } \zeta \in (2/3, 1]$$

Experiments

Objective: Brain age gap prediction in HC (healthy) and AD+ (Alzheimer's) cohorts from VNNs trained on 100-feature dataset [Sihag et al., NeurIPS, 2024 and JSTSP 2024]



- ROIs contributing to elevated brain age gap in AD+ across different resolutions



- Brain age gap is elevated in AD+ w.r.t HC cohort in 100-feature dataset
- Results on brain age gap retained after transferring VNN to 300 and 500-feature datasets

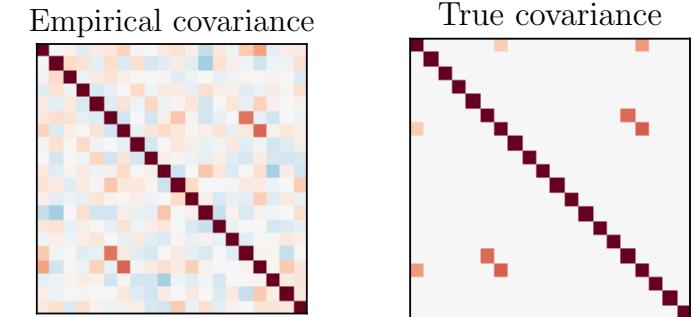
Variants of VNNs

Are VNNs enough?

- **Limitations of VNNs**
 - Sample covariance could be poor quality in **low data, high dimensionality setting**
 - High computational cost (quadratic in size of matrix for dense covariance)
 - No considerations of **temporal, evolving** data

Sparse VNNs

- **Sparse VNNs (S-VNN)** rely on sparsification of sample covariance matrix
- Sparsification improves estimation quality
- Strategies to sparsify
 - **Hard thresholding**



$$\eta(\hat{\mathbf{C}})_{ij} = \hat{c}_{ij} \text{ if } |\hat{c}_{ij}| \geq \tau/\sqrt{n}, 0 \text{ otherwise}$$

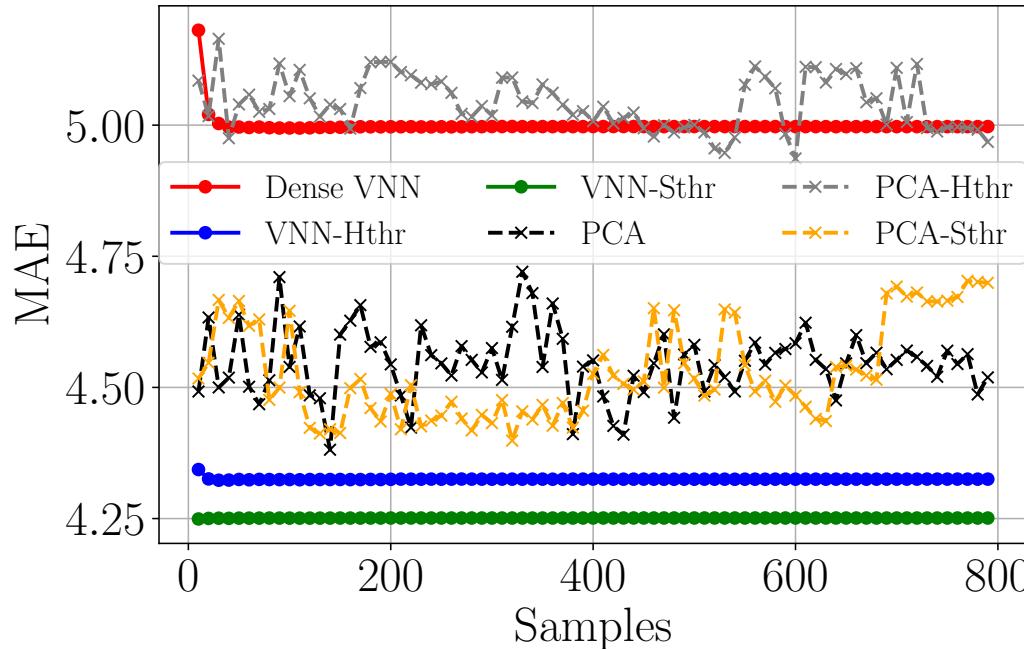
- **Soft thresholding**

$$\eta(\hat{\mathbf{C}})_{ij} = \hat{c}_{ij} - \text{sign}(\hat{c}_{ij})\tau/n \text{ if } |\hat{c}_{ij}| \geq \tau/\sqrt{n}, 0 \text{ otherwise}$$

- Both thresholding strategies preserve stability in S-VNNs [Cavallo et al., 2024]

Sparse VNNs: Numerical results

- Train VNNs/PCA on one covariance and test on another covariance estimated from less samples (synthetic dataset)



Results

- S-VNN (both soft and hard thresholding) outperform PCA and nominal VNNs
- VNNs more stable than PCA

Spatiotemporal VNNs

- VNN models discussed so far operate on *static* data
 - ➡ non-trivial modifications needed to handle temporal, non-stationary data
- **Spatio-temporal VNNs (STVNNs)**
 - **Model design**

1. Online covariance matrix estimate

$$\hat{\mathbf{C}}_{t+1} = \zeta_t \hat{\mathbf{C}}_t + \beta_t (\mathbf{x}_{t+1})(\mathbf{x}_{t+1})^\top$$

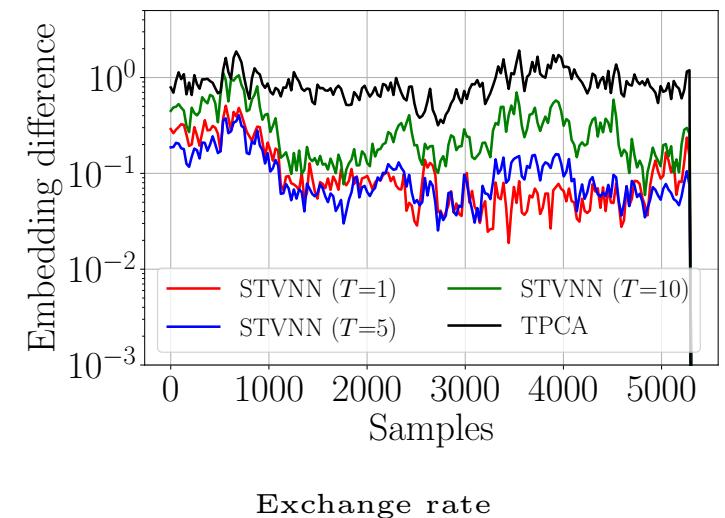
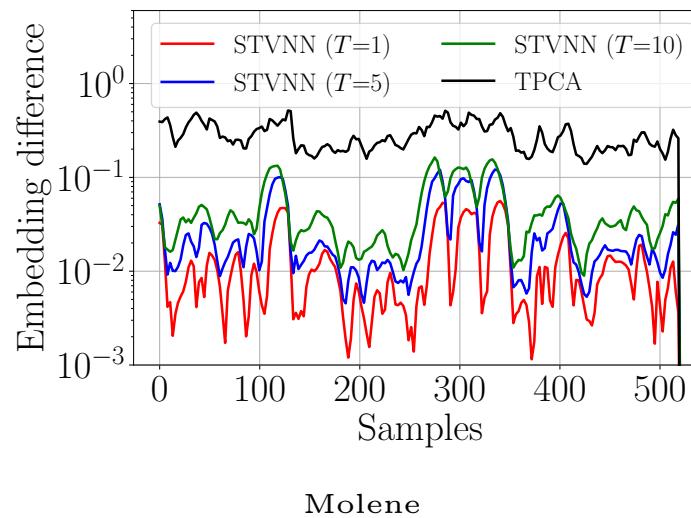
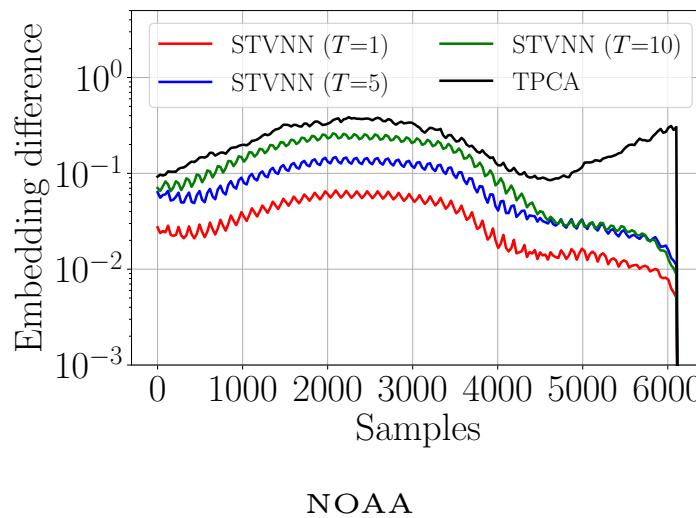
2. Spatio-temporal coVariance filter

$$\mathbf{z}_t := \mathbf{H}(\hat{\mathbf{C}}_t, \mathbf{h}_t, \mathbf{x}_{T:t}) = \sum_{t'=0}^{T-1} \sum_{k=0}^K h_{kt'} \hat{\mathbf{C}}_t^k \mathbf{x}_{t-t'}$$

Spatial and temporal convolution

Spatiotemporal VNNs

- STVNNs are stable to estimation errors in covariance [*]
- Numerical results
 - Time series forecasting task (weather data and currency exchange rates)
 - Train with one covariance, test with another estimated from fewer samples



[*] Cavallo et al., 2024

Concluding Remarks

- **Learning with covariance matrices**
 - Covariance matrices encode redundancies within dataset
 - their eigenvectors (principal components) inform the directions of maximum variance
 - PCA-driven methods can be **unstable**
 - PCA operates **restricted** to datasets of same dimensionality
- **CoVariance neural networks (VNNs)**
 - VNNs provide GSP-motivated implementation of PCA
 - Stable outcomes, transference across multiscale datasets

Concluding Remarks

- **Emerging areas** we did not cover in detail
 - **Sparse VNNs:** sparsifying covariance matrix [Cavallo et al., 2024]
 - **Spatiotemporal VNNs:** temporal datasets [Cavallo et al., 2024]
 - **Fair VNNs:** unbiased outcomes with VNNs [Cavallo et al., 2025]
 - **Optimality of covariance matrices:** suitability of covariance to learning task
[Khalafi et al., 2024]
 - **Application to brain age gap prediction** [Sihag et al., 2024; 2025]

References

Sihag, Saurabh, Mateos, Gonzalo, C. McMillan, and Ribeiro, Alejandro, "coVariance neural networks," in Proc. Conference on Neural Information Processing Systems, Nov. 2022.

Saurabh Sihag, Gonzalo Mateos, C. McMillan, and Alejandro Ribeiro, "Explainable brain age prediction using covariance neural networks," in Proc. Conference on Neural Information Processing Systems, 2023.

Saurabh Sihag, Gonzalo Mateos, and Alejandro Ribeiro, "Disentangling neurodegeneration with brain age gap prediction models: A graph signal processing perspective," in IEEE Signal Processing Magazine, 2025 (to appear).

Sihag, Saurabh, Mateos, Gonzalo, C. McMillan, and Ribeiro, Alejandro, "Transferability of covariance neural networks," IEEE Journal of Selected Topics in Signal Processing, pp. 1–16, 2024.

S. Khalafi, Saurabh Sihag, and Alejandro Ribeiro, "Neural tangent kernels motivate cross-covariance graphs in neural networks," in Forty-first International Conference on Machine Learning, 2024.

Sihag, Saurabh, Mateos, Gonzalo, and Ribeiro, Alejandro, "Explainable brain age gap prediction in neurodegenerative conditions using covariance neural networks," IEEE International Symposium on Biomedical Imaging, 2025.

A. Cavallo, Z. Gao, and Elvin Isufi, "Sparse covariance neural networks," arXiv:2410.01669, vol. cs.LG, 2024.

Cavallo, Andrea, et al. "Fair covariance neural networks." ICASSP 2025-2025 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). IEEE, 2025.

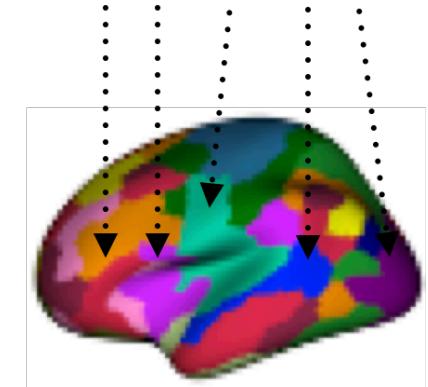
A. Cavallo, M. Sabbaqi, and Isufi, Elvin, "Spatiotemporal covariance neural networks," in Joint European Conference on Machine Learning and Knowledge Discovery in Databases, pp. 18–34, Springer, 2024.

Principled brain age gap prediction with VNNs

Neuroimaging Data: Basics

- Data sample corresponds to measurement associated with brain (cortical) surface

$$\mathbf{x} = [x_1, \dots, x_m]$$



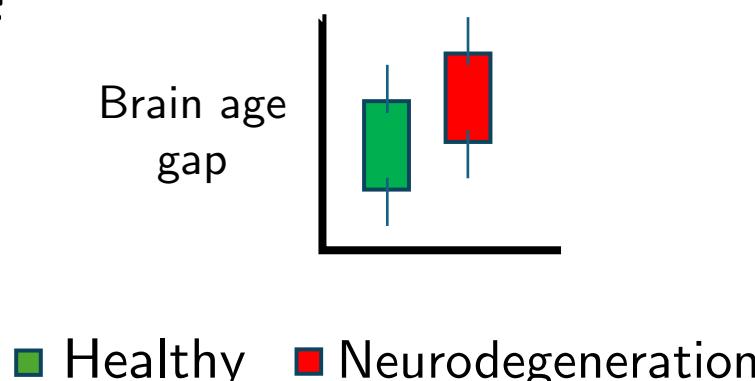
Anatomic features

- Brain surface is divided according to **brain atlases**
 - ➡ datasets may have **distinct** dimensionalities

- **Multi-resolution** brain atlas discretizes brain surface at multiple resolutions
(for e.g., Schaefer's atlas has resolutions 100-1000)

Brain age gap is a marker of neurodegeneration

- Individual rate of “aging” is different from chronological rate of aging
 - Driven by environment, genetics, behavior, **neurodegeneration**
- **Brain age** provides a biological estimate brain age, derived from brain imaging modalities
- The **brain age gap** is the deviation between brain age and chronological age

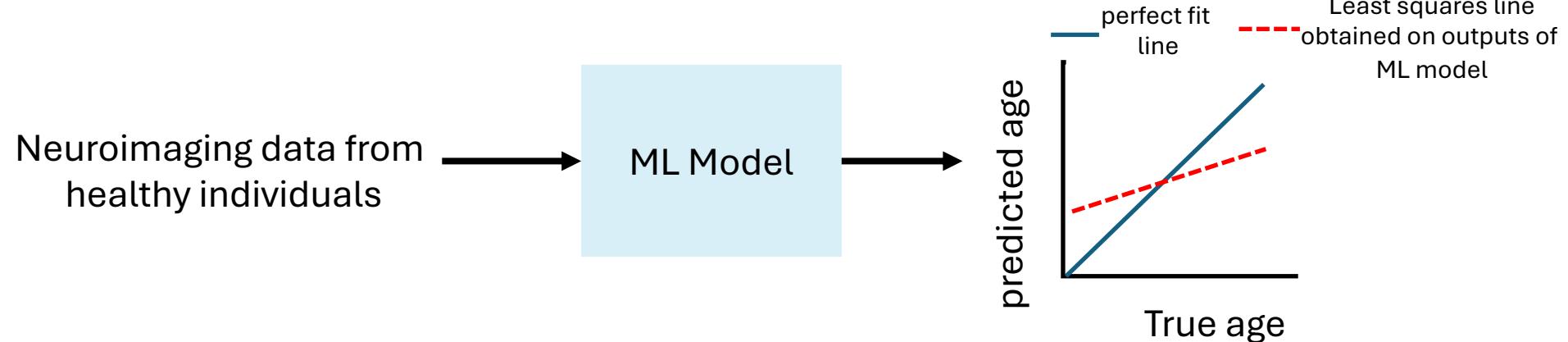


individual risks for neurological,
Brain age gap \propto neuropsychiatric
and neurodegenerative diseases

■ Healthy ■ Neurodegeneration

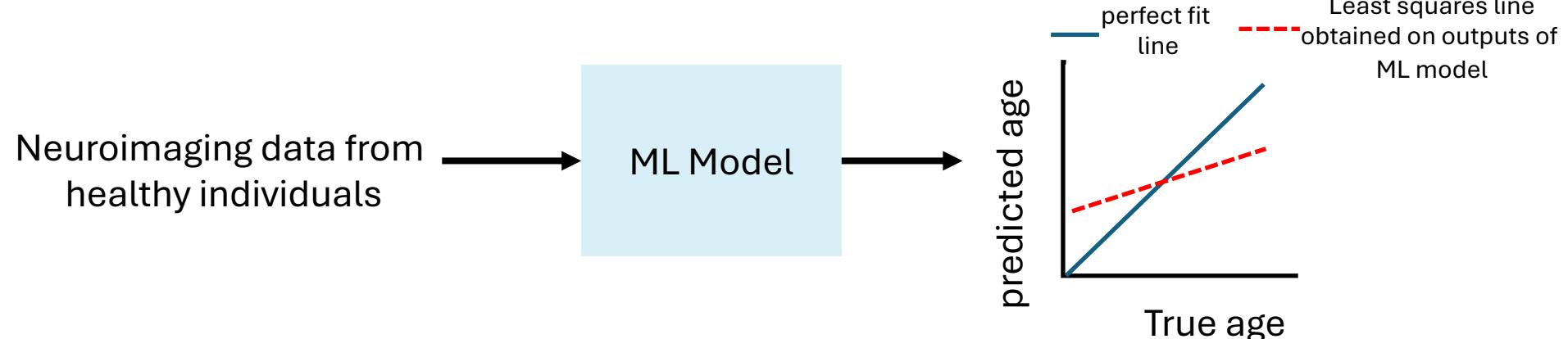
Brain age gap evaluation using ML

Step 1. Train ML model to predict chronological age for healthy controls from cortical thickness features



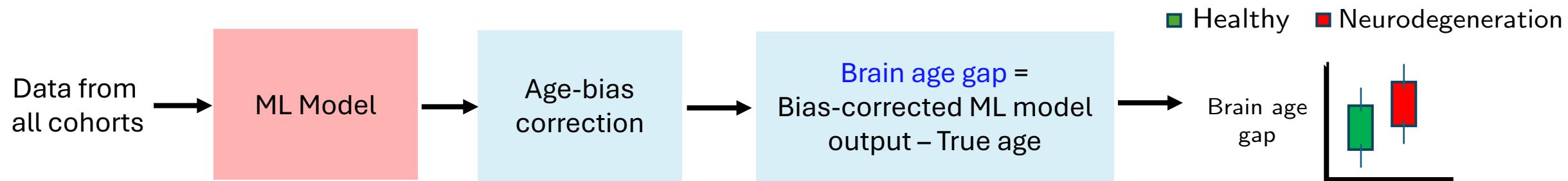
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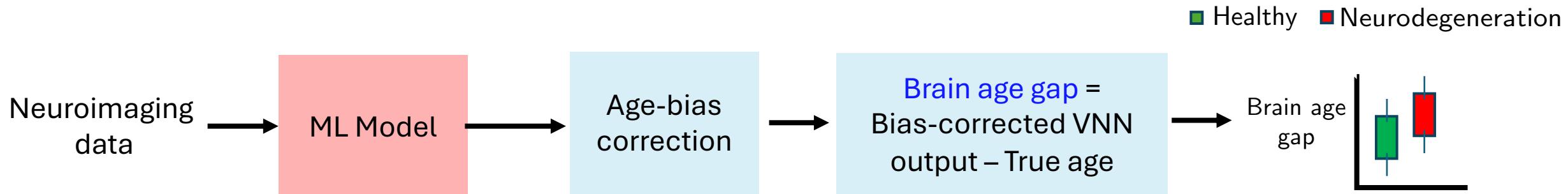


Step 2. Linear regression-based age-bias correct for outputs of ML model

Step 3. Obtain **brain age gap** for healthy controls and individuals with neurodegenerative condition.



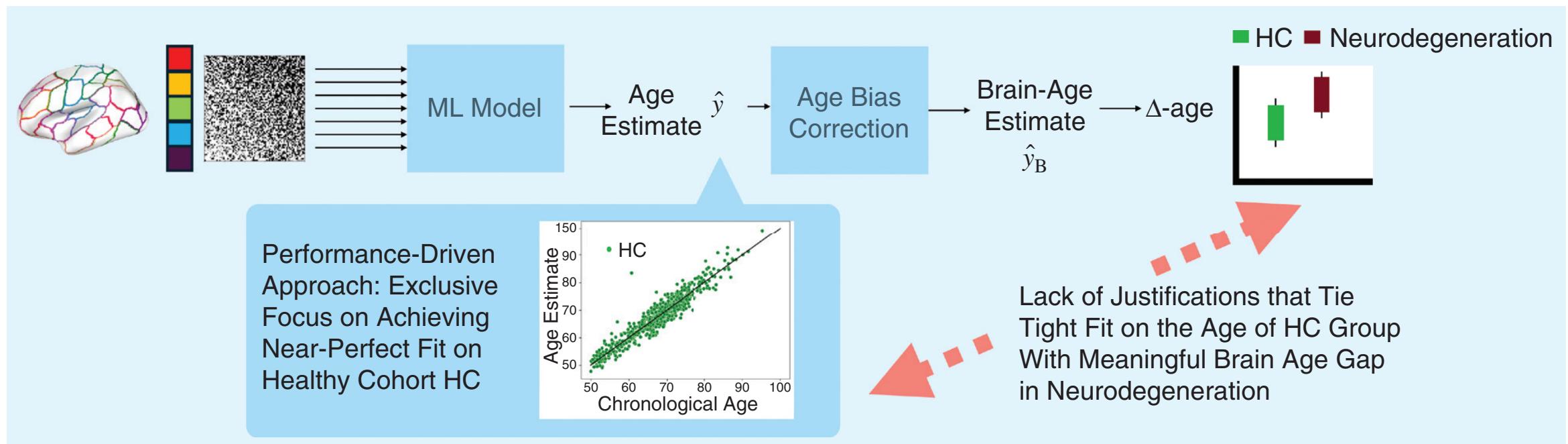
Choice of learning parametrization



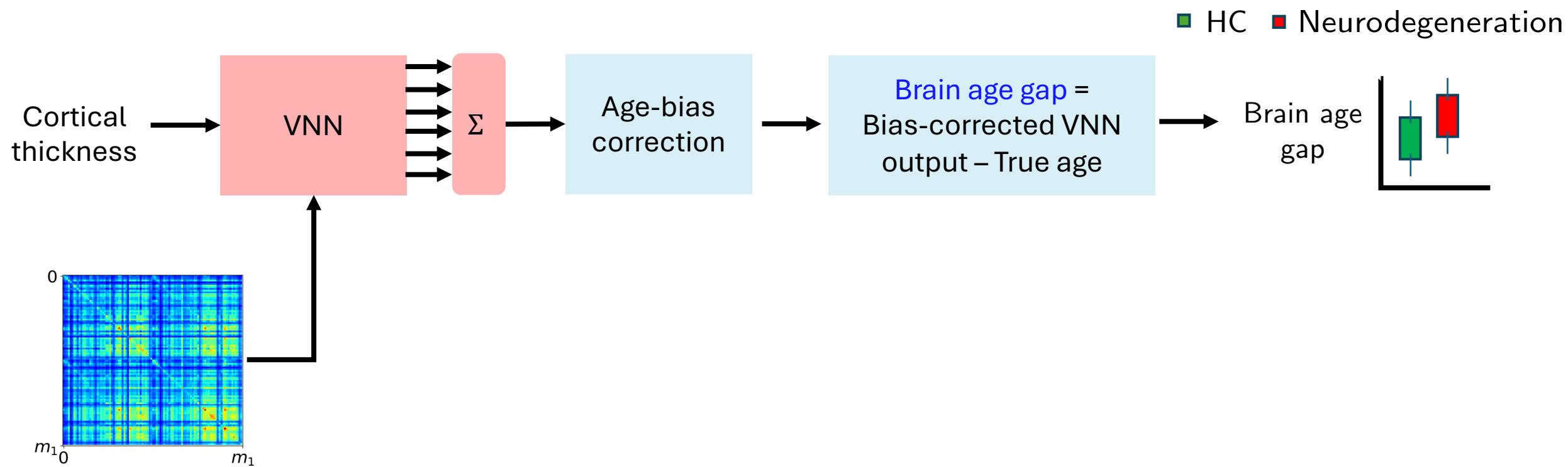
- Choice of ML model determines how information is leveraged to gauge brain age
- Prevalent approaches leverage neural networks as **ML model** to achieve best fit on healthy population: **Performance-driven approach**
- **Performance-driven approaches** do not necessarily lead to a ‘**meaningful**’ brain age gap

Choice of learning parametrization

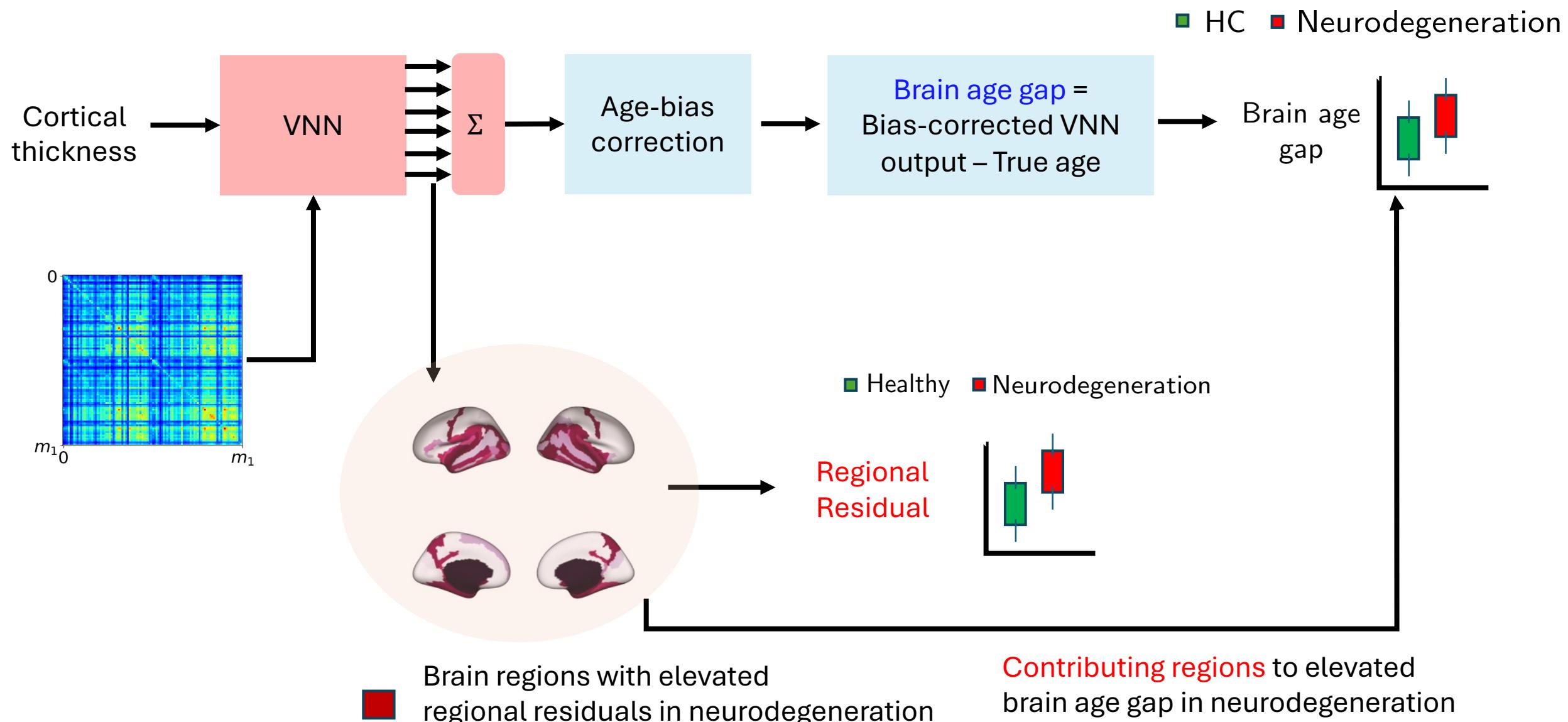
- Neural networks are prevalent in performance-driven approaches
- A Neural Network may **not be interpretable** and prone to **overfitting**
 - ➡ **methodological obscurity** in brain age gap prediction pipeline



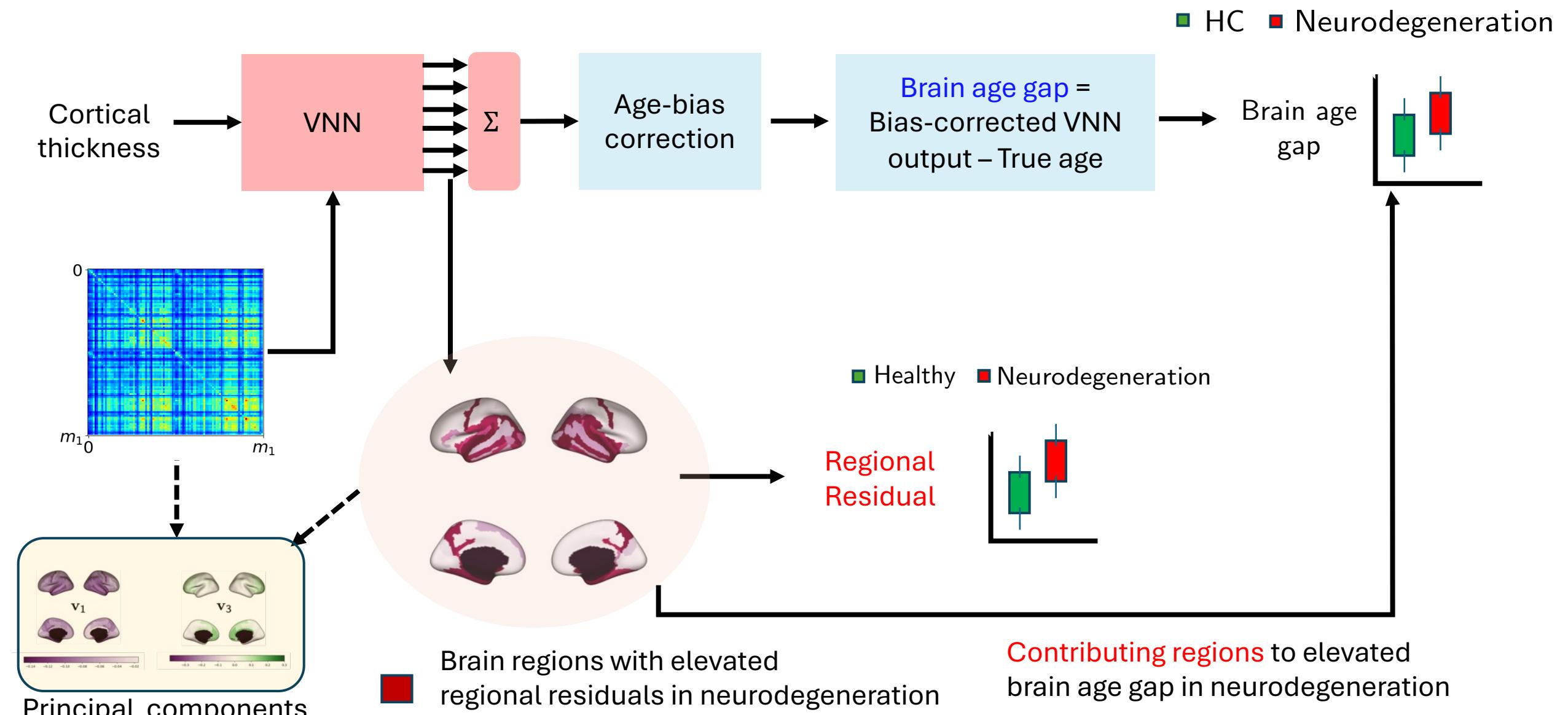
VNNs provide an anatomically interpretable and explainable brain age gap



VNNs provide an anatomically interpretable and explainable brain age gap



VNNs provide an anatomically interpretable and explainable brain age gap



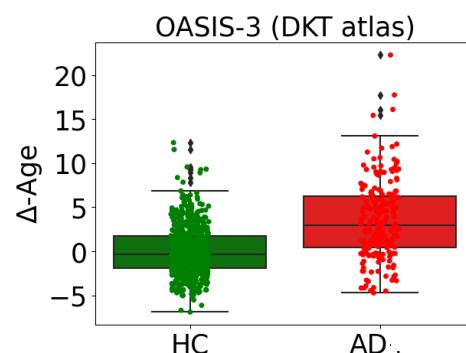
Experiments

- Participants from OASIS-3 dataset [*], 148 cortical thickness features per individual
(Distrieux brain atlas)

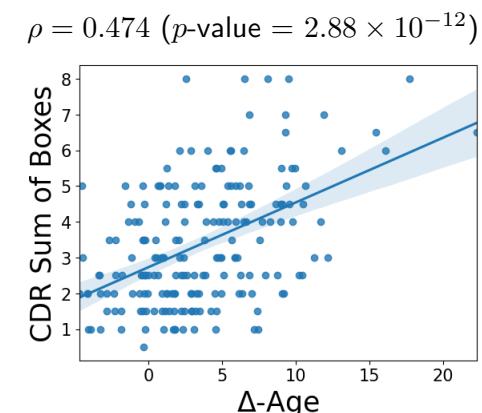
	HC	AD
Number	611	194
Age	68.38 (7.62)	74.72 (7.02)
Sex (m/f)	260/351	100/94
CDR sum of boxes	0	3.45 (1.74)

HC group: cognitively normal
AD group: AD diagnosis
CDR: Clinical dementia rating

- Brain age gap is elevated in AD group and correlated with CDR sum of boxes



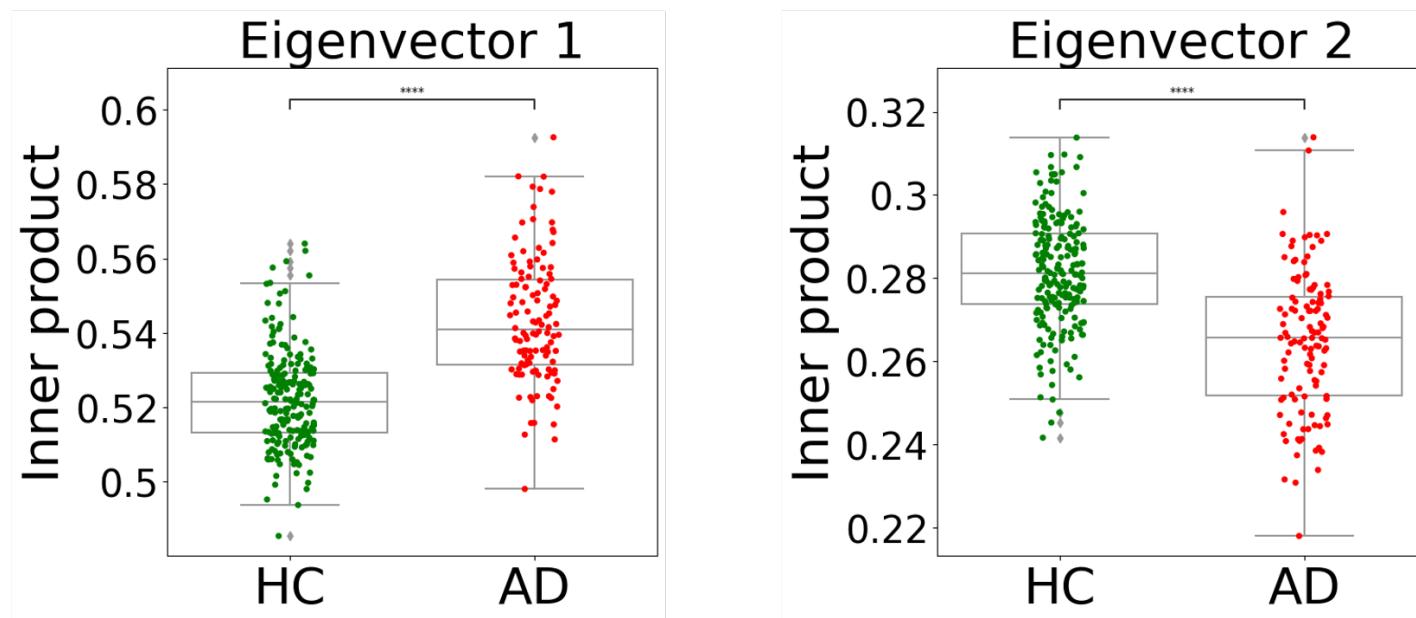
Anatomical interpretability



[*] Pamela J LaMontagne, et al. OASIS-3: longitudinal neuroimaging, clinical, and cognitive dataset for normal aging and Alzheimer disease. MedRxiv, 2019

Experiments

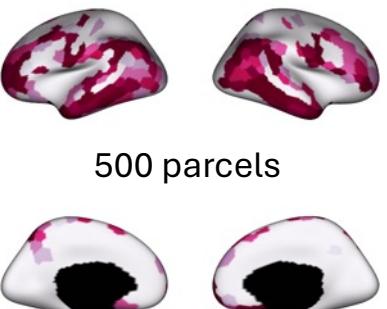
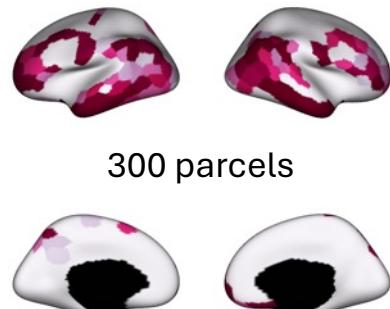
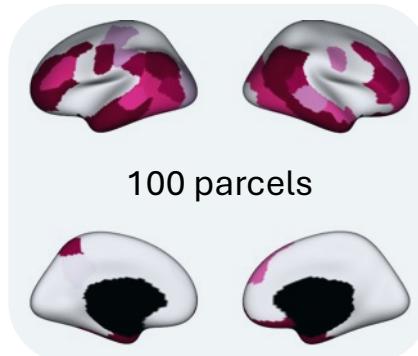
- VNN **distinctly** exploits eigenvectors in AD and HC groups



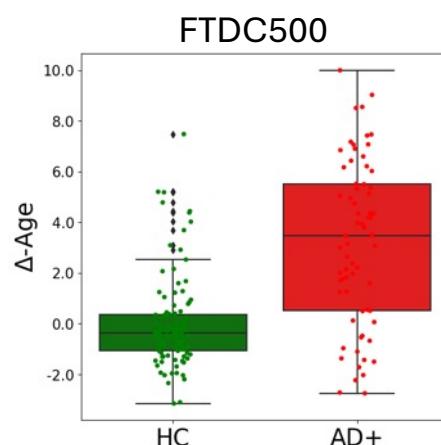
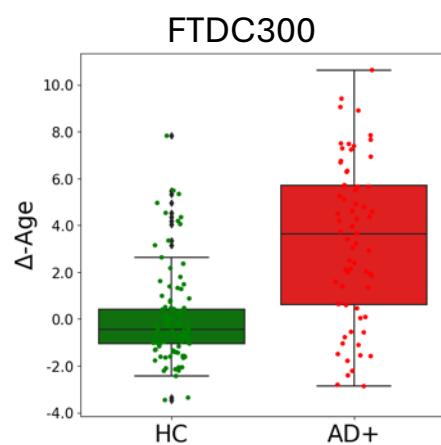
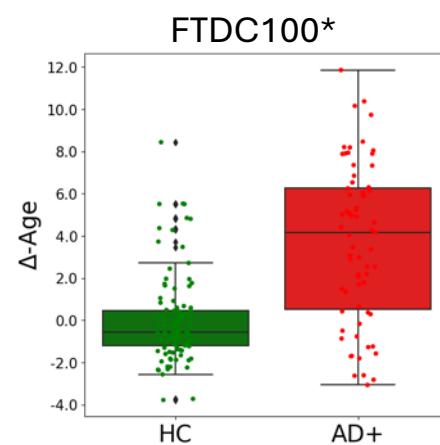
➡ explains anatomical interpretability of brain age gap in AD

Recap: Transferability of VNNs cross-validates brain age gap in multi-resolution setting

Objective: Brain age gap prediction in HC (healthy) and AD+ (Alzheimer's) cohorts from VNNs trained on 100-feature dataset



- ROIs contributing to elevated brain age gap in AD+ across different resolutions



- Brain age gap is elevated in AD+ w.r.t HC cohort in 100-feature dataset
- Results on brain age gap retained after transferring VNN to 300 and 500-feature datasets